The Higgs Mechanism
Electroweak Symmetry Breaking

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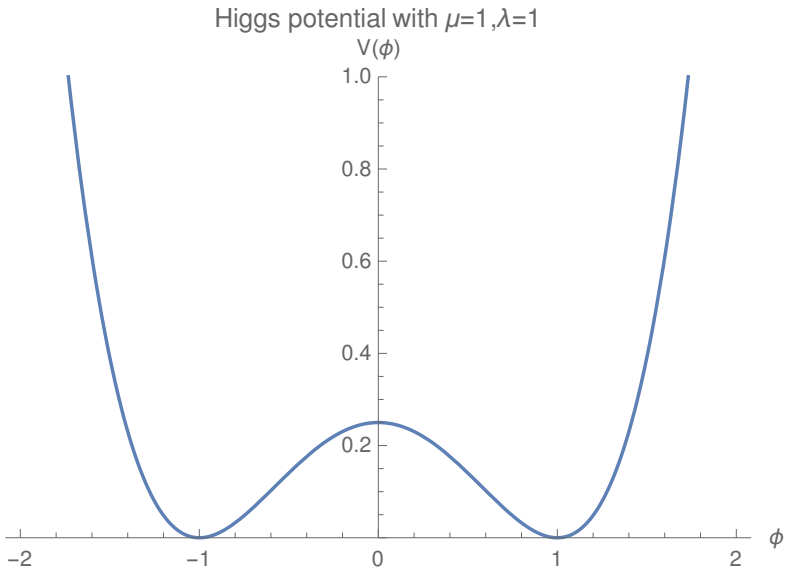
Lagrangian

The Lagrangian $\mathcal{L}$ is the zero-temperature analogue of the free energy $\mathcal{F}$.

$$\mathcal{L} = \frac{1}{2}\dot{\phi}(x)^2 - \frac{1}{2}\left(\nabla\phi(x)\right)^2 - \frac{1}{2}\mu^2\phi(x)^2 + \frac{1}{4}\lambda\phi(x)^4$$

$$V(\phi) = -\frac{1}{2}\mu^2\phi(x)^2 + \frac{1}{4}\lambda\phi(x)^4$$

Higgs Potential



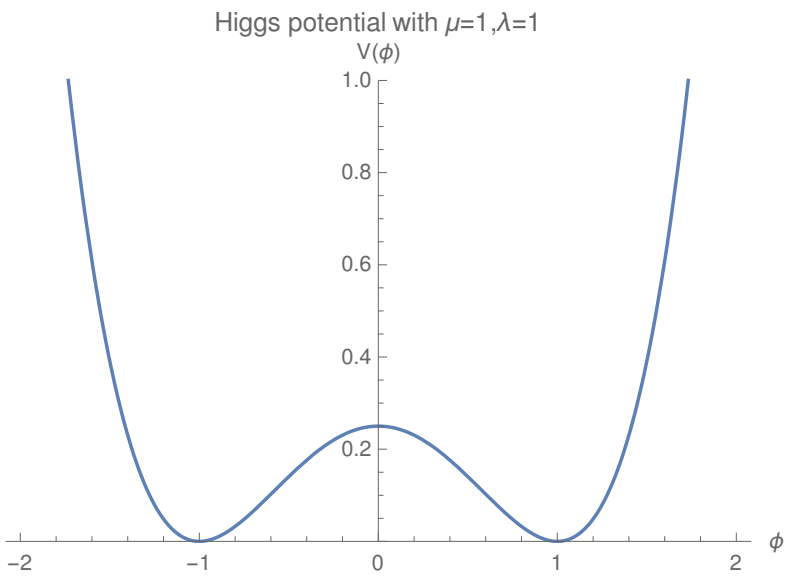
$$V(\phi) = -\frac{1}{2}\mu^2\phi(x)^2 + \frac{1}{4}\lambda\phi(x)^4$$

Initially the system has the $\mathbb{Z}_2$ symmetry $\phi \leftrightarrow -\phi$.

