Polygamy, co-wives' complementarities, and intra-household inequality

Ana Reynoso*

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Abstract

Regulating polygamy to induce more productive investments and improve women's rights requires anticipating changes in marital choices and welfare. I develop and estimate a novel theory of polygamy that incorporates an empirical feature previously overlooked: co-wives interact in a *senior-junior* hierarchy. In equilibrium, single, monogamous, and polygamous households emerge. Optimal female sorting generates co-wives inequality: high skilled women become senior wives in polygamous households with wealthy males and low skilled juniors. The estimated model reproduces the observed equilibrium. A novel policy implication is that in the absence of better outside options from marriage, banning polygamy harms all women.

*Department of Economics, University of Michigan, 611 Tappan Av., Ann Arbor, MI 48103, areynoso@umich.edu. I am extremely grateful to my Phd dissertation advisors Joseph Altonji, Costas Meghir, Larry Samuelson, and Chris Udry for their guidance and support. I am also grateful to Pierre André Chiappori, Scott Kominers, Chiara Margaria, Esteban Peralta, and Bernard Salanié for many productive discussions at various points of this project. I also thank Martha Bailey, Hoyt Bleakley, Tibor Heumann, Sylvie Lambert, Corinne Low, Robert Pollak, Tommaso Porzio, Meredith Startz, Mel Stephens, Fernando Tohme, Alessandra Voena, and seminar participants at Yale University, Columbia University, the Jerusalem School in Economic Theory, and University of Michigan for helpful comments, discussions, and suggestions. All errors are mine.

1 Introduction

Polygamy, the union of one man with multiple women, is a form of marriage that remains popular in most developing countries.[1] Academic research shows that polygamy is associated with underinvestment in physical assets (Tertilt, 2005), underinvestment in human capital (Gould, Moav, and Simhon (2008) and Behaghel and Lambert (2017)), the comparative advantage of female and child labor over capital in agriculture (Boserup (2007) and Jacoby (1995)), and wealth inequality between the sexes and between men (De La Croix and Mariani, 2015).

The policy debate, moreover, emphasizes issues of unequal property rights, especially for higher order polygamous wives who usually do not have access to marital rights in countries were polygamy is widely practiced but illegal. In attention to this problematic, many African countries have recently passed laws or proposed bills that either legalize or strengthen the regulations of polygamy. For example, The Kenyan Marriage Act 2014 (Kenya, 2014) legalizes polygamy and reduces the gap in marital rights between polygamous and monogamous wives.[2] As another example, the Ghanaian Spousal Rights to Property Bill (Ghana, 2009) proposes a more equal division of property upon divorce for women including those in polygamous marriages. Also since 2000 various Nigerian states started legalizing polygamy.

The correlation between polygamy, underdevelopment, and inequality calls for the design of policies that encourage more productive investments and improve the welfare of women. However, these designs need to anticipate the policy impacts on individual marital choices and welfare. A first fundamental step in the policy debate is to understand the incentives that drive women and men to form polygamous households and the characteristics of the households that emerge in equilibrium.

In this paper I develop and estimate a novel theory of polygamy that incorporates a stylized fact previously overlooked: that co-wives within polygamous families interact in a hierarchy of senior-junior co-wives. After showing that my model reproduces the observed marriage market equilibrium, I use it to evaluate what policies are welfare improving in these marriage markets.

[1] For example, polygamy is legal or accepted in more than 70% of African countries (UN, 2011), in about 30 countries in Asia (UN, 2011), and is practiced in 73% of the 1170 societies in Murdock's Ethnographic Atlas (Gould, Moav, and Simhon, 2008). Nowadays, the percent of married males that have more than one wife reaches levels as high as 63% in Mali, 35% in Burkina Faso, or 24% in Nigeria, as I show in column 5 in table 1 (table 1 summarizes household data from the Living Standard Measurment Study - Integrated Surveys on Agriculture (The World Bank, 2010-2014)).

[2] These include property division and child custody upon divorce and inheritance.

By taking into account co-wives interactions, my model provides a suitable tool to study women's selection into the senior or the junior roles and the gains from polygamy for women. Interestingly, in a model where co-wives complement each other, female's productivity in each household role and female's gains from marriage are increasing on the traits of potential co-wives. As a result, restrictions to polygamy may have unintended negative impacts for women if not accompanied by improvements in outside options from marriage.

I start by establishing the empirical relevance of co-wives interaction in polygamous marriages. First, table 1 (section 2), shows that co-wives co-habitation is very frequent. For example, more than 80% of polygamous households have co-wives living in the same dwelling in countries like Burkina Faso, Nigeria, and Niger. In addition, co-wives are observed to cooperate in farming (Akresh, Chen, and Moore (2016)) and divide their farming and household labor to cope with the workload (Boserup (2007)). This interaction occurs in spite of conflict and rivalry (Rossi (2018)), which suggests the existence of dominating complementarities within teams of co-wives. The empirical literature also notes that co-wives organize in a hierarchy of *senior wife- junior co-wives*. The senior wife controls more resources, has higher social and family status, and performs better in health, fertility, and child quality indicators (Matz (2016) surveys previous work and shows new evidence on co-wives inequality within the household).

My framework describes a marriage market that allows for bigamy, the most frequent form of polygamy. Females are valued by their skills and males are valued by their wealth. A marriage between a man and at most two women produces family welfare according to a household production function that has three inputs: husband's wealth, the skill of one wife in the junior position, and the skill of another wife in the senior position. A novel feature of this model is that all spouses' traits are complementary in producing the marital output, implying that household output cannot be decomposed into the sum of what the husband produces with each wife separately. The gain from marriage to an individual is the value of his or her share of the family output. The objective of any man is to choose the team of senior and junior wives that maximizes his gains from marriage. Similarly, the objective of any wife is to choose their household role and the team of husband-co-wife that maximizes her welfare.

I show that there exists a marriage market equilibrium where single, monogamous, and polygamous households emerge and is fully characterized by *thresholds* of women's skill levels.

The optimal sorting of women into household roles implies that the most skilled women become senior wives in polygamous households with wealthy males and low skilled juniors. Because the equilibrium utility gain from marriage is increasing in female skill, the model endogenously produces a differential status of co-wives within the household, a characteristic of polygamous families previously documented in the empirical literature but that is a novel theoretical implication.

In the empirical section of the paper, I develop a strategy to identify the thresholds of female skill levels that fully characterize the equilibrium. I use data from the Living Standards Measurement Study from Nigeria, a country with over 35% of polygamous males in the northern states as of 2010. Using various measures of performance in household production, I estimate the skill and wealth distribution of women and men by a principal component approach. I then estimate the parameters of the model by the method of moments. I contrast the estimated equilibrium implied by the model with the observed distribution of marriages and conclude that the threshold equilibrium with spousal complementarities and hierarchy of co-wives accurately reproduces the data.

The model allows me to investigate the welfare consequences of outlawing polygamy. While previous papers have focused on studying the gains from polygamy for men, the effects of polygamy for women are not well understood, especially in environments where co-wives work together. A novel implication of my model is that monogamy may be harmful to all women. One reason for this result is that monogamy prevents females from boosting their productivity when they work with a co-wife. Another reason for the negative welfare effect of monogamy on women comes from equilibrium conditions. In the equilibrium under polygamy, low skilled women are married to the wealthiest males, who find them attractive because they are willing to take the (less rewarded) junior wife position and make the higher skilled senior more productive. Under monogamy, on the contrary, low skill women are only accepted by the least wealthy men, depressing the welfare of low skilled wives. Interestingly, the worse marital prospects of low skilled women spill over to the higher skilled through competition in the marriage market, thus depressing all wives' equilibrium payoffs.

This paper contributes to our understanding of the economics of polygamy by extending the models of Becker (1973), De La Croix and Mariani (2015), Gould, Moav, and Simhon

(2008), Jacoby (1995), and Tertilt (2005) to allow for co-wives to complement each other (as well as their husband) in household production. These previous contributions treat polygamous families as a set of separate monogamous households where the husband produces the output of each nuclear family with each wife separately. Because in these models there is no relationship between co-wives (except through the husband's choice of number of partners), non of these models are able to explain why women marry a particular co-wife, what determines women's status within the household, and what would be the consequences of regulating polygamy for females. Unlike these papers, my model is suitable to understand women selection into types of marriages and household roles and derive policy implications for females.

Moreover, my model builds on the literature that investigates who marries whom and what are the gains from marriage. In his seminal work, Becker (1973) introduces the idea of modeling the process of household formation as a matching game both under monogamy and polygamy. I build on Becker's work and the literature that followed, in particular, the marriage market models under monogamy of Chiappori, Iyigun, and Weiss (2009a), Chiappori, Iyigun, and Weiss (2009b), Chiappori, Iyigun, and Weiss (2009c), Chiappori, Iyigun, Lafortune, and Weiss (2017a), Chiappori and Oreffice (2008), Chiappori, Oreffice, and Quintana-Domeque (2018), Chiappori, Salanié, and Weiss (2017b), and Low (2014). Even though my model extends these previous matching models by allowing males to match with more than one woman, I source on the techniques from these papers to solve my model.

More generally, the model in this paper also makes a contribution to the literature on many-to-one matching with complementarities. The non separability of the production function in co-wives' inputs implies that the characteristics of one wife affect the marginal productivity of the co-wife. For example, from the point of view of the husband, the attractiveness of any potential senior (junior) wife depends on the skill of a potential junior (senior) wife. That is, formally, the preference of husbands over teams of co-wives are *non-substitutable* (Roth and Sotomayor, 1990). Substitutability of preferences has received a lot of attention in the literature of matching with contracts, because when preferences are non-substitutable, existence of equilibria is not guaranteed (see, for example, Hatfield, Kominers, Nichifor, Ostrovsky, and Westkamp (2013)).[3]

[3]Intuitively, equilibria may fail to exist because teams of a husband and two co-wives may never agree to marry. For example, there could be situations in which male y finds female s attractive as a senior wife only when female s' joins as a junior wife, but female s' would only marry male y as his junior wife if he marries s'' as his senior wife.

In this paper, I exploit the observed hierarchical organization of co-wives to impose a structure on the household production function that allows me to show existence of equilibria even in the presence of non-substitutable preferences.

To the best of my knowledge, this is the first paper to apply and solve a model of many-to-one matching with a supply chain structure (Ostrovsky (2008), Sun and Yang (2006)) to family formation, allowing for non-substitutability of preferences. This application is by no means restricted to marriage markets. An interesting use of this model would be to study matching between heterogeneous firms and workers and wage inequality among co-workers in environments where workers sort into different but complementary occupations within the firm. In this sense, I extend the matching models of one firm to many workers of Eeckhout and Kircher (2018) and Kelso and Crawford (1982) to allow for complementarities between co-workers (in addition to the usual complementarity between workers and firms).

The paper is organized as follows. Section 2 presents descriptive evidence about polygamy and co-wives' interaction. Section 3 presents and solves the model. Section 4 describes the empirical strategy. Section 5 takes the model to the data. Section 6 investigates the effects of banning polygamy. Finally, section 7 concludes.

2 Polygamous families as extended hierarchical households

The model I present below is motivated by two aspects of polygamous families that have so far been overlooked: that they act as a joint extended family and that co-wives organize in a hierarchy of different household roles. In this section I provide more detailed evidence of these two features.

A possible indication that polygamous families are not a set of separate monogamous households is the frequency of co-wives' co-habitation. I explore data from seven African country surveys conducted by the Living Standard Measurment Study - Integrated Surveys on Agriculture (The World Bank, 2010-2014). Table 1 lists the country studies.

Columns (1) to (3) describe polygamous households that have a male head. Two facts become evident from this table. First, consistent with other data sources (for example, the

Table 1: Polygamy and co-wives interaction in the LSMS-ISA data

Country	Year	N	Polygamous households			
			Male head			Female head
			(1)	(2)	(3)	(4)
			N	as % of males	% Co-wives cohabit	as % of N
Mali	2014	2399	61	63	51	0.3
B. Faso	2014	6540	31	35	82	3.7
Nigeria	2010	3380	21	24	86	0.3
Niger	2011	2430	19	22	94	0.5
Uganda	2010	2206	15	20	10	5.4
Malawi	2010	10038	7.6	10	0.3	1
Ethiopia	2011	3427	4.2	5.5	8.3	0.8

Notes: *LSMS-ISA* stands for *Living Standard Measurment Study - Integrated Surveys on Agriculture*. The LSMS-ISA country surveys are publicly available from the World Bank **(click here for access)**. *N* refers to the total number of rural households in the country survey.

Demographic and Health Surveys analyzed in a related work by Fenske (2015)), polygamy is most frequent in Western Africa. Column (1) in the table shows the percent of male headed polygamous households in the data. Polygamy rates in Western countries vary between 19% and 61%. As a percent of male heads of households (column (2)), between 22% and 63% of males are polygamous in the West. Second, co-wives' cohabitation is very common in the countries with the highest polygamy rates but is less frequent in countries where polygamy rates are lowest. The percent of polygamous males that cohabit with his multiple wives is shown in column (3).[4] For example, in Niger, in 94% of polygamous families co-wives live in the same dwelling. A related statistic is the percent of households where the head is a polygamous female (column (4)). These would include households where one of the multiple wives of a polygamous male lives alone, separately from the husband and his other wives. As column (4) reveals, the frequency of these households is very low, reinforcing the picture that polygamous co-wives most frequently co-habit, at least in the Western countries.

In addition, empirical studies discuss whether polygamous families jointly produce the household output. Co-wives' cooperation and division of labor in household and farm labor was one of the main reasons Ester Boserup proposed for the existence of high polygamy rates in Western Sub Saharan Africa (Boserup, 2007). These rural economies are characterized by

[4]A man is observed to co-habit with multiple wives if his marital status is reported as "married (polygamous)" and more than one female is listed in the household roster as "spouse". Co-habitation follows from the fact that the LSMS-ISA defines a household as *"a group of people who have usually slept in the same dwelling and share their meals together."*

technological conditions that favor female and child work. Hence, women in these markets are highly valued for their farming skills and their fertility (an idea also present in Jacoby (1995)). Despite conflict, a co-wife is welcomed to help with the workload implied by housework and farming.[5] Recent studies discuss whether co-wives (together with their husbands) cooperate in agricultural production. For example, Akresh, Chen, and Moore (2016) find evidence that co-wives share farming inputs efficiently in Burkina Faso, and Dauphin (2013) arrives at similar conclusions in Burkina Faso and Benin, but rejects efficiency of polygamous families in Senegal. We can also observe whether co-wives work together in the farm in the LSMS-ISA data. For example, in Nigeria (the country that will be the focus of the empirical section of this paper), co-wives work together in farming in over 45% of polygamous households.

The second feature that usually arises in studies of polygamous households is that co-wives organize in a hierarchy of senior-junior wives, two different household roles that imply very different status within the household. Previous empirical studies find that first wives are healthier, have higher social status, are more productive, and have higher quality children, relative to lower-order wives (see for example Matz (2016), Gibson and Mace (2007), and Strassmann (1997)).

3 A model of polygamy with spousal complementarity and female household roles

In this section I propose and solve a new model of family formation under polygamy that is motivated by the two facts noted in the previous section. The first fact (that polygamous families function as a joint extended family) motivates the modeling of spouses having complementary traits in the household production function. The second fact (that co-wives organize in a hierarchy) motivates the distinction in the production function between the senior wife and the junior wife household roles.

The novel implication in the model with respect to previous models of polygamy is that the gains from marriage for women depend not only on the traits of their potential husband (as in previous models) but also on the traits of potential co-wives. This implies that there is a non-

[5]The trade off between market and household labor has also been noted in the context of monogamous marriage markets in developed countries, as in Low (2014).

trivial problem of optimally sorting women into monogamous versus polygamous marriages, and into the senior versus the junior wife position within polygamous families. The model, hence, rationalizes the selection of women into polygamous marriages and their occupation within the household. Importantly, the differential welfare status of co-wives that has been noted empirically arises endogenously in the equilibrium of my model.

3.1 Marriage market populations

A marriage market consists of two populations: males and females. Households that form in this marriage market can be single-headed (the head of the household is an unmarried man or woman), monogamous (the head of the household is a man married to one and only one wife), or polygamous (the head of the household is a man married to two wives).

Females are characterized by their endowment of skill, s, distributed according to continuous distribution F on $[\underline{S}, \overline{S}]$. Males are characterized by their wealth, y, distributed according to continuous distribution G on $[\underline{Y}, \overline{Y}]$.[6] I assume that the sex ratio is 1.[7]

People meet in the marriage market to produce household income according to a marital output that is a function of spouses traits, s and y. A marriage consists of a husband and at most two wives.

3.2 Marital Output

A woman endowed with skill level s can be employed in the household in the role of the *senior* wife or in the role of the *junior* wife. To reflect the virilocal aspect of polygamous societies, I assume that a male endowed with wealth y owns the household technology, H^y, which I present next.

Consider any coalition of a male y, a junior wife s', and a senior wife s, $(y, s', s) \in [\underline{Y}, \overline{Y}] \times [\underline{S}, \overline{S}]^2$. The marital output that team (y, s', s) would produce shall they be together is

$$H^y = h(y, s', s)$$

[6] To allow for some agents to choose to be single or to marry monogamously, the set of females is augmented by including a point $\emptyset_s$ to denote the "dummy" co-wife of any monogamous household or the "dummy" wife of any single male, and the set of males is augmented similarly with point $\emptyset_y$ to denote the "dummy" husband of any single woman (Chiappori, McCann, and Nesheim, 2010).

[7] The sex ratio is defined as the ratio between the mass of females to the mass of males.

That is, household income is produced according to technology H^y with male wealth y, the skill of a junior wife s', and the skill of a senior wife s. The marital output depends on the male trait to reflect the fact that in virilocal societies men inherit land which value (quality and quantity) depends on male wealth. The marital surplus depends on the role of females to reflect the fact that in polygamous societies co-wives perform different roles. All in all, this model reflects two important features of polygamous societies: division of labor across gender and division of labor within gender.

I make the following assumptions on the marital output, $h(y, s', s)$, where I denote $h_i(y, s', s)$ the partial derivative of $h(y, s', s)$ with respect to the i^{th} input.

