Polygamy, co-wives' complementarities, and intrahousehold inequality

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Abstract

I develop and test a novel theory of polygamy that incorporates an empirical feature previously overlooked: Co-wives interact in a *senior-junior* hierarchy. In equilibrium, single, monogamous, and polygamous households emerge. Optimal female sorting generates co-wives' inequality: High-skilled women become senior wives in polygamous households with wealthy men and low-skilled juniors. Monogamous couples are in the middle of the attractiveness distributions. Three sets of nonparametric tests using various data sets confirm the model's predictions.

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1 Introduction

Polygamy, the union of one man with multiple women, is a marriage form that remains popular in many developing countries.[1] Academic research shows that polygamy is associated with underinvestment in physical assets (Tertilt, 2005); underinvestment in human capital (Gould, Moav, and Simhon (2008) and Behaghel and Lambert (2017)); the comparative advantage of female and child labor over capital in agriculture (Boserup (1970) and Jacoby (1995)); and wealth inequality between the sexes and between men (De La Croix and Mariani, 2015).

The policy debate emphasizes issues related to unequal property rights. In particular, polygamous co-wives who join as second or later wives usually do not have access to marital rights in countries where polygamy is widely practiced but illegal. In response, many African countries have recently passed laws or proposed bills that either legalize or strengthen the regulations of polygamy. For example, the Kenyan Marriage Act 2014 (Kenya, 2014) legalizes polygamy and reduces the gap in marital rights between polygamous and monogamous wives.[2] As another example, the Ghanaian Spousal Rights to Property Bill (Ghana, 2009) proposes a more equal division of property upon divorce for women, including those in polygamous marriages. Also, since 2000, various Nigerian states have started to legalize polygamy.

The correlation between polygamy, lower development, and inequality calls for the design of policies that encourage more productive investments and improve the welfare of women. However, these designs need to anticipate the policy impacts on individual marital choices and welfare. A first fundamental step in the policy debate is to understand the incentives that drive women and men to form polygamous households and the characteristics of the households that emerge in equilibrium.

In this paper I develop and test a novel theory of polygamy that incorporates a stylized fact previously overlooked: Co-wives within polygamous families interact in a hierarchy of senior-junior co-wives. I study who marries whom both in monogamous and polygamous marriages and women's selection into the senior and junior roles based on their skills. Interestingly, in a model where co-wives complement each other in a hierarchy, optimal sorting of women implies skill inequality between co-wives. Using data from various countries I show that my model reproduces key features of marriage markets with polygamy.

I start by establishing the empirical relevance of co-wives' interaction in polygamous marriages. First, I show that co-wives' co-residence is frequent. For example, in more than 80% of polygamous households, co-wives live in the same dwelling in countries such as Burkina Faso, Nigeria, and Niger. In addition, co-wives cooperate in farming (Akresh, Chen, and Moore (2016)) and divide their farming and household labor to cope with the workload (Boserup

[1]For example, polygamy is legal or accepted in more than 70% of African countries (UN, 2011), in about 30 countries in Asia (UN, 2011), and is practiced in 73% of the 1,170 societies in Murdock's Ethnographic Atlas (Gould, Moav, and Simhon, 2008). Nowadays, the percentage of married males who have more than one wife reaches as high as 63% in Mali, 35% in Burkina Faso, and 24% in Nigeria, as I show in column 5 in Table 1 (Table 1 summarizes household data from the Living Standards Measurement Study - Integrated Surveys on Agriculture (The World Bank, 2010-2014)).

[2]These include property division and child custody upon divorce and inheritance.

(1970)). This interaction occurs despite conflict and rivalry (Rossi (2018) and Boltz and Chort (2019)), which suggests the existence of dominating complementarities within teams of co-wives. I also show evidence that co-wives organize in a hierarchy of *senior wife—junior co-wives*. The senior wife specializes in managerial tasks while the junior wife specializes in household chores and childcare.

My framework describes a marriage market that allows for bigamy, the most frequent form of polygamy. Females are valued by their skills and males are valued by their wealth. A marriage between a man and at most two women produces family welfare according to a household production function that has three inputs: husband's wealth, the skill of one wife in the junior position, and the skill of another wife in the senior position. A novel feature of this model is that all spouses' traits are complementary in producing the marital output, which implies that household output cannot be decomposed into the sum of what the husband produces with each wife separately. The gain from marriage to an individual is the value of his or her share of the family output. The objective of any man is to choose the team of senior and junior wives that maximizes his gains from marriage. Similarly, the objective of any wife is to choose their household role and the team of husband-co-wife that maximizes her welfare.

I show that there exists a marriage market equilibrium in which single, monogamous, and polygamous households emerge and is fully characterized by *thresholds* of women's skill levels. The optimal sorting of women into household roles implies that the most skilled women in the market become senior wives in polygamous households with the wealthiest men and the lowest skilled women in the market as their juniors. Women and men in the middle of the attractiveness distributions form monogamous marriages. Therefore, senior and junior co-wives in polygamous marriages differ significantly in their skill levels and monogamous wives fill the gap in the middle. Because the equilibrium utility gain from marriage is increasing in female skill, the model endogenously produces a differential status of co-wives within polygamous households—a characteristic of polygamous families that has previously been documented in the empirical literature but is a novel theoretical implication.

In the empirical section of the paper, I use different data sources from different countries to perform three sets of nonparametric tests of the model and find robust support for it. I use data from the Living Standards Measurement Study from Nigeria, a country in which more than 35% of men are polygamous in the northern states as of 2010. Additionally, I use data from the Demographic and Health Surveys from various countries in Sub-Saharan Africa (the area known as "the polygamy belt" due to the high incidence of polygamy). Using various measures of attractiveness at the time of marriage, I estimate the skill and wealth distributions of women and men. I then use the estimated attractiveness indexes to test various features of the model.

First, I test for the prediction of the model whereby the attractiveness distribution of senior polygamous wives dominates the attractiveness distribution of monogamous wives, which in turn dominates the attractiveness distribution of junior polygamous wives. I find support for the sorting patterns implied by my model across many African countries.

Second, I develop a novel test to evaluate whether co-wives have significantly different skills at marriage under the null hypothesis of equally skilled co-wives and reject the null hypothesis of no inequalities among co-wives within polygamous households.

Finally, I test the model prediction that matching is positive assortative between men and women and reject random matching.

Based on these tests that confirm the main predictions of my model in the data, this paper provides novel insights about the nature of polygamous households and contributes to our understanding of the economics of polygamy. I build on and extend the models of Becker (1973); Grossbard (1976); De La Croix and Mariani (2015); Gould, Moav, and Simhon (2008); Jacoby (1995); and Tertilt (2005) to allow for co-wives to complement each other (as well as their husband) in household production. These previous studies treat polygamous families as a set of separate monogamous households in which the husband produces the output of each nuclear family with each wife separately. In these models there is no relationship between co-wives (except through the husband's choice of number of partners), and thus these models are unable to explain why women marry a particular co-wife, what determines women's status within the household, and what would be the consequences of regulating polygamy for females. Unlike these papers, my model is able to demonstrate women's selection into types of marriages and household roles and derive policy implications for women.

By focusing on *household formation* in marriage markets that allow for polygamy, my paper complements an important literature that concentrates on the decisions within polygamous families *after* marriage. For example, Rossi (2018) focuses on co-wives' interactions in fertility decisions and shows, proposing a theory that is empirically supported, that women increase their fertility in response to their co-wife's having more children. Regarding intrahousehold resource allocation, Behaghel and Lambert (2017) study investments in child quality and find that polygamy is associated with lower children's education in Senegal. On the income-generating side, Akresh, Chen, and Moore (2016) and Jacoby (1995) study co-wives' interactions in agricultural production, interestingly finding evidence of co-wives' cooperation in farming. Building on this evidence of significant complementarities between co-wives after marriage, my paper studies the marriage patterns, women's sorting into household roles, and inequality within the household that endogenously arise in the marriage market equilibrium.

Moreover, my model builds on the literature that investigates who marries whom and what are the gains from marriage. In his seminal work, Becker (1973) introduces the idea of modeling the process of household formation as a matching game under both monogamy and polygamy. I build on Becker's work and the literature that followed; in particular, the marriage market models under monogamy of Chiappori, Iyigun, and Weiss (2009a); Chiappori, Iyigun, and Weiss (2009b); Chiappori, Iyigun, and Weiss (2009c); Chiappori, Iyigun, Lafortune, and Weiss (2017a); Chiappori and Oreffice (2008); Chiappori, Oreffice, and Quintana-Domeque (2018); Chiappori, Salanié, and Weiss (2017b); and Low (Forthcominga). Although my model extends these previous matching models by allowing males to match with more than one woman, I draw on the techniques in these papers to solve my model.

More generally, the model in this paper also contributes to the literature on many-to-one matching with complementarities. The non-separability of the production function in co-wives' inputs implies that the characteristics of one wife affect the marginal productivity of the co-wife. For example, from the point of view of the husband, the attractiveness of any potential senior (junior) wife depends on the skill of a potential junior (senior) wife. That is, formally, the preference of husbands over teams of co-wives is *non-substitutable* (Roth and Sotomayor, 1990). Substitutability of preferences has received a lot of attention in the literature on matching with contracts, because when preferences are non-substitutable, the existence of equilibria is not guaranteed (see, for example, Hatfield, Kominers, Nichifor, Ostrovsky, and Westkamp (2013)).[3] In this paper, I exploit the observed hierarchical organization of co-wives to impose a structure on the household production function that allows me to show the existence of equilibria even in the presence of non-substitutable preferences.

Moreover, I contribute to the literature on team formation by endogenizing the partition of women who will perform the different tasks in the household. For example, Carlier and Ekeland (2010) show the existence of stable teams in models in which the groups from which the team draws its members are exogenously given. In my model, women are drawn from the same population of skills, and the disjoint groups of seniors and juniors are endogenously determined in equilibrium.

To the best of my knowledge, this is the first paper to apply and solve a model of many-to-one matching with a supply chain structure (Ostrovsky (2008), Sun and Yang (2006)) to family formation, allowing for non-substitutability of preferences. This application is by no means restricted to marriage markets. An interesting use of this model would be to study matching between heterogeneous firms and workers and wage inequality among coworkers in environments in which workers sort into different but complementary occupations within the firm. In this sense, I extend the matching models of one firm to many workers of Eeckhout and Kircher (2018) and Kelso and Crawford (1982) to allow for complementarities between coworkers (in addition to the usual complementarity between workers and firms).

The paper is organized as follows. Section 2 presents descriptive evidence on polygamy and co-wives' interaction. Section 3 presents and solves the model. Section 4 describes the data and performs three sets of nonparametric tests of the model. Finally, Section 5 concludes.

2 Polygamous families: extended hierarchical households

The two novel features of my model—that co-wives organize in a hierarchy of different and complementary household roles—are salient in the data.

[3]Intuitively, equilibria may fail to exist because teams of a husband and two co-wives may never agree to marry. For example, there could be situations in which male y finds female s attractive as a senior wife only when female s' joins as a junior wife, but female s' would only marry male y as his junior wife if he marries s'' as his senior wife.

Women's labor and polygamy. Co-wives' cooperation and division of labor in household and farm labor is one of the main reasons Boserup (1970) proposes for the existence of high polygamy rates in Western Sub-Saharan Africa. These rural economies are characterized by technological conditions that favor female and child work over technification. Hence, women in these markets are highly valued for their farming skills and fertility (an idea also present in Jacoby (1995)). In the empirical section of this paper I draw on this descriptive evidence and measure female attractiveness as premarital skills (proxied by height and parental education) and number of fertile years at marriage. Despite potential conflict, a co-wife is welcomed to help with the workload implied by housework and farming.[4] This interaction in time use is observed in modern African data, as I show next.

Co-wives' interactions. Co-wives in polygamous families are observed to interact both in home production and farming. A possible indication that polygamous families are not a set of separate monogamous households is the frequency of co-wives' cohabitation. I explore data from seven African country surveys conducted by the Living Standards Measurement Study-Integrated Surveys on Agriculture (henceforth LSMS-ISA, The World Bank (2010-2014)). Table 1 lists the country studies.

Table 1: Polygamy and co-wives' interaction in the LSMS-ISA data

Country	Year	N	Polygamous households			
			Man head			Woman head
			(1)	(2)	(3)	(4)
			N	as % of males	% Co-wives cohabit	as % of N
Mali	2014	2399	61	63	51	0.3
Burkina Faso	2014	6540	31	35	82	3.7
Nigeria	2010	3380	21	24	86	0.3
Niger	2011	2430	19	22	94	0.5
Uganda	2010	2206	15	20	10	5.4
Malawi	2010	10038	7.6	10	0.3	1
Ethiopia	2011	3427	4.2	5.5	8.3	0.8

Notes: *LSMS-ISA* stands for *Living Standards Measurement Study-Integrated Surveys on Agriculture*. *Year* refers to the survey year. N refers to the total number of rural households in the country survey.

Columns (1) to (3) describe polygamous households that have a male head. Two facts are evident from this table. First, consistent with other data sources—for example, the Demographic and Health Surveys (henceforth DHS) analyzed in a related work by Fenske (2015)—polygamy is most frequent in Western Africa. Column (1) in the table shows the percentage of male-headed polygamous households in the data. Polygamy rates in West Africa vary between 19% and 61%. As a percentage of male heads of households (column (2)), between 22% and 63% of males are polygamous in the West. Second, co-wives' cohabitation is common in countries with the highest polygamy rates, but less frequent in countries where the polygamy rates are lowest.

[4]The trade-off between market and household labor has also been noted in the context of monogamous marriage markets in developed countries, as in Low (Forthcominga).

The percentage of polygamous males who cohabit with their multiple wives is shown in column (3).[5] For example, in Niger, in 94% of polygamous families co-wives live in the same dwelling. A related statistic is the percentage of households in which the head is a polygamous female (column (4)). These include households in which one of the multiple wives of a polygamous male lives alone, separate from the husband and his other wives. As column (4) shows, the frequency of these households is very low, reinforcing the picture that polygamous co-wives most frequently cohabit, at least in the Western countries.

In addition, empirical studies examine whether polygamous families jointly produce the household output—for instance, whether co-wives (together with their husbands) cooperate in agricultural production. Akresh, Chen, and Moore (2016) find evidence that co-wives share farming inputs efficiently in Burkina Faso, and Dauphin (2013) arrives at similar conclusions in Burkina Faso and Benin, but rejects efficiency of polygamous families in Senegal. We can also observe whether co-wives work together on the farm in the LSMS-ISA data. For example, in Nigeria (the country that will be the focus of the empirical section of this paper), co-wives work together in farming in over 45% of polygamous households.

The senior-junior hierarchy. The second feature that usually arises in studies of polygamous households is that co-wives organize in a hierarchy of senior-junior wives. The senior wife takes the role of the household's *head* wife and the junior wife takes the role of a *helper* or *secondary* wife. The literature finds that senior and junior wives have very different status within the household: Senior wives are healthier, have higher social status, are more productive, and have higher quality children, relative to lower-ranked wives (Matz (2016); Gibson and Mace (2007); and Strassmann (1997)).

Complementing this evidence, in Table 2 I show evidence of the hierarchical interaction between senior wives—who provide management for the family—and junior wives—who provide labor. Using data from the Nigerian LSMS-ISA (Panel A) and from 11 African DHS studies pooled together (Panel B), I report the coefficient on *senior* in regressions of household tasks within polygamous households.[6] All specifications include household fixed effects. The results show that senior wives are significantly more likely to be managers of the household's finances and businesses and more likely to have decision power on the household's large purchases and income (including husband's earnings), while junior wives are significantly more likely to fully specialize in housework and childcare.

[5] A man is observed to cohabit with multiple wives if his marital status is "married (polygamous)" and more than one female is listed in the household roster as "spouse." Cohabitation follows from the fact that the LSMS-ISA defines a household as "a group of people who have usually slept in the same dwelling and share their meals together."

[6] I define the senior wife as the one the husband identifies as the head wife. More details are provided in Section 4.

Table 2: Hierarchies in polygamous households

Panel A: Nigeria LSMS-ISA, 2010			
	Manages Household		Fully specializes
	Finances	Enterprises	in housework
Senior	0.0336	0.0444	-0.0205
	(0.0094)	(0.0149)	(0.0097)
Constant	0.6089	0.3572	0.2189
	(0.0068)	(0.0094)	(0.0060)
Observations	1217	1333	1305
Panel B: DHS—Countries in African Polygamy Belt pooled			
	Decides on Household		Fully specializes
	Large purchases	Income	in housework
Senior	0.0287	0.0544	-0.0635
	(0.0095)	(0.0116)	(0.0098)
Constant	0.2643	0.4331	0.3278
	(0.0057)	(0.0070)	(0.0059)
Observations	8110	8126	8102

Notes: *LSMS-ISA* stands for *Living Standards Measurement Study-Integrated Surveys on Agriculture*. *DHS* stands for *Demographic and Health Surveys*. The sample consists of all wives in polygamous households in rural Nigeria (Panel A) and rural Benin, Burkina Faso, Gambia, Ghana, Guinea, Mali, Niger, Nigeria, Senegal, Sierra Leone, and Togo (Panel B). The table shows estimated coefficients from regression models at the wife i and household h level $Task_{ih} = \beta_0 + \beta_1 Senior_{ih} + \delta_h + u_{ih}$. Tasks are Manages Household *Finances* (a dummy that takes value one if the wife is involved in financial decisions for the household); *Enterprises* (a dummy that takes value one if the wife is the manager of at least one farm or non-farm business run by the family); *Fully specializes in housework* (a dummy that takes value one if the wife is not working due to housework and/or childcare responsibilities); Decides on Household *Large purchase* (a dummy that takes value one if the wife is involved in decisions on large purchases); and *Income* (a dummy that takes value one if the wife is involved in deciding what to do with own or husband's earnings). *Senior* is a dummy variable that takes value one for wives whose husbands identify as the head wife. δ_h is a vector of household fixed effects. Robust standard errors in parentheses.