$$m_\phi^2 = \frac{d^2V(\phi)}{d\phi^2} = -\mu^2 + 3\lambda\phi^2 = -\mu^2$$

$$V_{min} = v = \frac{\mu}{\sqrt{\lambda}}$$

Symmetry Breaking



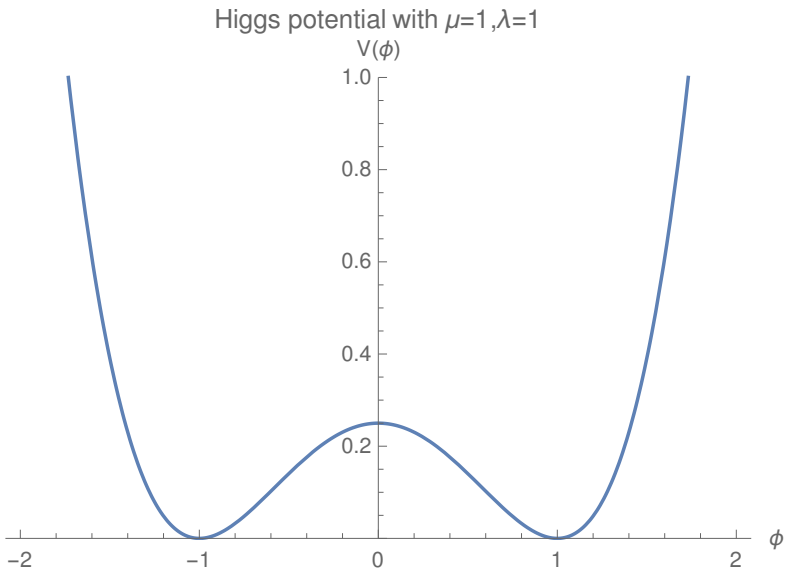
Let $\phi(x) = v + \sigma(x)$.

$$V(\sigma) = \mu^2\sigma(x)^2 + \sqrt{\lambda}\mu\sigma(x)^3 + \frac{1}{4}\lambda\sigma(x)^4$$

Does not have symmetry $\sigma \leftrightarrow -\sigma$. This symmetry is broken spontaneously.

$$m_\sigma^2 = 2\mu^2$$

Acquiring a mass



If there is another field ψ in $\mathcal{L}$ then it gets a mass even if it is initially massless.

$$V(\phi,\psi) = V(\phi) + V(\psi) + g\phi\psi^2$$

$$m_\psi^2 = 0$$

$$V(\sigma,\psi) = V(\sigma)+V(\psi)+gv\psi^2+g\sigma\psi^2$$

$$m_\psi^2 = 2gv$$

Higgs Mechanism

The field ϕ gives other fields masses when the $\phi \leftrightarrow -\phi$ symmetry breaks. We call σ the Higgs field. The process by which other fields get masses is called the Higgs mechanism.

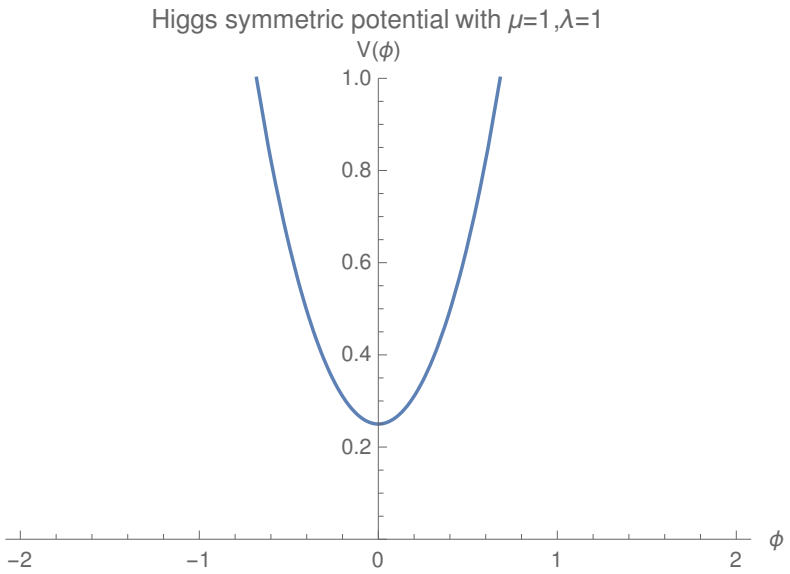
Free Energy

$$\mathcal{F} = \mathcal{H} - T\mathcal{S}$$
$$= \frac{1}{2}\dot{\phi(x)}^2 - \frac{1}{2}\left(\nabla\phi(x)\right)^2 - \frac{1}{2}\mu^2\phi(x)^2 + \frac{1}{4}\lambda\phi(x)^4 + \frac{1}{24}m_\phi^2 T^2 - \frac{\pi^2}{90}T^4$$

