Stress-energy tensor in soluble models of spherically symmetric charged black hole evaporation

Kamran Diba* and David A. Lowe

Department of Physics, Brown University, Providence, Rhode Island 02912

We study the decay of a near-extremal black hole in AdS$_2$, related to the near-horizon region of $(3+1)$-dimensional Reissner-Nordström spacetime, following Fabbri, Navarro, and Navarro-Salas. Back reaction is included in a semiclassical approximation. Calculations of the stress-energy tensor of matter coupled to the physical spacetime for an affine null observer demonstrate that the black hole evaporation proceeds smoothly and the near-extremal black hole evolves back to an extremal ground state, until this approximation breaks down.

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I. INTRODUCTION

The fate of near-extremal black holes during quantum evaporation has been of much interest because they present an excellent laboratory for investigating the information paradox. These black holes possess a stable ground state, namely the extremal black hole, and are able to avoid some of the problems which plague uncharged black holes during evaporation. For example, in the well-studied linear dilaton black hole model of Callan, Giddings, Harvey and Strominger (CGHS) [1], as the black hole evaporates away, the outer horizon encounters a naked singularity [2]. Charged black holes, on the other hand, possess a double horizon structure, with an inner and outer apparent horizon. In the extremal limit, where the black hole mass approaches the black hole charge in the appropriate units, the distance between the horizons is zero and they rest at some finite value of the radius (the extremal radius). The singularity at the center of the black hole thus lies safely behind the horizons, and there is no risk of encountering a naked singularity, even at the end point of evaporation. This fact, as well as their frequent appearance in string theory, makes them particularly appealing for investigation.

Jacobson has suggested that the semiclassical evolution of near-extremal black holes may break down while still far from extremality [3]. Using adiabatic arguments, Jacobson claims that in-falling photons created at the outer apparent horizon during Hawking evaporation will unavoidably fall through the inner horizon as well. If the photons encounter a large buildup of energy behind the inner horizon then the inner horizon is unstable and the semiclassical approximation is invalid. Otherwise, the photons will eventually pile up behind the outer horizon, causing it to become unstable. In either scenario, the semiclassical approximation may break down long before one would expect based on thermodynamic and/or statistical mechanics arguments [4].

String theory, on the other hand, suggests that the evaporation should proceed in a smooth way, with the excited black hole returning to its ground state. In particular, extremal black holes with Ramond-Ramond charge in type II string theories are described by configurations of D-branes, which have exact conformal field theory descriptions. At least at weak string coupling, it can be verified that the decay back to the extremal state is regular.

To investigate this issue, we study the behavior of the stress-energy tensor for a freely falling observer during the evaporation process. We calculate this and other observables for an extremal black hole after a shock perturbation of mass is sent in. In particular, we look for signs of instability and energy buildup behind the inner and outer horizons. We will use the semiclassical approximation and verify its validity close to the end point of evaporation, where it breaks down according to the criterion of [4]. In this paper, we utilize a model set forth by Fabbri, Navarro, and Navarro-Salas [5–7].

The Reissner-Nordström black hole line element is

$$ds^2 = -\left(1 - \frac{2l^2m}{r} + \frac{l^2q^2}{r^2}\right)dt^2 + \left(1 - \frac{2l^2m}{r} + \frac{l^2q^2}{r^2}\right)^{-1}dr^2 + r^2d\Omega^2 \tag{1}$$

$$= -\frac{(r-r_+)(r-r_-)}{r^2}dv^2 + 2drdv + r^2d\Omega^2, \tag{2}$$

with $r_\pm = l^2m \pm l\sqrt{l^2m^2-q^2}$. Setting $\phi = r^2/4l^2$ and $l = \sqrt{G_N}$, we can conformally rescale the metric by $d\tilde{s}^2 = \sqrt{\phi}ds^2$ and describe its two-dimensional reduction using the following action:

$$S = \int d^2x\sqrt{-g}[R\phi + l^{-2}V(\phi)], \tag{3}$$

where $V(\phi) = (4\phi)^{-1/2} - q^2(4\phi)^{-3/2}$. The extremal black hole radius corresponds to when $V(\phi_0) = 0$ [8], giving $\phi_0 = q^2/4$. A trapped surface, in a metric of the form

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + \phi^2d\Omega^2, \tag{4}$$

corresponds to when the two-sphere at x^+, x^- is decreasing in both null directions: $\partial_\pm\phi < 0$. Asymptotically, in these solutions, $\partial_-\phi < 0$ and $\partial_+\phi > 0$. Therefore an apparent horizon, which is the outer boundary of a trapped region, occurs at $\partial_+\phi = 0$.

*Email address: diba@het.brown.edu

The action above, Eq. (3), can also be derived from the Reissner-Nordström action considered in [9] and [10]

$$S = \int d^2x \sqrt{-g}[e^{-2\phi}R + 2e^{-2\phi}(\nabla\phi)^2 + 2 - 2q^2e^{2\phi}], \tag{5}$$

which, in the conformal gauge $(ds^2 = -e^{2\rho}dx^+dx^-)$, is equal to

$$S = \int d^2x[4\partial_+\partial_-\rho e^{-2\phi} - 4\partial_+\phi\partial_-\phi e^{-2\phi} + e^{2\rho}$$
$$- q^2e^{2\rho+2\phi}]. \tag{6}$$

By letting $\rho \rightarrow \rho + \phi/2$, which amounts to a conformally rescaling of the metric by $d\tilde{s}^2 = e^{\phi}ds^2$, we can rewrite this as

$$S = \int d^2x[4\partial_+\partial_-\rho e^{-2\phi} + e^{2\rho}e^{\phi} - q^2e^{2\rho}e^{3\phi}], \tag{7}$$

or

$$S = \int d^2x \sqrt{-g}[Re^{-2\phi} + V(\phi)], \tag{8}$$

with $V(\phi) = 2e^{\phi} - 2q^2e^{3\phi}$. Now redefining $e^{-2\phi}$ as ϕ the action can be expressed as

$$S = \int d^2x \sqrt{-g}[R\phi + V(\phi)], \tag{9}$$

with $V(\phi) = 2/\phi^{1/2} - 2q^2/\phi^{3/2}$, and $\phi = r^2$. This is equivalent to Eq. (10) with $l^2 = 1/4$ and q^2 rescaled to $2q^2$.

Returning to Eq. (3), performing an expansion of ϕ around ϕ_0 to first order ($\phi = \phi_0 + \tilde{\phi}$) in the action yields an effective near-extremal action

$$S = \int d^2x \sqrt{-g}[R\tilde{\phi} + 4\lambda^2\tilde{\phi}]. \tag{10}$$

We must keep that approximation in mind when making statements derived from this action.

When a shock mass is added to the black hole mass at $v = v_0$, $r_\pm$ in the metric (1) becomes modified

$$r_\pm = l^2m + l^2\Delta m \pm l\sqrt{l^2(m + \Delta m)^2 - q^2} \tag{11}$$

$$\approx l^2m \pm l\sqrt{l^2m^2 + 2l^2m\Delta m - q^2}, \tag{12}$$

to lowest order in Δm. In the extremal case when $m = q/l$, we get

$$r_\pm = \pm lq \pm l^2\sqrt{2m\Delta m}. \tag{13}$$

Letting $r_0 = lq$, this translates into

$$ds^2 = -\frac{(r - r_0 - l\sqrt{2r_0\Delta m})(r - r_0 + l\sqrt{2r_0\Delta m})}{r_0^2}dv^2$$

$$+ 2ldrdv \tag{14}$$

$$= -\frac{(r - r_0)^2 - 2lr_0\Delta m}{r_0^2}dv^2 + 2ldrdv \tag{15}$$

$$= -\left(\frac{\delta r^2}{r_0^2} - \frac{2l\Delta m}{r_0}\right)dv^2 + 2ldrdv \tag{16}$$

$$= -\left(\frac{\tilde{\phi}^2}{q^4} - \frac{2\Delta m}{q}\right)dv^2 + 2l\frac{d\phi}{\sqrt{\phi}}dv, \tag{17}$$

which leads to

$$d\tilde{s}^2 = \sqrt{\phi}ds^2 = -\left(2\frac{\tilde{\phi}^2}{q^3} - lm_S(v)\right)dv^2 + 2ld\tilde{\phi}dv, \tag{18}$$

where $m_S(v)$ is the shock mass perturbing the black hole written as a function of the null coordinate v (and equal to zero for $v < v_0$).