3 A model of polygamy with spousal complementarity and female household roles

In this section I propose and solve a new model of family formation under polygamy motivated by the two facts stated in the previous section. The first fact (that polygamous families function as a joint extended family) motivates the modeling of spouses having complementary traits in the household production function. The second fact (that co-wives organize in a hierarchy) motivates the distinction in the production function between the senior wife's and the junior wife's household roles.

The novel implication in the model with respect to previous models of polygamy is that the gains from marriage for women depend not only on the traits of their potential husband (as in previous models), but also on the traits of potential co-wives. This implies that there is a nontrivial problem of optimally sorting women into monogamous versus polygamous marriages, and into the senior versus junior wife position within polygamous families. The model, hence, rationalizes the selection of women into polygamous marriages and their occupation within the household. Importantly, the differential welfare status of co-wives that has been noted empirically arises endogenously in the equilibrium of my model.

3.1 Marriage market populations

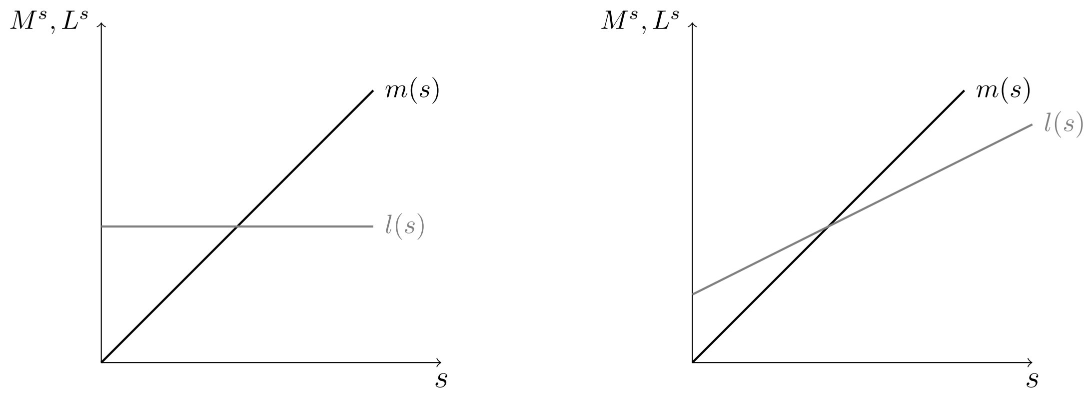
The marriage market consists of an equal mass of men and women. Women are distinguished by their endowment of skill, s, and men by their wealth, y, distributed according to continuous distribution F on $[\underline{S}, \overline{S}]$ and G on $[\underline{Y}, \overline{Y}]$, respectively.[7]

Households that form in this marriage market can be single-headed (the head of the household is an unmarried man or woman); monogamous (the head of the household is a man married to one and only one wife); or polygamous (the head of the household is a man married to two wives). A marriage consists of a husband and at most two wives.

3.2 Wives roles

Women can use their exogenous skill level s to perform the roles of the *senior* wife or of the *junior* wife.[8] In the empirical section, I specify skills as a single index that combines premarital ability (measured as height or parental education) and fertile years at marriage (also interpreted as youth). In the light of the evidence presented in Table 2 I interpret that women in the senior role use their bundle of ability and youth to perform managerial tasks within the household according to an increasing production function $M^s = m(s)$. Meanwhile, women in the junior wife role use their combined ability and youth to perform housework tasks according to an increasing production function $L^s = l(s)$. I assume that higher skilled women have both, comparative and absolute advantage in producing the *senior wife role* relative to lower skilled females. Figure 1 below illustrates two possible scenarios of how different female skill levels perform each of the household roles.

Figure 1: Production of household wives' roles



Panel A: Constant junior productivity Panel B: Increasing junior productivity

Panel A depicts the case in which all levels of female skills produce the same in the junior

[7]To allow for some agents to choose to be single or to marry monogamously, the set of females is augmented by including a point $\emptyset_s$ to denote the "dummy" co-wife of any monogamous household or the "dummy" wife of any single male, and the set of males is augmented similarly with point $\emptyset_y$ to denote the "dummy" husband of any single woman (Chiappori, McCann, and Nesheim, 2010).

[8]I build on the seminal work of Grossbard (1976) who models the demand for "wives services" in marriage markets with polygamy and extend it to consider two types of services provided by wives.

wife position, while panel B considers the case in which the production of the junior wife role is increasing in the skills of the woman performing that role. The distinction matters for tractability, and while I solve for both cases, the case depicted in panel A reflects the data better (as discussed in empirical section 4) and it is the simplest with which results obtain.

3.3 Marital Output

A marriage between a man and a team of junior-senior co-wives jointly produce the household output which I interpret as the total household utility value of the combination of income from family businesses or agricultural plots and children's welfare.

The family of a man type y, a junior wife type s', and a senior wife type s, $(y, s', s) \in [\underline{Y}, \overline{Y}] \times [\underline{S}, \overline{S}]^2$ produces the marital output according to household technology H^y—indexed by the male trait to reflect the virilocal aspect of polygamous societies:

$$H^y = H(y, l(s'), m(s)) = h(y, s', s).$$

The marital output depends on the male trait to reflect that in virilocal societies, men inherit land, the value of which (quality and quantity) depends on male wealth. In turn, co-wives contribute to producing the household income by performing their roles. All in all, this model reflects two important features of polygamous societies: The division of labor across gender and the division of labor within women.

I make the following assumptions on the *reduced form* marital output, $h(y, s', s)$, where I denote $h_i(y, s', s)$ as the partial derivative of $h(y, s', s)$ with respect to the i^{th} input.

Assumption 1 *The marital surplus $h(y, s', s)$ satisfies:*

1. **Differentiability**. *Household production function is twice continuously differentiable.*
2. **Monotonicity**. *Household output is strictly monotone in male wealth and in the skill of the senior wife, and weakly monotone in the skill of the junior wife: $h_1(y, s', s) > 0$, $h_2(y, s', s) \geq 0$, and $h_3(y, s', s) > 0$.*
3. **Supermodularity/Female Complementarity**. *Total social output when the most attractive individuals are together and the least attractive individuals are together is higher than when households are mixed. Formally, for any two input vectors $z = (y, s', s)$ and $\hat{z} = (\hat{y}, \hat{s'}, \hat{s})$, $h(z \vee \hat{z}) + h(z \wedge \hat{z}) \geq h(z) + h(\hat{z})$, where "$\vee$" and "$\wedge$" denote the joint and the meet of the vectors, respectively.*
4. **Hierarchy of Female Roles**.
   (a) *Higher skilled women are more productive in the senior wife role: For all $s > \hat{s}$, for all y, $h_3(y, \hat{s}, s) > h_2(y, s, \hat{s})$.*
   (b) *The marginal productivity of women's skills in the senior wife position is always higher than the marginal productivity of women's skills in the junior wife position: For all $y > \hat{y}$, $t > \hat{t}$, $s > \hat{s}$, $h_3(\hat{y}, \hat{t}, \hat{s}) > h_2(y, t, s)$.*

5. **Essential Coalitions**.

   (a) *The value of being single is zero:* $h(y, \emptyset_{s'}, \emptyset_s) = h(\emptyset_y, s', \emptyset_s) = h(\emptyset_y, \emptyset_{s'}, s) = 0.$

   (b) *All marriages must include the husband:* $h(\emptyset_y, s', s) = 0.$

   (c) *Marriages can be monogamous:* $h(y, \emptyset_{s'}, s) \geq 0$ *&* $h(y, s', \emptyset_s) \geq 0.$

Part 1 of Assumption 1 is standard and introduced only to simplify exposition.

Part 2 is also standard: All else equal, better inputs produce higher output. Marital surplus is weakly increasing in the junior wife role to consider the case in which all skills employed in the junior position contribute the same to producing household output.[9]

Part 3 means that any two spouses' traits are complements to produce household output. The complementarity between males and females is standard in family models, since it gives rise to the observed positive assortative matching between the sexes. The complementarity between co-wives is the novel contribution of this paper to the literature of household formation with polygamy. The assumption implies that the extended household production function, $h(y, s', s)$, is not separable in the wives' inputs or, more specifically, that $h(y, s', s) > h(y, \emptyset_{s'}, s) + h(y, s', \emptyset_s)$. This complementarity between co-wives captures the idea that the presence of a co-wife, despite conflict, helps to achieve the desired household goals in terms of fertility and income generation.

Part 4 is introduced to give meaning to the idea that there is a hierarchy of wives within polygamous marriages. This is captured in the model as the differential importance of women's roles in producing household output. First, part 4(a) means that a household that can afford two women will position the highest skilled of them in the senior wife position and the least skilled in the junior wife position, since this is the female sorting that yields the highest household output. Second, part 4(b) means that the productivity of skills in the senior wife role is significantly higher than the productivity of skills in the junior wife position. This is true to the extent that increasing the skill of the senior wife, even in a household in which all spouses have lower traits, is more profitable than increasing the skills of the junior wife in a household in which all spouses have higher traits. This assumption reflects the idea that one of the two roles is more important than the other in producing household output. Without loss of generality, that role is the *senior wife role*. Importantly, note that in the case in which the marital surplus is constant in the skill of the junior wife, $h_2(y, s', s) = 0$, part 4 is automatically satisfied.

Finally, part 5 specifies which types of households are possible in this marriage market. Households can be formed by single individuals (which value is normalized to zero), by marriages of a male and only one female, or by marriages of a male and two females. Single individuals earn the lowest possible value in the market. Households with no husband will not form, since two women together gain nothing with respect to splitting and becoming two single households. Last, parts 4 and 5(c) imply that monogamous wives contribute more in the senior role.

[9] This particular case yields roughly the same conclusions as the more general case, but entails much simpler mathematical derivations. The main results of this paper will be developed under this case of constant productivity of junior wives. In Appendix B I show all results under the more general case.

Taken together, while supermodularity is the key assumption in proving the assortativeness of the equilibrium, the hierarchy of roles is the key assumption in proving the patterns of women's sorting into household roles.

3.4 Marriage Market Equilibrium

The main objective of the modeling part of the paper is to characterize two features of the marriage market with polygamy and hierarchy of wives. First, the equilibrium sorting of women into household roles: In equilibrium, which women are employed as junior wives and which women are employed as senior wives? Second, the shape of the equilibrium matching between males and females: Who marries whom, and what do they gain in equilibrium? Just by assuming a marital surplus with complementarities and different productivities of female roles, I am able to show that there exists an equilibrium in the marriage market with positive selection of women into roles, and positive assortative matching between males and females within roles.

3.4.1 The problem of household formation

The marriage market is competitive and the marital output is perfectly transferable. This means that at the moment of choosing their partners, females and males face given prices $\{u(s)\}$ for all women $s \in [\underline{S}, \overline{S}]$ and $\{v(y)\}$ for all men $y \in [\underline{Y}, \overline{Y}]$ in the market. The marital output produced by any potential coalition is known at the moment of matching and is perfectly divided among potential spouses, according to the given sharing rule $((v(y), u(s'), u(s)))$.

The objective of male y is to form the team of wives that maximizes his profits, subject to being accepted:

$$v(y) \equiv \max_{s,s'} \quad h(y, s', s) - u(s') - u(s). \tag{1}$$

Because the marital output is transferable, in equilibrium it has to be the case that the argmax women (s, s') agree to marry y at the equilibrium prices.

3.4.2 Definitions

Before introducing the equilibrium concept, I need to define *outcome* and *stability* in the marriage market with polygamy and female roles.

Definition 1 *An* ***outcome*** *in the marriage market with polygamy and female roles is defined as a tupla* $(\mathcal{M}, \mathcal{L}, \mu, v^{\mu}, u^{\mu})$ *where:*

1. $\mathcal{M}$ **and** $\mathcal{L}$ **are, respectively, the set of seniors and the set of juniors such that they form a partition of the set of women***:*

$$\mathcal{M} \subseteq [\underline{S}, \overline{S}] \text{ and } \mathcal{L} \subseteq [\underline{S}, \overline{S}] \text{ such that } \mathcal{M} \cup \mathcal{L} = [\underline{S}, \overline{S}] \text{ and } \mathcal{M} \cap \mathcal{L} = \emptyset$$

2. μ **is a pure matching***: a non-degenerate measure on* $\mathcal{M} \times \mathcal{L} \times [\underline{Y}, \overline{Y}]$*, the marginals of which coincide with the measure of each set and where:*
   - $\mu_3(y) : [\underline{Y}, \overline{Y}] \to [\underline{S}, \overline{S}]$ *denotes the senior wife of male* y*,*

- $\mu_2(y) : [\underline{Y}, \overline{Y}] \to [\underline{S}, \overline{S}]$ *denotes the junior wife of male y, and*
- $\mu_1(s) : [\underline{S}, \overline{S}] \to [\underline{Y}, \overline{Y}]$ *denotes the husband of woman s.*

3. v^μ **and** u^μ **are feasible payoff functions associated to** μ*:* $\forall y, s, s' \in Spt(\mu)$

$$v^\mu : [\underline{Y}, \overline{Y}] \to \Re \text{ and } u^\mu : [\underline{S}, \overline{S}] \to \Re \text{ such that}$$
$$v^\mu(y) + u^\mu(s') + u^\mu(s) = h(y, s', s).$$

Intuitively, an outcome in the marriage market is a particular grouping of males and females into a set of households and a scheme of associated payoffs: Part 1 of Definition 1 indicates which women become senior wives and which become junior wives; part 2 indicates who matches with whom—that is, who is the senior wife, the junior wife, and the husband in each household; and part 3 indicates the gains from marriage for each individual.

Note that there are many outcomes in a given marriage market because there are many ways of grouping individuals, many ways of splitting the surplus generated by each group in a feasible way, and two possible roles for each woman. However, not all of these possible outcomes are *equilibrium* outcomes.

An equilibrium in the marriage market is an outcome such that all individuals maximize their preferences as indicated in problem 1 above. The equilibrium concept is that of *stable outcome* or *stable matching.*

Definition 2

1. *An outcome* $(\mathcal{M}, \mathcal{L}, \mu, v^\mu, u^\mu)$ *is **stable** if*

$$h(y, s', s) = v^\mu(y) + u^\mu(s') + u^\mu(s) \text{ ; } \forall y, s, s' \in Spt(\mu)$$
$$h(y, s', s) \leq v^\mu(y) + u^\mu(s') + u^\mu(s) \text{ ; otherwise}$$

2. *A matching* μ *is **stable** if there exist numbers* $v^\mu(y), u^\mu(s')$*, and* $u^\mu(s)$ *such that the outcome* $(\mathcal{M}, \mathcal{L}, \mu, v^\mu, u^\mu)$ *is stable. When a matching is stable, we say that there is no coalition that* blocks *the matching.*

Intuitively, a matching μ is stable if there is no individual or group of individuals that would agree to form a household (possibly of a single) that is not in society μ. If we take any coalition of individuals that are not matched together under μ, stability requires that together they cannot produce more than the sum of what each is earning under μ. If what they would produce if they deviate together is more than the sum of what they are earning, they can split the surplus and agree to deviate from society μ.

The definition above is standard in the literature of matching with transferable utility except that in addition to requiring that coalitions not block assignment μ, it also requires that women optimally choose their role. The fact that the partition of females into the two household roles is determined endogenously in equilibrium is a key output of this model, and constitutes one of the paper's main contributions to the literature. To see this, note that in Definition 2 there is no prerequisite for the support of μ: So long as the sets $\mathcal{M}$ and $\mathcal{L}$ partition the skills set, a woman $s \in [0, \bar{S}]$ can be either the second or the third argument in the marital surplus function. Hence, the identity of senior and junior wives matters in a nontrivial way for the definition of stability. For example, suppose that (y, s', s) and $(\hat{y}, \hat{s'}, \hat{s})$ are two households under μ. In this society, women s' and $\hat{s'}$ perform the role of the junior wife and women s and $\hat{s}$ perform the role of the senior wife. Now suppose that husband y and senior s together with woman $\hat{s}$ (a senior under μ) in the role of junior could produce together more than the sum of the individual profits under μ:

$$h(y, \hat{s}, s) > v^{\mu}(y) + u^{\mu}(\hat{s}) + u^{\mu}(s).$$

Since the excess product $h(y, \hat{s}, s) - (v^{\mu}(y) + u^{\mu}(\hat{s}) + u^{\mu}(s))$ can be used to increase the payoffs of all the members of this blocking coalition, husband y and senior s would like to match with $\hat{s}$ and employ her as a junior wife, and woman $\hat{s}$ would find it profitable to deviate from being the senior in household $(\hat{y}, \hat{s'}, \hat{s})$ to being the junior in household $(y, \hat{s}, s)$. Hence, the coalition $(y, \hat{s}, s)$ would block μ and μ would not be a stable matching. In sum, the novel contribution of this paper to the literature is that the solution of the model will endogenously determine not only who matches with whom, but also which women sort into being senior wives and which sort into being junior wives.