Effective potential

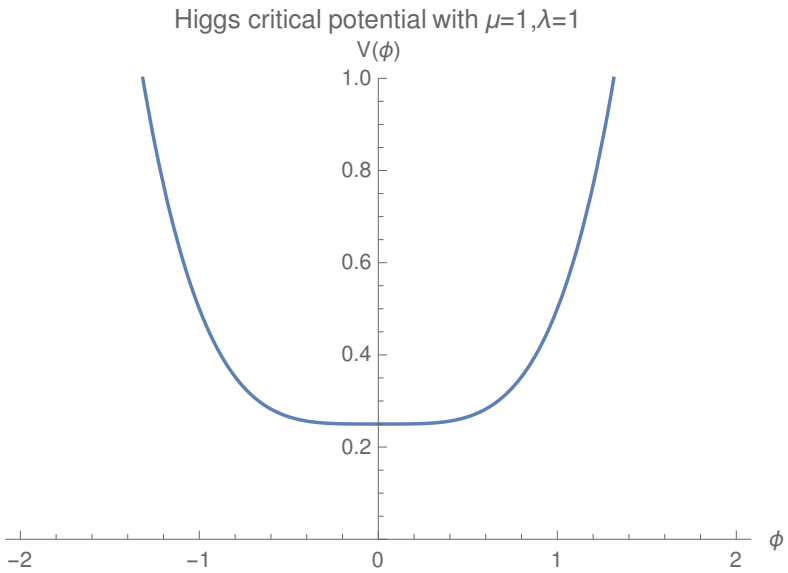
$$V(\phi) = -\frac{1}{2}\mu^2\phi(x)^2 + \frac{1}{4}\lambda\phi(x)^4 + \frac{1}{24}m_\phi^2 T^2 - \frac{\pi^2}{90}T^4$$
$$= -\frac{1}{2}\mu^2\phi(x)^2 + \frac{1}{4}\lambda\phi(x)^4 - \frac{1}{24}\mu^2 T^2 + \frac{1}{8}\lambda\phi^2 T^2 - \frac{\pi^2}{90}T^4$$

Symmetric State



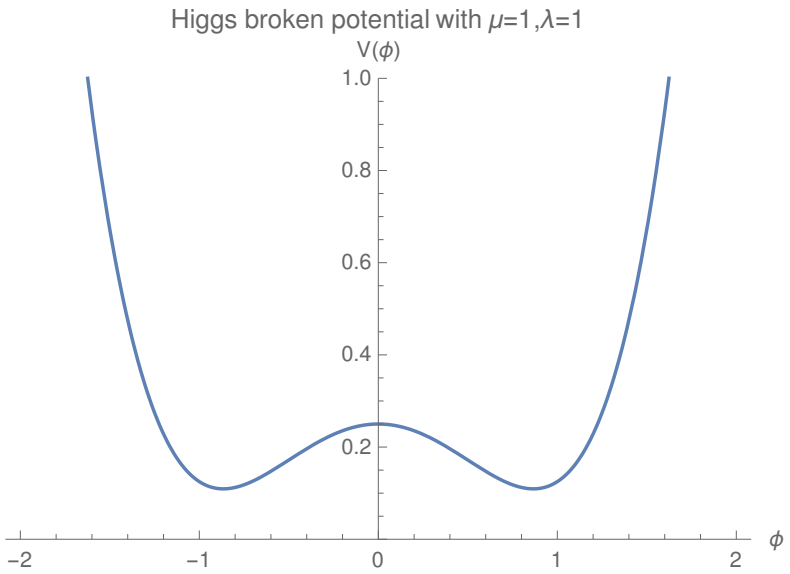
At high temperatures early in the universe (<10s), the Higgs potential was symmetric and the Higgs had a positive mass.