Assumption 1 *The marital surplus $h(y, s', s)$ satisfies:*

1. **Differentiability**. *Household production function is twice continuously differentiable.*
2. **Monotonicity**. *Household output is strictly monotone in male wealth and in the skill of the senior wife and weakly monotone in the skill of the junior wife: $h_1(y, s', s) > 0$, $h_2(y, s', s) \geq 0$, and $h_3(y, s', s) > 0$.*
3. **Supermodularity/Female Complementarity**. *Total social output when the most attractive individuals are together and the least attractive individuals are together is higher than when households are mixed. Formally, for any two input vectors $z = (y, s', s)$ and $\hat{z} = (\hat{y}, \hat{s'}, \hat{s})$, $h(z \vee \hat{z}) + h(z \wedge \hat{z}) \geq h(z) + h(\hat{z})$, where "$\vee$" and "$\wedge$" denote the joint and the meet of the vectors, respectively.*
4. **Hierarchy of Female Roles**.
   (a) *Higher skilled women are more productive in the senior wife role: for all $s > \hat{s}$, for all y, $h_3(y, \hat{s}, s) > h_2(y, s, \hat{s})$.*
   (b) *The marginal productivity of women skills in the senior wife position is always higher than the marginal productivity of women skills in the junior wife position: for all $y > \hat{y}$, $t > \hat{t}$, $s > \hat{s}$, $h_3(\hat{y}, \hat{t}, \hat{s}) > h_2(y, t, s)$.*
5. **Essential Coalitions**.
   (a) *The value of being single is zero: $h(y, \emptyset_{s'}, \emptyset_s) = h(\emptyset_y, s', \emptyset_s) = h(\emptyset_y, \emptyset_{s'}, s) = 0$.*
   (b) *All marriages must include the husband: $h(\emptyset_y, s', s) = 0$.*
   (c) *Marriages can be monogamous: $h(y, \emptyset_{s'}, s) \geq 0$ & $h(y, s', \emptyset_s) \geq 0$.*

Part 1 of assumption 1 is standard and introduced just to simplify exposition.

Part 2 is also standard: all else equal, better inputs produce higher output. I leave open the possibility that marital surplus is weakly increasing in the junior wife role to consider the case where all skills employed in the junior position contribute the same to producing household output. This particular case yields roughly the same conclusions as the more general case but entailing much simpler mathematical derivations. The main results of this paper will be developed under this case of constant productivity of junior wives. In appendix C I show all results under the more general case.

Part 3 means that any two spouses' traits are complements to produce household output. The complementarity between males and females is standard in family models since it gives rise to the observed positive assortative matching between the sexes. The complementarity between co-wives is the novel ingredient of this paper to the literature of household formation with polygamy. The assumption implies that the extended household production function, $h(y, s', s)$, is not separable in the wives' inputs or more specifically, that $h(y, s', s) > h(y, \emptyset_{s'}, s) + h(y, s', \emptyset_s)$. This complementarity between co-wives captures the idea that the presence of a co-wife, despite conflict, helps to achieve the desired household goals in terms of fertility and income generation.

Part 4 is introduced to give meaning to the idea that there is a hierarchy of wives within polygamous marriages. This is captured in the model as a differential importance in the roles of women in producing household output. First, part 4(a) means that a household that can afford two women will position the highest skilled of them in the senior wife position and the least skilled in the junior wife position since this is the female sorting that entails the highest household output. Second, part 4(b) means that the productivity of skills in the senior wife role is significantly higher than the productivity of skills in the junior wife position. This is true to the extent that increasing the skill of the senior wife even in a household where all spouses have lower traits is more profitable than increasing the skills of the junior wife in a household where all spouses have higher traits. This assumption reflects the idea that one of the two roles is more important than the other in producing household output. Without loss of generality, that role is the one called the *senior wife role*. Importantly, note that in the case that the marital surplus is constant in the skill of the junior wife, $h_2(y, s', s) = 0$, part 4 is automatically satisfied.

Finally, part 5 specifies which types of households are possible in this marriage market. Households can be formed by single individuals (which value is normalized to zero), by marriages of a male and a only one female, or by marriages of a male and two females. Single individuals earn the lowest possible value in the market. Households with no husband will not form since two women together gain nothing with respect to splitting and becoming two single households. Last, note that part 5(c) together with part 4 implies that monogamous families will employ their only wife in the senior wife position.

All in all, while supermodularity will be the key assumption to prove assortativeness of the equilibrium, hierarchy of roles will be the key assumption to prove the sorting patterns of females into household roles.

3.3 Marriage Market Equilibrium

The main objective of the modeling part of the paper is to characterize two features of the marriage market with polygamy and hierarchy of wives. First, the equilibrium sorting of women into household roles: in equilibrium, which women are employed as junior wives and which women are employed as senior wives? Second, the shape of the equilibrium matching between males and females: who marries whom and what do they gain in equilibrium? Just by assuming a marital surplus with complementarities and different productivities of female roles, I am able to show that there exists an equilibrium in the marriage market with positive selection of women into roles, and positive assortative matching between males and females within role.

3.3.1 The problem of household formation

The marriage market is competitive and the marital output perfectly transferable. This means that at the moment of choosing their partners, females and males face given prices $\{u(s)\}$ for all women $s \in [\underline{S}, \overline{S}]$ and $\{v(y)\}$ for all men $y \in [\underline{Y}, \overline{Y}]$ in the market. The marital output produced by any potential coalition is known at the moment of matching and it is perfectly divided among potential spouses according to the given sharing rule $((v(y), u(s'), u(s)))$.

The objective of male y is to form the team of wives that maximizes his profits, subject to being accepted.

$$v(y) \equiv \max_{s,s'} \quad h(y, s', s) - u(s') - u(s) \tag{1}$$

Because the marital output is transferable, in equilibrium it has to be the case that the argmax women (s, s') agree with marrying y at the equilibrium prices (that is, the household formation problem can be solved by a single decision maker).

3.3.2 Definitions

Before introducing the equilibrium concept, I need to introduce the definitions of *outcome* and of *stability* in the marriage market with polygamy and female roles.

Definition 1 *An **outcome** in the marriage market with polygamy and female roles is defined as a tupla $(\mathcal{M}, \mathcal{L}, \mu, v^{\mu}, u^{\mu})$ where:*

1. $\mathcal{M}$ **and** $\mathcal{L}$ **are, respectively, the set of seniors and the set of juniors such that they form a partition of the set of females** *:*

$$\mathcal{M} \subseteq [\underline{S}, \overline{S}] \text{ and } \mathcal{L} \subseteq [\underline{S}, \overline{S}] \text{ such that } \mathcal{M} \cup \mathcal{L} = [\underline{S}, \overline{S}] \text{ and } \mathcal{M} \cap \mathcal{L} = \emptyset$$

2. μ **is a pure matching***: a non-degenerate measure on $\mathcal{M} \times \mathcal{L} \times [\underline{Y}, \overline{Y}]$, the marginals of which coincide with the measure of each set and where:*
   - $\mu_3(y) : [\underline{Y}, \overline{Y}] \to [\underline{S}, \overline{S}]$ *denotes the senior wife of male y,*
   - $\mu_2(y) : [\underline{Y}, \overline{Y}] \to [\underline{S}, \overline{S}]$ *denotes the junior wife of male y, and*
   - $\mu_1(s) : [\underline{S}, \overline{S}] \to [\underline{Y}, \overline{Y}]$ *denotes the husband of woman s.*

3. v^{μ} **and** u^{μ} **are feasible payoff functions associated to** μ*: $\forall y, s, s' \in Spt(\mu)$*

$$v^{\mu} : [\underline{Y}, \overline{Y}] \to \Re \text{ and } u^{\mu} : [\underline{S}, \overline{S}] \to \Re \text{ such that}$$
$$v^{\mu}(y) + u^{\mu}(s') + u^{\mu}(s) = h(y, s', s)$$

Intuitively, an outcome in the marriage market is a particular grouping of males and females into a set of households and a scheme of associated payoffs: part 1 of definition 1 indicates which women become senior wives and which women become junior wives; part 2 indicates who matches with whom, that is, who is the senior wife, the junior wife, and the husband in each household; and part 3 indicates the gains from marriage to each individual.

Note that there are many outcomes in a given marriage market because there are many ways of grouping individuals, many ways of splitting the surplus generated by each group in a feasible way, and two possible roles for each woman. However, not all these possible outcomes are *equilibrium* outcomes.

An equilibrium in the marriage market is an outcome such that all individuals maximize their preferences as indicated in problem 1 above. The equilibrium concept is that of *stable outcome* or *stable matching.*

Definition 2

1. *An outcome* $(\mathcal{M}, \mathcal{L}, \mu, v^\mu, u^\mu)$ *is* ***stable*** *if*

$$h(y, s', s) = v^\mu(y) + u^\mu(s') + u^\mu(s) \ ; \forall y, s, s' \in Spt(\mu)$$
$$h(y, s', s) \leq v^\mu(y) + u^\mu(s') + u^\mu(s) \ ; \ otherwise$$

2. *A matching* μ *is* ***stable*** *if there exist numbers* $v^\mu(y), u^\mu(s')$*, and* $u^\mu(s)$ *such that the outcome* $(\mathcal{M}, \mathcal{L}, \mu, v^\mu, u^\mu)$ *is stable. When a matching is stable, we say that there is no coalition that* blocks *the matching.*

Intuitively, a matching μ is stable if there is no individual or group of individuals that would agree to form a household (possibly of a single) that is not in society μ. If we take any coalition of individuals that are not matched together under μ, stability requires that together they cannot produce more than the sum of what each is earning under μ. If what they would produce shall they deviate together is more than the sum of what they are earning, they can split the surplus and agree to deviate from society μ.

The definition above is standard in the literature of matching with transferable utility except that in addition to requiring coalitions not to block assignment μ, it also requires women to optimally choose their role. The fact that the partition of females into the two household roles is determined endogenously in equilibrium is a key output of this model, constituting one of the main contributions to the literature. To see this, note that in definition 2 there is no prerequisite for the support of μ: so long as the sets $\mathcal{M}$ and $\mathcal{L}$ partition the skills set, a woman $s \in [0, \bar{S}]$ can be either the second or the third argument in the marital surplus function. Hence, the identity of senior and junior wives matter in a nontrivial way for the definition of stability. For example, suppose that (y, s', s) and $(\hat{y}, \hat{s'}, \hat{s})$ are two households under μ. In this society, women s' and $\hat{s'}$ perform the role of the junior wife and women s and $\hat{s}$ perform the role of the senior wife. Now suppose that husband y and senior s together with woman $\hat{s}$ (a senior under μ) in the role of junior could produce together more than the sum of the individual profits under μ:

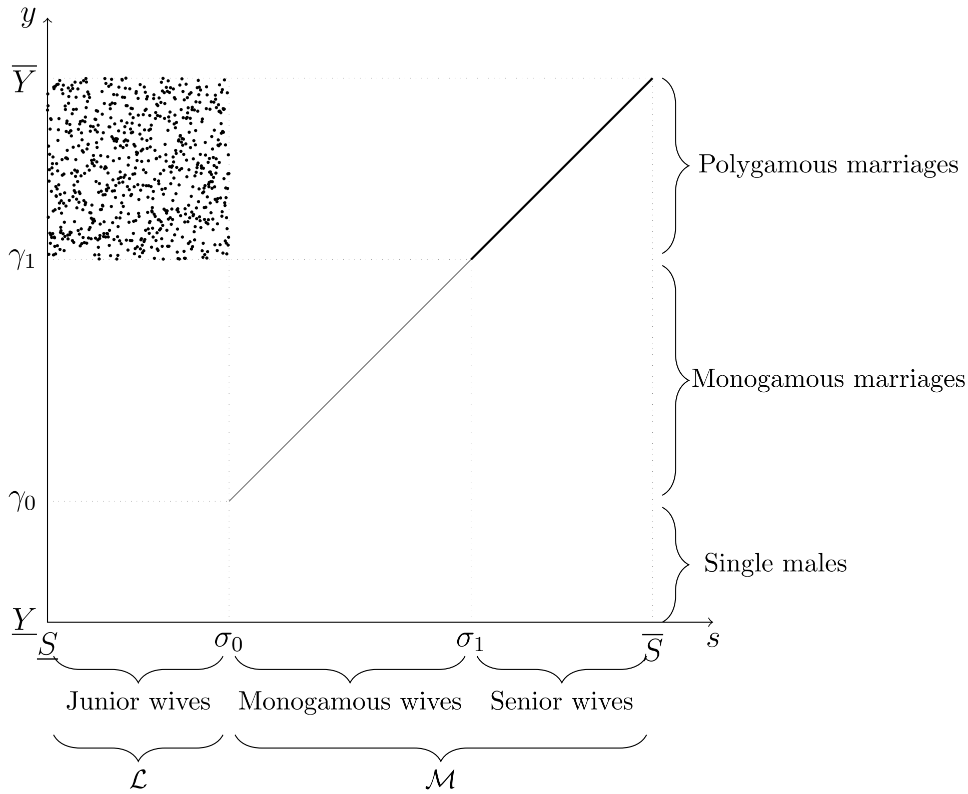
$$h(y, \hat{s}, s) > v^{\mu}(y) + u^{\mu}(\hat{s}) + u^{\mu}(s)$$

Since the excess product $h(y, \hat{s}, s) - (v^{\mu}(y) + u^{\mu}(\hat{s}) + u^{\mu}(s))$ can be used to increase the payoffs of all the members of this blocking coalition, husband y and senior s would like to match with $\hat{s}$ and employ her as a junior wife, and woman $\hat{s}$ would find it profitable to deviate from being the senior in household $(\hat{y}, \hat{s'}, \hat{s})$ to being the junior in household $(y, \hat{s}, s)$. Hence, the coalition $(y, \hat{s}, s)$ would block μ and μ would not be a stable matching. In sum, the novel contribution of this paper to the literature is that the solution of the model will endogenously determine not only who matches with whom, but also what women sort into being senior wives and what women sort into being junior wives.

3.3.3 Main result

In this section I characterize and prove the existence of a stable matching in the marriage market with polygamy and two female household roles for the case in which all females produce the same fixed output, K, when employed in the junior wife position. That is, in this section I show the equilibrium when the household output is $h(y, s', s) = h(y, K, s)$ such that it satisfies assumption 1 with $h_2(y, s', s) = 0$. The main result of this paper is illustrated in figure 1 below. In the figure, the vertical axis indicates male wealth and the horizontal axis displays female skills and the partition of skills into the junior and senior wife positions, $\mathcal{L}$ and $\mathcal{M}$, respectively. The solid upward sloped line indicates the equilibrium match between women with skills in $\mathcal{M}$ and husbands, while the scattered points in the top left area indicate the matching between women with skills in $\mathcal{L}$ and husbands. The figure can be summarized as follows: the equilibrium in this marriage market exhibits a threshold shape with positive assortative matching between males and females and positive sorting of females into household roles such that least skilled women take the junior wife role and most skilled women take the senior wife role. The relevant thresholds to note are σ_0 and σ_1 in the set of female skills, and γ_0 and γ_1 in the set of male wealth, all of which are endogenously determined in equilibrium.

Figure 1: A stable matching



There are three salient features of this equilibrium.

The first is the structure of polygamous households. The highest skilled women (women with skills above σ_1) are so skilled that they face the highest opportunity cost of being monogamous and forego the increment in their productivity that results from having a complementary co-wife. Hence, households with high skilled women will try to hire a junior wife. However, because of the higher productivity of skills in the senior role relative to the junior role, only the lowest skilled females (females with skills below σ_0) face an outside option low enough to be willing to take the less rewarded position of a junior wife in polygamous households. In effect, women below σ_0 trade off being the junior wife to rich polygamous households against being the senior wife to poor males with wealth below γ_0. As regards to matching patterns, because of supermodularity, the match between senior wives and husbands in polygamous households is positive assortative: higher skills contribute more to household output the higher the wealth of the husband. Because the marginal contribution of female skills in the junior wife position to household output is zero, matching between husbands and junior wives is random: any woman who accepts to be a junior wife brings the same value to any polygamous household. All in all, in polygamous households senior wives and husbands are alike in terms of their attractiveness, but co-wives differ a great deal in their skills, the seniors being top in the skill distribution and the juniors being bottom.

The second characteristic to notice is that some households end up being monogamous, even when co-wives complement each other. The reason is that there is a threshold σ_0 from which female skills are high enough that women with skills above this threshold prefer to be senior wives to middle wealth husbands than junior wives in richer polygamous households. Of course, women with skills between σ_0 and σ_1 would like to hire a junior wife, but all women willing to be junior wives prefer to be married to wealthier couples that outbid them. Hence, these marriages end up being monogamous, with positive assortative matching between husbands and wives given the supermodularity of the surplus. Note that assumption 1 implies that women in monogamous households take the senior wife position.

The last feature to notice is that poor males with wealth below γ_0 end up being single, which results from the assumption that the sex ratio is one, not high enough to compensate for the fact that some males have two wives.

The novel aspect of this characterization of marriage markets with polygamy is that all these relevant thresholds are endogenous: a solution to the model exists such that in equilibrium all women choose their role optimally and households optimally arise to be single-headed, monogamous, or polygamous. The next proposition formalizes this result: it establishes the existence of a threshold-shaped stable matching and characterizes the stable outcome. A sketch of the proof is provided subsequently in the text, while the formal proof is provided in appendix A. Appendix C, in turn, generalizes the result to the case where the contribution of skills in the junior wife position to household output is strictly positive ($h_2(y, s', s) > 0$).