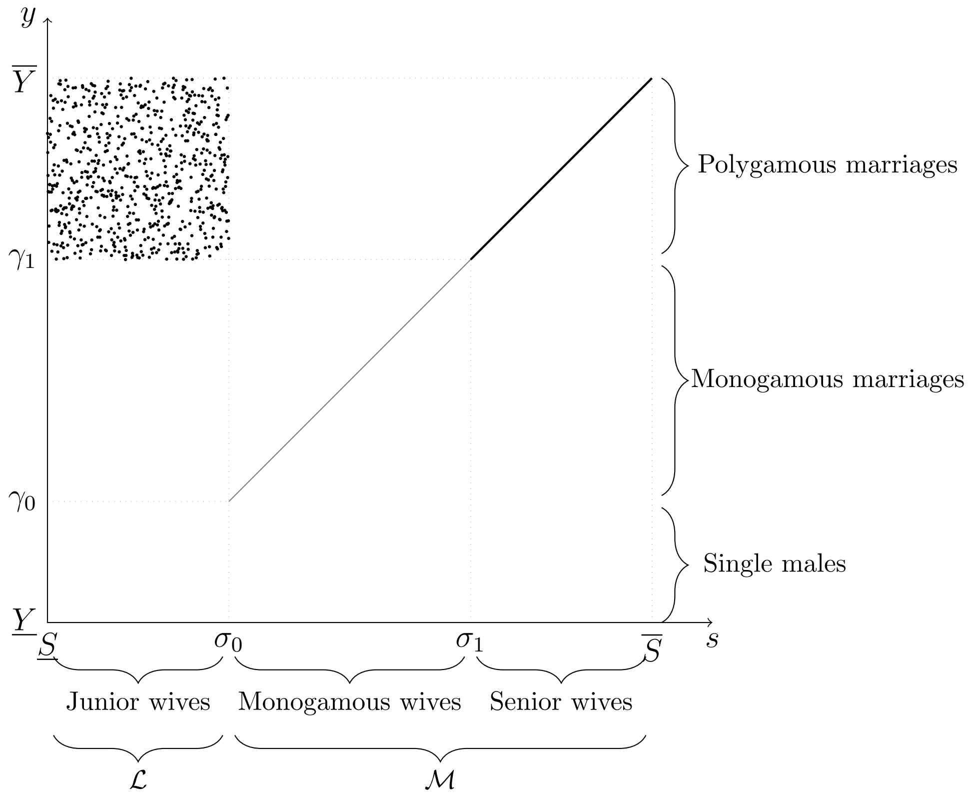
3.4.3 Main result

In this section I characterize and prove the existence of a stable matching in the marriage market with polygamy and two female household roles for the case in which all females produce the same fixed output, K, when employed in the junior wife position. That is, in this section I show the equilibrium when the household output is $h(y, s', s) = H(y, K, m(s))$ such that it satisfies Assumption 1 with $h_2(y, s', s) = 0$. The main result of this paper is illustrated in Figure 2. The vertical axis indicates male wealth and the horizontal axis displays female skills and the partition of skills into the junior and senior wife positions, $\mathcal{L}$ and $\mathcal{M}$, respectively. The solid upward-sloped line indicates the equilibrium match between women with skills in $\mathcal{M}$ and husbands, while the scattered points in the top left area indicate the matching between women with skills in $\mathcal{L}$ and husbands. The figure can be summarized as follows: The equilibrium in this marriage market exhibits a threshold shape with positive assortative matching between men and women and positive sorting of women into household roles, such that the least skilled women take the junior wife role and the most skilled women take the senior wife role. The relevant thresholds to note are σ_0 and σ_1 in the set of female skills, and γ_0 and γ_1 in the set of male wealth, all of which are endogenously determined in equilibrium.

There are three salient features of this equilibrium.

The first is the structure of polygamous households. The highest skilled women (with skills above σ_1) are so skilled that they face the highest opportunity cost of being monogamous and forgoing the increment in their productivity that results from having a complementary co-wife. Hence, households with high-skilled women will try to hire a junior wife. However, because of the higher productivity of skills in the senior role relative to the junior role, only

Figure 2: A stable matching



the lowest skilled women (with skills below σ_0) face an outside option low enough to be willing to take the less rewarded position of a junior wife in polygamous households. In effect, women below σ_0 trade off being the junior wife to rich polygamous households against being the senior wife to poor men with wealth below γ_0. With regard to matching patterns, because of supermodularity, the match between senior wives and husbands in polygamous households is positive assortative: Higher skills contribute more to household output the higher the wealth of the husband. Because the marginal contribution of female skills in the junior wife position to household output is zero, matching between husbands and junior wives is random: Any woman who accepts being a junior wife brings the same value to any polygamous household. All in all, in polygamous households senior wives and husbands are alike in terms of their attractiveness ranking, but co-wives differ a great deal in their skills, with the senior wife being at the top in the skill distribution and the junior wife being at the bottom.

The second characteristic is that some households end up being monogamous, even when co-wives complement each other. The reason is that there is a threshold σ_0 at which female skills are high enough that women with skills above this threshold prefer to be senior wives to middle-wealth husbands than junior wives in richer polygamous households. Of course, women with skills between σ_0 and σ_1 would like to hire a junior wife, but all women willing to be junior wives prefer to be married to wealthier couples that outbid them. Hence, these marriages end up being monogamous, with positive assortative matching between husbands and wives given the supermodularity of the surplus. Note that Assumption 1 implies that women in monogamous households take the senior wife position.

The last feature to notice is that poor men with wealth below γ_0 end up being single, which results from the assumption that the sex ratio is not high enough to compensate for the fact that some men have two wives.

The novel aspect of this characterization of marriage markets with polygamy is that all of these relevant thresholds are endogenous: A solution to the model exists such that in equilibrium, all women choose their role optimally and households optimally arise to be single-headed, monogamous, or polygamous. The next proposition formalizes this result: It establishes the existence of a threshold-shaped stable matching and characterizes the stable outcome. A sketch of the proof is provided in the text, and the formal proof is provided in Appendix A. Appendix B, in turn, generalizes the result to the case in which the contribution of skills in the junior wife position to household output is strictly positive ($h_2(y, s', s) > 0$).

Proposition 1 *The marriage market with populations $s \sim F[\underline{S}, \overline{S}]$ and $y \sim G[\underline{Y}, \overline{Y}]$ and marital output $h(y, s', s) = H(y, K, m(s))$ satisfying Assumption 1 with $h_2(y, s', s) = 0$ has a stable outcome, $(\mathcal{M}, \mathcal{L}, \mu, v^\mu, u^\mu)$, characterized by:*

1. *Unique thresholds $\sigma_0 \in [\underline{S}, F^{-1}(0.5)]$, $\sigma_1 = F^{-1}[1 - F(\sigma_0)]$, $\gamma_0 = G^{-1}[F(\sigma_0)]$, and $\gamma_1 = G^{-1}[F(\sigma_1)]$.*
2. *The partition of female skills into junior and senior wife roles, $\mathcal{L} = [\underline{S}, \sigma_0)$ and $\mathcal{M} = [\sigma_0, \overline{S}]$.*
3. *Matching function*

$$\mu = (y, \mu_2(y), \mu_3(y)) = \begin{cases} (y, \emptyset_s, \emptyset_s), & \forall y \in [\underline{Y}, \gamma_0) \\ (y, \emptyset_s, F^{-1}[G(y)]), & \forall y \in [\gamma_0, \gamma_1) \\ (y, s', F^{-1}[G(y)]), & \forall y \in [\gamma_1, \overline{Y}], s' \in [\underline{S}, \sigma_0). \end{cases}$$

4. *Feasible payoff functions*

$$u^\mu(s) = \begin{cases} h(\gamma_0, 0, \sigma_0), & \forall s \in [\underline{S}, \sigma_0) \\ h(\gamma_0, 0, \sigma_0) + \int_{\sigma_0}^{s} h_3\Big(\mu_1(t), 0, t\Big)\, \mathrm{d}t & \forall s \in [\sigma_0, \sigma_1) \\ h(\gamma_0, 0, \sigma_0) + \int_{\sigma_0}^{s} h_3\Big(\mu_1(t), \mu_2(t), t\Big)\, \mathrm{d}t & \forall s \in [\sigma_1, \overline{S}] \end{cases}$$

$$v^\mu(y) = \begin{cases} 0, & \forall y \in [\underline{Y}, \gamma_0) \\ \int_{\gamma_0}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\, \mathrm{d}t, & \forall y \in [\gamma_0, \gamma_1) \\ \int_{\gamma_0}^{y} h_1\Big(t, \mu_2(t), \mu_3(t)\Big)\, \mathrm{d}t, & \forall y \in [\gamma_1, \overline{Y}]. \end{cases}$$

3.4.4 Sketch of Proof

I follow Chiappori, Oreffice, and Quintana-Domeque (2018) and prove Proposition 1 using a direct approach. First, I conjecture the equilibrium is as described in Proposition 1 and derive a complete characterization in terms of threshold σ_0 from the *local* stability conditions. Second, I show existence and uniqueness of σ_0. Third, I prove that the characterized assignment is stable. The proof can be sketched as follows:

1. Characterization of the conjecture in terms of σ_0 (Section A.1):
   (a) From the distributions of males' and females' traits, I derive the **thresholds** and the **matching function** as functions of σ_0.
   (b) From the first-order necessary conditions, I obtain **indirect utilities** as functions of σ_0.
2. Proof of the existence of a unique threshold σ_0 from the indifference conditions of threshold individuals (Section A.2).
3. Check that the conjecture satisfies stability conditions (Section A.3):
   (a) Taking the sorting of women as given by threshold σ_0, I prove that there is no coalition of one, two, or three individuals that blocks the characterized assignment. This part of the proof relies heavily on the supermodularity of $h(y, s', s)$.
   (b) Show that women are sorting optimally: This part of the proof follows from Part 4 of Assumption 1, which guarantees that there is no blocking of any coalition in which females take a role different from the one prescribed by their position relative to σ_0.

In the absence of complementarities, the existence and uniqueness of stable outcomes in many-to-one marriage problems has been proved by equivalence with total surplus maximization or optimal transportation (Hatfield, Kominers, Nichifor, Ostrovsky, and Westkamp (2013); Shapley and Shubik (1971); and Chiappori, McCann, and Nesheim (2010)). The presence of co-wives' complementarity and endogenous sorting into household roles in my model imply that I cannot rely on these strategies to prove existence of stable outcomes and need to use the direct approach. First, the fact that women choose their role endogenously renders local optimality conditions only necessary, but not sufficient, for stability. In particular, one needs to disprove blocking coalitions in which women change their role. Second, there exist parameterizations of this model such that stable outcomes do not maximize total surplus.[10]

3.5 Assumptions and their testable implications

The distinctive features of this theory of polygamy—that is, co-wives' complementarities (Assumption 1, part 3) and the hierarchy of women's roles (Assumption 1, part 4)—drive three main testable predictions I examine in Section 4.

1. The skill distribution of seniors dominates the skill distribution of monogamous wives, which in turn dominates the skill distribution of juniors. First, complementarities between co-wives mean that the boost in productivity from having a junior co-wife is greater the higher the skill level of the senior. As a result, women who select into the senior wife position will

[10]Examples available upon request.

have the highest skills in the market. Second, the gap in the relative contributions of women's roles to production (hierarchies) implies that the lowest skilled women select into the junior role and the highest skilled into the senior role, segmenting the market. Intuitively, the lower the contribution of skills in the junior role the more skills are "wasted" in this role, which induces families to fill the junior position with as low a skill as possible.[11] Similarly, the higher the contribution of skills in the senior role the more profitable it is for families to fill the senior position with as high a skill as possible. Finally, as I expand on below, the gap in the returns to seniority versus the benefit of having a co-wife imply that monogamous women are of middle skills. Intuitively, low-skilled women benefit from being a junior to a high-skilled senior, but as skills increase, the returns to seniority (even without a co-wife) dominate.

2. Intrahousehold skill inequality between co-wives. By the assumption of hierarchies, families optimally position their highest skilled wife as senior and their lowest skilled as the junior. The higher the difference in the returns to skill in each role, the higher the intrahousehold skill inequality.

3. Assortative matching and men selection. A supermodular household production function (Assumption 1, part 3) implies positive assortative matching between men and wives within roles, a well-known result in the literature. As a corollary, the wealthiest men are polygamous and the less wealthy are monogamous or single, since it is more costly for wealthier men to become monogamous.

4. Equilibria with both monogamy and polygamy. The assumption of hierarchies in combination with that of co-wives' complementarities is also responsible for the feature whereby both monogamy and polygamy emerge in equilibrium. However, corner equilibria (all married individuals are monogamous or all are polygamous) are possible. Whether we have corner or interior equilibria depends on the trade-off between complementartities between co-wives' roles and the skill premium of the senior position relative to the junior. For example, as the contribution of the junior role in household output declines and the returns to seniority increase, more low-skilled women select into being a monogamous senior. In general, the higher the gap between co-wives' complementarties and the skill premium of the senior role, the higher the incidence of monogamy.

4 Nonparametric tests of the model

4.1 Strategy

In this section I use data from the Nigerian LSMS-ISA survey and the DHS from 11 countries in Sub-Saharan Africa and perform three sets of nonparametric tests to show empirical support for my model with polygamy and co-wives' inequalities.

This empirical exercise requires the observation of women's and men's traits at the time

[11] In the limiting case in which all skills contribute the same to production, the opportunity cost of filling the junior position with a wife of high skill is big, implying that couples of men and seniors are attracted to low-skilled women as their junior.

of marriage, the type of marriage they form (monogamous or polygamous), and wives' ranks within polygamous households (senior or junior).

I start by constructing indexes of premarital attractiveness for men and women to measure the traits on which individuals match in the marriage market. Importantly, and as I describe in Section 4.2.2, I only use variables that are observed at the time of marriage and that cannot change due to the distribution of resources within the household after marriage.

Next, I use the observation of whether individuals are married polygamous or monogamous to identify the type of marriage of each man and woman in the data.

The last input in testing the model is the assignment of polygamous wives to the senior or junior roles. I use wife's rank as reported by the husband, which I observe in all data sets.

With these data I perform three sets of nonparametric tests.

First, I test for the sorting patterns of women based on their attractiveness by type of marriage and wife rank. The prediction of the model is that the attractiveness distribution of senior polygamous wives dominates the attractiveness distribution of monogamous wives, which in turn dominates the attractiveness distribution of junior polygamous wives. I test these statistical dominance patterns under the null hypotheses that all women are drawn from the same population of attractiveness or that the distributions are equal across women in these three groups. I find strong support for the selection patterns implied by my model (Section 4.5 shows results using the LSMS-ISA data and Section 4.9 shows results using the DHS data).

Second, I zoom into marriages and test for the equilibrium shape under various null hypotheses. I develop a novel test to evaluate the equilibrium with hierarchies between co-wives under the null hypothesis that polygamous males marry equally skilled co-wives. My test rejects equality of skills between co-wives.

Finally, I test the model prediction that matching is positive assortative between men and women, and I reject random matching.

4.2 Data and construction of key variables

The main empirical analysis of the paper is conducted using the Nigerian LSMS-ISA (The World Bank, 2010-2014). This data set contains all the necessary inputs—pre-marital traits for men and women and wives' rank—to perform all tests. The LSMS-ISA-Nigeria is a nationally representative household panel survey of 4997 agricultural households[12] interviewed in two seasons per wave: the post-planting season and the post-harvest season. In this paper I use the 2010-2011 wave and restrict attention to agricultural households in the Northern states where polygamy is most prevalent.[13] The final sample includes 527 polygamous households, and 994 monogamous households.

I provide additional evidence in favor of my model using DHS data from all countries in Sub-Saharan Africa with at least a 20% share of polygamous men: Senegal, Mali, Niger, Guinea,

[12]Agricultural households are those that manage or own at least one agricultural plot.

[13]This restriction is motivated by the fact that polygamy is only legal in Northern states that are ruled by Sharia law. However, all of my results are robust to including Southern states.

Burkina Faso, Sierra Leone, Ghana, Togo, Benin, Nigeria, and Gambia.

4.2.1 Summary statistics

Table 3 shows summary statistics of the main data source measured for all households and for polygamous households, and the difference in statistics between polygamous and monogamous households (columns labeled "P-M"). The first three rows show characteristics of the marriage market. Women in this economy tend to marry young, with mean age at marriage around 17. Women in polygamous unions marry younger on average. Moreover, the age difference between husbands and wives is about 14 years; this gap increases by over 5 years on average in polygamous marriages relative to monogamous marriages.