Critical State



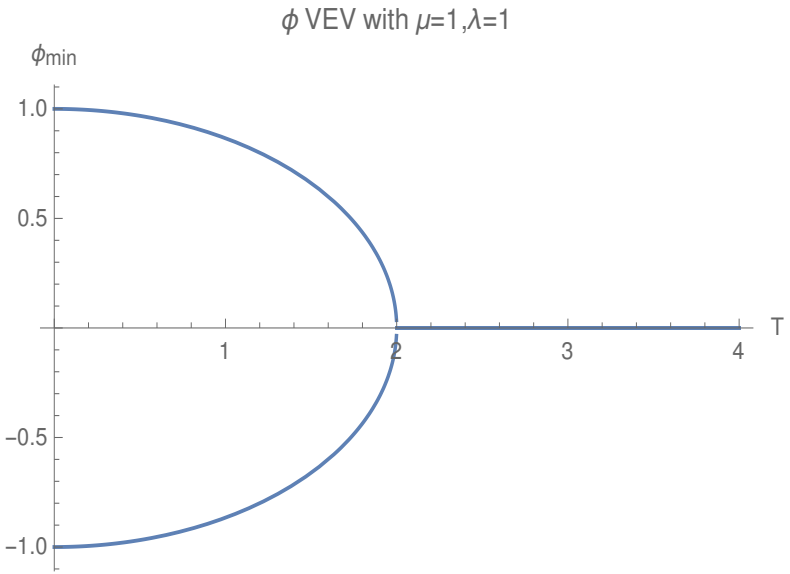
At the critical temperature ($\approx$10s), the second order phase transition occurs and the Higgs is massless.

Broken State



After the phase transition, the temperature continues to fall and the Higgs (σ) has a positive mass. The ground state moves away from $\phi = 0$ and any particles interacting with the Higgs acquire a mass. At zero temperature we reach the final state.

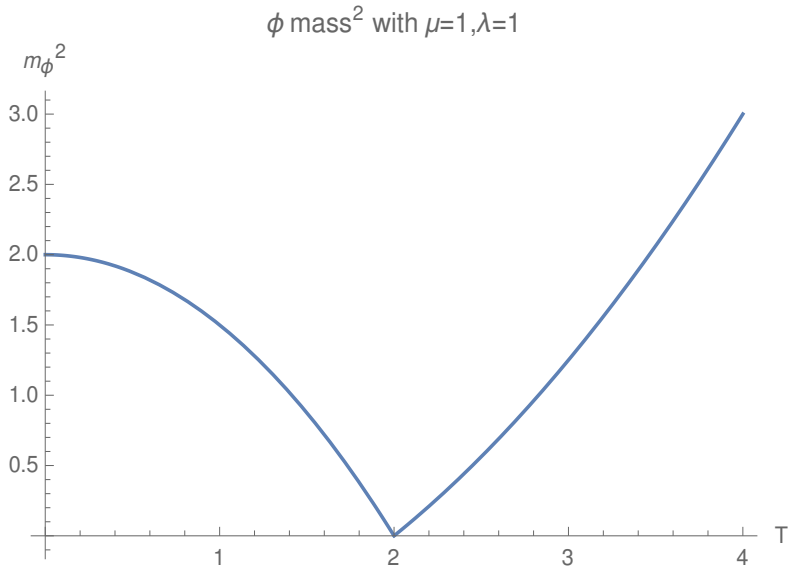
Order Parameter



$$\phi_{min}(T) = \begin{cases} 0 & T \geq 2v \\ \sqrt{v^2 - \frac{1}{4}T^2} & T < 2v \end{cases}$$

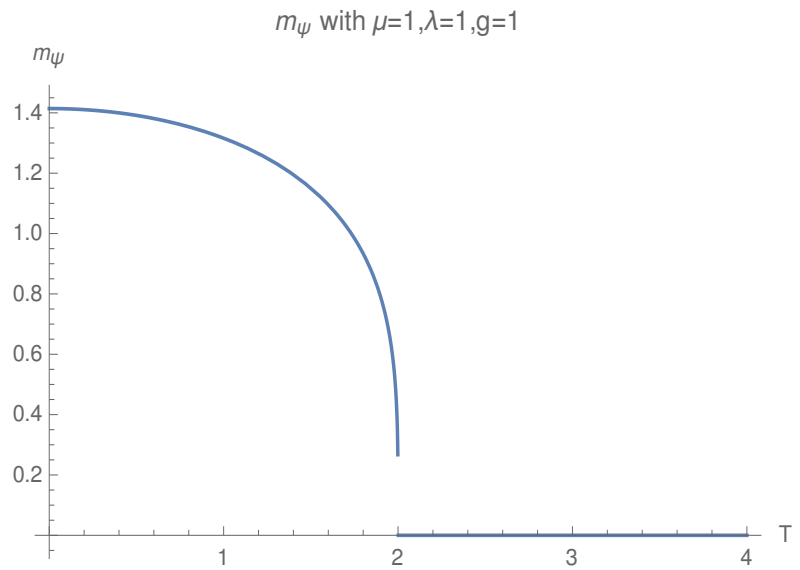
ϕ_{min} is an order parameter for this system!