Proposition 1 *The marriage market with populations* $s \sim F[\underline{S}, \overline{S}]$ *and* $y \sim G[\underline{Y}, \overline{Y}]$, *and marital output* $h(y, s', s) = h(y, K, s)$ *satisfying assumption 1 with* $h_2(y, s', s) = 0$ *has a stable outcome,* $(\mathcal{M}, \mathcal{L}, \mu, v^{\mu}, u^{\mu})$, *characterized by:*

1. *Thresholds* $\sigma_0 \in [\underline{S}, F^{-1}(0.5)]$, $\sigma 1 = F^{-1}[1-F(\sigma_0)]$, $\gamma_0 = G^{-1}[F(\sigma_0)]$, *and* $\gamma_1 = G^{-1}[F(\sigma_1)]$, *all of which are unique.*
2. *The partition of female skills into junior and senior wife roles,* $\mathcal{L} = [\underline{S}, \sigma_0)$ *and* $\mathcal{M} = [\sigma_0, \overline{S}]$
3. *Matching function*

$$\mu = (y, \mu_2(y), \mu_3(y)) = \begin{cases} (y, \emptyset_s, \emptyset_s), & \forall y \in [\underline{Y}, \gamma_0) \\ (y, \emptyset_s, F^{-1}[G(y)]), & \forall y \in [\gamma_0, \gamma_1) \\ (y, s', F^{-1}[G(y)]), & \forall y \in [\gamma_1, \overline{Y}], s' \in [\underline{S}, \sigma_0) \end{cases}$$

4. Feasible payoff functions

$$u^\mu(s) = \begin{cases} h(\gamma_0, 0, \sigma_0), & \forall s \in [\underline{S}, \sigma_0) \\ h(\gamma_0, 0, \sigma_0) + \int_{\sigma_0}^{s} h_3\Big(\mu_1(t), 0, t\Big)\, \mathrm{d}t & \forall s \in [\sigma_0, \sigma_1) \\ h(\gamma_0, 0, \sigma_0) + \int_{\sigma_0}^{s} h_3\Big(\mu_1(t), \mu_2(t), t\Big)\, \mathrm{d}t & \forall s \in [\sigma_1, \overline{S}] \end{cases}$$

$$v^\mu(y) = \begin{cases} 0, & \forall y \in [\underline{Y}, \gamma_0) \\ \int_{\gamma_0}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\, \mathrm{d}t, & \forall y \in [\gamma_0, \gamma_1) \\ \int_{\gamma_0}^{y} h_1\Big(t, \mu_2(t), \mu_3(t)\Big)\, \mathrm{d}t, & \forall y \in [\gamma_1, \overline{Y}] \end{cases}$$

3.3.4 Sketch of Proof

The complete proof of proposition 1 is in appendix A. I follow Chiappori, Oreffice, and Quintana-Domeque (2018) and prove proposition 1 by a "direct approach." This approach consists of three main parts. First, I assume the equilibrium is as described in proposition 1 and derive a complete characterization of it from the *local* stability conditions. This characterization includes thresholds, the matching function, and payoffs as a function of threshold σ_0. Second, I provide proof that there exist values of parameters of the model such that threshold σ_0 exists and is unique. Third, I prove that the characterized assignment in fact satisfies *global* stability conditions: there is no coalition that blocks the assignment and females are sorting optimally into household roles.

The steps for proving the proposition can be sketched as follows:

1. Full characterization of the conjecture in terms of threshold σ_0 (section A.1):

   (a) From the distributions of males' and females' traits, I derive the **thresholds** and the **matching function** as functions of σ_0.

   (b) From the first order necessary conditions I obtain **indirect utilities** as functions of

σ_0.

2. Proof of existence of a unique threshold σ_0 (section A.2):

   (a) From indifference conditions of threshold individuals, I show that a unique solution for σ_0 exists.

3. Take the above characterization as given and check stability conditions (section A.3):

   (a) Taking the sorting of women as given by threshold σ_0, I prove that there is no coalition of one, two, or three individuals that blocks the characterized assignment. This part of the proof relies heavily on the supermodularity of $h(y, s', s)$.

   (b) Show that women are sorting optimally: This part of the proof follows from part 4 of assumption 1, which guarantees that there is no blocking of any coalition in which females take a role different than the one prescribed by their position relative to σ_0.

In standard marriage problems that are two-sided one-to-one or many-to-one with substitutable preferences in the sense of Hatfield, Kominers, Nichifor, Ostrovsky, and Westkamp (2013), there would be no need for part 3 (checking stability conditions), because existence of the stable outcome would be guaranteed and we could arrive at its characterization either by following steps 1 and 2 or by solving the surplus maximization linear program. However, in this marriage market, complementarity of co-wives and their endogenous sorting into household roles make those general methods inapplicable. First, the fact that women choose their role endogenously makes local optimality conditions only necessary, but not sufficient for the outcome to be stable; hence, there is a need to prove global optimality by disproving any potential blocking coalition. Second, surplus maximization may fail to be a linear program since the household production function is not separable in the co-wives.

In this section I have proved the main result of the paper for general distributions of traits and production functions satisfying assumption 1 with $h_2(y, s', s) = 0$. The more general case where $h_2(y, s', s) > 0$ is presented in appendix C. Taking particular distributions and production functions allows arriving at a closed form characterization of the model that can be taken to the data and used for policy counterfactual simulations. In the next section, I present the closed form characterization of the model for the particular case of uniformly distributed traits and Cobb-Douglas household production function.

3.4 Example: Uniform-Cobb Douglas

While all that is required to prove existence of the equilibrium that exhibits co-wives' inequality is assumption 1 (essentially supermodularity and hierarchies), it is convenient to parameterize the model and solve it in closed form. In this subsection, I obtain the full characterization of the stable outcome in this marriage market for the case that traits are uniformly distributed and household output is generated by a Cobb-Douglas production function. This characterization is general enough to embed all the cases that will be discussed in applications below. Let female skill and male wealth be uniformly distributed over the intervals $[0, \overline{S}]$ and $[0, \overline{Y}]$, respectively:

$$s \sim U[0, \overline{S}]$$
$$y \sim U[0, \overline{Y}]$$

Let the household production function be

$$h(y, s', s) = y(1 + K)^{\beta} s^{\alpha}$$

This production function satisfies assumption 1 and hence, by proposition 1, there exists a solution for the model parameterized this way. The next proposition characterizes the equilibrium.

Proposition 2 *The model parameterized with* $s \sim U[0, \overline{S}]$, $y \sim U[0, \overline{Y}]$, *and* $h(y, s', s) = y(1+K)^{\beta} s^{\alpha}$ *has a stable equilibrium* $(\mathcal{M}, \mathcal{L}, \mu, v^{\mu}, u^{\mu})$ *with threshold shape as depicted in figure 1, and is characterized in closed form by:*

1. *Threshold* $\sigma_0 \in \left[0, \frac{\overline{S}}{2}\right]$ *that is the unique solution to equation*

$$\overline{Y}(1 - \frac{\sigma_0}{\overline{S}})(\overline{S} - \sigma_0)^{\alpha}((K+1)^{\beta} - 1) - \sigma_0^{\alpha+1}\frac{\overline{Y}}{\overline{S}} = 0 \tag{2}$$

   for all $K \in (K^L, K^H) = (0, 2^{1/\beta} - 1)$

2. *The set of junior wives* $\mathcal{L} = [0, \sigma_0)$ *and of senior wives* $\mathcal{M} = [\sigma_0, \overline{S}]$

3. *Matching function*

$$
\mu = (y, \mu_2(y), \mu_3(y)) = \begin{cases} (y, \emptyset_s, \emptyset_s), & \forall y \in [0, \gamma_0) \\ (y, \emptyset_s, \frac{\overline{S}}{\overline{Y}} y), & \forall y \in [\gamma_0, \gamma_1) \\ (y, s', \frac{\overline{S}}{\overline{Y}} y), & \forall y \in [\gamma_1, \overline{Y}], s' \in [0, \sigma_0) \end{cases}
$$

4. *Equilibrium payoffs*

$$
u^{\mu}(s) = \begin{cases} \sigma_0^{\alpha+1} \frac{\overline{Y}}{\overline{S}}, & \forall s \in [0, \sigma_0) \\ \Big(\sigma_0^{\alpha+1} + \frac{\alpha}{\alpha+1}(s^{\alpha+1} - \sigma_0^{\alpha+1})\Big) \frac{\overline{Y}}{\overline{S}}, & \forall s \in [\sigma_0, \overline{S} - \sigma_0) \\ \Big(\sigma_0^{\alpha+1} + \frac{\alpha}{\alpha+1}((1-\sigma_0)^{\alpha+1} - \sigma_0^{\alpha+1}) + & \\ \quad + \frac{\alpha}{\alpha+1}(s^{\alpha+1} - (1-\sigma_0)^{\alpha+1})(1+K)^{\beta}\Big) \frac{\overline{Y}}{\overline{S}}, & \forall s \in [\overline{S} - \sigma_0, \overline{S}] \end{cases}
$$

$$
v^{\mu}(y) = \begin{cases} 0, & \forall y \in [0, \sigma_0 \frac{\overline{Y}}{\overline{S}}) \\ \frac{y^{\alpha+1} - \gamma_0^{\alpha+1}}{\alpha+1} \Big(\frac{\overline{S}}{\overline{Y}}\Big)^{\alpha}, & \forall y \in \Big[\sigma_0 \frac{\overline{Y}}{\overline{S}}, \overline{Y}\Big(1 - \frac{\sigma_0}{\overline{S}}\Big)\Big) \\ \Big(\frac{(1-\sigma_0)^{\alpha+1} - \sigma_0^{\alpha+1}}{\alpha+1} + & \\ + \frac{y^{\alpha+1} - (1-\sigma_0)^{\alpha+1}}{\alpha+1}(1+K)^{\beta}\Big) \Big(\frac{\overline{S}}{\overline{Y}}\Big)^{\alpha}, & \forall y \in \Big[\overline{Y}\Big(1 - \frac{\sigma_0}{\overline{S}}\Big), \overline{Y}\Big] \end{cases}
$$

In appendix B I show the derivation of this characterization and prove the existence of threshold σ_0 for this case.

Obtaining the closed form solution of the model for the Uniform Cobb-Douglas case is convenient for the rest of the paper. The next three sections will rely on this characterization to show how the shape of the polygamous marriage market equilibrium can be identified from observed matches and equilibrium behavior, to show that the equilibrium of this model reproduces the observed equilibrium very well, and to conduct counterfactual policy analysis.

4 Empirical strategy

The main objective of the empirical part of this paper is to estimate the traits of threshold individuals, σ_0, σ_1, γ_0, and γ_1, and to evaluate how a model of household formation with polygamy where co-wives complement each other fits the the matching and sorting patterns observed in the data. A second objective is to use the estimated model to study what policies are welfare improving in these marriage markets, especially for women who have got less attention in literature discussion about the welfare effects of polygamy.

There are a few interesting challenges in estimation. First, in most data sets we do not observe the premarital traits that people match on in the marriage market. Instead, we usually observe equilibrium matches and post-marital behavior. Second, we typically do not observe who is the senior or the junior wife within polygamous households. Even when we may observe order of marriage, seniority within the household may be determined by other factors like youth or parental wealth. Finally, thresholds in the model are the unique solution to equations like (2) that, to be taken to the data, require knowledge about the parameters of the distribution of women's skills and men's wealth which are not usually observed directly from the data.

My empirical strategy attends these challenges as follows. First, I estimate women skills and men wealth outside of the model using a principal component analysis (henceforth, PCA) approach based on observed *equilibrium* household characteristics. Second, I rely heavily on the predictions of my model to classify as *senior* the wife with the highest skill level within the household. Third, I construct a "performance" ranking of women and men in the marriage market by taking the probability distribution of the skills and wealth measures estimated through the PCA approach. Forth, I specify the empirical model as one in which the household technology is Cobb-Douglas and the distribution of (the ranking of) marital traits is uniform with support $[0, 1]$. Finally, I estimate the parameters of the specified production function and the threshold σ_0 by the generalized method of moments. My empirical strategy is based on Hagedorn, Law, and Manovskii (2017), who develop an identification argument for models of sorting in the labor market using percentiles of the distribution of *equilibrium* wages and other labor market outcomes as the relevant matching inputs.

In the remaining of this section I present the data, show the out-of-the-model estimation of the attractiveness indexes, specify the household technology and distribution of female and male traits, and, finally, discuss identification and present the estimator.

4.1 Data and summary statistics

The data comes from the Nigerian Living Standards Measurement Study: Integrated Surveys on Agriculture (The World Bank, 2010-2014), henceforth referred to as LSMS-ISA. The LSMS-ISA-

Nigeria is a nationally representative household panel survey of 4997 agricultural households[8] interviewed in two seasons per wave: the post-planting season and the post-harvest season. In this paper I use the 2010-2011 wave and restrict attention to agricultural households in the rural sector. Moreover, to avoid erroneously labeling households that have not yet acquired a second wife as monogamous, I exclude from the analysis households with one wife where the head is younger than 40 years old. All in all, the final sample includes 439 single households, 609 polygamous households, and 1151 monogamous households.

Table 2: Summary statistics, LSMS-ISA Nigeria, 2010

	All		Polygamous		P-M	
	Mean	SD	Mean	SD	Mean	SD
Marriage market						
Women age at marriage	17.878	6.714	17.082	6.101	-1.623	0.278
Men age at marriage	29.49	9.521	26.798	8.38	-4.241	0.448
Husband-wife age gap	14.491	8.216	16.009	8.738	3.276	0.32
Household structure						
# of wives	1.13	.794	2.189	0.45	1.189	0.013
# of children	3.789	2.863	5.864	3.063	2.274	0.127
# of domestic workers	.013	.165	0.002	0.041	-0.006	0.004
# of other family	.367	1.103	0.23	0.811	-0.055	0.041
Production and Labor						
Plot uses only family labor	0.8	0.4	0.701	0.458	-0.124	0.016
Employed	0.59	0.492	0.591	0.492	0.005	0.01
Works only for family	0.925	0.264	0.934	0.249	0.016	0.007

Notes: *LSMS-ISA* stands for *Living Standard Measurment Study - Integrated Surveys on Agriculture*. The LSMS-ISA country surveys are publicly available from the World Bank **(click here for access)**. *P-M* refers to the difference in the corresponding statistics between polygamous and monogamous households. *Marriage market* and *Household structure* statistics are reported at the household level. *Plot uses only family labor* is a dummy variable measured at the agricultural plot level that takes value one if all workers in the plot are household members and zero if some worker on the plot is an external hire. *Employed* is a dummy variable measured for all household members older than 5 years old that takes value one if the individual works on their own, on a family plot, or for a non-family member. *Works only for family* is a dummy variable that takes value one if *Employed* equals one and the individual does not work for members outside of the household.

Table 2 shows summary statistics measured for all households and for polygamous households, and the difference in statistics between polygamous and monogamous households (columns labeled "P-M"). The first three rows show characteristics of the marriage market. Women in this economy tend to marry very young with mean age at marriage around 18. Women in polygamous unions marry 1.6 years younger on average. Moreover, the age difference between husbands and wives is about 14.5 years, gap that increases by 3 years on average in polygamous marriages relative to monogamous marriages. The second set of four rows show that most households in rural Nigeria consist of spouses and kids. The average number of wives in

[8] Agricultural households are those that manage or own at least one agricultural plot.

polygamous unions is two and the average number of kids is almost six, over two kids more relative to monogamous families. Other family members live in the household a third of times on average and households do not typically employ domestic workers. The last three rows show that agricultural production is extremely dependent on labor from household members (typically wives and kids). First, about 80% of agricultural plots in the data only use family labor as opposed to external hires. Second, on average 92.5% of employed individuals report working only for family members.

In appendix E I present the same statistics for the sample that does not exclude monogamous households with young heads. Interestingly, even when in the main sample used in this paper monogamous households are older than polygamous households, summary statistics are very similar, reflecting that marriage, household, and production patterns do not differ significantly across close generations.

4.2 Female skill and male wealth

As said, a major challenge of the empirical strategy is that, in typical datasets, we do not observe the matching traits of individuals. To infer the female skill and male wealth *permanent* distributions (those relevant at the time of marriage), I rely on the observed equilibrium outcomes of individuals. I source heavily on the anthropological and demographic related literature to select the key variables available in the data that explain females' and males' attractiveness in polygamous marriage markets and construct an index of attractiveness through a principal component approach. Table 3 shows summary statistics of the measurements used and their correlation with the first principal component of women skills and men wealth. The table shows statistics separately for the North and the South regions. This division is motivated by the fact that northern and southern states in Nigeria follow different laws. In particular, religion and polygamy legality are different in the north and in the south. This implies that they can be considered as different marriage markets.

For women, the measurements used to estimate female skills are *fertile years*, calculated as 49 minus the age at marriage;[9] *first wife*, an indicator that takes value one if the female is the first to get married in the household; *% businesses* and *% plots*, the fraction of the total

[9] 49 is the last age at which women are asked questions about their fertility in the LSMS-ISA data, so I take this as the last age a woman is fertile.

Table 3: Female and male attractiveness index and measurements

	North				South			
Measurements	*Obs.*	*Mean*	*SD*	*Corr.*	*Obs.*	*Mean*	*SD*	*Corr.*
				Female skills				
Fertile years	1648	32	6.473	.58	683	29	6.91	.227
First wife	1784	.634	.482	.746	700	.863	.344	.551
% businesses	1844	.19	.303	.487	944	.2	.372	.552
% plots	1844	.044	.192	.078	944	.356	.461	.493
Father educ.	1797	1.042	1.993	.118	934	1.497	3.321	.558
				Male wealth				
Education	1164	4.268	5.933	.823	677	7.552	5.802	.814
Father educ.	1189	1.079	2.205	.717	675	1.428	3.174	.795
# plots	1218	2.04	1.28	.198	677	1.959	1.105	.182

Notes: *Obs.* is the number of non missing observations for the corresponding variable. *SD* stands for *standard deviation*. *Corr.* refers to the correlation coefficient of the variable with the first principal component. *Fertile years* is calculated as 49 minus the age at marriage. *First wife* is a dummy that takes value one if the female is the first to get married in the household. *% businesses* and *% plots* refer the fraction of the total household production units that are managed by the wife. *Education* and *Father educ.* capture the number of years of completed education of the individual and the individual's father, respectively. *# plots* is the total number of agricultural plots that the household owns.

household production units (businesses and agricultural plots, respectively) that are managed by the wife; and *father education*, the years of completed education of the wife's father.

These measurements capture the different dimensions that make women attractive in economies where we observe polygamy. *Fertile years* represents women's ability to produce children (Boserup, 2007). *First wife* reflects attractiveness insofar more desirable women are unraveled in the marriage market and wed first (Matz (2016), Gibson and Mace (2007), and Strassmann (1997)). The percent of productive units (agricultural plots and businesses) managed by wives within the household reflect the managerial ability of the wife (Jacoby (1995), Akresh, Chen, and Moore (2016), and Dauphin (2013)). Finally, *father education* captures a measure of pre-marital skills.

Looking at column labeled *Mean* in table 3, the data reveals that, on average, women in the northern regions have 32 years of fertility (or equivalently, that they marry at the age of 17 on average, consistent with the information in table 2). Women in the south marry later, therefore having fewer years of fertility on average (about 29 years). 63.4% and 86.3% of females in the north and south, respectively, are first wives, consistent with the picture of higher polygamy rates in the north. In both regions, about 20% of household-owned businesses are managed by a wife. The picture for agricultural plots is very different, with only 4.4% being managed by wives in the north against 35.6% in the south. Finally, in both regions the fathers of wives are almost non educated, with an average of 1 and 1.5 years of completed education, respectively.

The measurements used to construct the index of male wealth are standard: *education* and *father education* capture the years of completed education of the husband and his father, respectively, and *# plots* is the total number of agricultural plots the household owns. All these traits are meant to capture dimensions of male wealth at the time of marriage. Noticeably, males in the south have 7.6 years of completed education on average, while males in the north have an average of 4.3 years of education.

