Some practical definitions

* Aperture vs. pupil ?

Pupil → reimaged aperture (put the filters, etc. here)

* f-number: "diameter of incoming ray bundle" OR "angle of convergence"

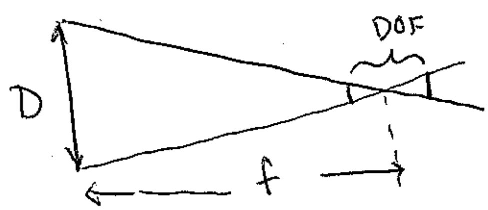
focal length → $\boxed{\dfrac{f}{D}}$ ← mirror/lens diameter

Large f-number: "slow"
Small f-number: "fast"

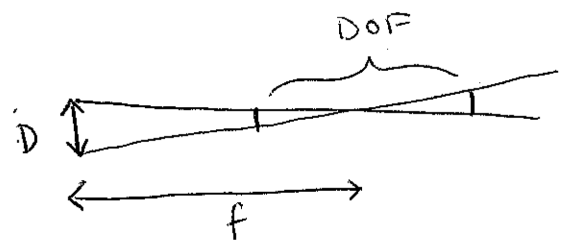
What happens when we put a <u>stop</u> in front of the pupil?

- D gets smaller (remember resolution gets worse)
beam becomes "slower" – but this can sharpen the image!

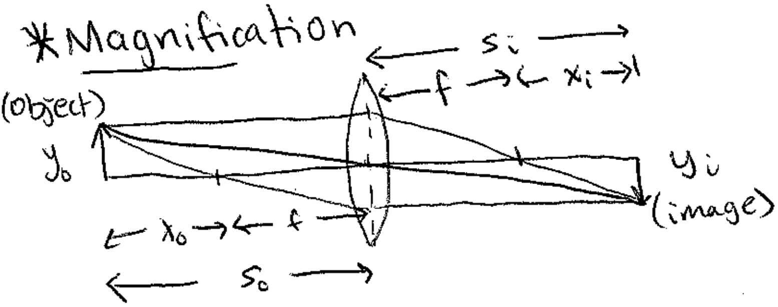
* Depth of field (or focus) – DOF



vs.



* <u>Magnification</u>



$$M \equiv \frac{y_i}{y_o} = -\frac{s_i}{s_o} \qquad x_o x_i = f^2$$

$$= \frac{x_i}{f} = \frac{f}{-x_o}$$

* Plate scale

→ Physical units to angular

$$M = \frac{\text{focal plane f-number}}{\text{primary mirror f-number}}$$ (remember, reimaging optics)

$$\boxed{f_{eq} = M f_{primary}}$$

$$\boxed{\theta_b\,[\text{rad}] = \frac{b}{f_{eq}}}$$ ← physical distance @ focal plane

$D\theta_{in} = d\,\theta_{out}$

What is the pixel size on the detector?
- we want to take advantage of telescope resolution

resolution elements: $\frac{\lambda}{D}$ in size

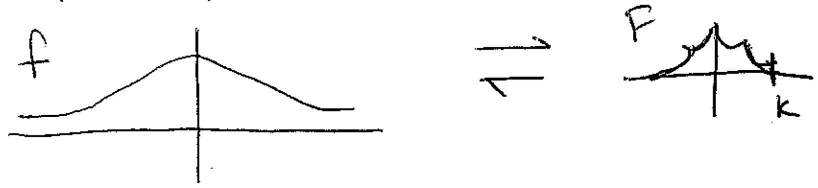
"How often should a signal be sampled so that all frequencies present are detected?" - J.M. Whittaker

highest frequency (spatial frequency) resolved is $\sim\frac{\lambda}{D}$
(thats the smallest detail the optical system resolves)

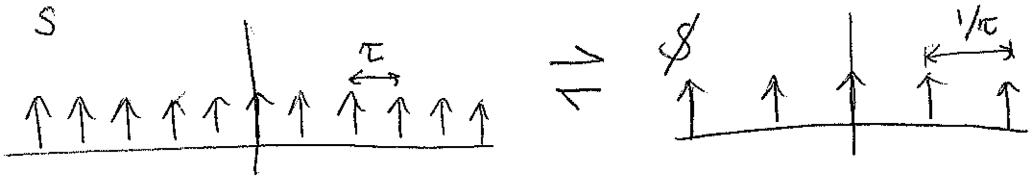
Answer: At least twice as often as the highest frequency.