Higgs mass



$$m_\phi^2(T) = \begin{cases} -\mu^2 + \frac{1}{4}\lambda T^2 & T \geq 2v \\ 2\mu^2 - \frac{1}{2}\lambda T^2 & T < 2v \end{cases}$$

Psi mass



$$V(\sigma,\psi) \supset g\phi_{min}\psi^2$$

$$m_\psi^2(T) = \begin{cases} 0 & T \geq 2v \\ 2g\sqrt{v^2 - \frac{1}{4}T^2} & T < 2v \end{cases}$$

Standard Model Electroweak Lagrangian

$$\mathcal{L}_{SM} \supset -V(\phi) + \left| \left(-\frac{g}{2} \sum_{i=1}^{3} W^i + \frac{g'}{2} B \right) \phi \right|^2 - \sum_{quarks} \lambda_q \bar{q} q \phi - \sum_{leptons} \lambda_l \bar{l} l \phi$$

Vector Bosons

When we take $\phi(x) = v + \sigma(x)$,

$$\mathcal{L}_{SM} \supset -V(\phi) + \left| \left(-\frac{g}{2} \sum_{i=1}^{3} W^i + \frac{g'}{2} B \right) \phi \right|^2 - \sum_{quarks} \lambda_q \bar{q} q \phi - \sum_{leptons} \lambda_l \bar{l} l \phi$$

$$\rightarrow \frac{1}{2} \left(\frac{1}{2} g v \right)^2 |W^1|^2 + \frac{1}{2} \left(\frac{1}{2} g v \right)^2 |W^2|^2 + \frac{1}{8} v^2 \begin{bmatrix} W^3 & B \end{bmatrix} \begin{bmatrix} g^2 & g g' \\ g g' & g'^2 \end{bmatrix} \begin{bmatrix} W^3 \\ B \end{bmatrix}$$

W^1 and W^2 bosons get a mass $m_W = \frac{1}{2} g v$, these are the charged vector bosons $W^{\pm}$ that mediate the weak interactions.

Vector Bosons

When we take $\phi(x) = v + \sigma(x)$,

$$\mathcal{L}_{SM} \supset -V(\phi) + \left| \left(-\frac{g}{2} \sum_{i=1}^{3} W^i + \frac{g'}{2} B \right) \phi \right|^2 - \sum_{quarks} \lambda_q \bar{q} q \phi - \sum_{leptons} \lambda_l \bar{l} l \phi$$

$$\rightarrow \frac{1}{2} \left(\frac{1}{2} g v \right)^2 |W^1|^2 + \frac{1}{2} \left(\frac{1}{2} g v \right)^2 |W^2|^2$$
$$+ \frac{1}{8} v^2 \begin{bmatrix} W^3 & B \end{bmatrix} \begin{bmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{bmatrix} \begin{bmatrix} W^3 \\ B \end{bmatrix}$$

Vector Bosons

When we take $\phi(x) = v + \sigma(x)$,

$$\mathcal{L}_{SM} \supset -V(\phi) + \left| \left(-\frac{g}{2} \sum_{i=1}^{3} W^i + \frac{g'}{2} B \right) \phi \right|^2 - \sum_{quarks} \lambda_q \bar{q} q \phi - \sum_{leptons} \lambda_l \bar{l} l \phi$$

$$\rightarrow \frac{1}{2} \left(\frac{1}{2} g v \right)^2 |W^1|^2 + \frac{1}{2} \left(\frac{1}{2} g v \right)^2 |W^2|^2$$
$$+ \frac{1}{8} v^2 \begin{bmatrix} A & Z \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & g^2 + g'^2 \end{bmatrix} \begin{bmatrix} A \\ Z \end{bmatrix}$$

W^3 and B turn into A, the massless photon field that mediates electromagnetism, and Z, the final massive vector boson that mediates weak interactions, with $m_Z^2 = \frac{1}{4} v^2 (g^2 + g'^2)$.