Using these traits I estimate the skill levels of women and the wealth of men by extracting the first principal component. This index of traits was estimated separately for the North and the South. The columns labeled *Corr.* in table 3 show the correlation between each measurement and the first principal component. In the north, the index of female attractiveness computed through the principal component mainly captures fertile years, order of marriage, and management of enterprises. In the south, the picture is somewhat different, with age at marriage being less important and father education more important. For males in both the north and the south marriage markets, the correlations between the principal component of male wealth and measurements mainly captures education and father's education.

After estimating the first principal component, I construct the ranking of attractiveness of males and females as the percentile each person represents in the principal component distribution. I define a wife in the data to be a *senior* wife if she is the wife with the highest estimated skill among all the wives of the husband in that household. The underlying assumption is that the distribution of the first principal component preserves the distribution of permanent traits observed by individuals in the marriage market.

4.3 Empirical specifications

In order to conduct counterfactual policy analysis and welfare implications it is necessary to impose some structure on the household production function and the distribution of traits. Given the estimated ranking of individuals, I estimate the parameters of the modified production function in which inputs are quantiles of spousal traits. The modified production function takes a Cobb-Douglas specification:

$$h(y, s', s) = h(p(y), p(s'), p(s)) = p(y)(1 + K)^{\beta} p(s)^{\alpha} \tag{3}$$

with percentiles uniformly distributed,

$$p(s) \sim U[0,1] \text{ and } p(y) \sim U[0,1].$$

4.4 Estimation

We are interested in estimating the vector of parameters $\theta = \{\alpha, \beta, K\} \in \Theta$ and the trait of threshold individuals σ_0, σ_1, γ_0, and γ_1 (depicted in figure 1). I next discuss identification and the estimator.

4.4.1 Identification

The identification argument is twofold. First, this model gives rise to a tight relationship between thresholds and parameters. Recall that equation (2) in proposition 2 shows that σ_0 depends on the model parameters and the traits distributions. Re-writing the equation for the empirical specifications presented above, for any given vector of parameters $\theta = \{\alpha, \beta, K\} \in \Theta$ σ_0 is the unique solution to equation

$$(1-\sigma_0)(1-\sigma_0)^{\alpha}((K+1)^{\beta}-1) - \sigma_0^{\alpha+1} = 0 \tag{4}$$

This relationship disciplines thresholds as a function of parameters.

The second step in the identification argument consists of showing that key moments produced by the model govern parameters. For every fixed $\theta \in \Theta$, the parametric distributions $U[0,1]$ for female and male traits together with household technology (3) allows me to solve for *who marries polygamous*, *who marries monogamous*, *which women select into the senior position*, and *which women become juniors* in closed form. Therefore, the model produces the fraction of polygamous marriages and means and variances of the attractiveness measures for women and men conditional on type of marriage and wife role as a function of model thresholds. These model moments (vector $mom^{model}(\theta)$) are obtained in closed form:

$mom^{model}(\theta) =$

$$\Big[\underbrace{\frac{1-\gamma_1(\theta)}{1-\gamma_0(\theta)}}_{\substack{\text{fraction}\\ \text{polygamous}}},\underbrace{\frac{\gamma_1(\theta)+\gamma_0(\theta)}{2},\frac{(\gamma_1(\theta)-\gamma_0(\theta))^2}{12}}_{\substack{\text{mean and variance of}\\ \text{monogamous husbands' wealth}}},\underbrace{\frac{\sigma_1(\theta)+\sigma_0(\theta)}{2},\frac{(\sigma_1(\theta)-\sigma_0(\theta))^2}{12}}_{\substack{\text{mean and variance of}\\ \text{monogamous wives' skills}}},$$

$$\underbrace{\frac{\gamma_1(\theta)+1}{2},\frac{(1-\gamma_1(\theta))^2}{12}}_{\substack{\text{mean and variance of}\\ \text{polygamous husbands' wealth}}},\underbrace{\frac{\sigma_1(\theta)+1}{2},\frac{(1-\sigma_1(\theta))^2}{12}}_{\substack{\text{mean and variance of}\\ \text{senior wives' skills}}},\underbrace{\frac{\sigma_0(\theta)}{2},\frac{\sigma_0(\theta)^2}{12}}_{\substack{\text{mean and variance of}\\ \text{junior wives' skills}}}\Big]$$

It is clear from the closed form expressions of model moments that they are tightly related to the model thresholds. Importantly, the model-generated moments are directly observed in the data, which implies that the data can be used to discipline the model thresholds (and, therefore, parameters through equation (4)).

All in all, for any given vector of parameters, one can solve for σ_0 and all other thresholds and for model moments mom^{model} that are observed in the data. It is worth remarking that uniqueness of the solution of σ_0 as a function of parameters is key for this identification argument. Recall that in appendix B I prove uniqueness and derive the relationship between σ_0 and the other thresholds (see equations (28) to (30)).

4.4.2 The method of moments

To estimate the vector of parameters $\theta \in \Theta$ I use the method of moments. The observation of equilibrium matches, household roles, and individual traits allows for the construction of the observed counterpart of vector of moments mom^{model}, denoted mom^{data}. I search for the vector of parameters $\theta = \{\alpha, \beta, K\}$ that minimizes the distance between the moments implied by the model and the same moments computed from the data:

$$\hat{\theta} = \underset{\theta\in\Theta}{\operatorname{argmin}} \quad [mom^{data} - mom^{model}(\theta)]'\mathcal{I}[mom^{data} - mom^{model}(\theta)]' \tag{5}$$

where $\mathcal{I}$ is a positive semi definite weighting matrix.

4.4.3 Estimation of thresholds using ranking of traits

In this section I demonstrate that with data from equilibrium performance *ranking* (instead of the original traits) the method of moments does an excellent job in estimating the thresholds of the original model. In effect, when the data generating process is

$$h(y,s',s) = y(1+K)^{\beta}s^{\alpha}$$
$$s \sim F[\underline{S},\overline{S}] \text{ and } y \sim G[\underline{Y},\overline{Y}]$$

the method of moments estimates σ_0, and when the data generating process is

$$h(y,s',s) = h(p(y),p(s'),p(s)) = p(y)(1+\tilde{K})^{\tilde{\beta}}p(s)^{\tilde{\alpha}}$$
$$p(s) \sim U[0,1] \text{ and } p(y) \sim U[0,1]$$

the method of moments estimates $\tilde{\sigma_0} = F(\sigma_0)$. That is, although the particular value of σ_0 depends on the particular specification of the production function and traits distributions, the threshold woman that is estimated when we observe a ranking of individuals and their occupation within the household is the same percentile as the threshold woman that would be estimated if we observed the true distribution of traits.[10]

I show this in appendix D where I perform a Monte Carlo exercise where I (i) specify three "true" models (household production function and distribution of traits), (ii) simulate data from these "true" models, (iii) construct the percentiles of marital traits distributions, and (iv) estimate the models using both data generated by the "true" models and by the percentiles of traits, by solving problem (5). When I compare both sets of estimates I conclude that when $\hat{\sigma_0}$ is the estimate from the original model, the estimate from the model specified with ranking of traits is extremely close to $F(\hat{\sigma_0})$.

[10]Note that if, for example, the original data generating process is characterized by distributions F and G being uniform, σ_0 is the unique solution to

$$\overline{Y}(1-\frac{\sigma_0}{\overline{S}})(\overline{S}-\sigma_0)^{\alpha}((K+1)^{\beta}-1) - \sigma_0^{\alpha+1}\frac{\overline{Y}}{\overline{S}} = 0$$

and the vector of model moments is

$$mom^{model}(\theta) = \Big[\underbrace{\frac{\overline{Y}-\gamma_1(\theta)}{\overline{Y}-\gamma_0(\theta)}}_{\substack{\text{fraction}\\\text{polygamous}}}, \underbrace{\frac{\gamma_1(\theta)+\gamma_0(\theta)}{2}, \frac{(\gamma_1(\theta)-\gamma_0(\theta))^2}{12}}_{\substack{\text{mean and variance of}\\\text{monogamous husbands' wealth}}}, \underbrace{\frac{\sigma_1(\theta)+\sigma_0(\theta)}{2}, \frac{(\sigma_1(\theta)-\sigma_0(\theta))^2}{12}}_{\substack{\text{mean and variance of}\\\text{monogamous wives' skills}}},$$
$$\underbrace{\frac{\gamma_1(\theta)+\overline{Y}}{2}, \frac{(\overline{Y}-\gamma_1(\theta))^2}{12}}_{\substack{\text{mean and variance of}\\\text{polygamous husbands' wealth}}}, \underbrace{\frac{\sigma_1(\theta)+\overline{S}}{2}, \frac{(\overline{S}-\sigma_1(\theta))^2}{12}}_{\substack{\text{mean and variance of}\\\text{senior wives' skills}}}, \underbrace{\frac{\sigma_0(\theta)}{2}, \frac{\sigma_0(\theta)^2}{12}}_{\substack{\text{mean and variance of}\\\text{junior wives' skills}}}\Big]$$

.

5 Evidence supporting the model

Does the equilibrium with positive assortative matching and co-wives' skill inequality resemble the distribution of spousal traits observed in marriages in the data? The main objective of this section is to explore whether there is empirical support for the threshold equilibrium described in proposition 1. First, I test one of the predictions of the model, namely, that matching is positive assortative between males and females within household role. Second, I estimate the model parameters by the method of moments. I conclude that even when the empirical specification of the model has no individual random components (usually required to fit the noise in the data), the equilibrium described in proposition 1 and illustrated in figure 1 reproduces the data patterns very well.

5.1 Assortativeness

Table 4 shows the degree of association between female and male traits ranking in both marriage markets, north and south, by type of spouse.

Table 4: Evidence on positive assortative matching

	Dependent variable: female skill					
	North			South		
	(1)	(2)	(3)	(4)	(5)	(6)
	Seniors	Juniors	Mono	Seniors	Juniors	Mono
Male wealth	0.138	0.053	0.074	0.261	0.069	0.276
	(0.034)	(0.026)	(0.034)	(0.084)	(0.036)	(0.033)
Observations	376	376	578	59	59	512

Notes: *Female skill* and *Male wealth* are the rankings of women's and men's attractiveness index, respectively, both constructed as described in section 4.2. *Mono* stands for *monogamous*. Robust standard errors in parentheses.

As implied by the model, the data suggests that the marriage market equilibrium exhibits positive assortative matching between males and females within household role. For example, in polygamous marriages, a higher wealth rank of the husband is associated with higher rank in female traits for both, the senior and the junior wives. The correlation is highly significant. The same conclusion can be drawn from monogamous marriages.

5.2 Estimation results and fit of the model

Even when the empirical model is so tightly specified, it fits the data very well. This is particularly true in markets with higher polygamy rates. Table 5 shows the results of the estimation of the parameters of model (3). The estimation of Cobb-Douglas parameters shown at the bottom of the table result from fitting targeted moments of the observed ranking of traits for males and females, conditional on type of marriage and female role.

Table 5: Model fitting to the data, rural Nigeria, 2010

Targeted moments	North		South	
	Data	Model	Data	Model
$N(poly)/N(married)$	0.46	0.55	0.14	0.35
$mean(y^{mono})$	0.49	0.50	0.51	0.50
$var(y^{mono})$	0.07	0.01	0.10	0.02
$mean(s^{mono})$	0.64	0.5	0.56	0.50
$var(s^{mono})$	0.05	0.01	0.07	0.02
$mean(y^{poly})$	0.50	0.82	0.42	0.87
$var(y^{poly})$	0.07	0.01	0.11	0.01
$mean(s^{senior})$	0.66	0.82	0.45	0.87
$var(s^{senior})$	0.04	0.01	0.04	0.01
$mean(s^{junior})$	0.17	0.18	0.09	0.13
$var(s^{junior})$	0.02	0.01	0.01	0.01
Estimated Parameters				
σ_0	0.36		0.26	
α	1.74		0.19	
β	0.45		0.58	
k	0.48		0.56	

Notes: *N(poly)* and *N(married)* refer to the number of polygamous and married males, respectively. *Data* refers to empirical moments. *Model* corresponds to moments produced by the model when parameters are set at the estimated levels. s and y refer to female skills and male wealth, respectively. *Mono* and *poly* stand for *monogamous* and *polygamous*, respectively. σ_0 is the female skill level at which a woman is indifferent between being a junior wife and a monogamous wife. The production function being estimated is $h(y, s', s) = y(1 + K)^{\beta} s^{\alpha}$.

If we believe that the relevant dimensions of female skills are captured by the measurements summarized in table 3, the estimation of the model supports the equilibrium with high inequality among co-wives particularly in the North. For example, in the data for the Northern region, we observe that the mean trait of junior wives is percentile 17, while the mean trait of senior wives is percentile 66. We hence observe a great deal of co-wives' attractiveness inequality in the data. The model fits this pattern very well: the mean attractiveness of junior wives implied by the model estimation corresponds to the 18^{th} percentile (almost equal to the analogous observed moment), while the mean attractiveness of senior wives is 82^{th} percentile (approximating the mean attractiveness of senior wives in the data). The model overestimates the observed mean attractiveness of senior wives, probably due to the fact that *first wife* is one of the most predictive factors in women's skills and all monogamous wives are trivially first wives. Hence, in the data the traits of the most skilled wife in polygamous households might be biased down towards the skills of monogamous wives. The model also fits polygamy rates very well: the estimation implies that 55% of married males are polygamous, against 46% in the data.

Additionally, the estimation of threshold σ_0 allows me to study how the model matches the percent of women that sort into different household roles (which are non-targeted moments). The threshold woman that is indifferent between being a junior and a monogamous wife is estimated as the 36^{th} percentile of the female skill distribution. This implies that the model classifies 36% of married females into the junior role position. In the data, based on the principal component analysis, the percent of junior wives is 29%. Note that the same comparison follows for senior polygamous wives, implying that the model underestimates the percent of married females that sort into monogamous marriages.[11] In marriage markets with low polygamy rates, like the South, the model performs worse.

Additionally, table A4 in appendix E shows that estimation results are very similar when the sample does not exclude young monogamous households. This may be due to the fact that *current* age plays no role in the model making the sample selection on age innocuous for estimation.

6 The welfare effects of outlawing polygamy

With the aim of improving marital rights for polygamous women, many countries have moved to increasing the regulation of polygamous marriages to make them more equal to monogamous marriages. Some examples include the legalization of polygamy in the northern states in Nigeria between 2000 and 2001 and the recent legalization of polygamy in Kenya in 2014.

A natural question is who benefits from polygamy being allowed as opposed to being banned. In this section, I take the preferred estimates of the model (the estimates for the North shown

[11]The implied estimation of σ_1 (0.64) results in that the model sorts 28% of married women as monogamous wives, while the fraction of monogamous wives in the data is 43% (of all married females for which data is not missing).

in table 5) and discuss this question by simulating the indirect utilities of males and females that are implied by the model under the polygamy regime and a counterfactual scenario in which polygamy is outlawed.

Figures 2 and 3 show the equilibrium utilities for women and men, respectively, that result from the household production function parameters taking the values estimated with data from the North in table 5. In both graphs, the dashed lines represent the model that allows for polygamy. The solid line represents the counterfactual utility levels females and males would attain if polygamy was banned. This counterfactual is constructed by assuming that for all couples, the junior wife productivity is zero, that is, $\tilde{K} = 0$ in model (3). This effectively means that now all couples have the production function of monogamous couples. Because it is assumed that the sex ratio is equal to one, the distribution of household output among the couple with the lowest types will be undetermined. To make the counterfactual as comparable as possible to the polygamy case, I assume that all the surplus from this first marriage is appropriated by the wife (as in the polygamy case).

Figure 2: The effect of banning polygamy on women's welfare.

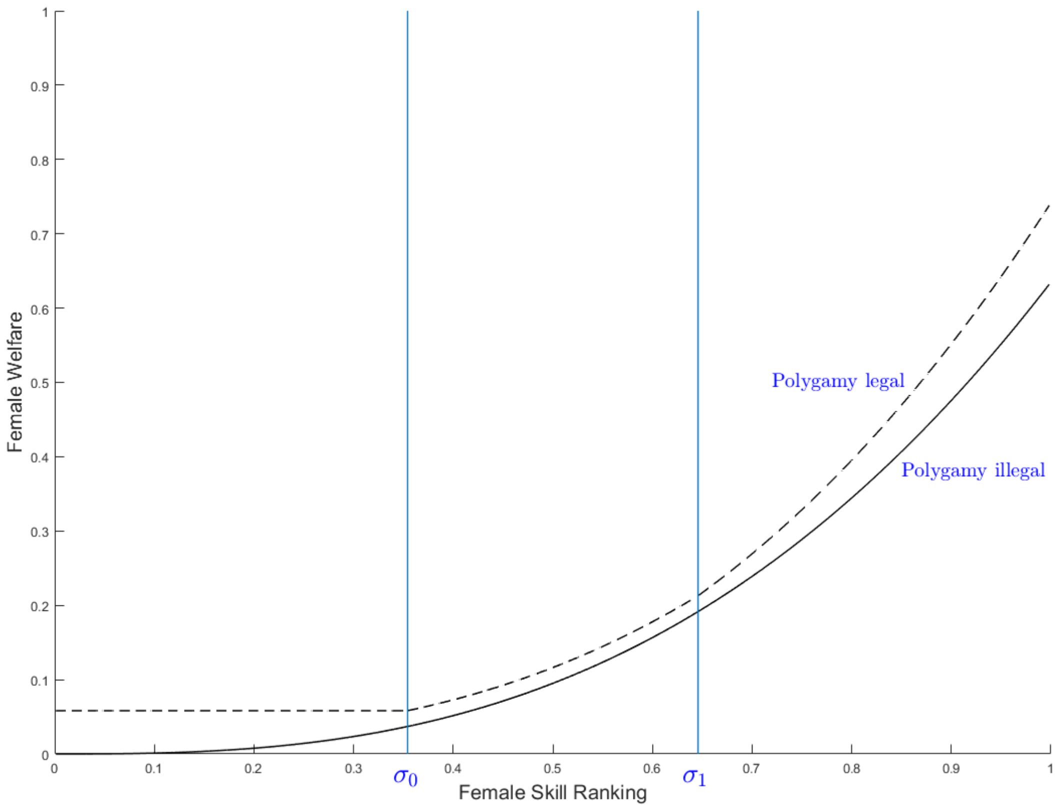
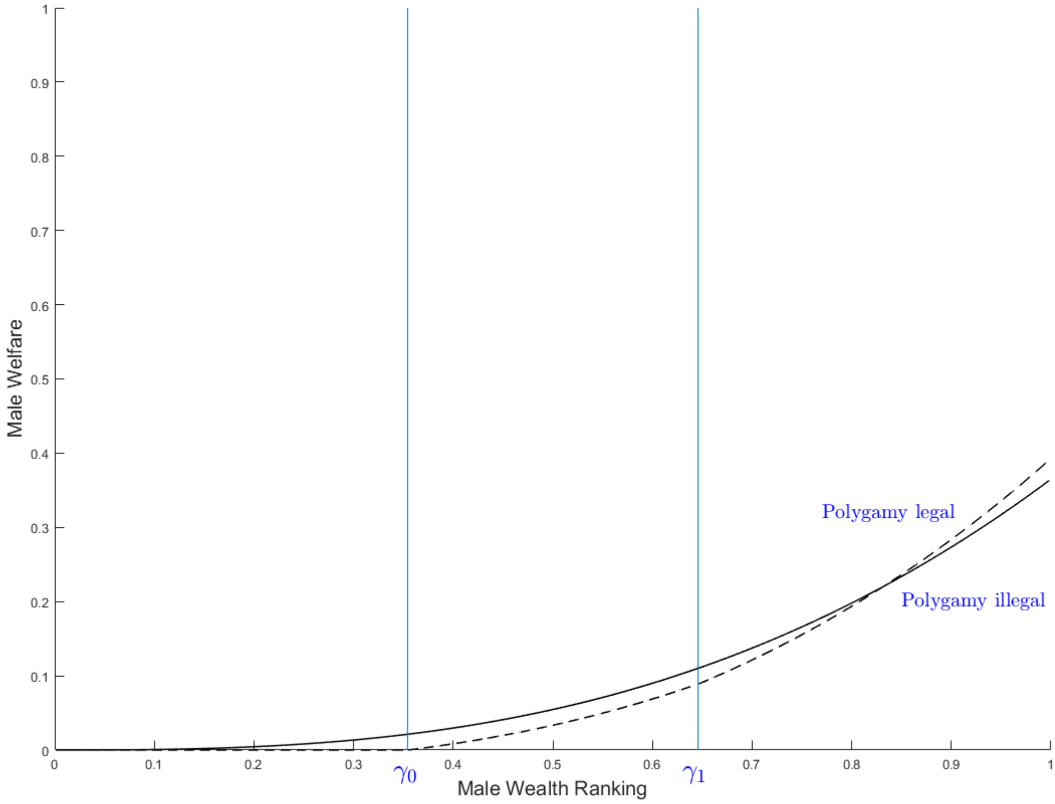


Figure 3: The effect of banning polygamy on men's welfare.