this limit is called the Nyquist limit

Graphical proof (from Bracewell):



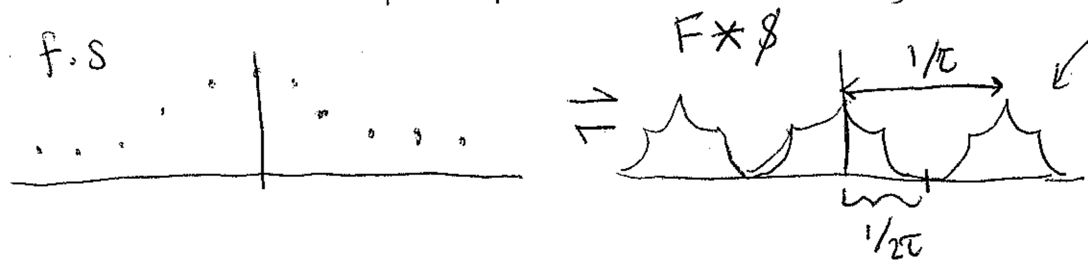
F is some function that is "band-limited", meaning finite, so it has a maximum, k. k represents the largest spatial frequency present in f



by the similarity theorem

A series of delta functions are used to sample f, period τ

(by the convolution theorem):



when $1/\tau$ is small enough these start to overlap. This is called "Aliasing"

in order to capture all spatial frequencies
$\frac{1}{2\tau} \geq k$ or the sampling frequency $\boxed{\tau \leq \frac{1}{2k}}$

The point: pixels should be $\frac{\lambda}{2D}$ or smaller to take advantage of resolution.

Quality of the optics & some examples of aberrations

What is a diffraction limited image?
- Qualitatively: can you see the Airy rings / fringes?
- Quantitatively: strehl ratio

definition: $\dfrac{\text{intensity at peak of seeing disk}}{\text{intensity at peak of true Airy disk}}$ ~ what you actually see ~ theoretical

Marechal approximation for strehl ratio, s:
* rms - root mean square

$$s \approx e^{-\left(\frac{2\pi\sigma}{\lambda}\right)^2}$$

σ... rms wavefront error (e.g. little manufacturing bumps on your mirror)

(Rieke p. 31)

$S \geq 0.8$ is considered diffraction-limited

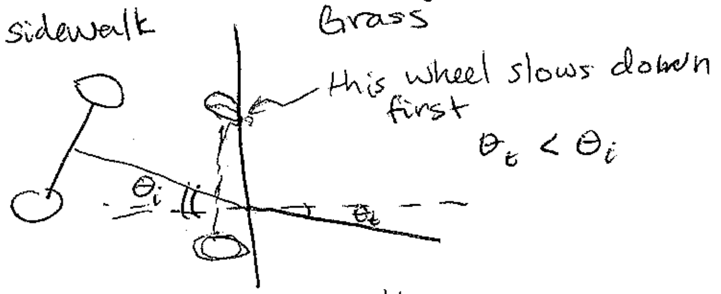
How do we bend light w/ optics?

- transmissive optics (Lenses, etc.)
- reflective optics (Mirrors)

Transmissive:

$n_i \sin\theta_i = n_t \sin\theta_t$

Lawn-mower analogy:



$n_{grass} > n_{sidewalk}$

From electromagnetic theory we can derive the reflection and transmission as a function of n and θ

$r = r(n, \theta)$ reflection

$t = t(n, \theta)$ transmission

Thinking about linear, isotropic, homogeneous materials n is derived from material properties

A closer look:

- n is complex

$$\tilde{n} = n_R - i n_I$$

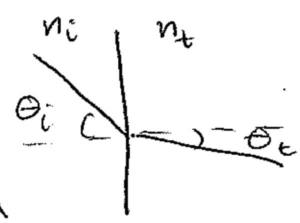
(real) (imaginary)

imaginary part describes absorption

- frequency (wavelength) dependent

$$n = n(\nu) \quad (\text{or } n(\lambda))$$

Therefore reflection, transmission and absorption are properties that can depend on frequency!