So far, we have been describing an "eternal" black hole. In order to study the Hawking radiation of these black holes, we must add dynamical matter fields to the action. Here, this is done by adding N minimally coupled scalar fields and studying the large N limit where the one-loop quantum correction adequately describes the effect of the Hawking radiation. This may not correspond to the most physically accurate way of describing the matter fields, but it is the most calculationally simple [11,12]. For this coupling of the matter fields, the effect of the back reaction on the spacetime geometry can be semiclassically included by adding a Liouville-Polyakov term [13].

$$I = \int d^2x \sqrt{-g}\left[R\tilde{\phi} + 4\lambda^2\tilde{\phi} - \frac{1}{2}\sum_{i=1}^N |\nabla f_i|^2\right]$$
$$- \frac{N\hbar}{96\pi}\int d^2x \sqrt{-g}R\square^{-1}R + \xi\frac{N\hbar}{12\pi}\int d^2x \sqrt{-g}\lambda^2. \tag{19}$$

Working in the conformal gauge where

$$d\tilde{s}^2 = -e^{2\rho}dx^+dx^-, \tag{20}$$

the equations of motion become

$$2\partial_+\partial_-\rho + \lambda^2e^{2\rho} = 0, \tag{21}$$

$$\partial_+\partial_-\tilde{\phi} + \lambda^2e^{2\rho}\left(\tilde{\phi} + (\xi - 1)\frac{N\hbar}{12\pi}\right) = 0, \tag{22}$$

$$\partial_+\partial_-f_i = 0, \tag{23}$$

$$-2\partial_\pm^2\tilde{\phi} + 4\partial_\pm\rho\partial_\pm\tilde{\phi} = T_{\pm\pm}^f - \frac{N\hbar}{12\pi}[(\partial_\pm\rho)^2$$
$$- \partial_\pm^2\rho + t_\pm(x^\pm)]. \tag{24}$$

ϕ can always be shifted to absorb the $\xi-1$ term, so without loss of generality, ξ is set equal to 1. The entire right-hand side of the final equation represents the full (classical plus quantum) matter stress-energy tensor. The functions, $t_\pm(x^\pm)$, are determined by the boundary conditions and depend on the vacuum choice. Under coordinate transformations, they transform according to

$$\left(\frac{d\tilde{x}}{dx}\right)^2 t_{\tilde{x}}(\tilde{x}) = t_x(x) - \frac{1}{2}\{\tilde{x},x\}, \qquad (25)$$

where $\{\tilde{x},x\}$ is the Schwarzian derivative defined by

$$\{f,x\} = \frac{2\partial_x^3 f \partial_x f - 3\partial^2 x f \partial_x^2 f}{2\partial_x f \partial_x f}. \qquad (26)$$

The non-tensor transformation of the functions, $t_\pm(x^\pm)$, arises as a direct consequence of the non-local nature of the Liouville-Polyakov term [1,11]. Among the other terms which appear in Eq. (24), $T_{\pm\pm}^f$, which is the classical part of the total stress energy, transforms as a tensor. $(\partial_\pm\rho)^2 - \partial_\pm^2\rho$, on the other hand, transforms according to the Schwarzian transformation equation (25). We can see this by letting $x^+ \rightarrow \tilde{x}^+$,

$$e^{2\rho}dx^+dx^- = e^{2\rho}\frac{dx^+}{d\tilde{x}^+}d\tilde{x}^+dx^-, \qquad (27)$$

i.e.

$$\rho \rightarrow \tilde{\rho} = \rho + \frac{1}{2}\log\left(\frac{dx^+}{d\tilde{x}^+}\right). \qquad (28)$$

Plugging $\tilde{\rho}$ in, we can confirm that

$$\left(\frac{d\tilde{x}}{dx}\right)^2 [(\partial_{\tilde{x}}\tilde{\rho})^2 - \partial_{\tilde{x}}^2\tilde{\rho}] = [(\partial_x\rho)^2 - \partial_x^2\rho] + \frac{1}{2}\{\tilde{x},x\}, \quad (29)$$

so that with the addition of the $t_\pm(x^\pm)$ the entire right-hand side of the stress-energy equations transforms as a tensor under a change of coordinate. As can be seen, this is also consistent with how the left-hand side transforms. The complete matter stress-energy tensor, which we denote simply by $T_{\pm\pm}$,

$$T_{\pm\pm} = T_{\pm\pm}^f - \frac{N\hbar}{12\pi}[(\partial_\pm\rho)^2 - \partial_\pm^2\rho + t_\pm(x^\pm)]$$

$$= -2\partial_\pm^2\tilde{\phi} + 4\partial_\pm\rho\partial_\pm\tilde{\phi}, \qquad (30)$$

transforms simply as a tensor under coordinate transformations for a given vacuum choice.

It is useful to define the vacuum in flat spacetime

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu. \qquad (31)$$

The scalar fields, f_i, can be decomposed into

$$f_i(x) = \sum_j [a_j u_j(x) + a_j^\dagger u_j^*(x)], \qquad (32)$$

where

$$u_j(x) = \frac{1}{\sqrt{4\pi\omega}}e^{ik\cdot x}, \quad k^0 = \omega, \qquad (33)$$

form a complete orthonormal set. The vacuum state, $|0\rangle$, is then defined such that

$$a_j|0\rangle = 0, \quad \forall\ j. \qquad (34)$$

If we wish to work with a conformally related spacetime

$$g_{\mu\nu}(x) = \Omega^2(x)\eta_{\mu\nu}, \qquad (35)$$

the scalar fields transform according to

$$f_i(x) = \Omega^{(2-n)/2}\sum_j [a_j u_j(x) + a_j^\dagger u_j^*(x)]. \qquad (36)$$

Now the vacuum state associated with the modes defined by Eq. (34) is known as the conformal vacuum.

In two dimensions, it is possible to express the stress energy of a spacetime conformally related to flat spacetime, $ds^2 = dudv$, by $d\bar{s}^2 = C(u,v)dudv$ [14]:

$$\langle T_\mu^\nu(g)\rangle = (-g)^{-1/2}\langle T_\mu^\nu(\eta)\rangle + \theta_\mu^\nu - (1/48\pi)R\delta_\mu^\nu \quad (37)$$

where

$$\theta_{uu} = -(\hbar/12\pi)C^{1/2}\partial_u^2 C^{-1/2} \qquad (38)$$

$$\theta_{vv} = -(\hbar/12\pi)C^{1/2}\partial_v^2 C^{-1/2} \qquad (39)$$

$$\theta_{uv} = \theta_{vu} \qquad (40)$$

for each scalar field. These θ terms give the Schwarzian derivatives of a function $h(u)$ when one makes the substitution $C(u,v) = \partial h/\partial u$. If the state used in evaluating the expectation value in flat spacetime is a vacuum state, then the state appearing in the curved spacetime expectation value is referred to as a conformal vacuum.

It is also possible to relate the stress-energy tensors defined in different flat spacetimes, $d\bar{s}^2 = d\bar{u}d\bar{v}$ and $ds^2 = dudv$ [15,14]. Following [15], consider a general metric

$$ds^2 = C(u,v)dudv, \qquad (41)$$

with

$$C(u,v) = A(\bar{u},\bar{v})\frac{d\bar{u}}{du}\frac{d\bar{v}}{dv}, \qquad (42)$$

where

$$v = \beta(\bar{v}) \qquad (43)$$

$$u = \beta(\bar{u} - 2R_0). \qquad (44)$$

Using Eq. (37), the stress-energy tensor with respect to the conformal vacuum is given by

$$T_{uu} = -F_u(C) \tag{45}$$

$$T_{vv} = -F_v(C), \tag{46}$$

where F denotes the function

$$F_x(y) \equiv (12\pi)^{-1} y^{1/2} \partial_x^2 y^{-1/2}. \tag{47}$$

For Eq. (42) in $\bar{u}, \bar{v}$ coordinates

$$T_{\bar{u}\bar{u}} = -F_{\bar{u}}(A) + F_{\bar{u}}(\beta') \quad \beta = \beta(\bar{u} - 2R_0) \tag{48}$$

$$T_{\bar{v}\bar{v}} = -F_{\bar{v}}(A) + F_{\bar{v}}(\beta') \quad \beta = \beta(\bar{v}). \tag{49}$$

However, since the first term on the right-hand side is equivalent to the stress energy with respect to the conformal vacuum of $A(\bar{u}, \bar{v}) d\bar{u} d\bar{v}$, this relation allows us to relate the stress-energy tensors expressed with respect to two different vacua. It can be summarized as

$$\left(\frac{du}{d\bar{u}} \right)^2 \langle 0|T_{uu}|0\rangle = \langle \bar{0}|T_{\bar{u}\bar{u}}|\bar{0}\rangle - \frac{N\hbar}{24\pi} \{u, \bar{u}\}. \tag{50}$$

Note that the last term on the right of Eq. (50) corresponds to the transformation of the stress-energy tensor when the conformal factor is $d\bar{u}/du$. That is to say, if we define the vacuum state with respect to the positive energy modes decomposed in $d\bar{s}^2 = d\bar{u}dv$ and transform to a conformally related spacetime, $ds^2 = dudv = (du/d\bar{u})d\bar{u}dv$, we obtain Eq. (50).