The novel implication of this model is that because of complementarity in female labor within the household, women's welfare goes down when polygamy is outlawed. This is because in the equilibrium under polygamy, low skilled women are married to the wealthiest men as junior wives. If polygamy is banned, those women are now only accepted by the least wealthy men, depressing low skill females' welfare. Interestingly, equilibrium conditions imply that this change in the marriage opportunities of low skilled women spills over reducing the equilibrium payoffs of all women.

The welfare implications for men are consistent with previous work. Only men at the top of the wealth distribution are better off with polygamy because acquiring a second wife yields the highest increments in household production for them. Men that are lower in the wealth distribution are harmed by polygamy because polygamy creates an "army" of single males that pushes equilibrium payoffs down with respect to the case where all males marry (even if the first man to get married earns nothing, by the extreme assumption I made that the first woman to get married appropriates all the surplus). When polygamy is banned, single men get married and so equilibrium conditions imply that all men's payoffs are pushed up.

6.1 Understanding welfare effects

There are two key features of the model that explain the negative effects of monogamy for women. First, the technological complementarity between co-wives implies that the presence of a co-wife in the household boosts the value of women in the marriage market. Intuitively, constraining the possibility to team up with a co-wife reduces household output more than proportionally and, hence, the share of output that is accrued by women. This feature of the model seems to accord to the empirical evidence presented in section 2 and be supported by the goodness of fit of the model to observed matching and sorting patterns in the data (table 5).

The second key feature is that the environment in this paper assumes that there are no outside options from marriage for women. For example, there is no possibility of supplying labor outside of the household or substituting the absence of a co-wife with hired labor. This feature of the model seems realistic too. The evidence presented in table 2 suggests that the economy is characterized by very weak labor markets implying that households rely to a great extent on family labor both for work on the plots and on the domestic chores.

In spite of the technological advantage of polygamy highlighted in this paper, many studies find that polygamous women have higher levels of unhappiness compared to monogamous women (see Dauphin (2013) and Rossi (2018) for a recent literature review), so a natural question is how to improve women's welfare in marriage markets that allow for polygamy. One way to answer this question is to understand what types of policies would lead to an equilibrium transition towards monogamy, so that when polygamy is a choice, households efficiently choose to have one wife. In this model, only technological changes lead to reductions in polygamy rates: decreasing the importance of human labor in the household technology or creating a path towards a fertility transition. The model would capture these changes through, for example, reductions in the productivity of the junior wife role in the household technology.

Interestingly on the contrary, in this environment with co-wives complementarities and poor outside options for women, education policies that improve women skills are not very effective in reducing polygamy rates. Intuitively, when education is only valuable in the marriage market improving the education of women will do nothing but redistribute welfare among co-wives within the household. However, the preference for marrying in a polygamous union will not decrease unless education becomes valuable outside of the household.

Finally, it is important to highlight that equilibrium transfers to the bride side in this model capture intra-household allocations that are appropriated by wives during marriage (a reduced form for utility from private consumption, leisure, or fertility). Polygamous societies, however, are also characterized by *bride price* payments (Boserup, 2007) from the groom to the *parents* of the bride at marriage (Ashraf, Bau, Nunn, and Voena, Forthcoming). One could think of an extension to a three-sided model adding the decisions of women's parents and where utility transfers to brides are shared between women and their fathers. The welfare discussion in this paper only concerns the utility transferred to wives within marriage, leaving the said extension for future research.

7 Conclusion

I propose a novel framework in the polygamy literature that captures the well-documented facts that polygamous families act as joint extended households where co-wives complement each other and organize in a hierarchy of senior-junior wives. Even though the emergence of polygamy seems to be related to the technological advantage of female labor, and even though polygamy affects women's well-being, the incentives of females to be part of polygamous families have been ignored so far. In this paper I fill this gap. To the best of my knowledge this is the first equilibrium model of marriage markets with legal polygamy that takes seriously the optimal sorting of females into polygamous versus monogamous families and into the senior or the junior wife roles.

The main result of the paper is that the equilibrium in the marriage market exhibits positive assortative matching between females and males, and positive sorting of females into household roles. The main and novel implication is that polygamous households show high levels of inequality among co-wives. Of course, the results follow from the assumptions on the household technology (in particular, supermodularity and hierarchical structure of female inputs). These assumptions are motivated by empirical regularities noted in the previous literature and lead to predictions that are supported by the data. The female inequality that arises endogenously from the model is consistent with previous empirical reduced form and descriptive studies that find that co-wives have differential welfare status within the household. In addition, the estimation of the model in the empirical section of this paper suggests that the distribution of households observed in the data resembles the distribution of equilibrium matches implied by the (estimated) model.

Two policy relevant model implications are worth discussing. First, the model sheds new light on the gender inequality issues associated with polygamy. Because the equilibrium is characterized by positive selection into household roles (the most attractive females become senior wives and the least attractive females become junior wives), the model predicts that marriage markets with polygamy have high levels of female inequality both within the household and across households. This aspect is new with respect to previous models of polygamy and adds to the model prediction of high male inequality (which has been highlighted in the previous literature).

The second remark is that the model provides a novel assessment of the welfare consequences of outlawing polygamy. Interestingly, a restriction on the number of wives reduces the welfare of all women (in addition to decreasing the welfare of the richest men in favor of the poorest men, as known from the previous literature). The negative effects of monogamy on women is explained by two features of the model, namely, that co-wives complement each other and that the economy lacks strong outside options from marriage, such as labor markets for women. These two features are consistent with the empirical evidence presented in this paper. Consequently, restricting the marriage prospects of women might not be the most successful instrument to improve welfare, at least in economies where polygamous families act as extended households that produce jointly.

All in all, this paper suggests that policies regulating polygamy with the purpose of fostering development and improving the welfare of women should be accompanied by improvements in the outside options of women, especially of the poorest ones. But in order to design successful policies it is necessary to have a precise knowledge of the economic forces that induce *both* women and men to form polygamous marriages. This paper advances in this knowledge.

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Appendix A Proof of Proposition 1

First, a brief note about notation.

Consider any coalition of a male, a female employed in the junior position, and a female employed in the senior position, $(y, s', s) \in [\underline{Y}, \overline{Y}] \times [\underline{S}, \overline{S}]^2$. The marital surplus that (y, s', s) would produce shall they be together is $h(y, s', s)$. Consider any matching μ (as defined in definition 1) under which (y, s', s) are together. Throughout the proof I will refer to the following objects in the following manner:

- $h_1\Big(y, \mu_2(y), \mu_1(y)\Big)$ is the partial derivative of the marital surplus with respect to male wealth evaluated at the marriage of y under matching μ.
- $h_2\Big(\mu_1(s'), s', \mu_3(s')\Big)$ is the partial derivative of the marital surplus with respect to the skill of the woman in the junior position evaluated at the marriage of s' under matching μ.
- $h_3\Big(\mu_1(s), \mu_2(s), s\Big)$ is the partial derivative of the marital surplus with respect to the skill of the woman in the senior position evaluated at the marriage of s under matching μ.

A.1 Full characterization in terms of σ_0

Thresholds and matching function

The assignment described in proposition 1 is positive assortative between males and females and by assumption female skills and male wealth are drawn from bounded, atomless, strictly increasing, and continuous distributions $F(s)$ and $G(y)$, respectively. These two features permit to express thresholds σ_1, γ_0, and γ_1 as closed form functions of σ_0. To see this, note that, for example, the mass of polygamous senior wives (females above σ_1) must be equal to the mass of junior wives (females below σ_0):

$$1 - F(\sigma_1)) = F(\sigma_0)$$

Hence,

$$\sigma 1 = F^{-1}[1 - F(\sigma_0)] \tag{6}$$

Similarly, according to proposition 1 the mass of married males (males above γ_0) must be equal to the mass of senior wives (females above σ_0):

$$1 - F(\sigma_0) = 1 - G(\gamma_0)$$

Hence,

$$\gamma_0 = G^{-1}[F(\sigma_0)] \tag{7}$$

Finally, the mass of polygamous males (males above γ_1) must be equal to the mass of polygamous seniors (females above γ_1):

$$1 - F(\sigma_1) = 1 - G(\gamma_1)$$
$$\gamma_1 = G^{-1}[F(\sigma_1)]$$

By (6)

$$\gamma_1 = G^{-1}[F(F^{-1}[1 - F(\sigma_0)])]$$

Hence,

$$\gamma_1 = G^{-1}[1 - F(\sigma_0)] \tag{8}$$

The same procedure can be used express any point in the males or females distribution as a function of their proposition 1 partner, that is, the conjectured matching function. To see this, note that according to the conjecture, if male y is married to female s employed in the senior wife position, the mass of males above y must be equal to the mass of females above s:

$$1 - F(s) = 1 - G(y)$$

Which provides a closed form expression of the spouse of y or of s:

$$y = G^{-1}[F(s)] = \mu_1(s) \quad \textit{and} \quad s = F^{-1}[G(y)] = \mu_3(y) \tag{9}$$

Finally, from the conjecture, the matching between male-senior wife couples and females in the junior wife position is random: for $s \geq \sigma_1$ and $y \geq \gamma_1$ such that $\mu_3(y) = s$:

$$\mu_2(s) = \mu_2(y) = s' \in [\underline{S}, \sigma_0] : \mu_3(s') = s \quad \textit{and} \quad \mu_1(s') = y \tag{10}$$

Indirect utilities

If the conjecture were to be stable, it must be the case that a monogamous husband solves

$$v(y) = \max_{s}(h(y, 0, s) - u(s))$$

and that the optimum is achieved at the spouses of y or s as described in (9). Then, from the first order conditions (evaluated at the optimum),

$$u_s(s) = h_3\Big(\mu_1(s), 0, s\Big) \tag{11}$$

By the envelope theorem,

$$v_y(y) = h_1\Big(y, 0, \mu_3(y)\Big) \tag{12}$$

Similarly a polygamous husband solves

$$v(y) = \max_{s,s':s>s'}(h(y, s', s) - u(s) - u(s'))$$

From first order conditions,

$$u_{s'}(s') = h_2\Big(\mu_1(s'), s', \mu_3(s')\Big) = 0 \tag{13}$$

$$u_s(s) = h_3\Big(\mu_1(s), \mu_2(s), s\Big) \tag{14}$$

And by the envelope theorem,

$$v_y(y) = h_1\Big(y, \mu_2(y), \mu_3(y)\Big) \tag{15}$$

Conditions 11 to 15 characterize the slopes of the payoff functions that must obtain in a stable equilibrium. Integrating these conditions over the assignment prescribed by matching μ in the corresponding segments (of monogamy and polygamy) characterizes the payoffs that each agent must be getting in a stable assignment as a function of each agents own type, unknown σ_0, and exogenous parameters. I obtain these expressions next.

Monogamy segment:

For all $s \in [\sigma_0, \sigma_1)$,

$$u(s) = u(\sigma_0) + \int_{\sigma_0}^{s} h_3\Big(\mu_1(t), 0, t\Big)\,\mathrm{d}t \tag{16}$$

For all $y \in [\gamma_0, \gamma_1)$,

$$v(y) = v(\gamma_0) + \int_{\gamma_0}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t \tag{17}$$

Polygamy segment:

For all $s \in [\sigma_1, \overline{S}]$,

$$u(s) = u(\sigma_0) + \int_{\sigma_0}^{s} h_2\Big(\mu_1(t), \mu_2(t), t\Big)\,\mathrm{d}t \tag{18}$$

For all $s \in [\underline{S}, \sigma_0)$,

$$u(s) = \int 0\,\mathrm{d}t = C \tag{19}$$

For all $y \in [\gamma_1, \overline{Y}]$,

$$v(y) = v(\gamma_0) + \int_{\gamma_0}^{y} h_1\Big(t, \mu_2(t), \mu_3(t)\Big)\,\mathrm{d}t \tag{20}$$

Note that in equilibrium all women sorted into the junior wife position must gain the same utility (C in expression (19)) since any skill level in that position would contribute the same to household output, so that couples are indifferent between any woman being employed as the junior wife.

A.2 Existence of solution for σ_0

Expressions (16) to (20) define equilibrium payoffs in terms each agent's type, exogenous parameters, and three unknowns: σ_0 (the type of the first woman that is a senior wife), C (they payoff that all junior wives receive in equilibrium), and $v(\gamma_0)$ (the payoff of the first male that gets married). To solve for these unknowns and arrive at the full characterization of the conjecture, I exploit another implication of stability and continuity of traits, namely, that individuals in the thresholds of two different segments in the assignment must be indifferent between being in one segment or in the other. I outline the steps below:

1. By assumption 1 the value of being single is zero:

$$for\ all\ y \in [\underline{Y}, \gamma_0) : v(y) = 0 \tag{21}$$

2. In a stable assignment, male γ_0 (the first male to get married) must be indifferent between being single and being married monogamously.[12] Hence,

$$v(\gamma_0) = 0 \tag{22}$$

3. Indifference conditions (21) and (22) imply that the wife of γ_0, monogamous wife σ_0, appropriates all the marital surplus:

$$u(\sigma_0) = h(\gamma_0, 0, \sigma_0) \tag{23}$$

4. In a stable assignment, female σ_0 must be indifferent between being a junior and being a senior wife.[13] Hence,

$$C = h(\gamma_0, 0, \sigma_0) \tag{24}$$

5. Similarly, in a stable assignment, female σ_1 and her husband, γ_1, must be indifferent between being monogamous and polygamous.[14] Hence,

$$C = h(\gamma_1, s', \sigma_1) - h(\gamma_1, 0, \sigma_1) = h(\gamma_1, K, \sigma_1) - h(\gamma_1, 0, \sigma_1) \tag{25}$$

[12] If γ_0 was strictly better off than singles, a single could improve his situation by agreeing to be paid less than γ_0 but more than zero and outbid γ_0's wife, creating a blocking pair. Conversely, if γ_0 was strictly worse off than a single, he would prefer to divorce his wife, and hence he would form a blocking coalition.

[13] If σ_0 was strictly better off than juniors, a junior could improve her situation by agreeing to be paid less than σ_0 but more than her payoff under the assignment and outbid σ_0's husband, creating a blocking pair. Conversely, if σ_0 was strictly worse off than a junior, she would prefer to join a marriage where she is the junior, creating a blocking coalition.

[14] If they were strictly better off than the next monogamous couple, this next monogamous couple could outbid (γ_0, σ_0)'s junior by offering her a higher payoff and still be able to improve their situation, which constitutes a blocking coalition. Conversely, if (γ_0, σ_0) were strictly worse off than the next monogamous couple, they would prefer to divorce their junior wife, creating a blocking pair.

where K is a parameter representing the constant contribution of junior wives to household output.

6. Equating indifference conditions (24) and (25),

$$h(\gamma_1, K, \sigma_1) - h(\gamma_1, 0, \sigma_1) = h(\gamma_0, 0, \sigma_0)$$

7. Finally, replacing γ_0, γ_1, and σ_1 by their expressions (6) to (8) in terms of σ_0, we arrive at an equation to solve for σ_0 as function of the parameters of distributions $F(s)$ and $G(y)$ and of production function $h(y, s', s)$:

$$\begin{aligned} E(\sigma_0) \quad = \quad & h(G^{-1}[1 - F(\sigma_0)], K, F^{-1}[1 - F(\sigma_0)]) \\ & -h(G^{-1}[1 - F(\sigma_0)], 0, F^{-1}[1 - F(\sigma_0)]) \\ & -h(G^{-1}[F(\sigma_0)], 0, \sigma_0) = 0 \end{aligned} \tag{26}$$

This is a non linear equation in σ_0. Note that a solution for the model with the shape described in proposition 1 imposes restriction on the *admissible* values that σ_0 can take. In particular, σ_0 (the first woman to marry as a senior) must satisfy:

$$\underline{S} \leq \sigma_0 \leq \sigma_1 \leq \overline{S}$$

This implies that $\sigma_0 \in [\underline{S}, F^{-1}(0.5)]$, where $F^{-1}(0.5)$ is the median female skill under F. When $\sigma_0 = \underline{S}$, all women marry as seniors and there is no polygamy in the market. When $\sigma_0 = \sigma_1$, which is satisfied for $\sigma_0 = F^{-1}(0.5)$, the first woman to marry as senior is the median woman, so that all seniors are polygamous.