Fermions

When we take $\phi(x) = v + \sigma(x)$,

$$\mathcal{L}_{SM} \supset -V(\phi) + \left| \left(-\frac{g}{2} \sum_{i=1}^{3} W^i + \frac{g'}{2} B \right) \phi \right|^2 - \sum_{quarks} \lambda_q \bar{q} q \phi - \sum_{leptons} \lambda_l \bar{l} l \phi$$

$$\rightarrow - \sum_{quarks} \lambda_q v \bar{q} q - \sum_{leptons} \lambda_l v \bar{l} l$$

So the fermions get masses $m_f^2 = 2\lambda_f v$.

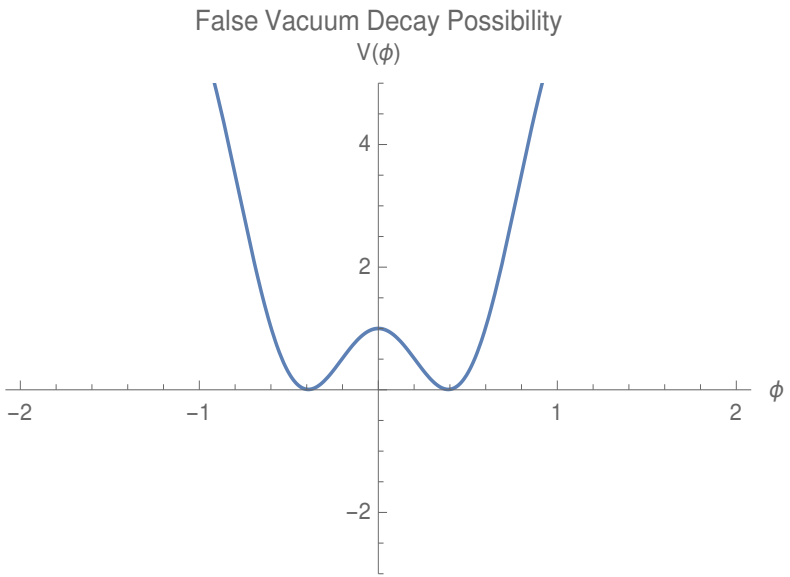
Electromagnetic and Weak Standard Model

We identify the field σ as the Standard Model Higgs boson that gives the vector bosons and the leptons their masses. The electroweak interaction has broken into separate Lagrangians for the electromagnetic interaction and the weak interaction.

$$\mathcal{L}_{EM} \supset -\sum_{quarks} m_q \bar{q} q - \sum_{leptons} m_l \bar{l} l$$

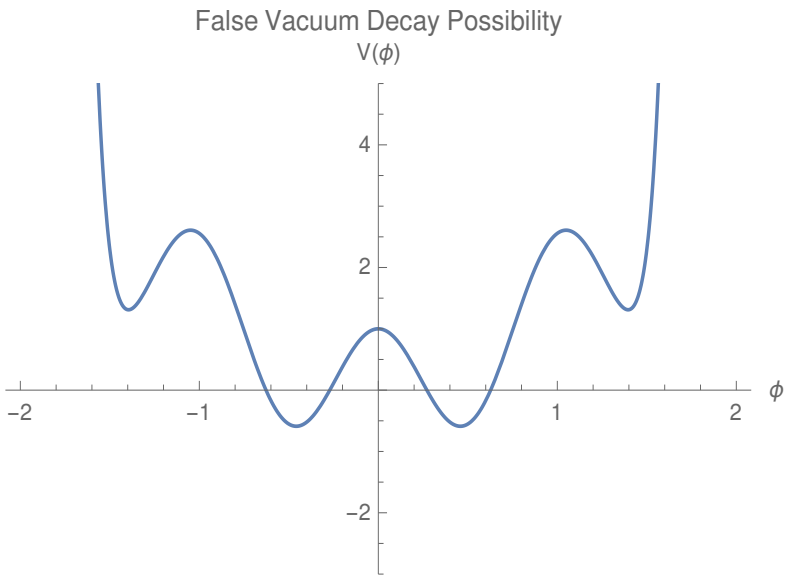
$$\mathcal{L}_{Weak} \supset \frac{1}{2}\left(\frac{1}{2}gv\right)^2 |W^+|^2 + \frac{1}{2}\left(\frac{1}{2}gv\right)^2 |W^-|^2 + \frac{1}{8}v^2(g^2+g'^2)\,|Z|^2$$