Table 3: Summary statistics, LSMS-ISA-Nigeria, 2010

	All		Polygamous		P-M	
	Mean	SD	Mean	SD	Mean	SD
Marriage market						
Women age at marriage	16.838	6.105	16.692	5.919	-.288	.272
Men age at marriage	26.963	8.499	26.436	8.27	-.804	.46
Husband-wife age gap	13.593	8.146	16.206	8.695	5.641	.331
Household structure						
# of wives	1.13	.794	2.189	0.45	1.189	0.013
# of children	3.789	2.863	5.864	3.063	2.274	0.127
# of domestic workers	.013	.165	0.002	0.041	-0.006	0.004
# of other family	.367	1.103	0.23	0.811	-0.055	0.041
Production and Labor						
Plot uses only family labor	0.8	0.4	0.701	0.458	-0.124	0.016
Employed	0.59	0.492	0.591	0.492	0.005	0.01
Works only for family	0.925	0.264	0.934	0.249	0.016	0.007
Women skill at marriage						
Fertile years	32.168	6.1	32.308	5.919	.288	.272
Father's education	1.144	2.221	1.119	2.013	-.052	.097
Men wealth at marriage						
Father's education	1.204	2.452	1.059	1.753	-.109	.121
Value of inherited land	6.494	39.858	9.027	62.823	4.1	2.182
# of plots inherited	1.461	1.3	1.512	1.309	.049	.071

Notes: *LSMS-ISA* stands for *Living Standard Measurement Study - Integrated Surveys on Agriculture*. *P-M* refers to the difference in the corresponding statistics between polygamous and monogamous households. *Marriage market* and *Household structure* statistics are reported at household level. *Plot uses only family labor* is a dummy variable measured at the agricultural plot level that takes value one if all workers in the plot are household members and zero if some worker on the plot is an external hire. *Employed* is a dummy variable that takes value one if the individual works on their own, on a family plot, or for a non-family member. *Works only for family* is a dummy variable that takes value one if *Employed* equals one and the individual does not work for members outside of the household.

The next four rows show that most households in rural Nigeria are nuclear. The average number of wives in polygamous unions is two and the average number of children is almost six—two more than in monogamous families. Other family members live in one-third of households on average and households do not typically employ domestic workers.

The next three rows show that agricultural production is extremely dependent on labor from household members (typically wives and children). First, 80% of agricultural plots in the data only use family labor as opposed to external hires. Second, on average, 92.5% of employed individuals report working only for family members.

4.2.2 Female skill and male wealth

The last five rows of Table 3 describe the measurements used to construct indexes of women's and men's attractiveness in the marriage market. Importantly, because I observe households at various points in their life cycle, I only select variables that would have been observed at the time of marriage, are not likely to be affected by post-marital allocation of resources within the household over the life cycle, and are recognized important traits in the marriage market.

For women, the measurements used to estimate female skills are *fertile years*, calculated as 49 minus the age at marriage,[14] and *father's education*, the years of completed education of the wife's father. While the former represents women's ability to produce children (Boserup, 1970), the later captures premarital skills or family wealth. On average, women have 32 years of fertility.[15] Moreover, the fathers of wives are almost non-educated, with an average of 1.14 years of completed education.

The measurements used to construct the index of male wealth are similar to those for women (*father education* and *age at marriage* to capture premarital ability) and add two components to capture husband's family wealth: Value and number of plots inherited by the husband.

I aggregate these factors into a single index for women and men using the first principal components. Because there are only two factors for women, the first and second principal components explain almost the same portion of the variation in the data. Therefore, I also construct the attractiveness index using the simple average of the standardized factors and verify that the results from all tests are identical.

4.2.3 Type of marriage and type of wife

I categorize all individuals in the data by type of marriage into polygamous or monogamous. Additionally, for women, I use the reported rank by the husband to categorize polygamous wives as senior or junior wives.[16] In the tests I perform below I use the categorization of men into

[14] Forty-nine is the last age at which women are asked questions about their fertility in the LSMS-ISA data, so I take this as the last age a woman is fertile.

[15] Previous studies in different contexts show that youth is valuable in the marriage market (Low (Forthcomingb), Choo and Siow (2006) are two recent examples with US data). In the majority of households in my sample junior wives are younger than their senior wife but, on average, junior wives marry older (the mean age at marriage is 18.13 for junior wives and 15.15 for senior wives). There are many explanations as of why some women marry older even though youth is valuable in the marriage market. For example, underlying ability together with search frictions may imply that lower skilled women take more time to find a match (Sautmann, 2017) and arrive older at marriage. Another possibility is that some low skilled women delay marriage to increase their skills, for example, through education—although this story is not consistent with the extremely low levels of women's education in the data. Even though my model is agnostic about the reasons behind age at marriage, these explanations are still consistent with the fact that older women are less attractive at the time of marriage for reasons determined *prior* to marriage production (for example, search frictions that interact with ability). It is in this sense that fertile years is a pre-marital trait exogenous to the value produced by the potential marriage.

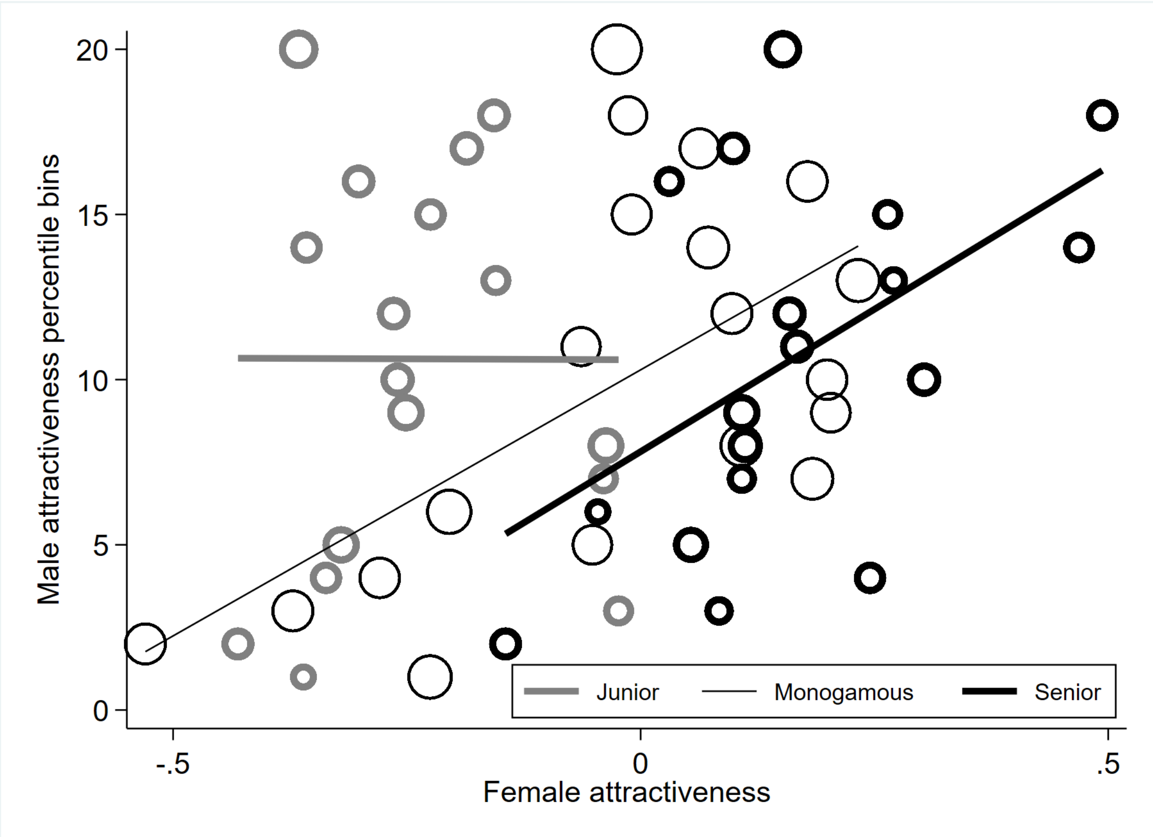
[16] In all of the datasets used, I consider the senior wife to be the first in the reported rank. Wife rank does not perfectly correlate with age: For example, in 5.8% of households in the LSMS-ISA data, the senior wife is

two groups, polygamous or monogamous, and of women in three groups, polygamous juniors, polygamous seniors, or monogamous. It is important to note that I estimate the attractiveness index using information for all men and all women irrespective of their categorization (that is, the estimation of premarital traits is blind to any marital outcome).

4.3 Marriage market equilibrium in the data

Figure 3 shows who marries whom in the data, providing the observed counterpart of Figure 2.

Figure 3: The Marriage Market Equilibrium in the Data



Notes: The size of hollow circles represents the number of observations. I smooth the data by taking the average female skills by percentile bin of men's type.

The horizontal axis displays women's attractiveness and the vertical axis displays percentile bins of male wealth. Each hollow circle in the figure represents the marriage of men in the corresponding percentile bin to the average skill of their wives. The lines in the graph represent the fitted regression of the male percentile bin on female attractiveness. Thick gray circles and lines represent marriages between polygamous men and wives reported as the junior in the household. Thick black circles and lines represent marriages between polygamous men and senior wives. Finally, thin black circles and lines represent monogamous marriages. The size of circles captures sample sizes when smoothing the data.[17]

younger than the junior wife.

[17] In producing this graph I exclude two outlier points: one point for the match between men and juniors above women's skill 0, and one point for the match between men and seniors below women's skill -0.5. The graph with all of the data is in Appendix D and shows similar conclusions in spite of the outlier matches.

Many features of the equilibrium in the data are consistent with the stable matching that arises from the stylized model (Figure 2). Most noticeably, monogamous matches lie in the middle between marriages of polygamous males to their junior and their senior wives. Second, males in the bottom-5 percentile bins of the wealth distribution are more likely to be monogamous, while males in the top-5 percentile bins are more likely to marry polygamous. On the other hand, there is monogamy all over the male wealth distribution so that, in general, we find both monogamous and polygamous men at any percentile bin. Finally, consistent with the model, monogamous marriages feature positive assortative matching, matching between polygamous males and senior wives is positive assortative, and there is random matching between polygamous men and junior wives.

4.4 Noise in attractiveness measures

Before introducing the empirical tests of the model, it is important to notice from Figure 3 that matching between men and women in terms of the measured attractiveness is positive assortative but not perfect, as the stylized model predicts. To reconcile the noisy data with the stylized model from Section 3, I introduce a stochastic structure in the measurement of women's skills. In the data I observe a man of wealth y married to women type

$$s(y) = \tilde{s}(y) + \epsilon(y), \tag{2}$$

where $\tilde{s}$ is the true skill and ϵ is a random component unobserved by me. I assume that $\epsilon(y)$ is identically and independently distributed Normal with parameters $(0, \sigma^2)$.

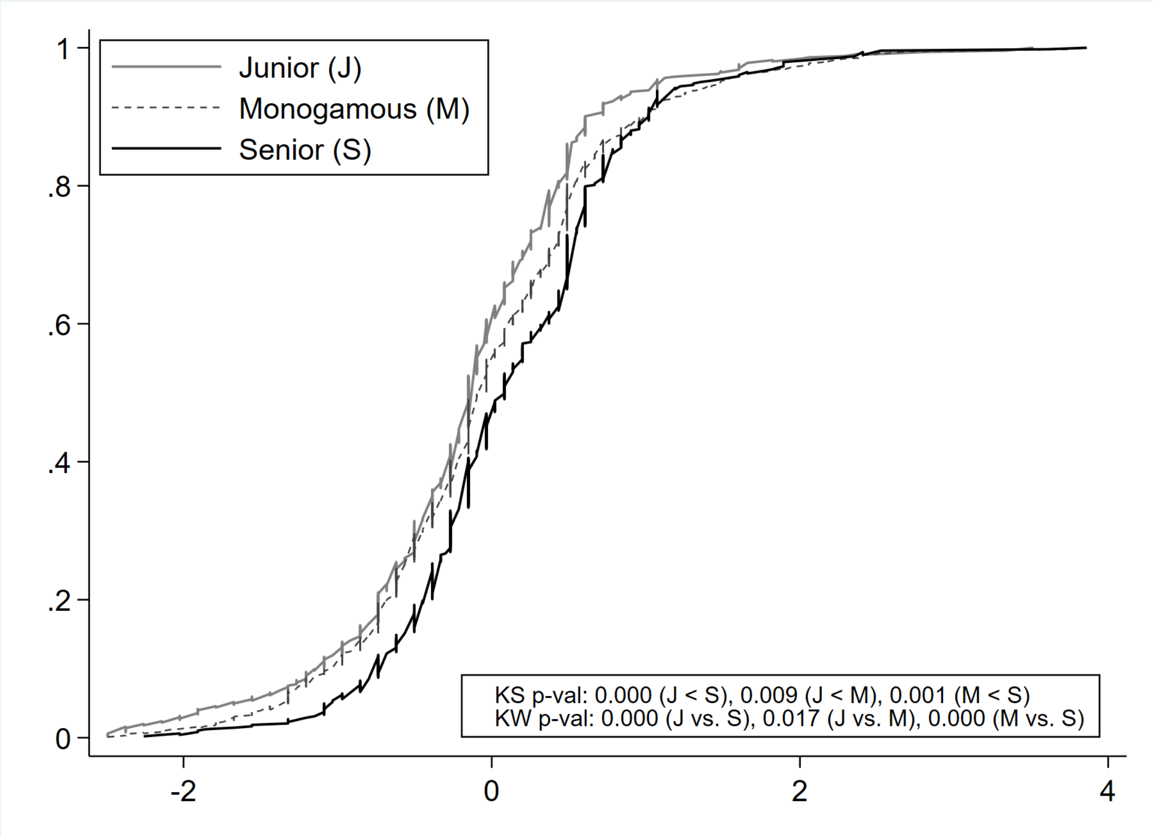
4.5 Tests of sorting by type of marriage and role

The first set of nonparametric tests shows that the data support the sorting patterns of individuals based on their marital traits into types of marriages and household roles for polygamous wives.

Figure 4 shows the distribution of attractiveness for women, broken down by type of wife. Junior wives are represented by the solid gray line, monogamous wives by the dashed line, and senior wives by the solid black line. As the model (augmented with the noise structure) predicts, the skills of polygamous wives of highest rank stochastically dominate the skills of all other wives, showing that senior wives are the highest skilled in the marriage market. Moreover, the distribution of monogamous wives stochastically dominates that of polygamous junior wives demonstrating that monogamous wives are in the middle of the female skill distribution between senior and junior polygamous wives.

To complement this evidence, I formally test for equality in attractiveness between any pair of groups of wives using two statistical tests, whose p-values are reported in the bottom right corner of the figure. The first line shows the Kolmogorov-Smirnov (KS) test of equality of distributions, which evaluates whether any two samples come from the same distribution. For

Figure 4: Distribution of women's skills by type of marriage and rank in the LSMS-ISA-Nigeria (2010) data



Notes: *LSMS-ISA* stands for *Living Standards Measurement Study-Integrated Surveys on Agriculture*. The horizontal axis displays the standardized index of female attractiveness at the time of marriage. The vertical axis shows the cumulative distribution. KW stands for the Kruskall-Wallis test of equality of distributions. KS stands for the Kolmogorov-Smirnov test of unique populations.

any two groups of women, I reject the null hypothesis that the two groups' skill distributions are equal against the alternative hypothesis implied by my model. For example, "0.000 ($J < S$)" means that I reject the null hypothesis that the skill distribution of junior wives (J) is equal to the skill distribution of senior wives (S) in favor of the hypothesis that juniors are significantly less skilled than seniors, with a p-value of 0. The second line shows the Kruskal-Wallis (KW) test of equality of populations, which evaluates whether any two distributions originate in the same population. Consistent with my model and the KS test, I reject that the distribution of skills across any two groups of women are equal.

I also test the model prediction that polygamous males are wealthier than monogamous males. I present the wealth distributions of men and the KW and KS tests in Appendix C. Consistent with my model the wealth distribution of polygamous men dominates that of monogamous men, though the differences are marginally significant.

These sets of tests use information on individual traits to test for equilibrium sorting into types of marriages and household roles without using information *at the household level*. In the next two subsections, I zoom into marriages and perform two sets of tests to answer: (i) are co-wives significantly different in their skills? and (ii) do men and women match assortatively?

4.6 Test of inequality between co-wives

I now develop a nonparametric test of the difference in skills of co-wives within polygamous families and reject alternative models of matching with polygamy in which polygamous men marry two wives of statistically equal attractiveness.

Under the null hypothesis of no differences between co-wives, the difference between the skills of the senior and junior wives of man y, denoted by $s(y)$ and $s'(y)$, respectively, amounts to the difference in the unobserved components introduced in equation (2):

$$s(y) - s'(y) = \epsilon(y) - \epsilon'(y).$$

As a result, the variance of the differences between co-wives across men must equal $2\sigma^2$ under equilibria with equal co-wives.

I use this feature of equilibria with no co-wives inequalities to develop the following test.

Under the null hypothesis of no co-wives inequalities in traits:

$$Var(s(y) - s'(y)) - 2\sigma^2 = 0.$$

Under the equilibrium proposed in this paper with co-wives' inequalities, however,

$$Var(s(y) - s'(y)) - 2\sigma^2 \neq 0.$$

The test statistic is

$$\frac{\sum(s(y) - s'(y))^2}{N_{poly}} - 2\hat{\sigma}^2,$$

where N_{poly} is the number of polygamous males in the data. I compute the first term of the test statistic, $\frac{\sum(s(y) - s'(y))^2}{N_{poly}}$, using information on the difference in skills between co-wives in all polygamous families in my data. Moreover, I compute the second term, $2\hat{\sigma}^2$, as twice the sample variance of the residuals of a regression of wife's skills on husband's wealth in the sample of *monogamous* couples.

Finally, the decision to reject or not the null hypothesis is made by comparing the empirical value of the test statistic with the percentiles of its distribution. I approximate the distribution of the test statistic by the Bootstrap, taking 1000 replications from the data.

I reject the null hypothesis that co-wives are equal at the 5% level. In effect, the empirical value of the statistic is 0.28, which lies between the 95th and the 99th percentiles of the distribution of the simulated distribution—0.22 and 0.34, respectively.