Let us see now what happens when we express Eq. (18) in null coordinates. The coordinate transformation $u = v + lq^3/\tilde{\phi}$ puts the metric into the form

$$d\tilde{s}^2 = -2\frac{\tilde{\phi}^2}{q^3} du dv \tag{51}$$

for $v < v_0$, and

$$d\tilde{s}^2 = -\left(2\frac{\tilde{\phi}^2}{q^3} - l\Delta m \right) d\bar{u} dv \tag{52}$$

for $v > v_0$ with

$$\bar{u} = v + \sqrt{\frac{2lq^3}{\Delta m}} \operatorname{arctanh}\left(\sqrt{\frac{2}{lq^3 \Delta m}} \tilde{\phi} \right). \tag{53}$$

Now we can see the relation between $\bar{u}$ and u:

$$u = v + \sqrt{\frac{2lq^3}{\Delta m}} \coth\left(\sqrt{\frac{\Delta m}{2lq^3}} (\bar{u} - v) \right). \tag{54}$$

Thus the outgoing flux in the null coordinate $\bar{u}$ can be calculated, since it is known that $T_{uu} = 0$ before the shock mass is introduced. Therefore

$$T_{\bar{u}\bar{u}} = \frac{N\hbar}{24\pi} \{\bar{u}, u\} = \frac{N\hbar}{24\pi lq^3} \Delta m, \tag{55}$$

which is a constant Hawking flux of radiation. We have not yet considered the effects of the back reaction, which will be done in a subsequent section. However, this preliminary examination demonstrates that we have indeed an evaporating black hole.

II. SOLUTIONS

The general solution to the stated equations of motion can be written in terms of four chiral functions, $A_\pm(x^\pm)$, and $a_\pm(x^\pm)$ [16,17] with

$$d\tilde{s}^2 = -\frac{\partial_+ A_+ \partial_- A_-}{\left(1 + \frac{\lambda^2}{2} A_+ A_-\right)^2} dx^+ dx^- \tag{56}$$

and

$$\tilde{\phi} = -\frac{1}{2} \left(\frac{\partial_+ a_+}{\partial_+ A_+} + \frac{\partial_- a_-}{\partial_- A_-} \right) + \frac{\lambda^2}{2} \frac{A_+ a_- + A_- a_+}{1 + \frac{\lambda^2}{2} A_+ A_-} \tag{57}$$

constrained by

$$\partial_+^2 \left(\frac{\partial_+ a_+}{\partial_+ A_+} \right) - \frac{\partial_+^2 A_+}{\partial_+ A_+} \partial_+ \left(\frac{\partial_+ a_+}{\partial_+ A_+} \right) = T_{++} \tag{58}$$

$$\partial_-^2 \left(\frac{\partial_- a_-}{\partial_- A_-} \right) - \frac{\partial_-^2 A_-}{\partial_- A_-} \partial_- \left(\frac{\partial_- a_-}{\partial_- A_-} \right) = T_{--}. \tag{59}$$

The first case initially studied by Fabbri, Navarro, and Navarro-Salas [5] consists of a shock mass, Δm, sent into the extremal black hole,

$$ds^2 = -\left(\frac{2\tilde{\phi}^2}{q^3} - l\Delta m\Theta(v - v_0) \right) dv^2 + 2ld\phi dv. \tag{60}$$

The gauge choice of $A_+ = x^+$ and $A_- = -2/\lambda^2 x^-$, with $\lambda^2 = l^{-2} q^{-3}$ yields

$$e^{2\rho} = \frac{2l^2 q^3}{(x^- - x^+)^2}, \tag{61}$$

$$ds^2 = -\frac{2l^2 q^3}{(x^- - x^+)^2} dx^+ dx^-. \tag{62}$$

This gauge fixes $[(\partial_\pm \rho)^2 - \partial_\pm^2 \rho]$ in the constraint equations to be identically zero everywhere. Thus, $t_\pm(x^\pm)$ represents the only quantum part of the stress-energy tensor. That is,
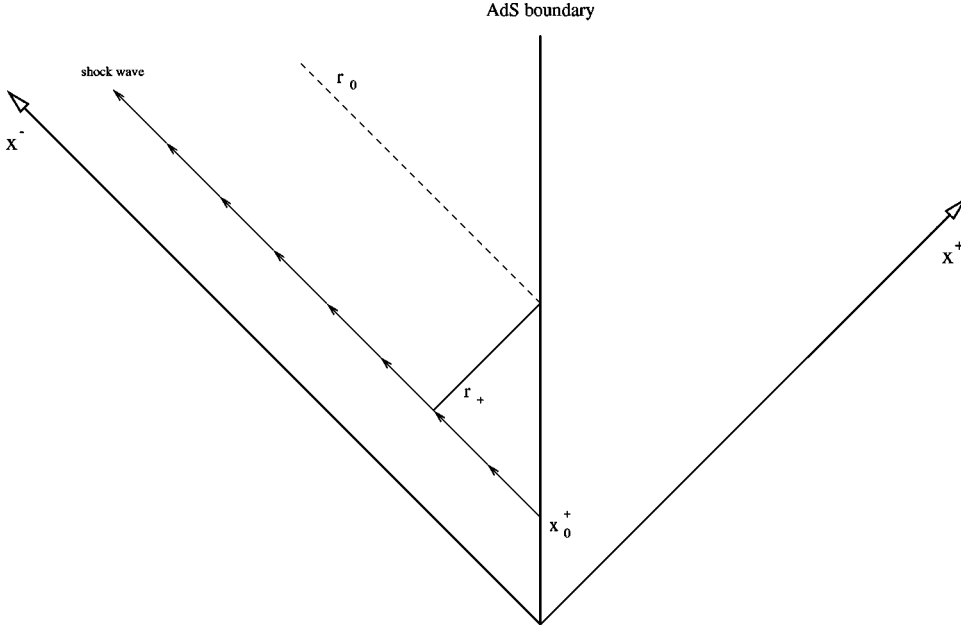
FIG. 1. Kruskal diagram for the static classical solution of the near-extremal RN black hole. The AdS$_2$ boundary is seen at $x^-=x^+$. r_+ represents the outer horizon, while r_-, the inner horizon, lies at $x^+\to\infty$. r_0, the extremal radius, does not meet with r_+ except at the AdS boundary, which represents timelike infinity.

$$T_{\pm\pm}=T_{\pm\pm}^f-\frac{N\hbar}{12\pi}t_\pm(x^\pm). \qquad (63)$$

An important consequence of this result is that the quantum nature of the solutions only manifests itself in the boundary conditions. The same solutions are obtained classically if the flux sent into the black hole coincides with the quantum boundary conditions. This will be discussed in more detail later on. The gauge choice itself corresponds to AdS$_2$ spacetime, with the AdS boundary occurring at the coordinate singularity $x^-=x^+$. The metric (60) can be brought into the gauge-fixed form by setting

$$a_+=-lq^3 \qquad (64)$$

$$a_-=0 \qquad (65)$$

for $v<v_0$. Requiring continuity at $v=v_0$, for $v>v_0$

$$a_+=-\frac{1}{2}\Delta mx_0^+(x^+-x_0^+)-lq^3, \qquad (66)$$

$$a_-=l^2q^3\Delta m\frac{x_0^+}{x^-}-l^2q^3\Delta m. \qquad (67)$$

Thus we have for $v<v_0$

$$\tilde{\phi}=\frac{lq^3}{x^--x^+}, \quad v=x^+, \qquad (68)$$

and for $v>v_0$

$$\tilde{\phi}=lq^3\frac{1-\dfrac{\Delta m}{2lq^3}(x^+-x_0^+)(x^--x_0^+)}{x^--x^+}, \qquad (69)$$

$$v=x_0^++\sqrt{\frac{2lq^3}{\Delta m}}\text{arctanh}\left[\sqrt{\frac{2lq^3}{\Delta m}}(x^+-x_0^+)\right]. \qquad (70)$$

These solutions break down as we approach $x^--x^+\sim4lq$ since the small $\tilde{\phi}$ approximations then becomes invalid. There is also a coordinate singularity in the metric that occurs when $x^-=x^+$. Equation (68) represents the vacuum of the solutions. The extremal radius, $\tilde{\phi}=0$, occurs at $x^--x^+\to\infty$. In the region below the AdS boundary, $x^-<x^+$, $\tilde{\phi}<0$. That is, this region corresponds in fact to the area behind the extremal black hole radius. The double-horizon structure manifests itself when we solve $\partial_+\tilde{\phi}=0$, giving a horizon at $x^--x^+\to\pm\infty$. So $x^->x^+$ and $x^-<x^+$ correspond to two different coordinate patches of the solutions, with $x^-<x^+$ corresponding to an area that actually lies behind $x^--x^+=\infty$. In analyzing these results, we study the area above the AdS boundary, $x^->x^+$.