Broken State



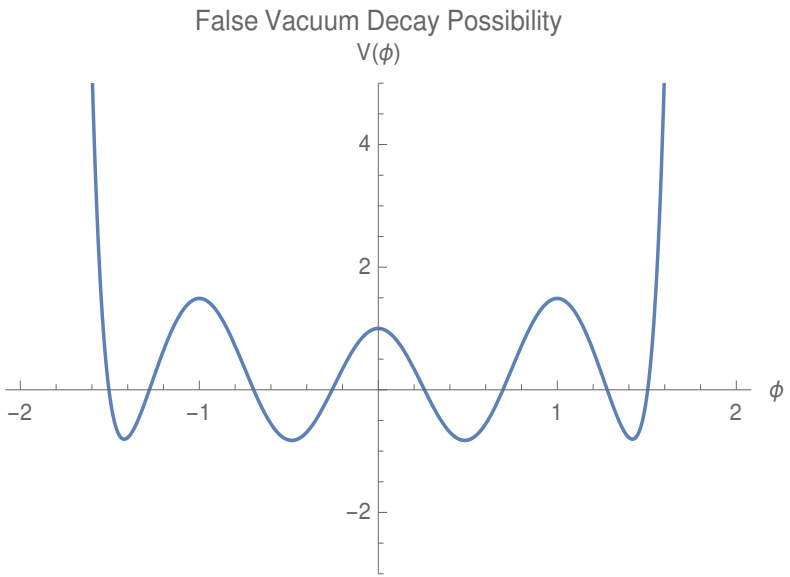
This is, to our best understanding, the Higgs potential today.

More Minima Appear



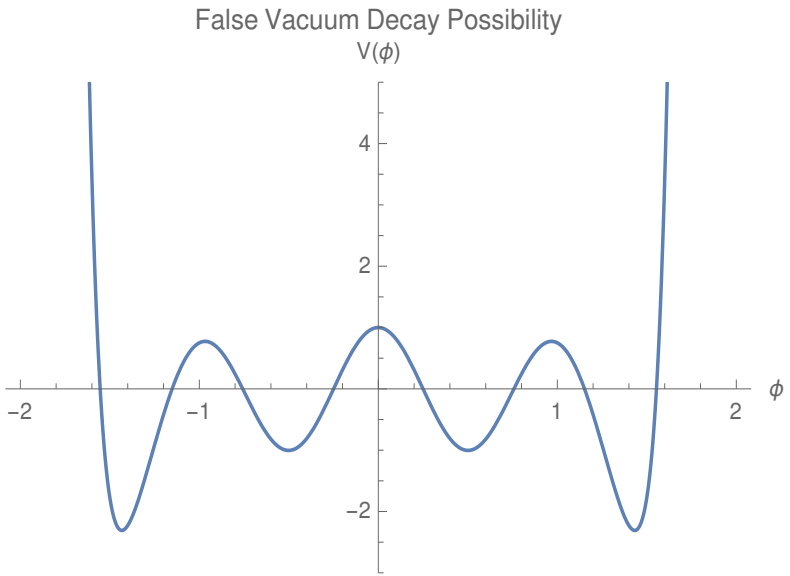
It could be that the Higgs has a different potential than we think, allowing for the possibility of a first order transition. After the electroweak symmetry breaking, the universe continued to cool and more minima may have appeared. In this plot, however, the Higgs is still in a global minimum.