In Snell's law: $n_i \sin\theta_i = n_t \sin\theta_t$

what if $\theta_t = 90°$? → Nothing is transmitted

"Total internal reflection"

there is a critical angle for T.I.R., θ_c

$$n_i \sin\theta_c = n_t \underbrace{\sin(90°)}_{1} \rightarrow \boxed{\sin\theta_c = \frac{n_t}{n_i}}$$

and $\boxed{n_i \geq n_t}$ must be true

Lenses vs. Mirrors:

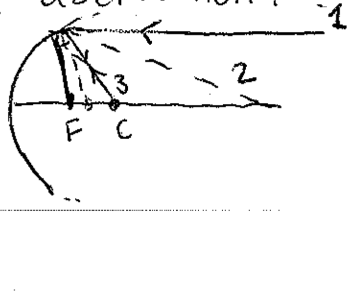
- Lenses have more serious "chromatic" effects, they respond differently to different wavelength because $n = n(\lambda)$ (Like how a prism can disperse white light into a rainbow)

Aberrations

stigmatism: "Any ray from one focus forms a perfect point image at the other"

→ that's the goal. All rays come to the same focus!

Spherical aberration:



Rays 1, 2, and 3 do not come to a focus.

Some more monochromatic aberrations:

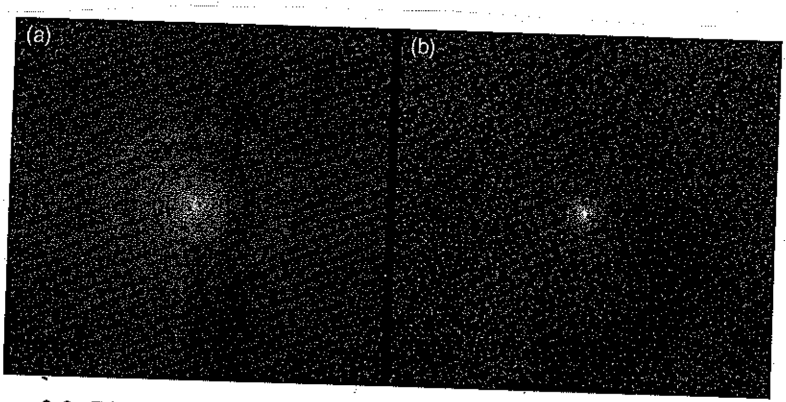


Figure 2.3. The most famous example of spherical aberration, the Hubble Space Telescope images before (left) and after (right) correction. From NASA Images.

Coma

optical sine theorem"

$$n_o y_o \sin\alpha_o = n_i y_i \sin\alpha_i$$

For zero coma:

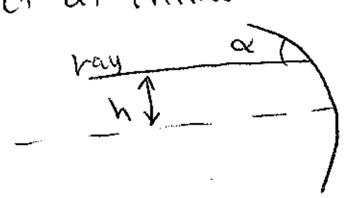
$$\frac{y_i}{y_o} = M_T = \text{constant}$$

so

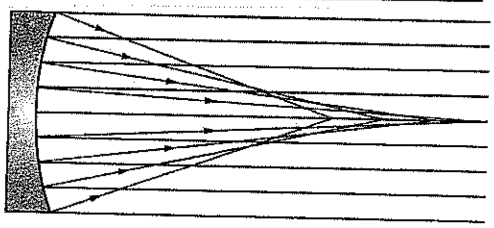
$$\frac{n_o \sin\alpha_o}{n_i \sin\alpha_i} = \text{const.}$$

and for an object at infinite distance

$$\frac{h}{\sin\alpha} = \text{constant}:$$



Coma



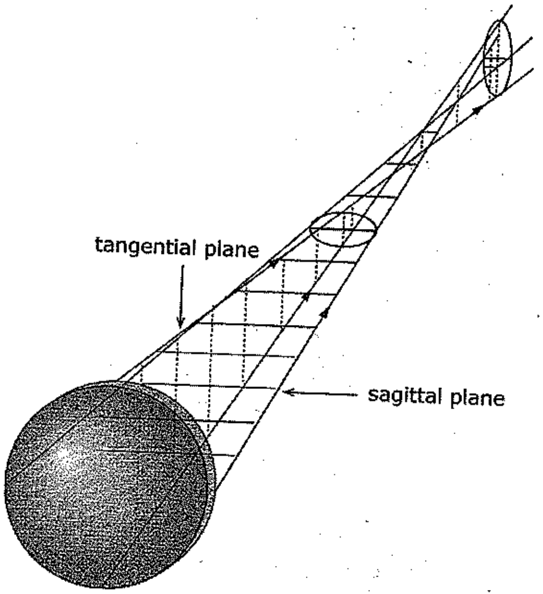
Astigmatism

different focal plane in one direction compared to perpendicular to it

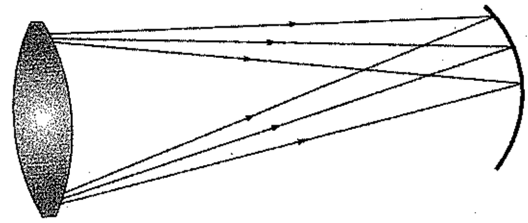
Curvature of field

focal plane is on a curved surface.