Equation (69) represents the classical solution shown in Fig. 1. Let us first consider this case: the extremal radius, $\tilde{\phi}=0$, occurs at $(x^+-x_0^+)(x^--x_0^+)=2lq^3/\Delta m$. The apparent horizons, $\partial_+\phi=0$, are given by $x^-=x_0^+\pm\sqrt{2lq^3/\Delta m}$. The AdS boundary represents spatial infinity, i.e., the region infinitely far away from the black hole where the radial variable ϕ becomes infinitely large, with the black hole itself lying above $x^->x^+$. We can see then that r_+ moves further out from the center of the black hole for larger values of the shock mass Δm, as expected. r_- can be understood to be at $x^-\to\infty$. r_0 and r_+ never meet (the apparent meeting point is actually at infinity), as one would expect without Hawking evaporation due to quantum effects.

We now wish to consider the semiclassical solutions. The key to solving these is picking the appropriate boundary conditions in a given vacuum state. The boundary condition is determined by the behavior of the stress-energy flux at regions far outside of the black hole, where statements can be made about the expected flux. This amounts to making an

appropriate choice for $t_\pm(x^\pm)$, as it represents the quantum part of the stress-energy tensor, T_{++} (recall that the conformal term involving ρ derivatives is zero in this gauge).

Fabbri, Navarro, and Navarro-Salas choose vacuum conformal to spacetime

$$ds^2 = -e^{2\rho}dv\,dx^- = -e^{2\rho}\frac{dv}{dx^+}dx^+dx^-, \qquad (71)$$

with $v(x^+)$ given by Eq. (70). That is, the mode decompositions discussed beginning with Eq. (32) are with respect to the flat spacetime, $dv\,dx^-$. This vacuum choice is not well explained in [5–7]. It is stated there that since in Eq. (51), before the shock mass is introduced, the stress energy is zero, then this represents the natural vacuum choice. However, a problem arises because while $v(x^+)$ is from Eq. (70), after the shock mass ($v > v_0$), in order for

$$ds^2 = -\left(2\frac{\tilde{\phi}^2}{q^3} - l\Delta m\Theta(v - v_0)\right)dv\,du, \qquad (72)$$

to be put into the form (20), u will also be a non-trivial function of x^-. In fact, it is given by

$$u = v_0 + \sqrt{\frac{2lq^3}{\Delta m}}\coth\left(\sqrt{\frac{\Delta m}{2lq^3}}(x^- - v_0)\right). \qquad (73)$$

Regardless, in arriving at Eq. (71), the vacuum choice of [5–7], necessarily requires $u = x^-$. Therefore, the choice of Eq. (71) as vacuum space is not consistent with Eq. (51).

In general, choosing $v = v(x^+)$ is a rather restrictive condition on these solutions. This can be seen by closer inspection: let us require that $v = v(x^+)$, i.e. $dv/dx^+ = f(x^+)$, or more conveniently,

$$\frac{dv}{dx^+} = \frac{lq^3}{F(x^+)}. \qquad (74)$$

To bring the metric (60) to the form (62) it is also necessary to require that

$$\partial_-\tilde{\phi} = -\frac{F(x^+)}{(x^- - x^+)^2}, \qquad (75)$$

which means

$$\tilde{\phi} = \frac{F(x^+)}{x^- - x^+} + G(x^+). \qquad (76)$$

Plugging this into the equation of motion for $\tilde{\phi}$ then leads to $G(x^+) = F'(x^+)/2$, so that

$$\tilde{\phi} = \frac{F(x^+)}{x^- - x^+} + \frac{F'(x^+)}{2}. \qquad (77)$$

Using this form of $\tilde{\phi}$ in the Eq. (30) we see that T_{--} will be zero everywhere in the solutions. Therefore, we realize now that we are confined to boundary conditions that give $t_-(x^-) = 0$.

As long as $v = v(x^+)$, then, the vacuum choice used corresponds to zero quantum flux in the spacetime of Eq. (71), which, while not of imperative physical interest, allows us to solve the equations of motion with relative ease:

$$t_-(x^-) = 0, \qquad (78)$$

$$t_+(x^+) = \frac{1}{2}\{v, x^+\}$$

$$= \frac{2lq^3\Delta m}{(2lq^3 - (x^+ - x_0^+)^2\Delta m)^2}. \qquad (79)$$

Again, there is no outgoing flux, T_{--}, being emitted from the black hole in the x^+ direction. The evaporation of the black hole proceeds simply through the negative flux entering the black hole. This means that in a sense the evaporation of the black hole is built into the solutions from the boundary conditions. As mentioned before, the classical solutions would have yielded the same result, given the same negative ingoing flux. The contribution of Eq. (79) to the stress-energy tensor is

$$T_{++} = -\frac{N\hbar}{12\pi}\frac{2lq^3\Delta m}{[2lq^3 - (x^+ - x_0^+)^2\Delta m]^2}, \qquad (80)$$

which does in fact correspond to a negative flux of energy that increases as $x^+ - x_0^+ \to \sqrt{2lq^3/\Delta m}$,

$$\partial_+^3 a_+ = -\frac{N\hbar}{12\pi}t_+(x^+)$$

$$= -\frac{N\hbar}{12\pi}\frac{2lq^3\Delta m}{[2lq^3 - (x^+ - x_0^+)^2\Delta m]^2}, \qquad (81)$$

$$6\partial_- a_- + 6x^-\partial_-^2 a_- + (x^-)^2\partial_-^3 a_-$$

$$= 0, \qquad (82)$$

can be integrated to solve for a_+ and a_- by requiring continuity of ϕ at $x^+ = x_0^+$, and by putting Eq. (60) in the form (20). The general solutions to Eq. (81) are

$$a_+ = -\frac{1}{2}\Delta mx_0^+(x^+ - x_0^+) - lq^3 + \frac{N\hbar}{\pi}P(x^+), \qquad (83)$$

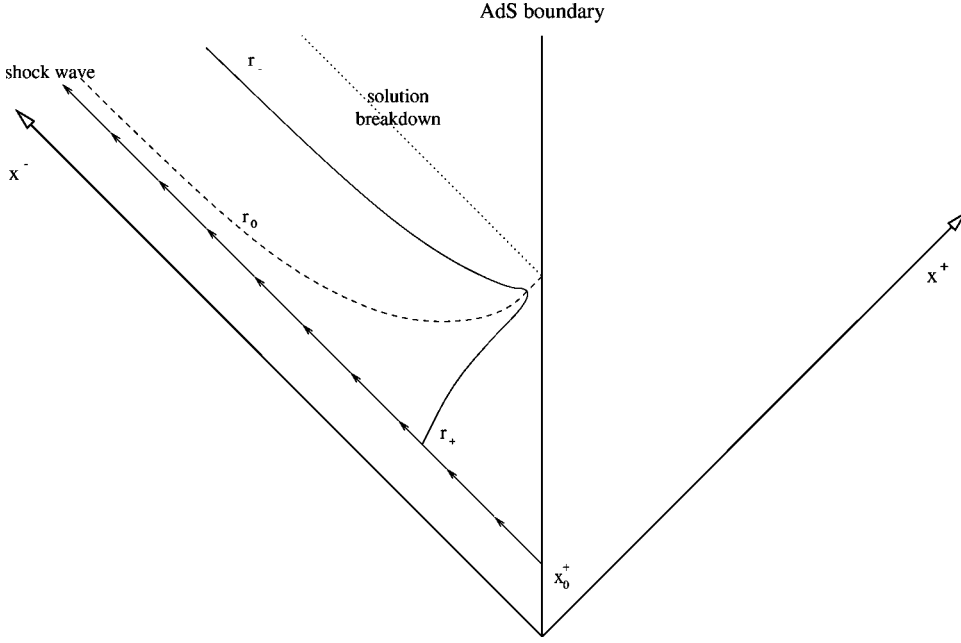$$a_- = l^2q^3\Delta m\frac{x_0^+}{x^-} - l^2q^3\Delta m, \qquad (84)$$

with

FIG. 2. Kruskal diagram for semiclassical solution of the near-extremal RN black hole. The AdS_2 boundary is again seen at $x^-=x^+$. r_+ and r_-, which are given by $\partial_+\phi=0$ evolve to meet at r_0 (ϕ_0), the extremal radius. Close to this point, the semiclassical approximation breaks down. In addition, the solutions become indeterminate as $x^+-x_0^+ \to \sqrt{2lq^3/\Delta m}$. The size of the shock mass Δm determines the degree to which the outer horizon moves out from $x^-=\infty$.

$$P(x^+)= \frac{(x^+-x_0^+)}{48} - \frac{\frac{2lq^3}{\Delta m}-(x^+-x_0^+)^2}{48\sqrt{\frac{2lq^3}{\Delta m}}}$$

$$\times \operatorname{arctanh}\left((x^+-x_0^+)\sqrt{\frac{\Delta m}{2lq^3}}\right). \qquad (85)$$

The resulting solution for $\tilde{\phi}$ is

$$\tilde{\phi}=lq^3 \frac{1-\dfrac{\Delta m(x^+-x_0^+)(x^--x_0^+)}{2lq^3}}{x^--x^+} + \frac{N\hbar}{\pi}\frac{P(x^+)}{x^--x^+}$$

$$+\frac{N\hbar}{2\pi}P'(x^+). \qquad (86)$$

Let us take a moment to carefully examine $P(x^+)$ and its properties. We may take note that $\operatorname{arctanh}(x)$ becomes logarithmically divergent as $x\to 1$. However, $(x^2-1)\operatorname{arctanh}(x) \to 0$ as $x\to 1$ remains finite. Therefore $P(x^+)$ becomes indeterminate for $x^+-x_0^+\gtrsim\sqrt{2lq^3/\Delta m}$ and $P'(x^+)$, which is logarithmically divergent, enlarges. Since ϕ represents the radial coordinate, and it depends directly on $P'(x^+)$, we can interpret $x^+-x_0^+\to\sqrt{2lq^3/\Delta m}$ as spatially being infinitely far from the black hole. It is not possible to evolve the solutions beyond this point. However, when the inner and outer horizon meet again at the extremal radius, at the end point of black hole evaporation, the semiclassical approximation has already broken down. This happens before we reach the divergence of $P'(x^+)$.