4.7 Test of positive assortative matching

I test the model's feature of assortative matching and reject random matching by looking at the regression coefficient in a model of male traits on female traits. Table 4 shows these coefficients and their robust standard errors by type of marriage and role of the spouse (if polygamous). Because men's wealth and women's skills are standardized, the coefficients are expressed in standard deviation units.

Table 4: Positive assortative matching

	Polygamous		Monogamous
	Male wealth	Male wealth	Male wealth
Junior's skills	-0.1072		
	(0.0731)		
Senior's skills		0.1831	0.0753
		(0.0941)	(0.0388)
Observations	355	355	926

Robust standard errors in parentheses.

As implied by the model, the data suggest that the marriage market equilibrium exhibits positive assortative matching between men and women within household role. For example, in polygamous marriages, a man one standard deviation wealthier marries a woman 18% of a standard deviation more skilled as their senior, with the correlation significant at the 5% level. Moreover, consistent with the version of the model that assumes that all women in the junior position produce the same fixed output, I cannot reject random matching between polygamous males and their junior wives. Thirdly, the correlation in spousal traits in monogamous marriages is positive and significantly different from zero.

4.8 Robustness checks: Accounting for marital transitions

A common challenge when testing and estimating matching models is that the marital history of some individuals in the data may not be complete. For example, in my context, some families that are observed to have married monogamous in 2010 might become polygamous in the future. As a result, some women and men who are categorized as monogamous should actually, according to the model, be treated as polygamous.

I perform three robustness checks to account for this possibility.

First, since for the main analysis I use the first wave of the LSMS-ISA panel, I follow families over the subsequent waves and identify the monogamous couples that incorporate a second wife in later waves. I add the information on the new wife to these households and treat them as polygamous. I then reproduce the full analysis (including a revision of the rank of the wives) under this new categorization of individuals and conclude that the results are unchanged.

Second, I also replicate the analysis excluding the monogamous families I observe transitioning, and the results are the same.

Finally, I run the analysis on the sample of households in which the husband is 40 years old or older (the sample most likely to have completed their marital history) and, once again, the results remain unchanged (although some tests have lower precision due to low sample size).

A second empirical challenge is that we only observe still-married couples, so that tests of marriage patterns based on this sample may be biased. In the rural Nigeria data, however, both the stock of divorced individuals and the fraction of individuals who report that a previous marriage ended in divorce are very low.[18] Nevertheless, I additionally run my main analysis on the sample of young couples[19] which is at lower risk of survival bias and all my main results are the same (although, again, some tests have lower precision due to low sample size). Because this sample of newest marriages is the most likely to experience (not-yet-observed) transitions from monogamy to polygamy, I also run the first two robustness checks described above on the sample of young couples and draw the same conclusions.

4.9 Additional evidence from the DHS

I complement my main analysis with DHS data to evaluate the testable predictions of the stochastic dominance patterns of women based on their attractiveness distributions.[20]

To construct the index of women's attractiveness I use *fertile years* (as in the main analysis), but since parental education is not available for adults not living with their parents I use *height* as a component of skills that captures physical stamina. Unlike other available measures of health (such as weight or BMI), height is an individual characteristic that does not vary significantly over the life cycle and is not affected by the intrahousehold allocation of resources.

I select countries in sub-Saharan Africa with at least 20% of polygamous men and where both *fertile years* and *health* are measured.

The tests of sorting of women by type of marriage and rank are reported in Figure 5 for all 11 countries combined in the first panel and for each individual country in the subsequent 11 panels. Each graph in the figure has the same structure as Figure 4 and shows the distribution of the attractiveness index for each type of wife, along with the KS and KW tests.

The first panel shows results for all countries combined, in which I residualize the attractiveness indexes after controlling for country fixed effects. Because I have a large enough sample, I restrict attention to households in which the head is at least 40 years old to maximize the likelihood that they have completed their marital history. Confirming the results from the main analysis using the LSMS-ISA data and in favor of my model, the distribution of senior women's skills significantly dominates the distribution of monogamous wives' skills which, in turn, dominates that of junior wives. I reject equality of distributions between any two groups of wives based on both the KS and the KW tests.[21]

I then present the results for each country separately.[22] First, in eight countries the statistical tests support the dominance patterns implied by my model in which the skill distribution of monogamous wives lies in the middle of the skill distribution of junior and senior polygamous wives, and the distribution of senior wives dominates all other distributions.

[18] In the sample used for my main analysis 1.14% of men and 0.23% of women are divorced or separated. Moreover, in the 2015 third wave of the data I observe that 6.31% of men and 2.68% of women had a previous marriage that ended in divorce.

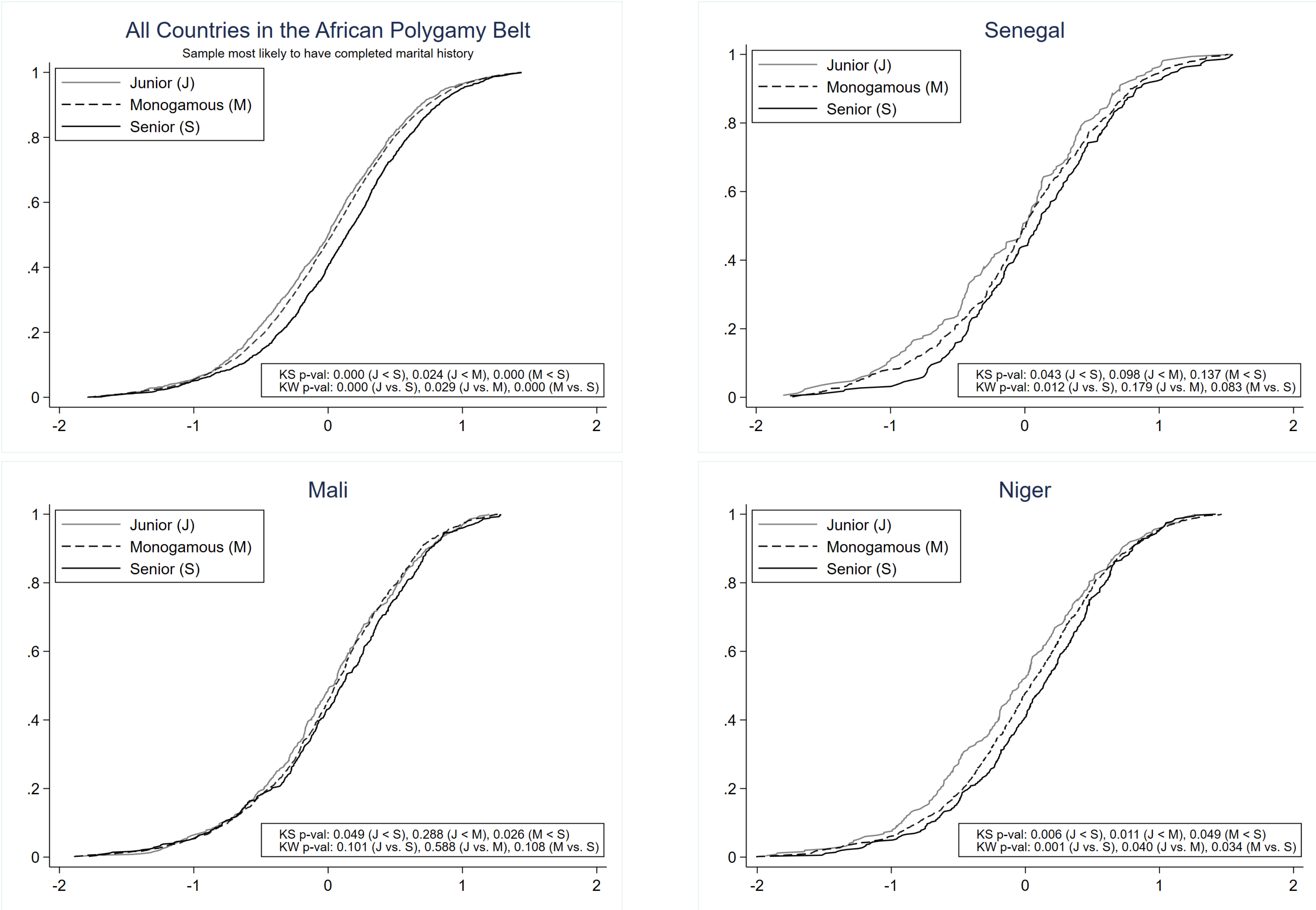
[19] Couples in which the head is less than 40 years old.

[20] The DHS includes the type of marriage, the rank of polygamous wives, and good measures of female attractiveness in the marriage market. However, it does not include relevant measures of male wealth like information on family wealth or landholdings.

[21] The plots in Figure 5 are smoother than those in Figure 4 due to the use of height—a continuous variable—instead of parental education—a discrete variable to construct the attractiveness index in the DHS.

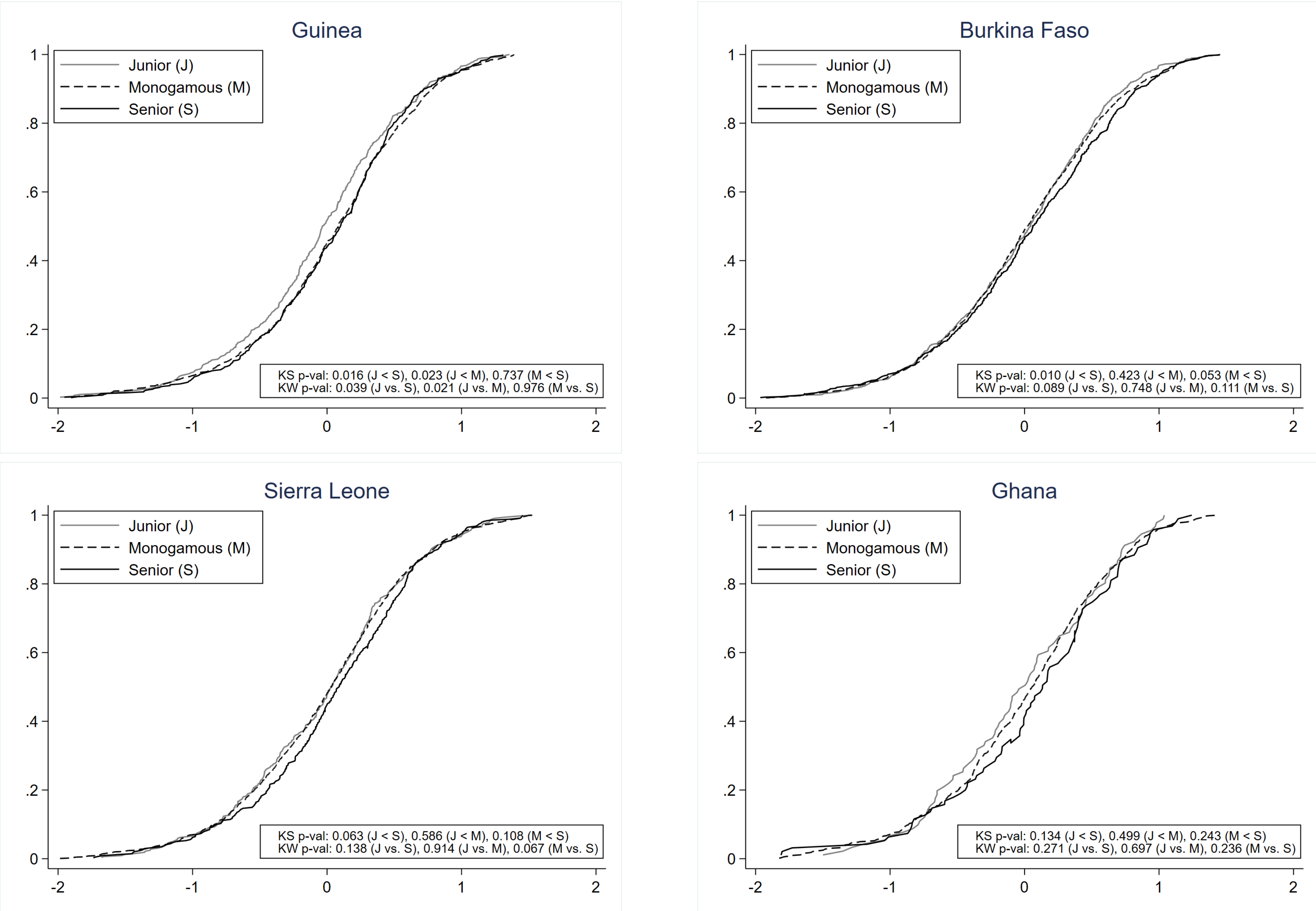
[22] Due to insufficient sample size, I do not select households based on age for the individual countries' analysis.

Figure 5 (part 1 of 3): Distribution of women's skills by type of marriage and rank in the DHS data



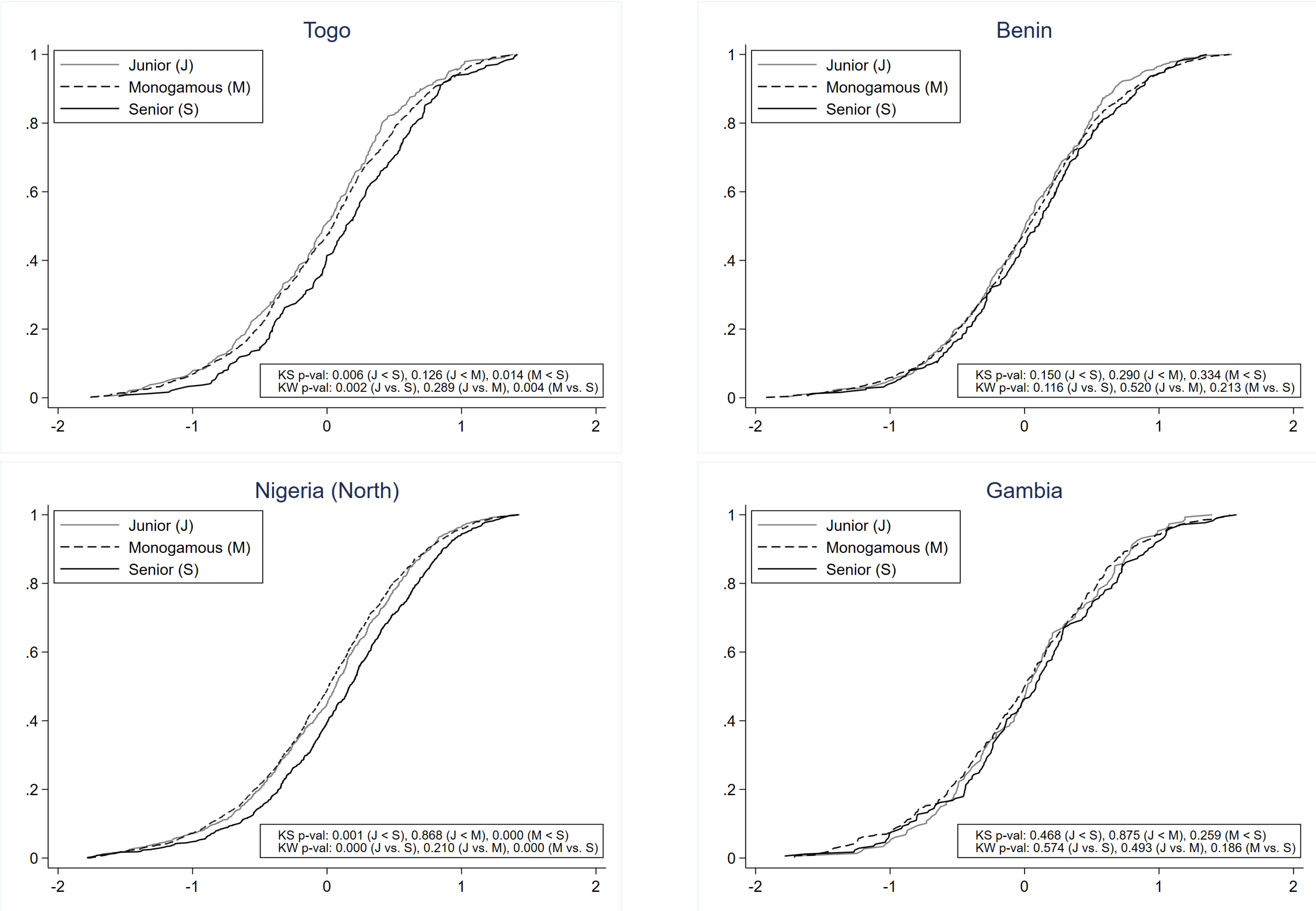
Notes: DHS stands for *Demographic and Health Survey*. The horizontal axes display the standardized index of female attractiveness at the time of marriage. The vertical axes show the cumulative distribution. KW stands for the Kruskall-Wallis test of equality of distribtuions. KS stands for the Kolmogorov-Smirnov test of unique populations.

Figure 5 (part 2 of 3): Distribution of women's skills by type of marriage and rank in the DHS data



Notes: DHS stands for *Demographic and Health Survey*. The horizontal axes display the standardized index of female attractiveness at the time of marriage. The vertical axes show the cumulative distribution. KW stands for the Kruskall-Wallis test of equality of distribtuions. KS stands for the Kolmogorov-Smirnov test of unique populations.