(And of course there are chromatic aberrations associated with transmissive optics because $n = n(\lambda)$)



Astigmatism.



Curvature of field.

Fundamentals Part 2: Detection

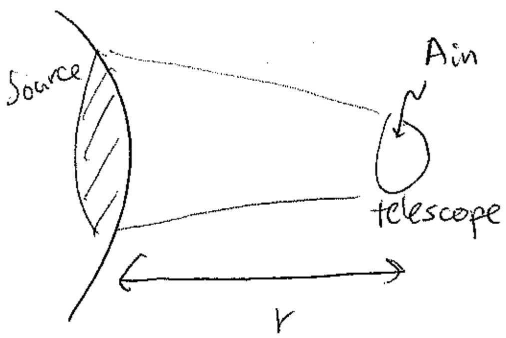
we'll discuss:
- detecting light
- noise
- different types of detectors

Thinking about: How do we detect (and quantify) the light?

but remember the science goals – what questions do we have?
- How big is the thing?
- How bright is the thing?
  - ↳ compared to its neighbors?
  - ↳ Does it change in time?
- what color is it?
- Is it smooth? Lumpy?
- etc.

Power recieved (Luminosity):

remember the poynting vector $\propto E^2$ (intensity)



solid angle

$$\Omega = \frac{A_{in}}{r^2}$$

what fraction of the source are you viewing..

What kinds of detectors are there and what are they physically detecting?

Photon Detectors
- Light interacts with material
  e.g. via photoelectric effect

Thermal Detectors
- Light absorbed, Temp. increases (x-ray, far IR, mm)

Coherent / Heterodyne Detectors
- Electric field interacts → measuring field (generally radio)

Interlude: Signal to Noise

two examples of ~~[illegible]~~ probability distributions the reflect physical noise phenomena:

Poisson

$$P(m,\mu) = \frac{e^{-\mu}\mu^{m}}{m!}$$

μ... mean

special property
standard deviation $\sigma = \sqrt{\mu} \leftarrow$ mean

m "successes" in μ observations
e.g. raindrops, nuclear decay - rare events in many trials

- A good representation of photon noise in high frequencies

Gaussian (Normal)

$$P(x,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$

σ... standard deviation
x...mean

F.T. (Gaussian) = Gaussian!

Photon statistics

in the right regime

$$\langle N^2 \rangle = n$$

noise
photon "counts" or # of photons recieved

so signal to noise $S/N = n/\sqrt{n}$
solution $\rightarrow$ get more counts!
reduce the noise

But actuall photons obey Bose - Einstein statistics

noise correction (from Rieke) $\langle N^2 \rangle = n\left[1 + \frac{\epsilon\tau\eta}{e^{h\nu/kT} - 1}\right]$

h - planck's constant
k - Boltzmann constant
ν - photon frequency
T - ~~[illegible]~~ ~~[illegible]~~ temperature

τ... transmittance
η... detector Quantum Efficiency (Q.E.)
ϵ... source emissivity

what if: $\nu \rightarrow$ big $(h\nu \gg kT)$

$\langle N^2 \rangle \approx n$ like we discussed

$\nu \rightarrow$ small $(h\nu \ll kT)$

$\langle N^2 \rangle$ blows up $\rightarrow$ need cold detectors in infrared

Correction is significant when $\lambda \gg \lambda_{\text{blackbody peak}}$

The point:

Some noise comes from the statistics

Some noise comes from an imperfect detector

might see

Detective Quantum Efficiency $= \dfrac{(S/N)^2_{out}}{(S/N)^2_{IN}}$ DQE

Example of Gaussian noise:

Random, uncorrelated pixel-to-pixel noise

More exposure doesn't bring this noise down

Characterizing this noise is important

(measuring & subtracting) Always some residual

Photo detectors

Old way $\rightarrow$ photographic plates

blackening of silver halides upon light interaction.

e.g. AgBr $Ag^+ Br^- +$ $\rightarrow Ag^+ + Br + e^- \rightarrow Ag + Br$

- Poor DQE

- non-linear, non-identical

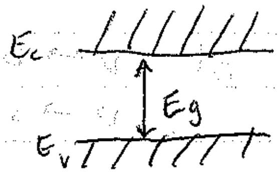
Now we are in the age of statistics!