Lemma 1 next establishes that for certain values of the parameters, a unique interior solution for σ_0 exists.

Lemma 1 *There exist values of K such that a unique interior solution for σ_0 exists for all $0 < \overline{S}$.*

Proof 1 *First, note that within the admissible set of solutions for σ_0 the derivative of $E(\sigma_0)$ with respect to σ_0, E_{σ_0}, is strictly negative:*

$$
\begin{aligned}
E_{\sigma_0} = {} & h_1(G^{-1}[1-F(\sigma_0)], K, F^{-1}[1-F(\sigma_0)]) \times \frac{d\gamma_1}{d\sigma_0} + \\
& + h_3(h(\overline{Y}, K, \overline{S}) - h(\overline{Y}, 0, \overline{S}) - h(\underline{Y}, 0, \underline{S})], K, F^{-1}[1-F(\sigma_0)]) \times \frac{d\sigma_1}{d\sigma_0} - \\
& - h_1(G^{-1}[1-F(\sigma_0)], 0, F^{-1}[1-F(\sigma_0)]) \times \frac{d\gamma_1}{d\sigma_0} - \\
& - h_3(G^{-1}[1-F(\sigma_0)], 0, F^{-1}[1-F(\sigma_0)]) \times \frac{d\sigma_1}{d\sigma_0} - \\
& - h_1(G^{-1}[F(\sigma_0)], 0, \sigma_0) \times \frac{d\gamma_0}{d\sigma_0} - \\
& - h_3(G^{-1}[F(\sigma_0)], 0, \sigma_0)
\end{aligned}
$$

Rearranging terms:

$$
\begin{aligned}
E_{\sigma_0} = {} & \Big(h_1(G^{-1}[1-F(\sigma_0)], K, F^{-1}[1-F(\sigma_0)]) - \\
& - h_1(G^{-1}[1-F(\sigma_0)], 0, F^{-1}[1-F(\sigma_0)]) \Big) \times \frac{d\gamma_1}{d\sigma_0} + \\
& + \Big(h_3(G^{-1}[1-F(\sigma_0)], K, F^{-1}[1-F(\sigma_0)]) - \\
& - h_3(G^{-1}[1-F(\sigma_0)], 0, F^{-1}[1-F(\sigma_0)]) \Big) \times \frac{d\sigma_1}{d\sigma_0} - \\
& - h_1(G^{-1}[F(\sigma_0)], 0, \sigma_0) \times \frac{d\gamma_0}{d\sigma_0} - h_3(G^{-1}[F(\sigma_0)], 0, \sigma_0)
\end{aligned} \tag{27}
$$

The first and second terms of (27) are strictly negative: the first factors in both lines are strictly positive by supermodularity of $h(y, s', s)$, while the second factors in both terms are strictly negative by the strict monotonicity and continuity of distributions $F(s)$ and $G(y)$. Moreover, the last term is strictly negative by monotonicity of $h(y, s', s)$ and properties of distributions $F(s)$ and $G(y)$. Hence, $E_{\sigma_0} < 0$.

Second, note that $E(\sigma_0)$ is a continuous function of σ_0.

Third, note that there exist a value of K, K^L, such that the function evaluated at the lower bound of the admissible values of σ_0 is strictly positive for all $K > K^L$:

$$
\begin{aligned}
E(\sigma_0 = \underline{S}) &= h(\gamma_1(\underline{S}), K, \sigma_1(\underline{S})) - h(\gamma_1(\underline{S}), 0, \sigma_1(\underline{S})) - h(\gamma_0(\underline{S}), 0, \underline{S}) \\
&= h(\overline{Y}, K, \overline{S}) - h(\overline{Y}, 0, \overline{S}) - h(\underline{Y}, 0, \underline{S}) \\
&> 0 \quad \forall K > K^L, \\
&\textit{with} \quad K^L : h(\overline{Y}, K^L, \overline{S}) - h(\overline{Y}, 0, \overline{S}) = h(\underline{Y}, 0, \underline{S})
\end{aligned}
$$

where the second equality follows from expressions 6 to 8 when $\sigma_0 = \underline{S}$,[15] *and the last inequality hols because of the existence of threshold* K^L *given the continuity and monotonicity of function* $h(y, s', s)$.

Fourth, note that there exist a value of K, K^H, *such that the function evaluated at the upper bound of the admissible values of* σ_0 *is strictly negative for all* $K < K^H$:

$$
\begin{aligned}
E(\sigma_0 = F^{-1}(0.5)) &= h(\gamma_1(F^{-1}(0.5)), K, \sigma_1(F^{-1}(0.5)) \\
&- h(\gamma_1(F^{-1}(0.5)), 0, \sigma_1(\underline{S})) - h(\gamma_0(F^{-1}(0.5)), 0, F^{-1}(0.5)) \\
&= h(G^{-1}(0.5), K, F^{-1}(0.5)) - \\
&- h(G^{-1}(0.5), 0, F^{-1}(0.5)) - h(G^{-1}(0.5), 0, F^{-1}(0.5)) \\
&< 0 \quad \forall K < K^H,
\end{aligned}
$$

$$
\textit{with} \quad K^H : h(G^{-1}(0.5), K^H, F^{-1}(0.5)) = 2 \times h(G^{-1}(0.5), 0, F^{-1}(0.5))
$$

Finally, note that by assumption 1, $K^L < K^H$:

By monotonicity of $h(y, s', s)$,

$$
h(\underline{Y}, 0, \underline{S}) < h(G^{-1}(0.5), 0, F^{-1}(0.5))
$$

From the expressions of K^L *and* K^H, *it then follows that*

[15] $\gamma_1(\underline{S}) = G^{-1}[1 - F(\underline{S})] = G^{-1}[1] = \overline{Y}$, $\sigma 1(\underline{S}) = F^{-1}[1 - F(\underline{S})] = F^{-1}[1] = \overline{S}$, and $\gamma_0(\underline{S}) = G^{-1}[F(\underline{S})] = G^{-1}[0] = \underline{Y}$.

$$h(\overline{Y}, K^L, \overline{S}) - h(\overline{Y}, 0, \overline{S}) < h(G^{-1}(0.5), K^H, F^{-1}(0.5)) - h(G^{-1}(0.5), 0, F^{-1}(0.5))$$

which can be expressed, by the Fundamental Theorem of Calculus, as

$$\int_0^{K^L} h_2\Big(\overline{Y}, r, \overline{S}\Big)\mathrm{d}r < \int_0^{K^H} h_2\Big(G^{-1}(0.5), r, F^{-1}(0.5)\Big)\mathrm{d}r$$

But by supermodularity of $h(y, s', s)$, the integrand in the left hand side is greater than the integrand in the right hand side, $h_2\Big(\overline{Y}, r, \overline{S}\Big)\mathrm{d}r > h_2\Big(G^{-1}(0.5), r, F^{-1}(0.5)\Big)\mathrm{d}r$. So, it must be the case that the integration area is greater on the right hand side. Hence, $K^L < K^H$.

In conclusion, since $E(\sigma_0)$ is strictly decreasing and continuous, σ_0 lies in compact set $[\underline{S}, F^{-1}(0.5)]$ and for values of parameters such that $K^L < K < K^H$, it is the case that $E(\underline{S}) > 0$ and $E(F^{-1}(0.5)) < 0$, there is a unique value of σ_0 for which $E(\sigma_0) = 0$.

$\square$

A.3 Stability

Taking the characterization of the assignment as given, I now show that there is no coalition that blocks this assignment. That is, that the assignment satisfies global stability conditions.

Lemma 2 *Take the characterization of assignment μ in this marriage market given by*

- *The solution for σ_0 from equation (26)*
- *Thresholds (6) to (8)*
- *Matching functions (9) and (10)*
- *Payoff functions (16) to (20)*

If the marital surplus $h(y, s', s)$ satisfies assumption 1 with $h_2(y, s', s) = 0$, then:

- **Part 1.** *Taking the role of females in assignment μ as given, there is no coalition of three that blocks this assignment.*
- **Part 2.** *Taking the role of females in assignment μ as given, there is no coalition of two that blocks this assignment.*
- **Part 3.** *Taking the role of females in assignment μ as given, there is no individual that blocks this assignment.*

- **Part 4.** *No female wants to change their role with respect to her role in assignment μ.*

Proof 2 (Part 1) *Note that it suffices to show that no* essential *coalition of three blocks the assignment, given that, by definition, inessential coalitions cannot do better than any essential coalitions.*

Essential coalitions of three are any group of a male, a junior wife, and a senior wife.

1. ***No polygamous male and his senior wife divorce their junior and get another junior***

Intuitively, any junior is equally productive so a couple of a male and his senior is indifferent between any woman that is willing to be employed as a junior, so they have no reason to undo the outcome of the random matching between them and the junior. Formally:

$\forall y \geq \gamma_1$, $\forall s \geq \sigma_1 : \mu_1(s) = y$, $\forall \hat{s} < \sigma_0 : \mu_2(y) = \mu_2(s) = s' \neq \hat{s}$ *suppose coalition* $(y, \hat{s}, s)$ *blocks* μ*. Then, it must be the case that*

$$h(y, \hat{s}, s) > v^{\mu}(y) + u^{\mu}(\hat{s}) + u^{\mu}(s)$$
$$h(y, \hat{s}, s) > h(y, s', s) - u^{\mu}(s') + u^{\mu}(\hat{s})$$
$$0 > 0, \text{ a contradiction that proves the statement.}$$

2. ***No polygamous male and any junior wife marry down to a lower senior***

For all $y \geq \gamma_1$*, for all* $\sigma_1 \leq \hat{s} \leq s : s = \mu_3(y)$*, and for any* $s' < \sigma_0$*, suppose coalition* $(y, s', \hat{s})$ *blocks* μ*. Then, it must be the case that*

$$h(y, s', \hat{s}) > v^{\mu}(y) + u^{\mu}(s') + u^{\mu}(\hat{s})$$

By the efficiency of the assignment and given that any skill produces the same amount of labor,

$$h(y, s', \hat{s}) > h(y, s', s) - u^{\mu}(s) + u^{\mu}(\hat{s})$$

Replacing by the payoffs under μ *and rearranging terms,*

$$h(y, s', s) - h(y, s', \hat{s}) < u^{\mu}(\hat{s}) - u^{\mu}(\hat{s}) + \int_{\hat{s}}^{s} h_3\Big(\mu_1(t), \mu_2(t), t\Big), \mathrm{d}t$$

By the Fundamental Theorem of Calculus and by the fact that any skills produce the same labor output,

$$\int_{\hat{s}}^{s} h_3\Big(y, s', t\Big)\mathrm{d}t < \int_{\hat{s}}^{s} h_3\Big(\mu_1(t), \mu_2(t), t\Big)\mathrm{d}t = \int_{\hat{s}}^{s} h_3\Big(\mu_1(t), s', t\Big)\mathrm{d}t \Longrightarrow$$

$$\Longrightarrow \int_{\hat{s}}^{s} h_3\Big(y, s', t\Big)\mathrm{d}t - \int_{\hat{s}}^{s} h_3\Big(\mu_1(t), s', t\Big)\mathrm{d}t < 0 \Longrightarrow$$

$$\Longrightarrow \int_{\hat{s}}^{s} \int_{\mu_1(t)}^{y} h_{31}\Big(r, s', t\Big)\mathrm{d}r\mathrm{d}t < 0$$

a contradiction with supermodularity of $H(y, L, M)$ that proves that $(y, s', \hat{s})$ cannot block μ.

Note that this implies that no polygamous senior, s, can marry up to a husband $y > \mu_1(s)$.

3. ***No polygamous senior and any junior marry down to a lower male***

For all $s \geq \sigma_1$, for all $\gamma_1 \leq \hat{y} \leq y : y = \mu_1(s)$, and for any $s' < \sigma_0$, suppose coalition $(\hat{y}, s', s)$ blocks μ. Then, it must be the case that

$$h(\hat{y}, s', s) > v^{\mu}(\hat{y}) + u^{\mu}(s') + u^{\mu}(s)$$

As in point 3 above, the efficiency of the assignment, the fact that any skills produce the same labor output, and the payoffs that characterize μ imply that

$$h(y, s', s) - h(\hat{y}, s', s) < \int_{\hat{y}}^{y} h_1\Big(t, \mu_2(t), \mu_3(t)\Big)\mathrm{d}t = \int_{\hat{y}}^{y} h_1\Big(t, s', \mu_3(t)\Big)\mathrm{d}t$$

Applying the FTC twice,

$$\int_{\hat{y}}^{y} \int_{\mu_3(t)}^{s} h_{13}\Big(t, s', r\Big)\mathrm{d}r\mathrm{d}t < 0$$

a contradiction with supermodularity of $H(y, L, M)$ that proves that $(\hat{y}, s', s)$ cannot block μ.

Note that this implies that no polygamous male, y, can marry up to a senior $s > \mu_3(y)$.

4. ***No monogamous couple can get a junior wife***

For all $\sigma_0 \leq s < \sigma_1$, for all $\gamma_0 \leq y < \gamma_1 : y = \mu_1(s)$, and for any $s' < \sigma_0$, suppose coalition (y, s', s) blocks μ. Then, it must be the case that

$$h(y, s', s) > v^{\mu}(y) + u^{\mu}(s') + u^{\mu}(s)$$

By efficiency,

$$h(y, s', s) > h(y, 0, s) + u^{\mu}(s')$$

Rearranging terms and substituting $u^{\mu}(s')$ by its expression in (25),

$$h(y, s', s) - h(y, 0, s) > h(\gamma_1, s', \sigma_1) - h(\gamma_1, 0, \sigma_1) \Longrightarrow$$
$$\Longrightarrow h(y, s', s) + h(\gamma_1, 0, \sigma_1) > h(\gamma_1, s', \sigma_1) + h(y, 0, s)$$

a contradiction with supermodularity of $H(y, L, M)$ that proves that (y, s', s) cannot block μ.

The fact that no monogamous couple can afford a junior will imply that no coalition of a male and a senior that are not married under μ will be able to afford a junior. I show this in conditions 5 and 6 below.

5. ***No monogamous senior can get a junior by marrying down to a lower male***

For all $\sigma_0 \leq s < \sigma_1$, for all $\gamma_0 \leq \hat{y} < \gamma_1 : \hat{y} < \mu_1(s) = y$, and for any $s' < \sigma_0$, suppose coalition $(\hat{y}, s', s)$ blocks μ. Then, it must be the case that

$$h(\hat{y}, s', s) > v^{\mu}(\hat{y}) + u^{\mu}(s') + u^{\mu}(s)$$

Substituting $v^{\mu}(\hat{y})$ by its expression given by (17), and since by point 4 above (y, s', s) does not block μ,

$$h(\hat{y}, s', s) > v^{\mu}(y) - \int_{\hat{y}}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t + u^{\mu}(s') + u^{\mu}(s) \geq h(y, s', s) - \int_{\hat{y}}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t$$

These inequalities imply that

$$h(\hat{y}, s', s) > h(y, s', s) - \int_{\gamma_0}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t$$

Rearranging terms and using the FTC we arrive at a contradiction,

$$0 > \int_{\hat{y}}^{y} h_1\Big(t, s', s\Big)\,\mathrm{d}t - \int_{\hat{y}}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t > \int_{\hat{y}}^{y} h_1\Big(t, 0, s\Big)\,\mathrm{d}t - \int_{\hat{y}}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\,\mathrm{d}t =$$

$$= \int_{\hat{y}}^{y} \int_{\mu_3(t)}^{s} h_{13}\Big(t, 0, r\Big)\, \mathrm{d}r\mathrm{d}t > 0$$

where the last inequality obtains from supermodularity of $H(y, L, M)$.

6. ***No monogamous male can get a junior by marrying down to a lower senior***

For all $\gamma_0 \leq y < \gamma_1$, for all $\sigma_0 \leq \hat{s} < \sigma_1 : \hat{s} < \mu_3(y) = s$, and for any $s' < \sigma_0$, suppose coalition $(y, s', \hat{y})$ blocks μ. Then, it must be the case that

$$h(y, s', \hat{y}) > v^{\mu}(y) + u^{\mu}(s') + u^{\mu}(\hat{s})$$

By a similar argument as in point 5 above, I substitute $u^{\mu}(\hat{s})$ by its expression given by (16), and use the fact that (y, s', s) does not block μ to arrive at a contradiction to supermodularity of $H(y, L, M)$:

$$h(y, s', \hat{s}) > v^{\mu}(y) + u^{\mu}(s') + u^{\mu}(s) - \int_{\hat{s}}^{s} h_3\Big(\mu_1(t), 0, t\Big)\, \mathrm{d}t \geq$$
$$\geq h(y, s', s) - \int_{\hat{s}}^{s} h_3\Big(\mu_1(t), 0, t\Big)\, \mathrm{d}t$$

$$\Longrightarrow$$

$$0 > \int_{\hat{s}}^{s} h_3\Big(y, s', t\Big)\, \mathrm{d}t - \int_{\hat{s}}^{s} h_3\Big(\mu_1(t), 0, t\Big)\, \mathrm{d}t >$$
$$> \int_{\hat{s}}^{s} h_3\Big(y, 0, t\Big)\, \mathrm{d}t - \int_{\hat{s}}^{s} h_3\Big(\mu_1(t), 0, t\Big)\, \mathrm{d}t = \int_{\hat{s}}^{s} \int_{\mu_1(t)}^{y} h_{13}\Big(t, 0, r\Big)\, \mathrm{d}r\mathrm{d}t$$

a contradiction to supermodularity of $H(y, L, M)$.

$\square$

Proof 3 (Part 2) .

Note, as in part 1, that it suffices to show that no essential *coalition of two blocks the assignment.*

Essential coalitions of two are: any coalition of a male and a junior wife and any coalition of a male and a senior wife.

7. ***No coalition of a male and a junior blocks*** μ

For all $y \in [\underline{Y}, \bar{Y}]$ *and for any* $s' < \sigma_0$*, suppose coalition* $(y, s', 0)$ *blocks* μ*. Then, it must be the case that*

$$h(y, s', 0) = 0 > v^{\mu}(y) + u^{\mu}(s') > 0$$

a contradiction that proves the statement.