Figure 5 (part 3 of 3): Distribution of women's skills by type of marriage and rank in the DHS data



Notes: DHS stands for *Demographic and Health Survey*. The horizontal axes display the standardized index of female attractiveness at the time of marriage. The vertical axes show the cumulative distribution. KW stands for the Kruskall-Wallis test of equality of distribtuions. KS stands for the Kolmogorov-Smirnov test of unique populations.

In three of these eight countries (Senegal, Niger, and Togo) all of the hypotheses tests reject the equality of distributions for any pair of groups of wives. Moreover, in five countries (Mali, Guinea, Burkina Faso, Sierra Leone, and Nigeria) two sets of hypotheses support my model: In all of these countries, the distribution of senior wives' skills significantly dominates that of junior wives and either the distribution of senior wives significantly dominates that of monogamous wives or the distribution of monogamous wives significantly dominates that of junior wives. Finally, for three countries (Ghana, Benin and Gambia), I am not able to reject the equality of distributions for any pair of types of wives, even though graphically the patterns go in the direction predicted by the model.

Overall, the evidence suggests that countries in the polygamy belt show a consistent pattern of selection into polygamy and types of roles within households, in which there is significant skill inequality between co-wives within polygamous households.

4.10 Rejection of alternative models based on the evidence

Key observational distinctions with *dual-monogamous* models. The assumption of co-wives' complementatires produces matching and sorting patterns than differentiate my theory from the "dual-monogamous" model in which polygamous men set up two households and divide their time for producing with two co-wives who do not interact. To give the dual-monogamous model the greatest chance of producing sorting patterns similar to my model, suppose that hierarchies in women's roles and supermodularity are maintained.

First, if production across a man's families is identical, supermodularity implies that men match with many women of similar skills, in contrast to my model's prediction of co-wives' skill inequality (Eeckhout and Kircher, 2018). In effect, the results of my test of inequality between co-wives (Section 4.6) rejects this prediction of the dual-monogamy model.

Second, assume that husbands are technologically constrained to marry at most one senior. The assumption of hierarchies implies that seniors will be of higher skills than juniors, as in my model. However, without co-wives' complementarities, some of the highest skilled women may prefer a monogamous marriage to an exclusive husband, in contrast to my model's prediction that senior polygamous wives have the highest skills. Once again, the evidence that the skills of senior wives stochastically dominate the skills of monogamous wives rejects a prediction of the dual-monogamous model.

Finally, it is worth noting that in some economies, co-wives' cohabitation, division of labor within the household, and joint work in farming is very frequent, while polygamous males living in various dwellings is very infrequent (as I discuss in Section 2 and show in Tables 1 and 2).

Conflict and distaste for polygamy. Some studies show that women at higher risk of polygamy take protective strategies such as saving more (Boltz and Chort, 2019), suggesting a dislike for polygamy or potential conflict between co-wives (Jankowiak, Sudakov, and Wilreker, 2005). Even though for tractability my model abstracts from these elements, a more complex version could include idiosyncratic costs of polygamy as a (negative) shifter to marital value.

This cost may exist in addition to the *technological* advantage of co-wives' joint work for the family, and as long as the technological motive for polygamy dominates, the predictions of my model would remain. Note that in my empirical tests I allow for measurement error in skills, which could be interpreted as unobserved features affecting the productivity of skills within the household (such as the cost associated with polygamy). In addition, the evidence on co-wives co-habitation and hierarchical organization presented in Section 2 supports the importance of dominating productive interactions between co-wives, and the tests I perform in above in this section reject alternative models that assume away these productive interactions.

Sequential formulation. An interesting question is whether a sequential formulation of this model, in which men marry their first wife, spend some time in monogamy, and marry their second wife later, would have different implications than my "one-shot" marriage market model. I argue that it would not. The predicted matching patterns of the two models would differ if the attractiveness *rankings* of men and women significantly changed in between the two marriages. First, in the data, the median gap between a man's marriages is 10 years, and the median age gap between co-wives is 15 years, which may not be long enough for the female skill distribution to radically change. Second, under the assumptions of my model the wealth *ranking* of men would not change because of their period in monogamy. In effect, because production in monogamy is supermodular, men accumulate more wealth during their first marriage but the wealthier accumulate relatively more, leaving the relative position of men unchanged. Overall, I conclude that the sequential formulation would not change the distribution of attractiveness and the individual rankings, leading to the same equilibria.

5 Conclusion

I propose a novel framework in the polygamy literature that captures the fact whereby co-wives interact and organize in a hierarchy of senior-junior wives. I study the characteristics of households that emerge in equilibrium, in particular, the optimal sorting of women into polygamous versus monogamous marriages and into the role of the senior or junior wife, based on their skills.

The main result of the paper is that the equilibrium in the marriage market exhibits positive assortative matching between females and males, and positive sorting of females into household roles. The novel implication is that polygamous households show high levels of co-wives' inequality: Senior wives are the highest-skilled in the market and junior wives are the lowest-skilled. Monogamous wives fill the gap in between.

I perform three sets of nonparametric tests that support the predictions of the model. In particular, data from many countries in the African polygamy belt confirm the sorting patterns of women whereby the skill distribution of polygamous senior wives dominates that of monogamous wives, which in turn, dominates the skill distribution of junior polygamous wives.

From a policy perspective, the demonstration of significant inequality between co-wives sheds new light on the gender inequality issues associated with polygamy. Inequality between women in polygamous societies is new with respect to previous models of polygamy and adds to the model prediction of high male inequality (which has been highlighted in the literature).

An interesting open question is what the consequences of policies regulating polygamy are for women's welfare. This paper suggests that policies that regulate polygamy for the purpose of fostering development and improving the welfare of women should be accompanied by improvements in the outside options of women, especially of the poorest ones. But in order to design successful policies it is necessary to have precise knowledge of the economic forces that induce *both* women and men to form polygamous marriages and the nature of the matching patterns that emerge in equilibrium. This paper advances knowledge in this crucial area.

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Appendix A Proof of Proposition 1

First, a brief note about notation.

Consider any coalition of a male, a female employed in the junior position, and a female employed in the senior position, $(y, s', s) \in [\underline{Y}, \overline{Y}] \times [\underline{S}, \overline{S}]^2$. The marital surplus that (y, s', s) would produce shall they be together is $h(y, s', s)$. Consider any matching μ (as defined in Definition 1) under which (y, s', s) are together. Throughout the proof I will refer to the following objects in the following manner:

- $h_1\Big(y, \mu_2(y), \mu_3(y)\Big)$ is the partial derivative of the marital surplus with respect to male wealth evaluated at the marriage of y under matching μ.
- $h_2\Big(\mu_1(s'), s', \mu_3(s')\Big)$ is the partial derivative of the marital surplus with respect to the skill of the woman in the junior position evaluated at the marriage of s' under matching μ.
- $h_3\Big(\mu_1(s), \mu_2(s), s\Big)$ is the partial derivative of the marital surplus with respect to the skill of the woman in the senior position evaluated at the marriage of s under matching μ.

A.1 Full characterization in terms of σ_0

Thresholds and matching function

The assignment described in Proposition 1 is positive assortative between males and females and by assumption female skills and male wealth are drawn from bounded, atomless, strictly increasing, and continuous distributions $F(s)$ and $G(y)$, respectively. These two features permit to express thresholds σ_1, γ_0, and γ_1 as closed form functions of σ_0. To see this, note that, for example, the mass of polygamous senior wives (females above σ_1) must be equal to the mass of junior wives (females below σ_0):

$$1 - F(\sigma_1) = F(\sigma_0)$$

Hence,

$$\sigma 1 = F^{-1}[1 - F(\sigma_0)] \tag{3}$$

Similarly, according to Proposition 1 the mass of married males (males above γ_0) must be equal to the mass of senior wives (females above σ_0):

$$1 - F(\sigma_0) = 1 - G(\gamma_0)$$

Hence,

$$\gamma_0 = G^{-1}[F(\sigma_0)] \tag{4}$$

Finally, the mass of polygamous males (males above γ_1) must be equal to the mass of polygamous seniors (females above γ_1):

$$1 - F(\sigma_1) = 1 - G(\gamma_1)$$
$$\gamma_1 = G^{-1}[F(\sigma_1)]$$

By (3)

$$\gamma_1 = G^{-1}[F(F^{-1}[1 - F(\sigma_0)])]$$

Hence,

$$\gamma_1 = G^{-1}[1 - F(\sigma_0)] \tag{5}$$

The same procedure can be used to express any point in the males or females distribution as a function of their Proposition 1 partner, that is, the conjectured matching function. To see this, note that according to the conjecture, if male y is married to female s employed in the senior wife position, the mass of males above y must be equal to the mass of females above s:

$$1 - F(s) = 1 - G(y)$$

Which provides a closed form expression of the spouse of y or of s:

$$y = G^{-1}[F(s)] = \mu_1(s) \quad \textit{and} \quad s = F^{-1}[G(y)] = \mu_3(y) \tag{6}$$

Finally, from the conjecture, the matching between male-senior wife couples and females in the junior wife position is random: for $s \geq \sigma_1$ and $y \geq \gamma_1$ such that $\mu_3(y) = s$:

$$\mu_2(s) = \mu_2(y) = s' \in [\underline{S}, \sigma_0] : \mu_3(s') = s \quad \textit{and} \quad \mu_1(s') = y \tag{7}$$

Indirect utilities

If the conjecture were to be stable, it must be the case that a monogamous husband solves

$$v(y) = \max_s (h(y, 0, s) - u(s))$$

and that the optimum is achieved at the spouses of y or s as described in (6). Then, from the first order conditions (evaluated at the optimum),

$$u_s(s) = h_3\Big(\mu_1(s), 0, s\Big) \tag{8}$$

By the envelope theorem,

$$v_y(y) = h_1\Big(y, 0, \mu_3(y)\Big) \tag{9}$$

Similarly a polygamous husband solves

$$v(y) = \max_{s,s':s>s'} (h(y, s', s) - u(s) - u(s'))$$

From first order conditions,

$$u_{s'}(s') = h_2\Big(\mu_1(s'), s', \mu_3(s')\Big) = 0 \tag{10}$$

$$u_s(s) = h_3\Big(\mu_1(s), \mu_2(s), s\Big) \tag{11}$$

And by the envelope theorem,

$$v_y(y) = h_1\Big(y, \mu_2(y), \mu_3(y)\Big) \tag{12}$$

Conditions (8) to (12) characterize the slopes of the payoff functions that must obtain in a stable equilibrium. Integrating these conditions over the assignment prescribed by matching μ in the corresponding segments (of monogamy and polygamy) characterizes the payoffs that each agent must be getting in a stable assignment as a function of each agents own type, unknown σ_0, and exogenous parameters. I obtain these expressions next.

Monogamy segment:

For all $s \in [\sigma_0, \sigma_1)$,

$$u(s) = u(\sigma_0) + \int_{\sigma_0}^{s} h_3\Big(\mu_1(t), 0, t\Big)\,\mathrm{d}t \tag{13}$$

For all $y \in [\gamma_0, \gamma_1)$,

$$v(y) = v(\gamma_0) + \int_{\gamma_0}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t \tag{14}$$

Polygamy segment:

For all $s \in [\sigma_1, \overline{S}]$,

$$u(s) = u(\sigma_0) + \int_{\sigma_0}^{s} h_3\Big(\mu_1(t), \mu_2(t), t\Big)\,\mathrm{d}t \tag{15}$$

For all $s \in [\underline{S}, \sigma_0)$,

$$u(s) = \int 0\,\mathrm{d}t = C \tag{16}$$

For all $y \in [\gamma_1, \overline{Y}]$,

$$v(y) = v(\gamma_0) + \int_{\gamma_0}^{y} h_1\Big(t, \mu_2(t), \mu_3(t)\Big)\,\mathrm{d}t \tag{17}$$

Note that in equilibrium all women sorted into the junior wife position must gain the same utility (C in expression (16)) since any skill level in that position would contribute the same to household output, so that couples are indifferent between any woman being employed as the junior wife.

A.2 Existence of solution for σ_0

Expressions (13) to (17) define equilibrium payoffs in terms each agent's type, exogenous parameters, and three unknowns: σ_0 (the type of the first woman that is a senior wife), C (they payoff that all junior wives receive in equilibrium), and $v(\gamma_0)$ (the payoff of the first male that gets married). To solve for these unknowns and arrive at the full characterization of the conjecture, I exploit another implication of stability and continuity of traits, namely, that individuals in the thresholds of two different segments in the assignment must be indifferent between being in one segment or in the other. I outline the steps below:

1. By Assumption 1, the value of being single is zero:

$$for\ all\ y \in [\underline{Y}, \gamma_0) : v(y) = 0 \tag{18}$$

2. In a stable assignment, male γ_0 (the first male to get married) must be indifferent between being single and being married monogamously.[23] Hence,

$$v(\gamma_0) = 0 \tag{19}$$

3. Indifference conditions (18) and (19) imply that the wife of γ_0, monogamous wife σ_0, appropriates all the marital surplus:

$$u(\sigma_0) = h(\gamma_0, 0, \sigma_0) \tag{20}$$

4. In a stable assignment, female σ_0 must be indifferent between being a junior and being a senior wife.[24] Hence,

$$C = h(\gamma_0, 0, \sigma_0) \tag{21}$$

5. Similarly, in a stable assignment, female σ_1 and her husband, γ_1, must be indifferent between being monogamous and polygamous.[25] Hence, any surplus from marrying a junior wife must be appropriated by the junior—whose pay is C:

$$C = h(\gamma_1, s', \sigma_1) - h(\gamma_1, 0, \sigma_1) = h(\gamma_1, K, \sigma_1) - h(\gamma_1, 0, \sigma_1) \tag{22}$$

[23] If γ_0 was strictly better off than singles, a single could improve his situation by agreeing to be paid less than γ_0 but more than zero and outbid γ_0's wife, creating a blocking pair. Conversely, if γ_0 was strictly worse off than a single, he would prefer to divorce his wife, and hence he would form a blocking coalition.

[24] If σ_0 was strictly better off than juniors, a junior could improve her situation by agreeing to be paid less than σ_0 but more than her payoff under the assignment and outbid σ_0's husband, creating a blocking pair. Conversely, if σ_0 was strictly worse off than a junior, she would prefer to join a marriage where she is the junior, creating a blocking coalition.

[25] If they were strictly better off than the next monogamous couple, this next monogamous couple could outbid (γ_0, σ_0)'s junior by offering her a higher payoff and still be able to improve their situation, which constitutes a blocking coalition. Conversely, if (γ_0, σ_0) were strictly worse off than the next monogamous couple, they would prefer to divorce their junior wife, creating a blocking pair.

where K is a parameter representing the constant contribution of junior wives to household output.

6. Equating indifference conditions (21) and (22),

$$h(\gamma_1, K, \sigma_1) - h(\gamma_1, 0, \sigma_1) = h(\gamma_0, 0, \sigma_0)$$

7. Finally, replacing γ_0, γ_1, and σ_1 by their expressions (3) to (5) in terms of σ_0, we arrive at an equation to solve for σ_0 as function of the parameters of distributions $F(s)$ and $G(y)$ and of production function $h(y, s', s)$:

$$\begin{aligned} E(\sigma_0) \;\; = \;\; & h(G^{-1}[1-F(\sigma_0)], K, F^{-1}[1-F(\sigma_0)]) \\ & -h(G^{-1}[1-F(\sigma_0)], 0, F^{-1}[1-F(\sigma_0)]) \\ & -h(G^{-1}[F(\sigma_0)], 0, \sigma_0) = 0 \end{aligned} \tag{23}$$

This is a non linear equation in σ_0. Note that a solution for the model with the shape described in Proposition 1 imposes restrictions on the *admissible* values that σ_0 can take. In particular, σ_0 (the first woman to marry as a senior) must satisfy:

$$\underline{S} \leq \sigma_0 \leq \sigma_1 \leq \overline{S}$$

This implies that $\sigma_0 \in [\underline{S}, F^{-1}(0.5)]$, where $F^{-1}(0.5)$ is the median female skill under F. When $\sigma_0 = \underline{S}$, all women marry as seniors and there is no polygamy in the market. When $\sigma_0 = \sigma_1$, which is satisfied for $\sigma_0 = F^{-1}(0.5)$, the first woman to marry as senior is the median woman, so that all seniors are polygamous.

Lemma 1 next establishes that for certain values of the parameters, a unique interior solution for σ_0 exists.