1st Order Critical State



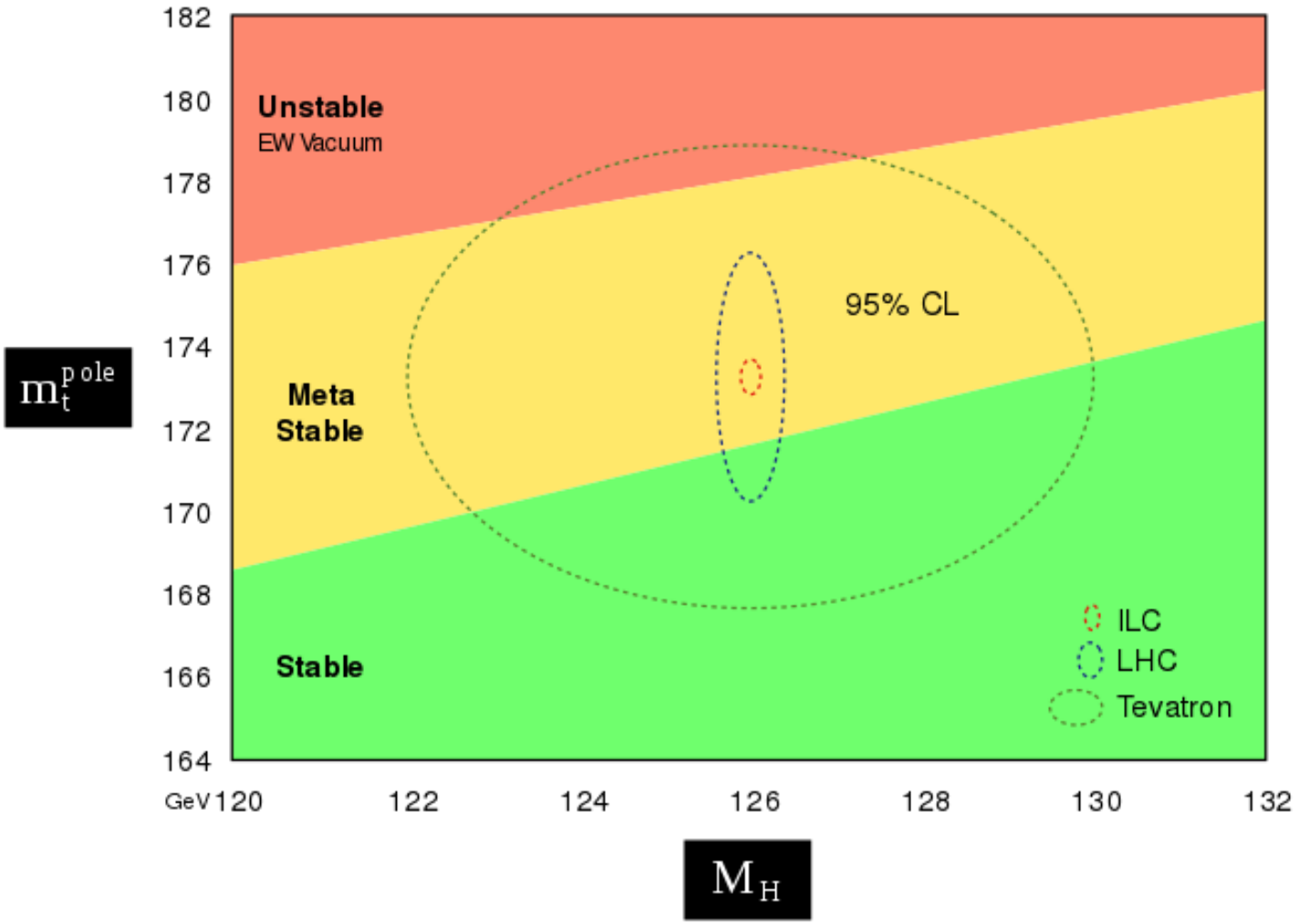
As the universe cools more, the extra minima keep falling. At the critical temperature, both minima have the same value.

1st Order Broken State



Now the extra minima are the global minima! The current Higgs state is only metastable and becomes the "false vacuum." It could quantum tunnel into the true vacuum at any time. When this happens the transition will release energy, and the walls will expand at the speed of light, leaving behind the true vacuum state. Most of the particle masses will be changed and the universe as we know it will cease to exist.

Current Bounds on the False Vacuum



References

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  - A Model of Leptons (S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264)
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  - QFT for the Gifted Amateur – Lancaster and Blundell
- Lecture Notes
  - Higgs Mechanism – Vadim Kaplunovsky
  - Cosmology II – Hannu Kurki-Suonio