8. ***No polygamous male and her senior divorce their junior***

For all $\gamma_1 \leq y \leq \bar{Y}$*, for all* $\sigma_1 \leq s \leq \bar{S} : \mu_3(y) = s$*, and for any* $s' < \sigma_0$*, suppose coalition* $(y, 0, s)$ *blocks* μ*. Then, it must be the case that*

$$h(y, 0, s) > v^{\mu}(y) + u^{\mu}(s) = h(y, s', s) - u^{\mu}(s') = h(y, s', s) - h(\gamma_1, s', \sigma_1) + h(\gamma_1, 0, \sigma_1)$$

Hence,

$$\begin{aligned} &h(y, 0, s) > h(y, s', s) - h(\gamma_1, s', \sigma_1) + h(\gamma_1, 0, \sigma_1) \Longrightarrow \\ &\Longrightarrow h(y, 0, s) + h(\gamma_1, s', \sigma_1) > h(y, s', s) + h(\gamma_1, 0, \sigma_1) \end{aligned}$$

which contradicts supermodularity of $H(y, L, M)$*.*

9. ***No woman wants to marry down monogamously***

For all $\sigma_0 \leq s < \bar{S}$*, for all* $\gamma_0 \leq \hat{y} < \bar{Y} : \hat{y} < \mu_1(s) = y$*, and for any* $s' < \sigma_0$*, suppose coalition* $(\hat{y}, 0, s)$ *blocks* μ*. Then, it must be the case that*

$$h(\hat{y}, 0, s) > v^{\mu}(\hat{y}) + u^{\mu}(s)$$

By efficiency, rearranging terms, and using the FTC:

$$h(\hat{y}, 0, s) > v^{\mu}(\hat{y}) + h(y, 0, s) - v^{\mu}(y)$$

$$v^{\mu}(y) - v^{\mu}(\hat{y}) > \int_{\hat{y}}^{y} h_1\Big(t, 0, s\Big)\, \mathrm{d}t \Longrightarrow$$

$$\Longrightarrow \int_{\hat{y}}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\, \mathrm{d}t > \int_{\hat{y}}^{y} h_1\Big(t, 0, s\Big)\, \mathrm{d}t \Longrightarrow$$

$$0 > \int_{\hat{y}}^{y} \int_{\mu_s(t)}^{s} h_{13}\Big(t, 0, r\Big)\, \mathrm{d}r\mathrm{d}t$$

which contradicts supermodularity of $H(y, L, M)$.

10. ***No male wants to marry down monogamously***

For all $\gamma_0 \leq y < \bar{Y}$, for all $\sigma_0 \leq \hat{s} < \bar{S} : \hat{s} < \mu_3(y) = s$, and for any $s' < \sigma_0$, suppose coalition $(y, 0, \hat{s})$ blocks μ. Then, it must be the case that

$$h(y, 0, \hat{s}) > v^{\mu}(y) + u^{\mu}(\hat{s})$$

Similarly as in point 9 above, by efficiency, rearranging terms, and using the FTC:

$$0 > \int_{\hat{s}}^{s} \int_{\mu_3(t)}^{y} h_{13}\Big(r, 0, t\Big)\, \mathrm{d}r\mathrm{d}t$$

which contradicts supermodularity of $H(y, L, M)$.

□

Proof 4 (Part 3) .

11. ***No married individual prefers to be single***

The production function is such that females and males produce zero as singles. All married individuals obtain a positive indirect utility in the match. Hence, no single blocks μ.

□

Proof 5 (Part 4) .

Up to now I have taken the role of women conjectured in μ fixed to disproof blocking coalitions. In this section of the proof I show that no woman wants to change her role.

Consider, first, women $s' < \sigma_0$ being employed as juniors under μ.

First, I showed in statement 2 that no polygamous male prefers to marry down a senior below her senior wife in μ. Hence, females $s' < \sigma_0$ are not desired in the senior wife position by polygamous males.

Second, I showed in statement 10 that no male prefers to marry monogamously a wife below her senior wife in μ. Hence, females $s' < \sigma_0$ are not desired in the senior wife position by monogamous males that stay monogamous.

Third, I showed in statement 6 that no monogamous male can become polygamous by marrying down to a lower female in the senior wife position. By this argument, a monogamous male cannot become polygamous by marrying woman $s' < \sigma_0$ as senior wife and woman $\hat{s}' < s' < \sigma_0$ as junior wife. Hence, females $s' < \sigma_0$ are not desired in the senior wife position by monogamous males trying to become polygamous.

All in all, it remains to be shown that females $s' < \sigma_0$ will not marry as seniors to single males.

12. ***No junior prefers to marry down as a senior to a single male***

For all $y < \gamma_0$, for all $s' < \sigma_0$, suppose coalition $(y, 0, s')$ blocks μ. Then, it must be the case that

$$h(y, 0, s') > v^{\mu}(y) + u^{\mu}(s')$$

Replacing by the payoffs of s' and y under μ

$$h(y, 0, s') > h(\gamma_0, 0, \sigma_0)$$

which contradicts monotonicity of $H(y, L, M)$.

Hence, females $s' < \sigma_0$ are optimally placed in the junior position.

Consider, next, women $s \geq \sigma_0$ being employed as seniors under μ. By statement 7 no coalition of a male and a junior block μ. Hence, women $s \geq \sigma_0$ can only be desired as junior wives in polygamous households.

13. ***No coalition of a male and a senior prefers to replace their junior for a woman in a senior wife position under μ***

The proof follows crucially from the fact that all women are equally productive in the junior wife position irrespective of their skills. This makes substitution of juniors unprofitable.

First, by statement 1 polygamous couples are indifferent between the junior assigned to them under μ and any other woman in the junior position. Hence, females $s \geq \sigma_0$ are not desired in the junior position by any polygamous couple of a male and a senior married under μ.

Second, I showed in statement 2 that no polygamous male prefers to marry down a senior below her senior wife under μ and any junior. Hence, females $s \geq \sigma_0$ are not desired in the junior wife position by any coalition of a male y and a senior $\hat{s} < \mu_s(y)$.

Finally, I showed in statement 3 that no polygamous senior wife prefers to marry down a man below her husband under μ and any junior. Hence, females $s \geq \sigma_0$ are not desired in the junior position by any coalition of a senior wife s and a man $\hat{y} < \mu_1(s)$.

All in all, women $s \geq \sigma_0$ are not desired as juniors by any coalition. Hence, females $s \geq \sigma_0$ are optimally placed in the senior wife position.

$\square$

Proof 6 (Proposition 1) .

Whenever a solution for σ_0 exists, by lemma 2 there is no essential coalition that blocks the assignment. Hence, it is stable.

$\square$

Appendix B Full characterization of the Uniform Cobb-Douglas example

This appendix shows the full characterization of the equilibrium in this marriage market with polygamy and two female household roles for the case that females' and males' attractiveness measures are uniformly distributed ($s \sim U[0, \overline{S}]$ and $y \sim U[0, \overline{Y}]$) and household production function is Cobb-Douglas with constant productivity of female skills in the junior wife role ($h(y, s', s) = y(1 + K)^{\beta} s$). Note that since this model satisfies assumption 1 by proposition 1 a threshold-shaped stable outcome exists. In this appendix I characterize it in closed form.

B.1 Full characterization in terms of σ_0

Thresholds and matching function

The assignment described in proposition 2 is positive assortative between males and females and by assumption female skills and male wealth are drawn from the uniform distribution. These two features of the equilibrium permit to express thresholds σ_1, γ_0, and γ_1 as closed form functions of σ_0. To see this, note that, for example, the mass of polygamous senior wives (females above σ_1) must be equal to the mass of junior wives (females below σ_0):

$$1 - U(\sigma_1) = U(\sigma_0)$$

Hence,

$$\sigma_1 = \overline{S} - \sigma_0 \tag{28}$$

Similarly, according to proposition 2 the mass of married males (males above γ_0) must be equal to the mass of senior wives (females above σ_0):

$$1 - U(\sigma_0) = 1 - U(\gamma_0)$$

Hence,

$$\gamma_0 = \sigma_0 \frac{\overline{Y}}{\overline{S}} \tag{29}$$

Finally, the mass of polygamous males (males above γ_1) must be equal to the mass of polygamous seniors (females above γ_1):

$$1 - U(\sigma_1) = 1 - U(\gamma_1)$$

Hence, by (28)

$$\gamma_1 = \overline{Y}(1 - \frac{\sigma_0}{\overline{S}}) \tag{30}$$

The same procedure can be used express any point in the males or females distribution as a function of their proposition 2 partner, that is, the conjectured matching function. To see this, note that according to the conjecture, if male y is married to senior wife s, the mass of males above y must be equal to the mass of females above s:

$$1 - U(s) = 1 - U(y)$$

Which provides a closed form expression of the spouse of y or of s in the segments $y \in (\gamma_0, \overline{Y}]$ and $s \in (\sigma_0, \overline{S}]$:

$$y = \frac{\overline{Y}}{\overline{S}} s = \mu_1(s) \quad \Rightarrow \quad s = \frac{\overline{S}}{\overline{Y}} y = \mu_3(y) \tag{31}$$

Finally, from the conjecture, the matching between male-senior wife couples and co-wives in the junior position is random: for $s \in (\sigma_1, \overline{S}]$ and $y \in (\gamma_1, \overline{Y}]$ such that $\mu_3(y) = s$:

$$\mu_2(s) = \mu_2(y) = s' \in [0, \sigma_0) : \mu_3(s') = s \quad \textit{and} \quad \mu_1(s') = y \tag{32}$$

This means that assortatively matched couple of senior wife s and a polygamous male y is matched to one woman randomly chosen from the set $[0, \sigma_0)$.

Indirect utilities

For the Cobb-Douglas case, in a stable assignment a monogamous husband solves

$$v(y) = \max_s (ys^\alpha - u(s)) \tag{33}$$

and a polygamous husband solves

$$v(y) = \max_{s,s':s>s'} (y(1+K)^\beta s^\alpha - u(s) - u(s')) \tag{34}$$

Moreover, in a stable matching the optimum must be achieved at the spouses of y, s', or s as described by the matching function (31) and (32).

First Order Conditions 11 to 15 obtained for the general case and matchin function 31 characterize the slopes of the payoff functions for the Cobb-Douglas example. For monogamous families:

$$u_s(s) = \mu_1(s)\alpha s^{\alpha-1} = \alpha \frac{\overline{Y}}{\overline{S}} s^\alpha \tag{35}$$

$$v_y(y) = \mu_3(y)^\alpha = \left(\frac{\overline{S}}{\overline{Y}}\right)^\alpha y^\alpha \tag{36}$$

For polygamous families:

$$u_{s'}(s') = 0 \tag{37}$$

$$u_s(s) = \mu_1(s)(1+K)^{\beta}\alpha s^{\alpha-1} = (1+K)^{\beta}\alpha\frac{\overline{Y}}{\overline{S}}s^{\alpha} \tag{38}$$

$$v_y(y) = (1+K)^{\beta}\mu_3(y)^{\alpha} = (1+K)^{\beta}\Big(\frac{\overline{S}}{\overline{Y}}\Big)^{\alpha}y^{\alpha} \tag{39}$$

Integrating conditions (35) to (39) over the conjectured assignment in the corresponding segments (of monogamy and polygamy) and replacing thresholds γ_0, γ_1, and σ_1 by their expressions (28) to (30) characterize the payoffs that each agent must be getting in a stable equilibrium as a function of each agents own type, unknown threshold σ_0, and exogenous parameters $\overline{S}$, $\overline{Y}$, K, α, and β. I obtain these expressions next.

Monogamy segment:

For all $s \in [\sigma_0, \sigma_1)$,

$$u(s) = u(\sigma_0) + \int_{\sigma_0}^{s}\alpha\frac{\overline{Y}}{\overline{S}}t^{\alpha}\,\mathrm{d}t$$

$$u(s) = u(\sigma_0) + \frac{\alpha}{\alpha+1}\frac{\overline{Y}}{\overline{S}}(s^{\alpha+1} - \sigma_0^{\alpha+1}) \tag{40}$$

For all $y \in [\gamma_0, \gamma_1)$,

$$v(y) = v(\gamma_0) + \int_{\gamma_0}^{y}\Big(\frac{\overline{S}}{\overline{Y}}\Big)^{\alpha}t^{\alpha}\,\mathrm{d}t$$

$$v(y) = v(\gamma_0) + \Big(\frac{\overline{S}}{\overline{Y}}\Big)^{\alpha}\frac{y^{\alpha+1} - \gamma_0^{\alpha+1}}{\alpha+1} \tag{41}$$

Polygamy segment:

For all $s \in [\sigma_1, \overline{S}]$,

$$u(s) = u(\sigma_0) + \int_{\sigma_0}^{\sigma_1}\alpha\frac{\overline{Y}}{\overline{S}}t^{\alpha}\,\mathrm{d}t + \int_{\sigma_1}^{s}\alpha(1+K)^{\beta}\frac{\overline{Y}}{\overline{S}}t^{\alpha}\,\mathrm{d}t$$

$$u(s) = u(\sigma_0) + \frac{\alpha}{\alpha+1}((1-\sigma_0)^{\alpha+1} - \sigma_0^{\alpha+1})\frac{\overline{Y}}{\overline{\overline{S}}} + \frac{\alpha}{\alpha+1}(s^{\alpha+1} - (1-\sigma_0)^{\alpha+1})\frac{\overline{Y}}{\overline{\overline{S}}}(1+K)^{\beta} \tag{42}$$

For all $s' \in [0, \sigma_0)$,

$$u(s') = \int 0 \, \mathrm{d}t$$

$$u(s') = C \tag{43}$$

For all $y \in [\gamma_1, \overline{Y}$,

$$v(y) = v(\gamma_0) + \int_{\gamma_0}^{\gamma_1} \left(\frac{\overline{S}}{\overline{\overline{Y}}}\right)^{\alpha} t^{\alpha} \, \mathrm{d}t + \int_{\gamma_1}^{y} (K+1)^{\beta} \left(\frac{\overline{S}}{\overline{\overline{Y}}}\right)^{\alpha} t^{\alpha} \, \mathrm{d}t$$

$$v(y) = v(\gamma_0) + \frac{(1-\sigma_0)^{\alpha+1} - \sigma_0^{\alpha+1}}{\alpha+1}\left(\frac{\overline{S}}{\overline{\overline{Y}}}\right)^{\alpha} + \frac{y^{\alpha+1} - (1-\sigma_0)^{\alpha+1}}{\alpha+1}\left(\frac{\overline{S}}{\overline{\overline{Y}}}\right)^{\alpha}(1+K)^{\beta} \tag{44}$$

B.2 Existence of solution for σ_0

As in the general case presented in appendix A, indifference conditions of threshold individuals can be exploited to solve for σ_0.

1. By assumption 1 the value of being single is zero:

$$for\ all\ y \in [\underline{Y}, \gamma_0) : v(y) = 0 \tag{45}$$

2. In a stable assignment, male γ_0 (the first male to get married) must be indifferent between being single and being married monogamously:

$$v(\gamma_0) = 0 \tag{46}$$

3. Indifference conditions (45) and (46) imply that the wife of γ_0, monogamous wife σ_0, appropriates all the marital surplus:

$$u(\sigma_0) = h(\gamma_0, 0, \sigma_0) = \sigma_0^{\alpha+1}\frac{\overline{Y}}{\overline{S}} \tag{47}$$

where the last equality follows from replacing γ_0 by its expression 29

4. In a stable assignment, female σ_0 must be indifferent between being a junior and being a senior wife:

$$C = \sigma_0^{\alpha+1}\frac{\overline{Y}}{\overline{S}} \tag{48}$$

5. Finally, in a stable assignment the first couple of senior wife and husband to become polygamous must be indifferent between being monogamous and polygamous:

$$C = \gamma_1(K+1)^{\beta}\sigma_1^{\alpha} - \gamma_1\sigma_1^{\alpha} \tag{49}$$

Equating indifference conditions (48) and (49) and replacing σ_1 and γ_1 by their expressions 28 and 30, I arrive at a non linear equation to solve for σ_0:

$$E(\sigma_0) = \overline{Y}(1 - \frac{\sigma_0}{\overline{S}})(\overline{S} - \sigma_0)^{\alpha}((K+1)^{\beta} - 1) - \sigma_0^{\alpha+1}\frac{\overline{Y}}{\overline{S}} = 0 \tag{50}$$

Note that the solution for σ_0 does not depend on the length of the males' traits distribution, $\overline{Y}$. Lemma 3 below establishes existence of a unique solution for σ_0 in the admissible set $[0, \frac{\overline{S}}{2}]$.

Lemma 3 *For all $K \in (0, 2^{1/\beta} - 1)$, there exists a unique interior solution for $\sigma_0 \in [0, \frac{\overline{S}}{2}]$.*

Proof 7 *First, note that within the admissible set of solutions for σ_0 the derivative of $E(\sigma_0)$ with respect to σ_0, E_{σ_0}, is strictly negative for $K > 0$:*

$$E_{\sigma_0} = -[\frac{1}{\overline{S}}(\overline{S} - \sigma_0)^{\alpha} + \alpha(1 - \frac{\sigma_0}{\overline{S}})(\overline{S} - \sigma_0)^{\alpha-1}][(1+K)^{\beta} - 1] - \frac{1}{\overline{S}}(\alpha+1)\sigma^{\alpha} < 0$$

Note, also, that $E(\sigma_0)$ is a continuous function of σ_0.

From lemma 1, K^L solves

$$h(\overline{Y}, K^L, \overline{S}) - h(\overline{Y}, 0, \overline{S}) = h(\underline{Y}, 0, \underline{S})$$
$$\overline{Y}(1+K^L)^{\beta}\overline{S}^{\alpha} - \overline{Y}\overline{S}^{\alpha} = 0$$

so $K^L = 0$.

Also from lemma 1 K^H *solves*

$$h(G^{-1}(0.5), K^H, F^{-1}(0.5)) = 2 \times h(G^{-1}(0.5), 0, F^{-1}(0.5))$$
$$\frac{\overline{Y}}{2}(1+K^H)^\beta(\frac{\overline{S}}{2})^\alpha = 2\frac{\overline{Y}}{2}(\frac{\overline{S}}{2})^\alpha$$

so $K^H = 2^{1/\beta} - 1$.