In the semiclassical solution, the inner and outer horizon come together and meet at the extremal $r=r_0$ radius (see Fig. 2), consistent with our picture of black hole evaporation.

If we consider the extremal black hole as the limit of a near-extremal black hole, it has a double horizon which becomes spatially separated with the introduction of a shock mass. A larger shock mass corresponds to a bigger separation of r_+ and r_-. However, as the black hole evaporates, the two apparent horizons should eventually approach each other and return to the extremal limit. The classical solution of the equations of motion (69) is recovered by taking the limit $\hbar\to 0$. These solutions do not demonstrate any outgoing flux, as a result of the imposed condition that $T_{--}=0$. All evaporation manifests itself in a negative ingoing flux $T_{++}<0$. We should also keep in mind that Eq. (20) is conformally related to the dimensional reduction of the physical metric (1), so it is necessary to make further calculations to understand what is really happening.

Note that there are approximations that have been made which need to be reexamined. Bringing the metric (60) into the conformal gauge form (20) necessarily requires that

$$2l\partial_-\tilde{\phi}\partial_+v(x^+)+e^{2\rho}\ll 1 \qquad (87)$$

and

$$\left(-2\frac{\tilde{\phi}^2}{q^3}+l\Delta m\right)[\partial_+v(x^+)]^2+2l\partial_+\tilde{\phi}\partial_+v(x^+)\ll 1. \qquad (88)$$

Putting solutions for $\tilde{\phi}$, Eq. (86), into Eq. (87) yields the constraint that

$$\frac{4l^2q^3\kappa P(x^+)}{(x^--x^+)^2[(x^+-x_0^+)^2\Delta m-2lq^3]}\ll 1 \qquad (89)$$

plus a more complicated constraint that we omit due to space and aesthetic considerations. We must monitor the quantities on the left-hand side of the above equations to ensure that

our approximations are valid in the regions of interest. This has been done for all ensuing discussion.

III. THE PHYSICAL METRIC

The analysis has thus far been incomplete because the stress-energy tensor considered does not correspond to an observer living in $3+1$ dimensions. One of our new contributions is to study components of the stress energy measured by an observer freely falling in the $(3+1)$-dimensional spacetime. This physical metric ds^2 is related to the metric studied in the previous section $d\widetilde{s}^2$ (20) by the conformal factor, $\sqrt{\phi}$:

$$ds^2 = \frac{1}{\sqrt{\phi}} d\widetilde{s}^2 = -\frac{e^{2\rho}}{\sqrt{\phi}} dx^+ dx^-. \tag{90}$$

To construct a stress energy tensor that couples to this metric (90) we must add extra matter fields to the action (19) that couple in a covariant way. This addition will not alter the previous equations of motion as long as the number of other matter fields N is large.

It is relevant to consider what happens to a freely falling observer coming in from far outside of the black hole. Since it is difficult to analytically describe the geodesic for an affinely parametrized freely falling observer, we consider the next best thing: a null in-falling observer. The Christoffel symbols for the physical metric (90) are

$$\Gamma^+_{++} = 2\partial_+\rho - \frac{1}{2}\partial_+\log\phi, \tag{91}$$

$$\Gamma^-_{--} = 2\partial_-\rho - \frac{1}{2}\partial_-\log\phi. \tag{92}$$

From the geodesic equation for x^-, we get

$$\frac{d^2 x^-}{d\tau^2} + \left(2\partial_-\rho - \frac{1}{2}\partial_-\log\phi\right)\frac{dx^-}{d\tau}\frac{dx^-}{d\tau} = 0, \tag{93}$$

with a similar equation for x^+. We consider the case where τ is an affinely parametrized null geodesic $\widetilde{x}^-$ such that

$$ds^2 = -\frac{e^{2\rho}}{\sqrt{\phi}}\frac{dx^+}{d\widetilde{x}^-}\frac{dx^-}{d\widetilde{x}^-} = 0. \tag{94}$$

This has solutions for fixed x^+, leaving

$$\widetilde{x}^- = \widetilde{x}^-(x^-). \tag{95}$$

Using

$$\frac{\partial\rho}{\partial x^-}\frac{dx^-}{d\widetilde{x}^-} = -\frac{\partial\rho}{\partial x^+}\frac{dx^+}{d\widetilde{x}^-} + \frac{d\rho}{d\widetilde{x}^-} \tag{96}$$

and the fact that we are working with a null geodesic, Eq. (93) reduces to

$$\frac{d^2 x^-}{d\widetilde{x}^{-2}} + \frac{d(2\rho - \frac{1}{2}\log\phi)}{d\widetilde{x}^-}\frac{dx^-}{d\widetilde{x}^-} = 0, \tag{97}$$

which is then solved to give

$$\frac{d\widetilde{x}^-}{dx^-} = C\frac{e^{2\rho(x^-)}}{\sqrt{\phi}}, \tag{98}$$

where C is a constant of integration which we can set equal to 1. Using the conformal factor arising from Eq. (90) in the relation (37) gives

$$T_{\pm\pm} = \frac{\hbar}{24}\left(\frac{3}{8}\frac{(\partial_\pm\phi)^2}{\phi^2} - \frac{1}{2}\frac{\partial^2_\pm\phi}{\phi} + \frac{\partial_+\rho\partial_+\phi}{\phi} + 2\partial^2_+\rho\right.$$
$$\left. - 2(\partial_+\rho)^2 - 2t_\pm(x^\pm)\right), \tag{99}$$

where again, $t_\pm(x^\pm)$ are determined by the boundary conditions (i.e., vacuum choice). Part of this tensor, $N\hbar/12\pi[\partial^2_+\rho - (\partial_+\rho)^2 - t_\pm]$, is the source term on the right-hand side of the equations of motion. This is because the matter fields couple to $\sqrt{-g} = e^{2\rho}/2$, and hence the above terms contribute to the back reaction. The stress energy above is not a source for the back reaction, but is what an observer traveling through the physical spacetime would measure.

In the affinely parametrized coordinates, the stress energy is

$$\widetilde{T}_{\pm\pm} = \left(\frac{dx^\pm}{d\widetilde{x}^\pm}\right)^2 T_{\pm\pm} \tag{100}$$

$$= \frac{\hbar}{24}\frac{\phi}{e^{4\rho}}\left(\frac{3}{8}\frac{(\partial_\pm\phi)^2}{\phi^2} - \frac{1}{2}\frac{\partial^2_\pm\phi}{\phi} + \frac{\partial_+\rho\partial_+\phi}{\phi} + 2\partial^2_+\rho\right.$$
$$\left. - 2(\partial_+\rho)^2 - 2t_\pm(x^\pm)\right), \tag{101}$$

where $\widetilde{T}$ is used to denote the stress-energy tensor with respect to $\widetilde{x}^\pm$. We consider the behavior in the weak back-reaction regime, where $N\hbar/(24\pi q^2)\ll 1$, where the adiabatic approximation should be valid. Far outside of the black hole, closer to the AdS boundary, when $\phi - \phi_0 \gg \phi_h - \phi$ (using ϕ_h to denote the radius at the horizon, $\partial_+\phi = 0$), while still within the validity of the near-horizon approximation ($\phi - \phi_0 \ll 1$) the flux in and out will have a more physically intuitive interpretation. We hope that since the contours of ϕ we consider are very close to the AdS boundary, they represent sufficiently well the behavior that occurs at "infinity," without actually leaving the near-horizon region of our calculations [18]. For example, consider the contour depicted in Fig. 3. Here, $\phi > \phi_0$, yet it is small enough to be consistent with the previous approximation, $\widetilde{\phi}/\phi_0 \ll 1$. We are interested in the behavior of the physical stress energies, as seen by an affine observer along the inner and outer horizon, in
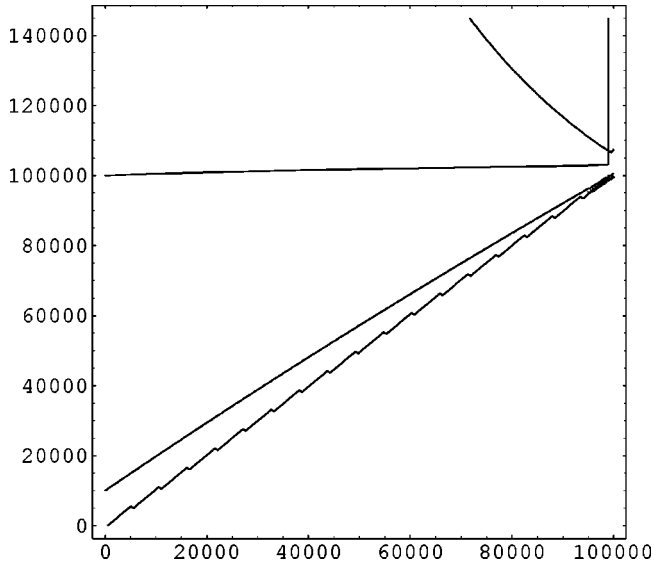
FIG. 3. The outer horizon is shown as it recedes to meet the inner horizon at ϕ_0. Also shown is the contour for fixed radius $\tilde{\phi} = 100$, near the AdS boundary, for which subsequent plots were made. [$l = 1, q = 100; \Delta m = .0002; N\hbar = 25\pi; \phi_0 = 2500.$]