Need to understand our noise as much as the signal

What do we use today to detect photons?

- Semiconductors (w/ bias voltage)

- illumination yields "photocurrent"



photon absorption "bumps" a valence electron to the conduction band

Must supply at least E_g to absorb a photon. cutoff $\lambda_c = hc/E_g$

Generally $E_g \sim$ UV - IR

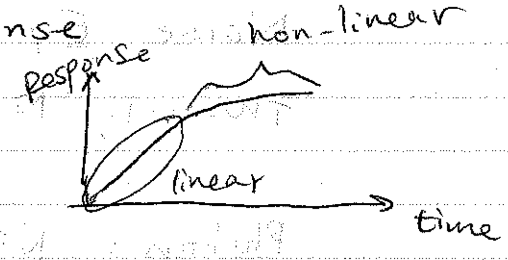
How can we change E_g?

"dope" semiconductors $\longrightarrow$ introduce impurities

like turning a knob, a whole trade

Other sources of noise:

- Johnson / Nyquist Noise
  "Brownian motion of charge carriers"
  $$\langle I_J^2 \rangle = \frac{4kTdf}{R}$$
  How to mitigate:
  Low temp, high frequency.
- "Read noise" Analog to Digital conversion
  often treated as random
  rms electron noise
  (also reset noise)
- Linear regime, time response
  - takes time to "discharge"
  return to state before exposure



The point: sophisticated electronics require sophisticated treatment of associated noise and understanding of underlying physical processes.

Rieke discusses many different types of detectors
→ could be a good mini project to explore one or more

CCDs, briefly
"Charge Coupled Devices"
- Grid of pixels → each an electron bucket
- photon interaction with the pixel semiconductor generates an electron
- Electrostatic field necessary to keep electron there
- "Bucket" should be deep enough for multiple
- Each bucket like a parallel plate capacitor
"MOS" structure - Metal oxide ~~structure~~ semiconductor
- Depletion region (size of bucket) controllable.

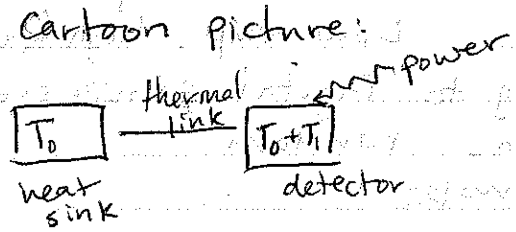
McLean p.253 bucket collection analogy
p.329 defective pixels in lines

Thermal Detectors

- Bolometers

photon energy absorbed & thermalized
thermometer produces an electric signal

Cartoon picture:



Noise **E**quivalent **P**ower

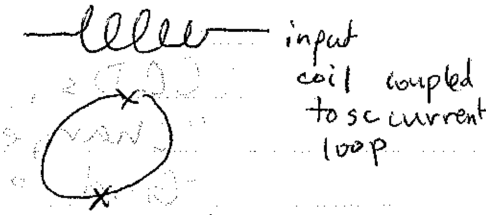
Thermal: $NEP_T = \sqrt{4kT^2G}\,/\eta$ $(T = T_0 + T_1)$ } we want a cold detector

Photon: $NEP_{ph} = \frac{hc}{\lambda}\sqrt{\frac{2\phi}{\eta}}$ ϕ... photons/s

In practice:

TES - Transition Edge sensors
  ΔT changes superconductivity - VERY sensitive
  - what temp. generally associated w/ s.c.?

SQUIDs - Superconducting Quantum Interference Devices
  Amplifies signal
  current modulation → large
  change in SQUID Current



MKIDs - Microwave Kinetic Inductance Detectors
  detector in resonant circuit
  flux changes resonant frequency
  read by HEMT High Electron Mobility Transistor
  (amplifier)

→ These are also good research topics for mini-project!

~~Theory~~: Coherent Detectors

"Heterodyne Receivers"

Antennae.

- Measure electric field (like picking up a signal with your radio)

e.g. with radio antennae and process the signal electronically..

Or can mix the source signal with a man-made signal and measure their ~~~~ interference at a more convenient frequency

A comment on the Quantum Limit:

Uncertainty:

- $\Delta E \underbrace{\Delta t}_{} \geq h/2\pi$

for time varying signal this is phase information about the wave

The counts recieved per unit energy in the optical/IR makes coherent detection extremely difficult in those wavelengths and higher energies.