Next, note that the function evaluated at the lower bound of the admissible values of σ_0 *is strictly positive for* $K > K^L = 0$*:*

$$E(0) = \overline{Y}\overline{S}^\alpha[(1+K)^\beta - 1] > 0 \textit{ for } K > 0,$$

and that the function evaluated at the upper bound of the admissible values of σ_0 *is strictly negative for* $K < K^H = 2^{1/\beta} - 1$*:*

$$E\Big(\frac{\overline{S}}{2}\Big) = \Big(\frac{\overline{S}}{2}\Big)^\alpha\Big(\frac{1}{2}[(K+1)^\beta - 2]\Big) < 0 \textit{ for } K < 2^{1/\beta} - 1.$$

Finally, note that $K^L < K^H$ *for all* $\beta > 0$. *This relationship between* K *and* β *restricts the particular parameters of the Cobb Douglas functions that can be taken to the data.*

All in all, since $E(\sigma_0)$ *is strictly decreasing and continuous in* σ_0, σ_0 *lies in compact set* $[0, \frac{\overline{S}}{2}]$, *and* $E(0) > 0$ *and* $E(\frac{\overline{S}}{2}) < 0$ *for* $0 < K < 2^{1/\beta} - 1$, *there is a unique value of* σ_0 *for which* $E(\sigma_0) = 0$.

$\square$

B.3 Proof of proposition 2

By lemma 3, there exists a threshold σ_0 that partitions the set of females into junior and senior wives as prescribed by proposition 2. Moreover, from indifference conditions 46, 47 and 48 the equilibrium payoff of single males is zero and the equilibrium payoff of all junior wives $s' \in \mathcal{L}$ is $u(s') = u(\sigma_0) = \sigma_0^{\alpha+1}\frac{\overline{Y}}{\overline{S}}$. Replacing these values in expressions 40 to 44 proves that equilibrium payoffs are as indicated in proposition 2. Finally, since the parametrization of the model in proposition 2 satisfies assumption 1, by proposition 1 the characterization satisfies stability conditions, so that there is no essential coalition that blocks the assignment.

$\square$

Appendix C The case with $h_2(y, s', s) > 0$

In this section I generalize the main result of the paper to the case of increasing productivity of female skills in the junior wife position. To make the exposition as comparable as possible to the case developed in the paper, consider the following household output, $\tilde{h}(y, s', s)$,

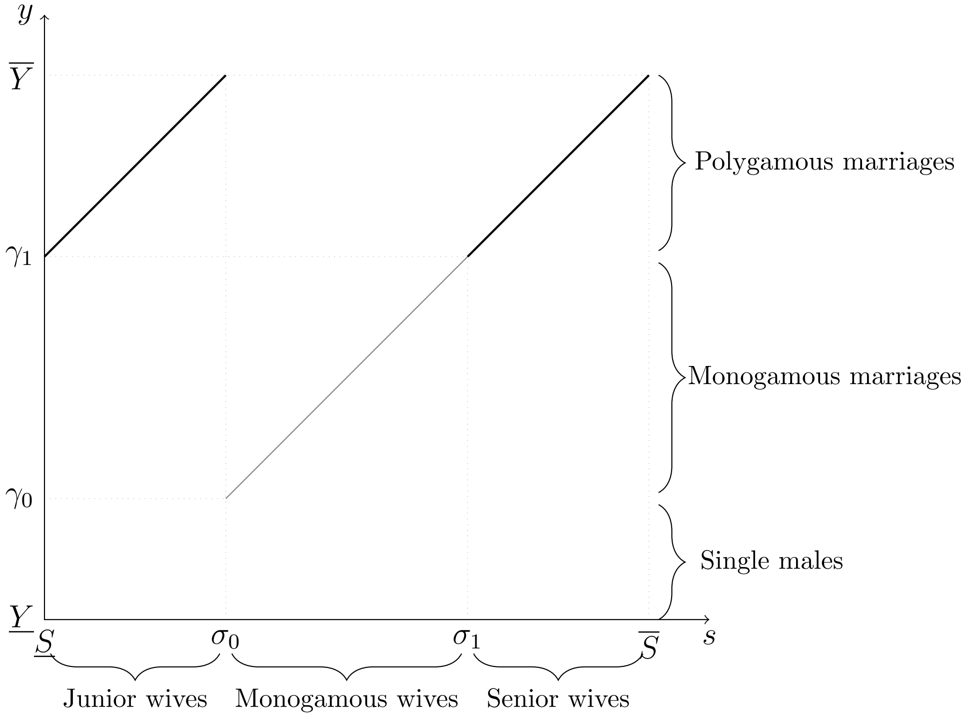
$$\tilde{h}(y, s', s) = h(y, K, s)(\epsilon s' + 1)$$

with $h(y, K, s)$ specified as in proposition 1. First, note that since $h(y, K, s)$ satisfies assumption 1, $\tilde{h}(y, s', s)$ satisfies parts 1, 2, 3, and 5 of assumption 1. In particular, notice that $\tilde{h}(y, s', s)$ is supermodular and that the marginal productivity of husband y and senior wife s is strictly increasing in the type of the junior wife s':

$$\tilde{h}_{i2}(y, s', s) = h_i(y, K, s)\epsilon > 0 \quad \textit{for all} \quad i = \{1, 3\} \tag{51}$$

Second, note that as $\epsilon \to 0$, $\tilde{h}_2(y, s', s) \to 0$ for all $(y, s', s) \in [\underline{S}, \overline{S}]^2 \times [\underline{Y}, \overline{Y}]$. Hence, by continuity there exists an ϵ small enough so that $\tilde{h}(y, s', s)$ satisfies part 4 of assumption 1. I consider such functions here. The stable outcome in this marriage market is depicted in figure 4. The shape of the equilibrium is as for the constant junior wife productivity case, except that because of (51), matching between husbands and junior wives is positive assortative instead of random.

Figure 4: Stable matching when $h_2(y, s', s) > 0$



Proposition 3 *The marriage market with populations* $s \sim F[\underline{S}, \overline{S}]$ *and* $y \sim G[\underline{Y}, \overline{Y}]$*, and marital output* $\tilde{h}(y, s', s) = h(y, K, s)(\epsilon s' + 1)$*, with* $h(y, K, s)$ *specified as in proposition 1 and for* ϵ *such that* $\tilde{h}(y, s', s)$ *satisfies assumption 1, has a stable outcome,* $(\mathcal{M}, \mathcal{L}, \mu, v^{\mu}, u^{\mu})$*, characterized by:*

1. *Thresholds* $\sigma_0 \in [\underline{S}, F^{-1}(0.5)]$, $\sigma 1 = F^{-1}[1 - F(\sigma_0)]$, $\gamma_0 = G^{-1}[F(\sigma_0)]$, *and* $\gamma_1 = G^{-1}[F(\sigma_1)]$, *all of which are unique.*

2. *The partition of female skills into junior and senior wife roles,* $\mathcal{L} = [0, \sigma_0)$ *and* $\mathcal{M} = [\sigma_0, \overline{S}]$

3. *Matching function*

$$\mu = (y, \mu_2(y), \mu_3(y)) = \begin{cases} (y, \emptyset_s, \emptyset_s), & \textit{for all } y \in [\underline{Y}, \gamma_0) \\ (y, \emptyset_s, F^{-1}[G(y)]), & \textit{for all } y \in [\gamma_0, \gamma_1) \\ (y, s', F^{-1}[G(y)]), & \textit{for all } y \in [\gamma_1, \overline{Y}], s' \in [\underline{S}, \sigma_0) \end{cases}$$

4. *Feasible payoff functions*

$$u^{\mu}(s) = \begin{cases} h(\gamma_0, 0, \sigma_0), & \textit{for all } s \in [0, \sigma_0) \\ h(\gamma_0, 0, \sigma_0) + \int_{\sigma_0}^{s} h_3\Big(\mu_1(t), 0, t\Big)\, \mathrm{d}t & \textit{for all } s \in [\sigma_0, \sigma_1) \\ h(\gamma_0, 0, \sigma_0) + \int_{\sigma_0}^{s} h_3\Big(\mu_1(t), \mu_2(t), t\Big)\, \mathrm{d}t & \textit{for all } s \in [\sigma_1, \overline{S}] \end{cases}$$

$$v^{\mu}(y) = \begin{cases} 0, & \textit{for all } y \in [0, \sigma_0) \\ \int_{\gamma_0}^{y} h_1\Big(t, 0, \mu_3(t)\Big)\, \mathrm{d}t, & \textit{for all } y \in [\gamma_0, \gamma_1) \\ \int_{\gamma_0}^{y} h_1\Big(t, \mu_2(t), \mu_3(t)\Big)\, \mathrm{d}t, & \textit{for all } y \in [\gamma_1, \overline{Y}] \end{cases}$$

Proof 8 (Proposition 3) *The proof of this case is very similar to the constant junior role developed in the body of the paper. The main difference is that it is not always the case that females sorted according to threshold σ_0 will not want to change their role. The proof follows closely the one developed in the constant junior role case. First, I obtain the characterization of the assignment in terms of σ_0. Then, I argue that for some ϵ small enough, a solution to σ_0 exists. Finally, I prove that this characterization satisfies global stability conditions by disproving potential blocking coalitions and resorting of females into different household roles. Note that part 4 of assumption 1 prevents the latter.*[16]

□

[16] The complete and detailed proof is available upon request.

Appendix D Monte Carlo excersise to demonstrate estimation of thresholds using ranking of traits

In this section I show that the method of moments does an excellent job in estimating model thresholds even when we observe the distribution of the ranking of marital traits instead of the distribution of the traits themselves.

I first generate simulated data from the three models outlined in table A1.

Table A1: Models

	Distributions		Parameters			Solution
	s	y	α	β	K	for σ_0
Model 1	$U[0, 0.8]$	$U[0, 1]$	0.2	0.6	0.5	0.2036
Model 2	$U[0, 0.1]$	$U[0, 1]$	0.2	0.6	0.5	0.0255
Model 3	$U[0, 0.8]$	$U[0, 0.5]$	0.2	0.6	0.5	0.2036

Notes: In all models, the production function is $h(y, s', s) = y(1 + K)^{\beta} s^{\alpha}$.

For example, Model 1 in table A1 captures a marriage market in which females' traits are uniformly distributed in the interval $[0, 0.8]$, males' traits are uniformly distributed in the interval $[0, 1]$, and the parameters of the household production function are $\theta = \{0.2, 0.6, 0.5\}$. Under these distributions and technological parameters, the equilibrium matching in the marriage market is such that the threshold woman σ_0 is at female skill value 0.2036. To generate a simulated dataset of marriages in this environment, I draw 10,000 males and females from their respective traits distributions and match them according to threshold $\sigma_0 = 0.2036$. The second step in the heuristic exercise is to assume that we do not observe the true value of σ_0 and produce the vector of moments from the simulated data. Finally, I use the method of moments to search for the values of σ_0 and θ that minimize the distance from model generated moments and the simulated data moments.

Table A2 shows the results of Monte Carlo simulation exercises of the three models summarized in table A1. For the three models, the column labeled *True distributions* shows the results of estimating σ_0 and θ from the method of moments when the data is generated by the models in table A1. The column labeled *Ranking* shows the analogous estimation results but when the data generated is not perfectly observed: instead of observing the true distribution of traits, we observe the cumulative distribution of traits (or the ranking of individuals). Note that marriages remain the same across both simulated datasets, the only difference across both dataset are the values of the individual traits.

Table A2 reveals that the method of moments does an excellent job in identifying the percentile of σ_0. To see this, note first that, for example, the estimated value of σ_0 is 0.1998 when the data is generated according to model 1 in table A1, very close to the model true value of σ_0: 0.2036. Second, note that the estimated value of σ_0 is when the data is generated according to model 1 but we observe the ranking of individuals 0.2499, a figure that is remarkably close to the percentile that threshold σ_0 estimated under the true distributions represents (which is 0.2498). This exercise constitutes a heuristic argument that suggests that the shape of the equilibrium is identified from the observation of individual rankings and equilibrium behavior.

Table A2: Models simulations and estimation of σ_0 with ranking of traits

	Model 1		Model 2		Model 3	
	True distributions	Ranking	True distributions	Ranking	True distributions	Ranking
Specification of traits:	$s \sim U[0, 0.8]$ and $y \sim U[0, 1]$	$s, y \sim U[0, 1]$	$s \sim U[0, 0.8]$ and $y \sim U[0, 1]$	$s, y \sim U[0, 1]$	$s \sim U[0, 0.8]$ and $y \sim U[0, 0.5]$	$s, y \sim U[0, 1]$
Thresholds						
σ_0	0.1998	0.2499	0.026	0.2601	0.1996	0.2499
$p(\sigma_0)$	0.2498	0.2499	0.26	0.2601	0.2495	0.2499
Parameters						
α	0.1890	0.1915	0.1929	0.1948	0.1885	0.1904
β	0.5532	0.5531	0.5697	0.5695	0.5530	0.5528
k	0.5412	0.5405	0.5570	0.5570	0.5407	0.5415

Moments	$m(\theta_T)$	$m(\hat{\theta}_T)$	$m(\theta_R)$	$m(\hat{\theta}_R)$	$m(\theta_T)$	$m(\hat{\theta}_T)$	$m(\theta_R)$	$m(\hat{\theta}_R)$	$m(\theta_T)$	$m(\hat{\theta}_T)$	$m(\theta_R)$	$m(\hat{\theta}_R)$
$N(poly)/N(married)$	.3314	.3328	.3314	.3332	.3518	.3511	.3518	.3516	.3314	.3323	.3314	.3332
$mean(y^{mono})$	.4997	.5	.4985	.5	.4997	.5	.4967	.5	.2498	.25	.4985	.5
$var(y^{mono})$	.0204	.0209	.0212	.0208	.0199	.0192	.0194	.0192	.0051	.0052	.0212	.0208
$mean(s^{mono})$	.3989	.4	.5015	.5	.0503	.05	.4961	.5	.3989	.4	.5015	.5
$var(s^{mono})$	.013	.0134	.0198	.0208	.0002	.0002	.0195	.0192	.013	.0134	.0198	.0208
$mean(y^{poly})$	.8728	.8752	.8752	.875	.8725	.8701	.8691	.8699	.4364	.4376	.8752	.875
$var(y^{poly})$	.0054	.0052	.0052	.0052	.0055	.0056	.0057	.0056	.0014	.0013	.0052	.0052
$mean(s^{senior})$	.6982	.7001	.8727	.875	.0872	.087	.8688	.8699	.6982	.7002	.8727	.875
$var(s^{senior})$	.0035	.0033	.0054	.0052	.0001	.0001	.0057	.0056	.0035	.0033	.0054	.0052
$mean(s^{junior})$	.101	.0999	.1288	.125	.0127	.013	.1273	.1301	.101	.0998	.1289	.125
$var(s^{junior})$	.0034	.0033	.0055	.0052	.0001	.0001	.0054	.0056	.0035	.0033	.0055	.0052

Notes: In all models the true female and male traits distributions are $s \sim U[0, \overline{S}]$ and $y \sim U[0, \overline{Y}]$, respectively, and the production function is $h(y, s', s) = y(1 + K)^{\beta} s^{\alpha}$. In all models, *Ranking* refers to the CDF of the traits distributions. σ_0 is the female skill level at which a woman is indifferent between being a junior wife and a monogamous wife. *N(poly)* and *N(married)* refer to the number of polygamous and married males, respectively. s and y refer to female skills and male wealth, respectively. *Mono* and *poly* stand for *monogamous* and *polygamous*, respectively.

Appendix E Summary statistics and estimates including all monogamous households

Table A3: Summary statistics, LSMS-ISA Nigeria, 2010 (including all monogamous households)

	All		Polygamous		P-M	
	Mean	SD	Mean	SD	Mean	SD
Marriage market						
Women age at marriage	17.684	6.38	17.082	6.101	-1.044	0.241
Men age at marriage	28.2	9.103	26.798	8.38	-1.979	0.412
Husband-wife age gap	13.234	8.165	16.009	8.738	5.03	0.294
Household structure						
# of wives	1.106	0.721	2.189	0.45	1.189	.011
# of children	3.584	2.712	5.864	3.063	2.552	0.114
# of domestic workers	0.01	0.149	0.002	.041	-0.004	0.004
# of other family	0.369	1.101	0.23	0.811	-0.083	0.042
Production and Labor						
Plot uses only family labor	0.793	0.405	0.701	0.458	-0.105	0.016
Employed	0.595	0.491	0.591	0.492	0.005	0.009
Works only for family	0.921	0.269	0.934	0.249	0.02	0.006

Notes: *LSMS-ISA* stands for *Living Standard Measurment Study - Integrated Surveys on Agriculture*. The LSMS-ISA country surveys are publicly available from the World Bank **(click here for access)**. *P-M* refers to the difference in the corresponding statistics between polygamous and monogamous households. *Marriage market* and *Household structure* statistics are reported at the household level. *Plot uses only family labor* is a dummy variable measured at the agricultural plot level that takes value one if all workers in the plot are household members and zero if some worker on the plot is an external hire. *Employed* is a dummy variable measured for all household members older than 5 years old that takes value one if the individual works on their own, on a family plot, or for a non-family member. *Works only for family* is a dummy variable that takes value one if *Employed* equals one and the individual does not work for members outside of the household.

Table A4: Model fitting to the data, rural Nigeria, 2010 (including all monogamous households)

Targeted moments	North		South	
	Data	Model	Data	Model
$N(poly)/N(married)$	0.35	0.47	0.11	0.35
$mean(y^{mono})$	0.51	0.50	0.51	0.50
$var(y^{mono})$	0.08	0.01	0.09	0.02
$mean(s^{mono})$	0.63	0.50	0.54	0.50
$var(s^{mono})$	0.05	0.01	0.07	0.02
$mean(y^{poly})$	0.48	0.84	0.39	0.87
$var(y^{poly})$	0.07	0.01	0.10	0.01
$mean(s^{senior})$	0.63	0.84	0.41	0.87
$var(s^{senior})$	0.04	0.01	0.04	0.01
$mean(s^{junior})$	0.14	0.16	0.08	0.13
$var(s^{junior})$	0.01	0.01	0.01	0.01
Estimated Parameters				
σ_0	0.32		0.26	
α	0.22		0.19	
β	0.67		0.57	
k	0.65		0.56	

Notes: *N(poly)* and *N(married)* refer to the number of polygamous and married males, respectively. *Data* refers to empirical moments. *Model* corresponds to moments produced by the model when parameters are set at the estimated levels. s and y refer to female skills and male wealth, respectively. *Mono* and *poly* stand for *monogamous* and *polygamous*, respectively. σ_0 is the female skill level at which a woman is indifferent between being a junior wife and a monogamous wife. The production function being estimated is $h(y, s', s) = y(1+K)^{\beta}s^{\alpha}$.