order to test the stability of the evaporating black hole. Figs. 4–6 illustrate the values of the stress energy along these contours. We note from these that the stress energy varies smoothly throughout the evaporation process. The stress energy tensors for an affinely parametrized observer evaluated at each point along the fixed radial contour are shown in Figs. 7 and 8. In each case, the flux approaches zero as the black hole evaporates, which is consistent with the evaporation of a near-extremal black hole, which should cease as the black hole returns to its extremal state. In order to make physical sense of the quantities, it may be useful to look at the difference $\tilde{T}_{--} - \tilde{T}_{++}$, Fig. 9. This is where we can have a reasonable interpretation of what is going on, as the
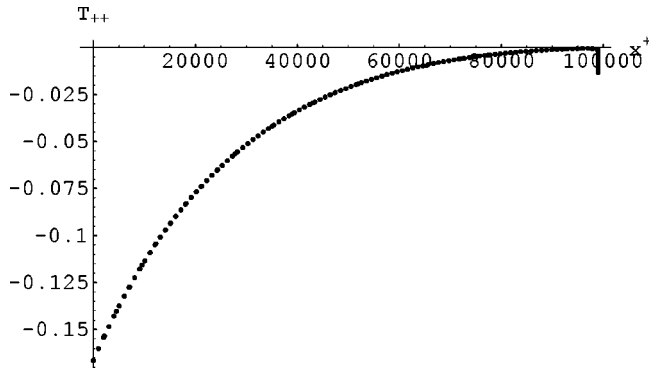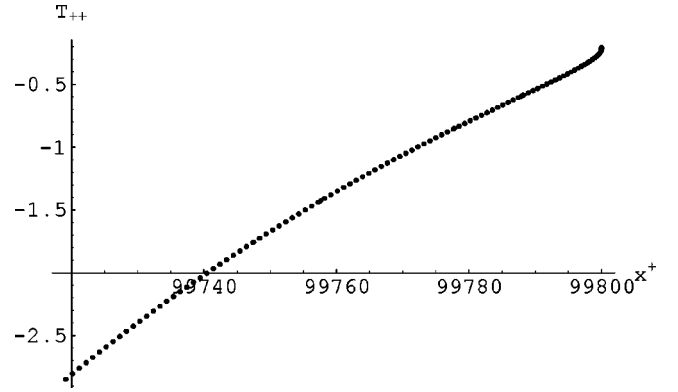

FIG. 4. $\tilde{T}_{++}$, the stress energy of an affine null observer in physical spacetime $ds^2 = -(e^{2\rho}/\sqrt{\phi})dx^+dx^-$, evaluated at points along the outer horizon, up to the point at which the two horizons meet again. Notably, the evolution of the stress is smooth. In this vacuum choice, $\tilde{T}_{++}$ increases quickly with increasing x^-. Therefore, as the outer horizon recedes to meet the inner horizon, there is a large increase in the stress energy toward the end of the evaporation. [$l = 1, q = 100; \Delta m = .0002; N\hbar = 25\pi; \phi_0 = 2500.$]

FIG. 5. $\tilde{T}_{++}$, the stress energy of an affine null observer in physical spacetime $ds^2 = -(e^{2\rho}/\sqrt{\phi})dx^+dx^-$, evaluated at points along the inner horizon, up to the point at which the two horizons meet again. $\tilde{T}_{++}$ decreases very quickly with x^-, as the inner horizon moves on a nearly null-like trajectory. [$l = 1; q = 100; \Delta m = .0002; N\hbar = 25\pi; \phi_0 = 2500.$]

difference will represent the net flux going through a surface of constant ϕ. The net flux going through a surface of fixed radius is positive and vanishes away with time. The integral of this quantity along the contour would give us the Arnowitt-Deser-Misner (ADM) mass, defined at spatial infinity outside of a black hole. The indication then is that the shock mass evaporates away, returning the black hole to its extremal state. We observe that the differential black hole ADM mass increases for a bit, before decreasing down close to zero. The temporary rise in mass before dying down may be consistent with observations by [7]. Nevertheless, the important quantity is its integral.

We can also consider the Bondi mass. This is generally defined at future null infinity $x^+ \to \infty$, giving $m(x^-)$. The AdS boundary of these solutions makes it difficult to use this definition. However, as discussed in [7,19,20], because T_{--} is chosen everywhere to be zero, it is possible to define a Bondi mass $m(x^+)$ for all x^- with
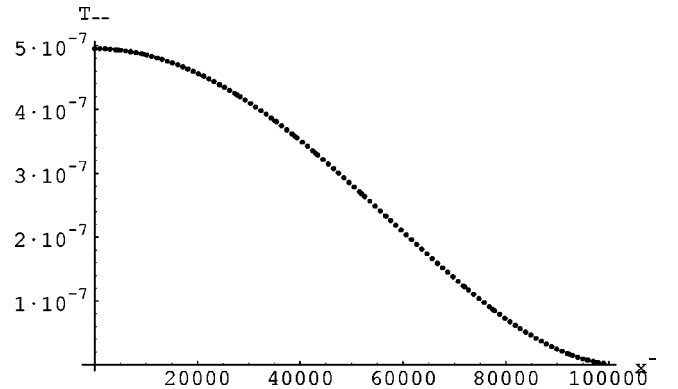

FIG. 6. $\tilde{T}_{--}$, the stress energy of an affine null observer in physical spacetime $ds^2 = -(e^{2\rho}/\sqrt{\phi})dx^+dx^-$, evaluated at points along the outer horizon, up to the point at which the two horizons meet again. The behavior of $\tilde{T}_{--}$ reflects the Hawking radiation leaving the black hole and reaching zero as the black hole returns to extremality. Along the inner horizon, $\tilde{T}_{--}$ is essentially zero. [$l = 1, q = 100; \Delta m = .0002; N\hbar = 25\pi; \phi_0 = 2500.$]
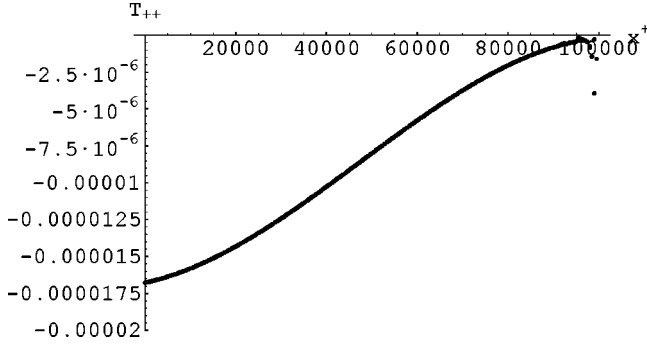
FIG. 7. $\tilde{T}_{++}$, the stress energy of an affine null observer in physical spacetime $ds^2 = -(e^{2\rho}/\sqrt{\phi})dx^+ dx^-$, shown at points along the fixed radius $\tilde{\phi} = 100$. $\tilde{T}_{++}$ is negative, a remnant of the boundary conditions which demonstrates that negative flux gets sent in to reduce the black hole mass. This flux goes to zero as the black hole returns to extremality. [$l=1, q=100; \Delta m = .0002; N\hbar = 25\pi; \phi_0 = 2500.$]

$$m_S(x^+) = m_{S0} - 2l \int dx^+ e^{-2\rho} \partial_- \tilde{\phi} T_{++}. \quad (102)$$

We want to verify that $\partial_- m_S = 0$. By applying the partial derivative with respect to x^- to the above we obtain

$$\partial_- m_S(x^+) = -2l \int dx^+ e^{-2\rho} T_{++} [-2\partial_- \rho \partial_- \tilde{\phi} + \partial_-^2 \tilde{\phi}], \quad (103)$$

where we used the fact that $\partial_- T_{++} = 0$. Further manipulation, using the equations of motion, gives

$$\partial_- m_S(x^+) = l \int dx^+ e^{-2\rho} T_{++} \left(T_{--}^f - \frac{N\hbar}{12\pi} t_-(x^-) \right). \quad (104)$$