Lemma 1 *There exist values of K such that a unique interior solution for σ_0 exists for all $0 < \overline{S}$.*

Proof 1 *First, note that within the admissible set of solutions for σ_0 the derivative of $E(\sigma_0)$ with respect to σ_0, E_{σ_0}, is strictly negative:*

$$\begin{aligned} E_{\sigma_0} = & \, h_1(G^{-1}[1-F(\sigma_0)], K, F^{-1}[1-F(\sigma_0)]) \times \frac{d\gamma_1}{d\sigma_0} + \\ & + h_3(h(\overline{Y}, K, \overline{S}) - h(\overline{Y}, 0, \overline{S}) - h(\underline{Y}, 0, \underline{S})], K, F^{-1}[1-F(\sigma_0)]) \times \frac{d\sigma_1}{d\sigma_0} - \\ & - h_1(G^{-1}[1-F(\sigma_0)], 0, F^{-1}[1-F(\sigma_0)]) \times \frac{d\gamma_1}{d\sigma_0} - \\ & - h_3(G^{-1}[1-F(\sigma_0)], 0, F^{-1}[1-F(\sigma_0)]) \times \frac{d\sigma_1}{d\sigma_0} - \\ & - h_1(G^{-1}[F(\sigma_0)], 0, \sigma_0) \times \frac{d\gamma_0}{d\sigma_0} - \\ & - h_3(G^{-1}[F(\sigma_0)], 0, \sigma_0) \end{aligned}$$

Rearranging terms:

$$\begin{aligned}E_{\sigma_0} = & \Big(h_1(G^{-1}[1-F(\sigma_0)], K, F^{-1}[1-F(\sigma_0)])- \\ & - h_1(G^{-1}[1-F(\sigma_0)], 0, F^{-1}[1-F(\sigma_0)])\Big) \times \frac{d\gamma_1}{d\sigma_0} + \\ & + \Big(h_3(G^{-1}[1-F(\sigma_0)], K, F^{-1}[1-F(\sigma_0)])- \\ & - h_3(G^{-1}[1-F(\sigma_0)], 0, F^{-1}[1-F(\sigma_0)])\Big) \times \frac{d\sigma_1}{d\sigma_0} - \\ & - h_1(G^{-1}[F(\sigma_0)], 0, \sigma_0) \times \frac{d\gamma_0}{d\sigma_0} - h_3(G^{-1}[F(\sigma_0)], 0, \sigma_0)\end{aligned} \tag{24}$$

The first and second terms of (24) are strictly negative: the first factors in both lines are strictly positive by supermodularity of $h(y, s', s)$, while the second factors in both terms are strictly negative by the strict monotonicity and continuity of distributions $F(s)$ and $G(y)$. Moreover, the last two terms are strictly negative by monotonicity of $h(y, s', s)$ and properties of distributions $F(s)$ and $G(y)$. Hence, $E_{\sigma_0} < 0$.

Second, note that by Assumption 1, $E(\sigma_0)$ is a continuous function of σ_0.

Third, note that there exist a value of K, K^L, such that the function evaluated at the lower bound of the admissible values of σ_0 is strictly positive for all $K > K^L$:

$$\begin{aligned}E(\sigma_0 = \underline{S}) &= h(\gamma_1(\underline{S}), K, \sigma_1(\underline{S})) - h(\gamma_1(\underline{S}), 0, \sigma_1(\underline{S})) - h(\gamma_0(\underline{S}), 0, \underline{S}) \\ &= h(\overline{Y}, K, \overline{S}) - h(\overline{Y}, 0, \overline{S}) - h(\underline{Y}, 0, \underline{S}) \\ &> 0 \quad \forall K > K^L, \\ & \textit{with} \quad K^L : h(\overline{Y}, K^L, \overline{S}) - h(\overline{Y}, 0, \overline{S}) = h(\underline{Y}, 0, \underline{S})\end{aligned}$$

where the second equality follows from expressions (3) to (5) when $\sigma_0 = \underline{S}$,[26] and the last inequality holds because of the existence of threshold K^L given the continuity and monotonicity of function $h(y, s', s)$.

Fourth, note that there exist a value of K, K^H, such that the function evaluated at the upper bound of the admissible values of σ_0 is strictly negative for all $K < K^H$:

[26] $\gamma_1(\underline{S}) = G^{-1}[1-F(\underline{S})] = G^{-1}[1] = \overline{Y}$, $\sigma 1(\underline{S}) = F^{-1}[1-F(\underline{S})] = F^{-1}[1] = \overline{S}$, and $\gamma_0(\underline{S}) = G^{-1}[F(\underline{S})] = G^{-1}[0] = \underline{Y}$.

$$
\begin{aligned}
E(\sigma_0 = F^{-1}(0.5)) &= h(\gamma_1(F^{-1}(0.5)), K, \sigma_1(F^{-1}(0.5)) \\
&- h(\gamma_1(F^{-1}(0.5)), 0, \sigma_1(\underline{S})) - h(\gamma_0(F^{-1}(0.5)), 0, F^{-1}(0.5)) \\
&= h(G^{-1}(0.5), K, F^{-1}(0.5)) - \\
&- h(G^{-1}(0.5), 0, F^{-1}(0.5)) - h(G^{-1}(0.5), 0, F^{-1}(0.5)) \\
&< 0 \quad \forall K < K^H, \\
&\textit{with} \quad K^H : h(G^{-1}(0.5), K^H, F^{-1}(0.5)) = 2 \times h(G^{-1}(0.5), 0, F^{-1}(0.5))
\end{aligned}
$$

Finally, note that by Assumption 1, $K^L < K^H$:
By monotonicity of $h(y, s', s)$,

$$
h(\underline{Y}, 0, \underline{S}) < h(G^{-1}(0.5), 0, F^{-1}(0.5))
$$

From the expressions of K^L and K^H, it then follows that

$$
h(\overline{Y}, K^L, \overline{S}) - h(\overline{Y}, 0, \overline{S}) < h(G^{-1}(0.5), K^H, F^{-1}(0.5)) - h(G^{-1}(0.5), 0, F^{-1}(0.5))
$$

which can be expressed, by the Fundamental Theorem of Calculus, as

$$
\int_0^{K^L} h_2\Big(\overline{Y}, r, \overline{S}\Big)\mathrm{d}r < \int_0^{K^H} h_2\Big(G^{-1}(0.5), r, F^{-1}(0.5)\Big)\mathrm{d}r
$$

But by supermodularity of $h(y, s', s)$, the integrand in the left-hand-side is greater than the integrand in the right-hand-side, $h_2\Big(\overline{Y}, r, \overline{S}\Big)\mathrm{d}r > h_2\Big(G^{-1}(0.5), r, F^{-1}(0.5)\Big)\mathrm{d}r$. So, it must be the case that the integration area is greater on the right hand side. Hence, $K^L < K^H$.

In conclusion, since $E(\sigma_0)$ is strictly decreasing and continuous, σ_0 lies in compact set $[\underline{S}, F^{-1}(0.5)]$ and for values of parameters such that $K^L < K < K^H$, it is the case that $E(\underline{S}) > 0$ and $E(F^{-1}(0.5)) < 0$, there is a unique value of σ_0 for which $E(\sigma_0) = 0$.

□

A.3 Stability

Taking the characterization of the assignment as given, I now show that there is no coalition that blocks this assignment. That is, that the assignment satisfies global stability conditions.

Lemma 2 *Take the characterization of assignment μ in this marriage market given by*

- *The solution for σ_0 from equation (23);*
- *Thresholds (3) to (5);*
- *Matching functions (6) and (7); and*
- *Payoff functions (13) to (17).*

If the marital surplus $h(y, s', s)$ satisfies Assumption 1 with $h_2(y, s', s) = 0$, then:

- **Part 1.** *Taking the role of females in assignment μ as given, there is no coalition of three that blocks this assignment.*
- **Part 2.** *Taking the role of females in assignment μ as given, there is no coalition of two that blocks this assignment.*
- **Part 3.** *Taking the role of females in assignment μ as given, there is no individual that blocks this assignment.*
- **Part 4.** *No female wants to change their role with respect to her role in assignment μ.*

Proof 2 (Part 1) *Note that it suffices to show that no* essential *coalition of three blocks the assignment, given that, by definition, inessential coalitions cannot do better than any essential coalitions.*

Essential coalitions of three are any group of a male, a junior wife, and a senior wife.

1. ***No polygamous male and his senior wife divorce their junior and get another junior***

Intuitively, any junior is equally productive so a couple of a male and his senior is indifferent between any woman that is willing to be employed as a junior, so they have no reason to undo the outcome of the random matching between them and the junior. Formally:

$\forall y \geq \gamma_1$, $\forall s \geq \sigma_1 : \mu_1(s) = y$, $\forall \hat{s} < \sigma_0 : \mu_2(y) = \mu_2(s) = s' \neq \hat{s}$ *suppose coalition* $(y, \hat{s}, s)$ *blocks* μ*. Then, it must be the case that*

$$h(y, \hat{s}, s) > v^\mu(y) + u^\mu(\hat{s}) + u^\mu(s)$$
$$h(y, \hat{s}, s) > h(y, s', s) - u^\mu(s') + u^\mu(\hat{s})$$
$$0 > 0, \text{ a contradiction that proves the statement.}$$

2. ***No polygamous male and any junior wife marry down to a lower senior***

For all $y \geq \gamma_1$, for all $\sigma_1 \leq \hat{s} \leq s : s = \mu_3(y)$, and for any $s' < \sigma_0$, suppose coalition $(y, s', \hat{s})$ blocks μ. Then, it must be the case that

$$h(y, s', \hat{s}) > v^\mu(y) + u^\mu(s') + u^\mu(\hat{s})$$

By the efficiency of the assignment and given that any skill produces the same amount of labor,

$$h(y, s', \hat{s}) > h(y, s', s) - u^\mu(s) + u^\mu(\hat{s})$$

Replacing by the payoffs under μ and rearranging terms,

$$h(y, s', s) - h(y, s', \hat{s}) < u^\mu(\hat{s}) - u^\mu(\hat{s}) + \int_{\hat{s}}^{s} h_3\Big(\mu_1(t), \mu_2(t), t\Big), \mathrm{d}t$$

By the Fundamental Theorem of Calculus (henceforth, FTC) and by the fact that any skills produce the same labor output,

$$\int_{\hat{s}}^{s} h_3\Big(y, s', t\Big)\mathrm{d}t < \int_{\hat{s}}^{s} h_3\Big(\mu_1(t), \mu_2(t), t\Big)\mathrm{d}t = \int_{\hat{s}}^{s} h_3\Big(\mu_1(t), s', t\Big)\mathrm{d}t \Longrightarrow$$

$$\Longrightarrow \int_{\hat{s}}^{s} h_3\Big(y, s', t\Big)\mathrm{d}t - \int_{\hat{s}}^{s} h_3\Big(\mu_1(t), s', t\Big)\mathrm{d}t < 0 \Longrightarrow$$

$$\Longrightarrow \int_{\hat{s}}^{s} \int_{\mu_1(t)}^{y} h_{31}\Big(r, s', t\Big)\mathrm{d}r\mathrm{d}t < 0$$

a contradiction with supermodularity of $h(y, s', s)$ that proves that $(y, s', \hat{s})$ cannot block μ.

Note that this implies that no polygamous senior, s, can marry up to a husband $y > \mu_1(s)$.

3. ***No polygamous senior and any junior marry down to a lower male***

For all $s \geq \sigma_1$, for all $\gamma_1 \leq \hat{y} \leq y : y = \mu_1(s)$, and for any $s' < \sigma_0$, suppose coalition $(\hat{y}, s', s)$ blocks μ. Then, it must be the case that

$$h(\hat{y}, s', s) > v^{\mu}(\hat{y}) + u^{\mu}(s') + u^{\mu}(s)$$

As in point 3 above, the efficiency of the assignment, the fact that any skills produce the same labor output, and the payoffs that characterize μ imply that

$$h(y, s', s) - h(\hat{y}, s', s) < \int_{\hat{y}}^{y} h_1\Big(t, \mu_2(t), \mu_3(t)\Big)\mathrm{d}t = \int_{\hat{y}}^{y} h_1\Big(t, s', \mu_3(t)\Big)\mathrm{d}t$$

Applying the FTC twice,

$$\int_{\hat{y}}^{y} \int_{\mu_3(t)}^{s} h_{13}\Big(t, s', r\Big)\mathrm{d}r\mathrm{d}t < 0$$

a contradiction with supermodularity of $h(y, s', s)$ that proves that $(\hat{y}, s', s)$ cannot block μ.

Note that this implies that no polygamous male, y, can marry up to a senior $s > \mu_3(y)$.

4. ***No monogamous couple can get a junior wife***

For all $\sigma_0 \leq s < \sigma_1$, for all $\gamma_0 \leq y < \gamma_1 : y = \mu_1(s)$, and for any $s' < \sigma_0$, suppose coalition (y, s', s) blocks μ. Then, it must be the case that

$$h(y, s', s) > v^{\mu}(y) + u^{\mu}(s') + u^{\mu}(s)$$

By efficiency,

$$h(y, s', s) > h(y, 0, s) + u^{\mu}(s')$$

Rearranging terms and substituting $u^{\mu}(s')$ by its expression in (22),

$$h(y, s', s) - h(y, 0, s) > h(\gamma_1, s', \sigma_1) - h(\gamma_1, 0, \sigma_1) \Longrightarrow$$
$$\Longrightarrow h(y, s', s) + h(\gamma_1, 0, \sigma_1) > h(\gamma_1, s', \sigma_1) + h(y, 0, s)$$

a contradiction with supermodularity of $h(y,s',s)$ that proves that (y,s',s) cannot block μ.

The fact that no monogamous couple can afford a junior will imply that no coalition of a male and a senior that are not married under μ will be able to afford a junior. I show this in conditions 5 and 6 below.

5. ***No monogamous senior can get a junior by marrying down to a lower male***

For all $\sigma_0 \leq s < \sigma_1$, for all $\gamma_0 \leq \hat{y} < \gamma_1 : \hat{y} < \mu_1(s) = y$, and for any $s' < \sigma_0$, suppose coalition $(\hat{y}, s', s)$ blocks μ. Then, it must be the case that

$$h(\hat{y}, s', s) > v^{\mu}(\hat{y}) + u^{\mu}(s') + u^{\mu}(s)$$

Substituting $v^{\mu}(\hat{y})$ by its expression given by (14), and since by point 4 above (y,s',s) does not block μ,

$$h(\hat{y}, s', s) > v^{\mu}(y) - \int_{\hat{y}}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t + u^{\mu}(s') + u^{\mu}(s) \geq h(y, s', s) - \int_{\hat{y}}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t$$

These inequalities imply that

$$h(\hat{y}, s', s) > h(y, s', s) - \int_{\gamma_0}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t$$

Rearranging terms and using the FTC we arrive at a contradiction,

$$0 > \int_{\hat{y}}^{y} h_1\Big(t, s', s\Big)\,\mathrm{d}t - \int_{\hat{y}}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t > \int_{\hat{y}}^{y} h_1\Big(t, 0, s\Big)\,\mathrm{d}t - \int_{\hat{y}}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t =$$

$$= \int_{\hat{y}}^{y} \int_{\mu_3(t)}^{s} h_{13}\Big(t, 0, r\Big)\,\mathrm{d}r\mathrm{d}t > 0$$

where the last inequality obtains from supermodularity of $h(y,s',s)$.

6. ***No monogamous male can get a junior by marrying down to a lower senior***

For all $\gamma_0 \leq y < \gamma_1$, for all $\sigma_0 \leq \hat{s} < \sigma_1 : \hat{s} < \mu_3(y) = s$, and for any $s' < \sigma_0$, suppose coalition $(y, s', \hat{y})$ blocks μ. Then, it must be the case that

$$h(y, s', \hat{y}) > v^{\mu}(y) + u^{\mu}(s') + u^{\mu}(\hat{s})$$

By a similar argument as in point 5 above, I substitute $u^{\mu}(\hat{s})$ by its expression given by (13), and use the fact that (y,s',s) does not block μ to arrive at a contradiction to supermodularity of $h(y,s',s)$:

$$h(y,s',\hat{s}) > v^{\mu}(y) + u^{\mu}(s') + u^{\mu}(s) - \int_{\hat{s}}^{s} h_3\Big(\mu_1(t),0,t\Big)\,\mathrm{d}t \geq$$
$$\geq h(y,s',s) - \int_{\hat{s}}^{s} h_3\Big(\mu_1(t),0,t\Big)\,\mathrm{d}t$$

$$\Longrightarrow$$

$$0 > \int_{\hat{s}}^{s} h_3\Big(y,s',t\Big)\,\mathrm{d}t - \int_{\hat{s}}^{s} h_3\Big(\mu_1(t),0,t\Big)\,\mathrm{d}t >$$
$$> \int_{\hat{s}}^{s} h_3\Big(y,0,t\Big)\,\mathrm{d}t - \int_{\hat{s}}^{s} h_3\Big(\mu_1(t),0,t\Big)\,\mathrm{d}t = \int_{\hat{s}}^{s}\int_{\mu_1(t)}^{y} h_{13}\Big(t,0,r\Big)\,\mathrm{d}r\mathrm{d}t$$

a contradiction to supermodularity of $h(y,s',s)$*.*

□

Proof 3 (Part 2) .

Note, as in part 1, that it suffices to show that no essential *coalition of two blocks the assignment.*

Essential coalitions of two are: any coalition of a male and a junior wife and any coalition of a male and a senior wife.

7. ***No coalition of a male and a junior blocks*** μ

For all $y \in [\underline{Y},\bar{Y}]$ *and for any* $s' < \sigma_0$*, suppose coalition* $(y,s',0)$ *blocks* μ*. Then, it must be the case that*

$$h(y,s',0) = 0 > v^{\mu}(y) + u^{\mu}(s') > 0$$

a contradiction that proves the statement.