The bracketed term, then, must be constrained to zero for the mass formula to be valid, which is indeed the case for these
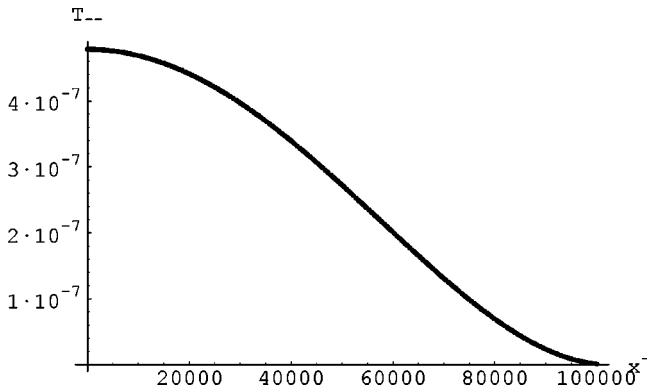

FIG. 8. $\tilde{T}_{--}$, the stress energy of an affine null observer in physical spacetime $ds^2 = -(e^{2\rho}/\sqrt{\phi})dx^+ dx^-$, shown at points along the fixed radius $\tilde{\phi} = 100$. This positive outward flux goes to zero as the black hole returns to extremality. [$l=1, q=100; \Delta m = .0002; N\hbar = 25\pi; \phi_0 = 2500.$]
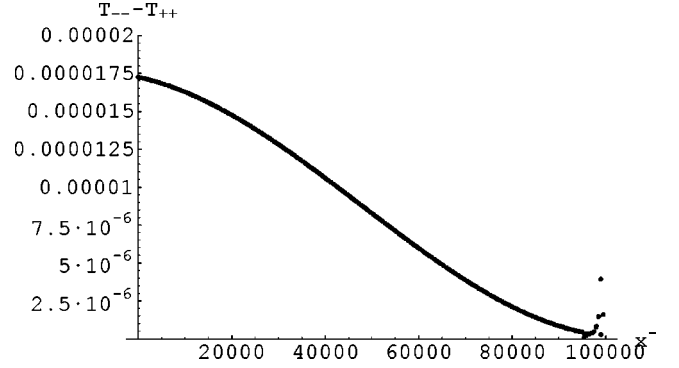

FIG. 9. $\tilde{T}_{--} - \tilde{T}_{++}$, the stress energy of an affine null observer in physical spacetime $ds^2 = -(e^{2\rho}/\sqrt{\phi})dx^+ dx^-$, shown at points along the fixed radius $\tilde{\phi} = 100$. This quantity is related to the differential ADM mass of the black hole. The area under the curve corresponds to the finite shock mass of the black hole. [$l=1, q=100; \Delta m = .0002; N\hbar = 25\pi; \phi_0 = 2500.$]

solutions. A calculation of this mass using the above derived values yields to first order in $N\hbar$

$$m_S(x^+) = \Delta m - \frac{N\hbar}{12\pi} \sqrt{\frac{\Delta m}{2lq^3}} \operatorname{arctanh}\left((x^+ - x_0^+) \sqrt{\frac{\Delta m}{2lq^3}} \right). \quad (105)$$

A plot of $m_S(x^+)$ (Fig. 10) shows that evaporation occurs slowly until close to the meeting of the horizons, at which point the mass significantly drops. The mass evaporates to zero at the same value of x^+ where the outer apparent horizon recedes back to the extremal radius. That is to say, when $m_S(x_f^+) = 0$, $r_0(x_f^+) = r_+(x_f^+)$. The overshooting of the zero-point suggests that the evaporation does not end once extremality is reached. However, the semiclassical description of black hole radiation is applicable only as long as

$$\left| T \left(\frac{\partial T}{\partial m_S} \right) \right| \ll |T|, \quad (106)$$
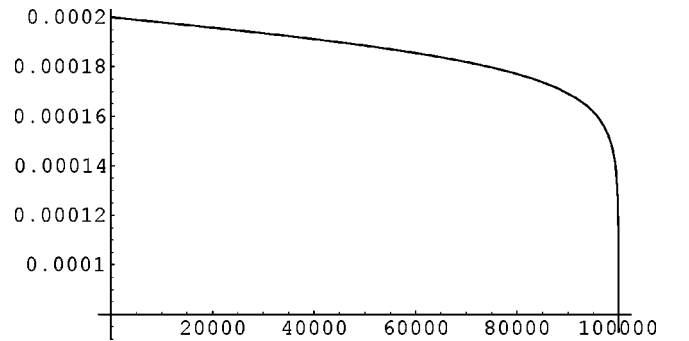

FIG. 10. The Bondi mass, evaluated for the conformally rescaled spacetime $ds^2 = -e^{2\rho} dx^+ dx^-$, does not indicate significant evaporation until close to the very end of black hole evaporation. This plot overshoots the zero mass point when the semiclassical approximation breaks down at the end point of evaporation. [$l=1, q=100; \Delta m = .0002; N\hbar = 75\pi; \phi_0 = 2500.$]

where T is the black hole temperature. The temperature fluctuations must remain small compared to the temperature itself [4]. This means that our solutions no longer describe the evolution of the black hole once it has returned to its extremal state. We can only trust our results up to $m_S(x^+) = 0$, when the outer horizon r_+ and the inner horizon r_- meet again at r_0.

IV. DYNAMICAL BOUNDARY SOLUTIONS

The other case considered by Fabbri, Navarro, and Navarro-Salas [6,7] involves a questionable choice of boundary conditions. Here, instead of sending in a shock mass and observing the evolution of the black hole, the shock mass is permitted "quantum corrections" and allowed to behave dynamically. That is, $m_S(v)$, the shock mass, is now allowed to vary with v, instead of being expressed simply by the $\Delta m \Theta(v - v_0)$ function. In this case, the solutions for $\tilde{\phi}$ cannot be found analytically. As stated before, it is required that $v = v(x^+)$, or more conveniently, we choose $v(x^+)$ to be of the form (74). Repeating the results following (74), and once again gauge fixing ρ so that Eq. (61) still holds, we see that

$$-2\partial_+^2 \tilde{\phi} + 4\partial_+ \rho \partial_+ \tilde{\phi} = -F'''(x^+) = T_{++}^f - \frac{N\hbar}{12\pi} t_+(x^+).$$
(107)

From the Schwarzian derivative we have

$$t_+(x^+) = \frac{1}{2}\{v, x^+\} = \frac{1}{2}\left(\frac{1}{2}\frac{F'(x^+)^2}{F(x^+)^2} - \frac{F''(x^+)}{F(x^+)}\right).$$
(108)

Thus

$$T_{++}^f = \Delta m \, \delta(x^+ - x_0^+)$$

$$= -F'''(x^+) + \frac{N\hbar}{24\pi}\left[\frac{1}{2}\left(\frac{F'(x^+)}{F(x^+)}\right)^2 - \frac{F''(x^+)}{F(x^+)}\right].$$
(109)

Continuity of $\tilde{\phi}$ requires that
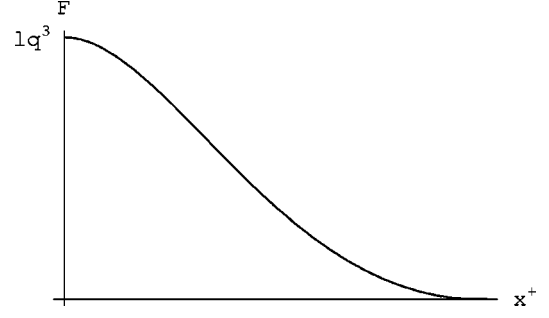
$$F(x_0^+) = lq^3,$$
(110)

FIG. 11. A plot of the behavior of the function $F(x^+)$ which approaches zero with increasing x^+. $F = 0$ signals the end point of evaporation, which occurs at the AdS boundary, $x^- = x^+$, representing timelike infinity.

$$F'(x_0^+) = 0.$$
(111)

It follows further from Eq. (109) that

$$F''(x_0^+) = -\Delta m.$$
(112)

It is now possible to numerically solve for $F(x^+)$, using Eq. (109) and the boundary conditions (110)–(112). Typical behavior for $F(x^+)$ can be seen in Fig. 11. We set

$$\left(-2\frac{\tilde{\phi}^2}{q^3} + l\Delta m\right)[\partial_+ v(x^+)]^2 + 2l\partial_+ \tilde{\phi}\partial_+ v(x^+) = 0,$$
(113)

which is required in order to eliminate off-diagonal components in Eq. (20), when making the coordinate transformations. This allows us to solve for

$$m_S(x^+) = \frac{F'^2(x^+) - 2F(x^+)F'(x^+)}{2lq^3}.$$
(114)

Using Eq. (109) this can be rewritten as

$$m_S(x^+) = \frac{24\pi}{N\hbar l q^3}F^2(x^+)F'''(x^+).$$
(115)

In a less straightforward manner, we could have also used Eqs. (75) and (61) to evaluate the mass based on the previous definition (102),

$$\partial_+ m_S(x^+) = -\frac{F(x^+)F'''(x^+)}{lq^3}$$
(116)

$$= -\frac{F(x^+)F'''(x^+) + F(x^+)F''(x^+) - F(x^+)F''(x^+)}{lq^3}$$
(117)

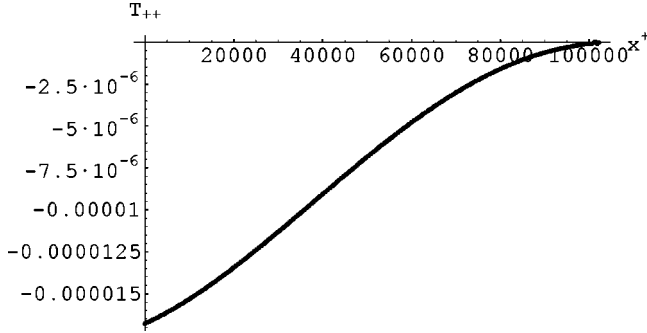$$= \partial_+\left(\frac{24\pi F(x^+)^2 F'''(x^+)}{N\hbar l q^3}\right),$$
(118)

FIG. 12. $\tilde{T}_{++}$, the stress energy of an affine null observer in physical spacetime $ds^2 = -e^{2\rho}/\sqrt{\tilde{\phi}}dx^+dx^-$, evaluated at points along the fixed radius $\tilde{\phi} = 100$. Again, the flux is negative, indicating that negative mass is being sent in as a result of the boundary conditions. This flux goes to zero as the black hole evaporation draws to an end. [$l = 1, q = 100; \Delta m = .0002; N\hbar = 75\pi; \phi_0 = 2500.$]

arriving at Eq. (114). We can see now that

$$\partial_+ m_S(x^+) = -\frac{N\hbar}{24\pi}\frac{m_S(x^+)}{F(x^+)}. \tag{119}$$

Multiplying by dx^+/dv

$$\partial_v m_S(v) = -\frac{N\hbar}{24\pi l q^3}m_S(v), \tag{120}$$

so that

$$m_S(v) = \Delta m e^{-(N\hbar/24\pi)[(v-v_0)/lq^3]}\Theta(v-v_0). \tag{121}$$

Using a complementary method, it is also possible to define instead a late time Bondi mass [21,22], which behaves as

$$\partial_u m_S(u) = -\frac{N\hbar}{24\pi l q^3}m_S(u). \tag{122}$$

We can now use the fact that the four-dimensional stress-energy tensor can be related back to the rate of change of the mass [7,23],

$$T_{vv}^{(4)} = \partial_v m_S(v) = \Delta m \delta(v-v_0) - \frac{N\hbar}{24\pi l q^3}m_S(v)\Theta(v-v_0), \tag{123}$$

to see that there is a negative flux of energy being sent in to reduce the black hole mass [21]. Generally, however, it does not make sense to allow the shock mass being sent in to vary as a function of x^+ for all values of x^-, including those corresponding to very large distances away, where quantum effects due to an "infinitely" far black hole ought to be negligibly small. Since $m_S(v)$ now behaves as shown above, as opposed to simply $m_S(v) = \Delta m\Theta(v-v_0)$, then this solution corresponds to sending in negative mass after the initial Δm. This is an artifact of the boundary condition requirement which forces $T_{--} = 0$ everywhere, so that basically the black hole evaporation occurs through flux being sent in from in-
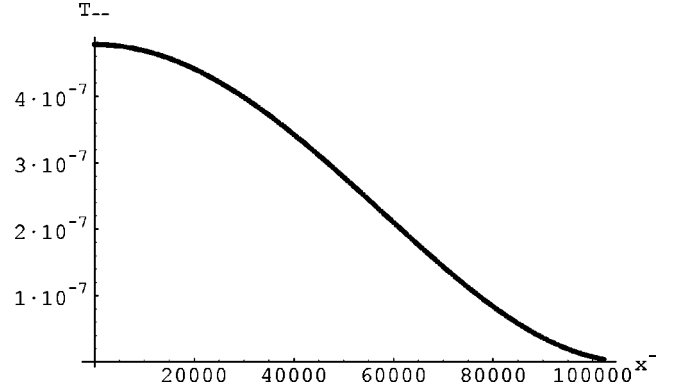

FIG. 13. $\tilde{T}_{--}$, the stress energy of an affine null observer in physical spacetime $ds^2 = -e^{2\rho}/\sqrt{\tilde{\phi}}dx^+dx^-$, evaluated at points along the fixed radius $\tilde{\phi} = 100$. This outward positive flux goes to zero as the black hole returns to extremality. [$l = 1, q = 100; \Delta m = .0002; N\hbar = 75\pi; \phi_0 = 2500.$]

finity outside. The previous boundary condition, however, was more natural, since the diminishing mass was arrived at, rather than put in by hand. Nonetheless, it may be argued that the difference between these two boundary conditions is minute. In both cases, the same general phenomena is occurring; $T_{--} = 0$ and $T_{++} < 0$ bring about the evolution of the black hole.

The stress-energy tensor can again be calculated for the affinely null coordinates. We proceed in the same manner as before, evaluating $\tilde{T}_{++}$ and $\tilde{T}_{--}$ for the physical metric (90). We see from the stress-energy tensors, $\tilde{T}_{++}$, in Fig. 12 that there is indeed a negative flux of energy entering the black hole. In order to bring about exponential decay of the shock mass, it was necessary to send in negative mass to bring it down. $\tilde{T}_{--}$, (Fig. 13) behaves also as expected. There is a flux of energy being emitted from the black hole which approaches zero as the black hole evaporates away. The differential ADM mass, as represented by $\tilde{T}_{--} - \tilde{T}_{++}$
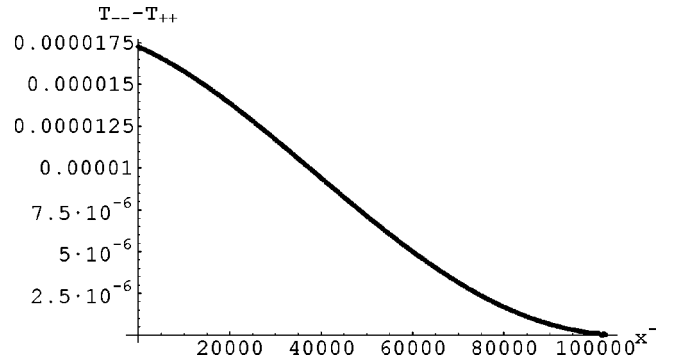

FIG. 14. $\tilde{T}_{--} - \tilde{T}_{++}$, the stress energy of an affine null observer in physical spacetime $ds^2 = -e^{2\rho}/\sqrt{\tilde{\phi}}dx^+dx^-$, shown at points along the fixed radius $\tilde{\phi} = 100$. This quantity is related to the differential ADM mass of the black hole. The area under the curve corresponds to the finite shock mass of the black hole. The boundary conditions were chosen so that this quantity goes to zero monotonically. [$l = 1, q = 100; \Delta m = .0002; N\hbar = 75\pi; \phi_0 = 2500.$]

(Fig. 14) decays down to zero, as a result of the combined effect of the negative energy being sent in and the natural black hole emission.

V. CONCLUSION

We have considered toy models for semiclassical charged black hole evaporation with back reaction included. The calculations of the physical stress-energy tensor of a freely falling observer allowed us to verify that an observer at a fixed distance outside of the black hole perceives a flux decaying to zero, as the black hole evaporates away the injected shock mass and returns to the extremal state. The affine stress energy also varies smoothly across the inner and outer apparent horizons, suggesting that there is no buildup of energy in places which would have undermined the validity of the semiclassical solutions. Hence, while in these solutions the black hole evaporation is largely dictated by the negative flux of energy entering the black hole, the vacuum choices discussed do indeed yield a picture consistent with charged black hole evaporation in string theory [24] without encountering the singularities feared by [3]. Nevertheless, there are some questions and difficulties of interpretation that would

be better answered by working in an asymptotically Minkowskian spacetime. Most notably, we are troubled by the apparent arbitrariness of the imposed boundary conditions. It would be more natural to choose a vacuum in a spacetime that corresponds to what an observer infinitely far from the black hole might observe. In addition, the evaporation of the black hole in these solutions is inherently dictated *a priori* by $t_\pm(x^\pm)$. It would be more satisfying if quantum effects did not have to be predicated, but rather were naturally manifested as the solutions evolved forward in time. More physically motivated boundary conditions render the equations of motion less easily solvable, making it necessary to solve a set of coupled partial differential equations instead of the ordinary differential equations used here. This would have to be done numerically. This work is currently being completed [25–27]. Preliminary results, however, seem to be consistent with those presented here [28].

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