8. ***No polygamous male and her senior divorce their junior***

For all $\gamma_1 \leq y \leq \bar{Y}$*, for all* $\sigma_1 \leq s \leq \bar{S} : \mu_3(y) = s$*, and for any* $s' < \sigma_0$*, suppose coalition* $(y,0,s)$ *blocks* μ*. Then, it must be the case that*

$$h(y,0,s) > v^{\mu}(y) + u^{\mu}(s) = h(y,s',s) - u^{\mu}(s') = h(y,s',s) - h(\gamma_1,s',\sigma_1) + h(\gamma_1,0,\sigma_1)$$

Hence,

$$h(y,0,s) > h(y,s',s) - h(\gamma_1,s',\sigma_1) + h(\gamma_1,0,\sigma_1) \Longrightarrow$$
$$\Longrightarrow h(y,0,s) + h(\gamma_1,s',\sigma_1) > h(y,s',s) + h(\gamma_1,0,\sigma_1)$$

which contradicts supermodularity of $h(y,s',s)$*.*

9. ***No woman wants to marry down monogamously***

For all $\sigma_0 \leq s < \bar{S}$, for all $\gamma_0 \leq \hat{y} < \bar{Y} : \hat{y} < \mu_1(s) = y$, and for any $s' < \sigma_0$, suppose coalition $(\hat{y}, 0, s)$ blocks μ. Then, it must be the case that

$$h(\hat{y}, 0, s) > v^\mu(\hat{y}) + u^\mu(s)$$

By efficiency, rearranging terms, and using the FTC:

$$h(\hat{y}, 0, s) > v^\mu(\hat{y}) + h(y, 0, s) - v^\mu(y)$$

$$v^\mu(y) - v^\mu(\hat{y}) > \int_{\hat{y}}^{y} h_1\Big(t, 0, s\Big)\, \mathrm{d}t \Longrightarrow$$

$$\Longrightarrow \int_{\hat{y}}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\, \mathrm{d}t > \int_{\hat{y}}^{y} h_1\Big(t, 0, s\Big)\, \mathrm{d}t \Longrightarrow$$

$$0 > \int_{\hat{y}}^{y} \int_{\mu_s(t)}^{s} h_{13}\Big(t, 0, r\Big)\, \mathrm{d}r\mathrm{d}t$$

which contradicts supermodularity of $h(y, s', s)$.

10. ***No male wants to marry down monogamously***

For all $\gamma_0 \leq y < \bar{Y}$, for all $\sigma_0 \leq \hat{s} < \bar{S} : \hat{s} < \mu_3(y) = s$, and for any $s' < \sigma_0$, suppose coalition $(y, 0, \hat{s})$ blocks μ. Then, it must be the case that

$$h(y, 0, \hat{s}) > v^\mu(y) + u^\mu(\hat{s})$$

Similarly as in point 9 above, by efficiency, rearranging terms, and using the FTC:

$$0 > \int_{\hat{s}}^{s} \int_{\mu_3(t)}^{y} h_{13}\Big(r, 0, t\Big)\, \mathrm{d}r\mathrm{d}t$$

which contradicts supermodularity of $h(y, s', s)$.

□

Proof 4 (Part 3) .

11. ***No married individual prefers to be single***

The production function is such that females and males produce zero as singles. All married individuals obtain a positive indirect utility in the match. Hence, no single blocks μ.

□

Proof 5 (Part 4) .

Up to now I have taken the role of women conjectured in μ fixed to disproof blocking coalitions. In this section of the proof I show that no woman wants to change her role.

Consider, first, women $s' < \sigma_0$ being employed as juniors under μ.

First, I showed in statement 2 that no polygamous male prefers to marry down a senior below her senior wife in μ. Hence, females $s' < \sigma_0$ are not desired in the senior wife position by polygamous males.

Second, I showed in statement 10 that no male prefers to marry monogamously a wife below her senior wife in μ. Hence, females $s' < \sigma_0$ are not desired in the senior wife position by monogamous males that stay monogamous.

Third, I showed in statement 6 that no monogamous male can become polygamous by marrying down to a lower female in the senior wife position. By this argument, a monogamous male cannot become polygamous by marrying woman $s' < \sigma_0$ as senior wife and woman $\hat{s}' < s' < \sigma_0$ as junior wife. Hence, females $s' < \sigma_0$ are not desired in the senior wife position by monogamous males trying to become polygamous.

All in all, it remains to be shown that females $s' < \sigma_0$ will not marry as seniors to single males.

12. ***No junior prefers to marry down as a senior to a single male***

For all $y < \gamma_0$, for all $s' < \sigma_0$, suppose coalition $(y, 0, s')$ blocks μ. Then, it must be the case that

$$h(y, 0, s') > v^{\mu}(y) + u^{\mu}(s')$$

Replacing by the payoffs of s' and y under μ

$$h(y, 0, s') > h(\gamma_0, 0, \sigma_0)$$

which contradicts monotonicity of $h(y, s', s)$.

Hence, females $s' < \sigma_0$ are optimally placed in the junior position.

Consider, next, women $s \geq \sigma_0$ being employed as seniors under μ. By statement 7 no coalition of a male and a junior block μ. Hence, women $s \geq \sigma_0$ can only be desired as junior wives in polygamous households.

13. ***No coalition of a male and a senior prefers to replace their junior for a woman in a senior wife position under μ***

The proof follows crucially from the fact that all women are equally productive in the junior wife position irrespective of their skills. This makes substitution of juniors unprofitable.

First, by statement 1 polygamous couples are indifferent between the junior assigned to them under μ and any other woman in the junior position. Hence, females $s \geq \sigma_0$ are not desired in the junior position by any polygamous couple of a male and a senior married under μ.

Second, I showed in statement 2 that no polygamous male prefers to marry down a senior below her senior wife under μ and any junior. Hence, females $s \geq \sigma_0$ are not desired in the junior wife position by any coalition of a male y and a senior $\hat{s} < \mu_s(y)$.

Finally, I showed in statement 3 that no polygamous senior wife prefers to marry down a man below her husband under μ and any junior. Hence, females $s \geq \sigma_0$ are not desired in the junior position by any coalition of a senior wife s and a man $\hat{y} < \mu_1(s)$.

All in all, women $s \geq \sigma_0$ are not desired as juniors by any coalition. Hence, females $s \geq \sigma_0$ are optimally placed in the senior wife position.

□

Proof 6 (Proposition 1) .

Whenever a solution for σ_0 exists, by lemma 2 there is no essential coalition that blocks the assignment. Hence, it is stable.

□

Appendix B The case with $h_2(y, s', s) > 0$

In this section I generalize the main result of the paper to the case of increasing productivity of female skills in the junior wife position. To make the exposition as comparable as possible to the case developed in the paper, consider the following household output, $\tilde{h}(y, s', s)$,

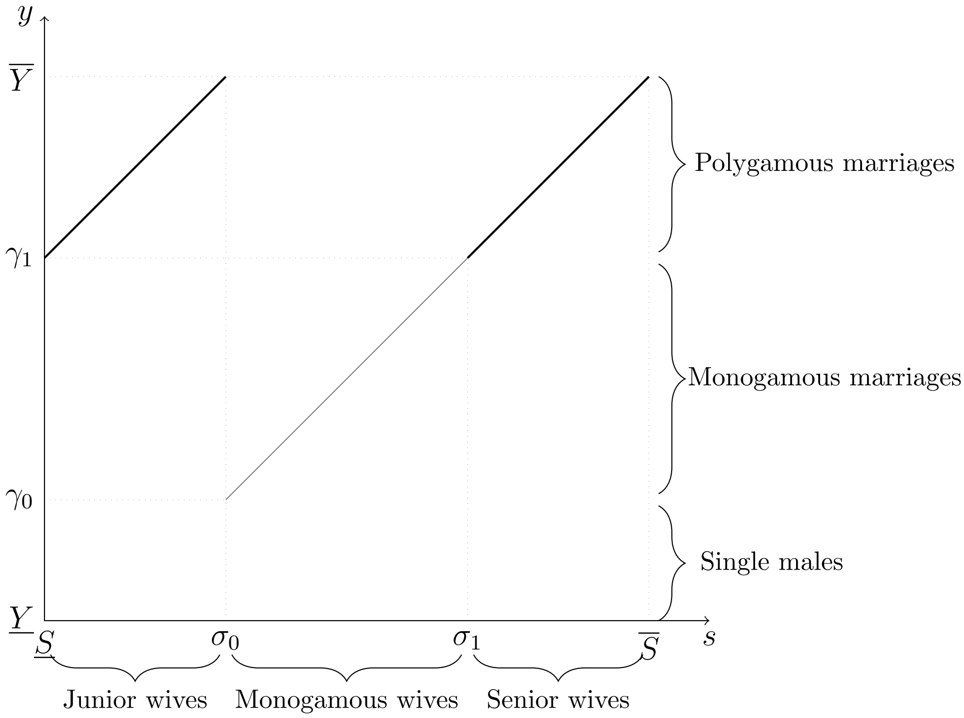
$$\tilde{h}(y, s', s) = h(y, K, s)(\epsilon s' + 1)$$

with $h(y, K, s)$ specified as in Proposition 1. First, note that since $h(y, K, s)$ satisfies Assumption 1, $\tilde{h}(y, s', s)$ satisfies parts 1, 2, 3, and 5 of Assumption 1. In particular, notice that $\tilde{h}(y, s', s)$ is supermodular and that the marginal productivity of husband y and senior wife s is strictly increasing in the type of the junior wife s':

$$\tilde{h}_{i2}(y, s', s) = h_i(y, K, s)\epsilon > 0 \quad \text{for all} \quad i = \{1, 3\} \tag{25}$$

Second, note that as $\epsilon \to 0$, $\tilde{h}_2(y, s', s) \to 0$ for all $(y, s', s) \in [\underline{S}, \overline{S}]^2 \times [\underline{Y}, \overline{Y}]$. Hence, by continuity there exists an ϵ small enough so that $\tilde{h}(y, s', s)$ satisfies part 4 of Assumption 1. I consider such functions here. The stable outcome in this marriage market is depicted in Figure A1. The shape of the equilibrium is as for the constant junior wife productivity case, except that because of (25), matching between husbands and junior wives is positive assortative instead of random.

Figure A1: Stable matching when $h_2(y, s', s) > 0$



Proposition 2 *The marriage market with populations $s \sim F[\underline{S}, \overline{S}]$ and $y \sim G[\underline{Y}, \overline{Y}]$, and marital output $\tilde{h}(y, s', s) = h(y, K, s)(\epsilon s' + 1)$, with $h(y, K, s)$ specified as in Proposition 1 and for ϵ such that $\tilde{h}(y, s', s)$ satisfies Assumption 1, has a stable outcome, $(\mathcal{M}, \mathcal{L}, \mu, v^\mu, u^\mu)$, characterized by:*

1. *Unique thresholds $\sigma_0 \in [\underline{S}, F^{-1}(0.5)]$, $\sigma_1 = F^{-1}[1 - F(\sigma_0)]$, $\gamma_0 = G^{-1}[F(\sigma_0)]$, and $\gamma_1 = G^{-1}[F(\sigma_1)]$, all of which are unique.*
2. *The partition of female skills into junior and senior wife roles, $\mathcal{L} = [0, \sigma_0)$ and $\mathcal{M} = [\sigma_0, \overline{S}]$*
3. *Matching function*

$$\mu = (y, \mu_2(y), \mu_3(y)) = \begin{cases} (y, \emptyset_s, \emptyset_s), & \textit{for all } y \in [\underline{Y}, \gamma_0) \\ (y, \emptyset_s, F^{-1}[G(y)]), & \textit{for all } y \in [\gamma_0, \gamma_1) \\ (y, s', F^{-1}[G(y)]), & \textit{for all } y \in [\gamma_1, \overline{Y}], s' \in [\underline{S}, \sigma_0) \end{cases}$$

4. *Feasible payoff functions*

$$u^\mu(s) = \begin{cases} h(\gamma_0, 0, \sigma_0), & \textit{for all } s \in [0, \sigma_0) \\ h(\gamma_0, 0, \sigma_0) + \int_{\sigma_0}^{s} h_3\Big(\mu_1(t), 0, t\Big)\, \mathrm{d}t & \textit{for all } s \in [\sigma_0, \sigma_1) \\ h(\gamma_0, 0, \sigma_0) + \int_{\sigma_0}^{s} h_3\Big(\mu_1(t), \mu_2(t), t\Big)\, \mathrm{d}t & \textit{for all } s \in [\sigma_1, \overline{S}] \end{cases}$$

$$v^{\mu}(y) = \begin{cases} 0, & \textit{for all } y \in [0, \sigma_0) \\ \int_{\gamma_0}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t, & \textit{for all } y \in [\gamma_0, \gamma_1) \\ \int_{\gamma_0}^{y} h_1\Big(t, \mu_2(t), \mu_3(t)\Big)\,\mathrm{d}t, & \textit{for all } y \in [\gamma_1, \overline{Y}] \end{cases}$$

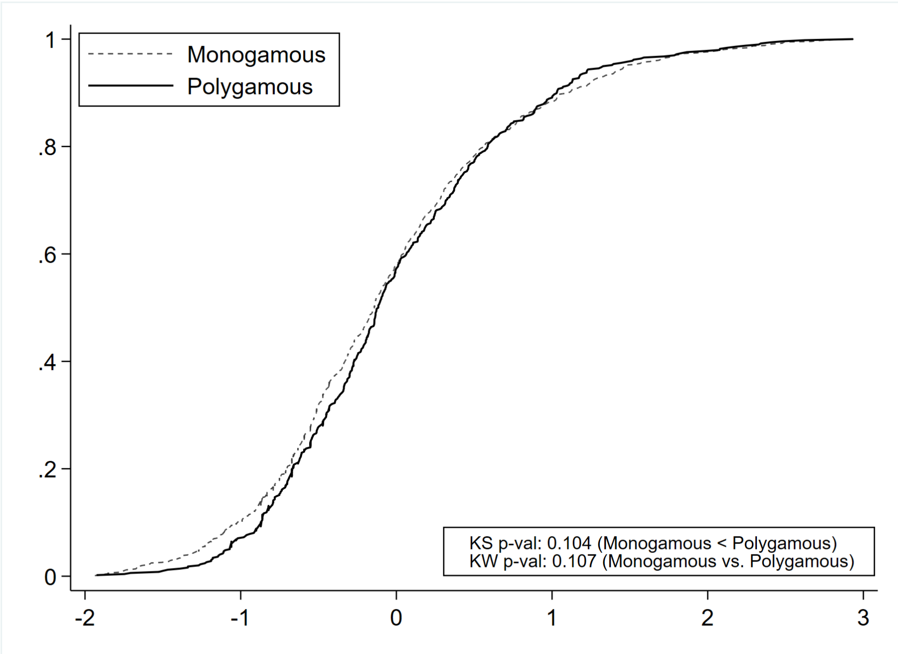
Proof 7 (Proposition 2) *The proof of this case is very similar to the constant junior role developed in the body of the paper. The main difference is that it is not always the case that females sorted according to threshold σ_0 will not want to change their role. The proof follows closely the one developed in the constant junior role case. First, I obtain the characterization of the assignment in terms of σ_0. Then, I argue that for some ϵ small enough, a solution to σ_0 exists. Finally, I prove that this characterization satisfies global stability conditions by disproving potential blocking coalitions and resorting of females into different household roles. Note that part 4 of Assumption 1 prevents the latter.*[27]

□

[27]The complete and detailed proof is available upon request.

Appendix C Test of selection of men by type of marriage

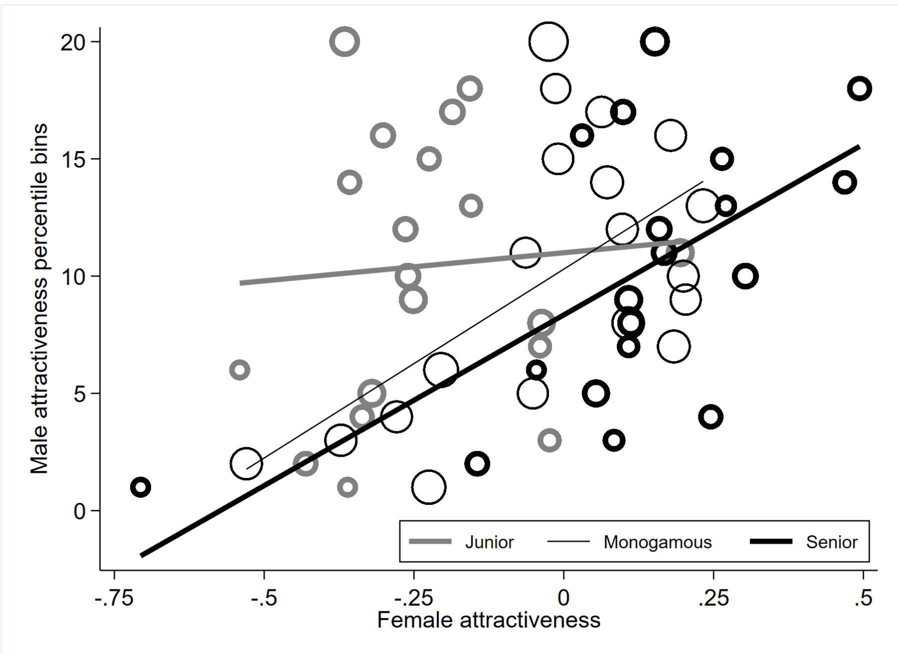
Figure A2: Distribution of men's wealth by type of marriage in the LSMS-ISA Nigeria (2010)



Notes: *LSMS-ISA* stands for *Living Standard Measurement Study - Integrated Surveys on Agriculture*. The horizontal axis display the standardized index of female attractiveness at the time of marriage. The vertical axes show the cumulative distribution. KW stands for the Kruskall-Wallis test of equality of distributions. KS stands for the Kolmogorov-Smirnov test of unique populations.

Appendix D Observed equilibrium including outliers

Figure A3: The Marriage Market Equilibrium in the Data including outliers



Notes: The size of hollow circles represents the number of observations. I smooth the data by taking the average female skills by percentile bin of men type.