

# Compactified M/string theory prediction (August 2011) of the Higgs boson mass and properties

$$\rightarrow M_h = 126 \pm 2 \text{ GeV, SM-like}$$

EXPLAIN

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Ultimately we would like an underlying predictive theory for physics beyond the SM, and derivation/explanations of SM itself – compactified M/string theory seems to provide a good framework – turns out can make some predictions, in particular  $M_h$

Surprising?

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Ann Arbor, February 1, 2013

- ❑ Introduction – making string theory predictions for data
  - *assumptions* – not directly related to Higgs sector
  - stabilizing moduli – crucial for derivation
  - $\mu$  in string theory
- ❑ Higgs mass derivation
- ❑ Results
- ❑ Implications
- ❑ Little hierarchy problem reduced
- ❑ Associated LHC predictions for gluinos, charginos
- ❑ Naturalness?
- ❑ Final remarks

**Goal: Understand the ground state of our M/string theory – we live there – M/string theory provides powerful framework Beyond the Standard Model**

***There has not been enough thought about what it means to make predictions, explanations from string theory for data – predictions, explanations should be based on generic projection of extra dimensional theories into 4D large spacetime, plus small dimensions***

***Non-generic → less explanatory, maybe risk contradictions***

## □ Philosophy to compute Higgs mass, properties:

**Divide all compactified string/M theories into two classes**

- **Some generically have TeV scale physics, EWSB, no contradictions with cosmology, etc – study all these -- if our world is described by a compactified string/M theory it will look like these – turns out it's easy to find them**
- **The rest**

*Find many – “compactified constrained string/M theories”*

**Calculate/derive  $M_h / M_Z$  for those solutions**

-- at end remark on absolute calculation of  $M_h$

# PAPERS ABOUT M-THEORY COMPACTIFICATIONS ON $G_2$ MANIFOLDS

(11 D – 7 small D = our 4D)

**Earlier work** (stringy, mathematical) :

- Review of supergravity work, Duff hep-th/0201062
- Papadopoulos, Townsend th/9506150, 7D manifold with  $G_2$  holonomy preserves **N=1 supersymmetry**
- Acharya, hep-th/9812205, **non-abelian gauge fields localized on singular 3 cycles**
- Acharya, hep-th/0011289
- Atiyah and Witten, hep-th/0107177
- Atiyah, Maldacena, Vafa, hep-th/0011256
- Acharya and Witten, hep-th/0109152, **chiral fermions supported at points with conical singularities**
- Witten, hep-ph/0201018 – shows **embedding MSSM probably ok**
- Beasley and Witten, hep-th/0203061, **Kahler form**
- Friedmann and Witten, th/0211269
- Lukas, Morris hep-th/0305078, **gauge kinetic function**
- **Acharya and Gukov, hep-th/0409101 – review – good summary of known results about singularities, holonomy and supersymmetry, etc – all  $G_2$  moduli geometric – gravity mediated because two 3-cycles won't interact directly in 7D manifold**

We started M/string compactification fall of 2005,  
interested in moduli stabilization, susy breaking, Higgs,  
since LHC coming

***Do the derivations here in M-theory case since those calculations effectively complete – results may hold in some or all other corners of string theory since they depend on only a few generic features of resulting soft-breaking Lagrangian (but  $\mu$ ,  $\tan\beta$ ?)***

## Our M-theory papers

--*Review arXiv:1204.2795 , Acharya, Kane, Kumar*

[**Acharya**, Kane, Piyush Kumar, Bobkov, Kuflik, Shao, Ran Lu, Watson, Bob Zheng]

- M-Theory Solution to Hierarchy Problem th/0606262
- **Stabilized Moduli, TeV scale, *squark masses = gravitino mass, heavy; gaugino masses suppressed*** 0701034
- **Spectrum, scalars heavy, wino-like LSP, large trilinears (no R-symmetry) 0801.0478**
- Study moduli, **Nonthermal** cosmological history– **generically moduli  $\gtrsim 30$  TeV so gravitino  $\gtrsim 30$  TeV, squarks  $\approx$  gravitino so squarks  $\geq 30$  TeV** 0804.0863
- CP Phases in M-theory (weak CPV OK) and EDMs 0905.2986
- **Lightest moduli masses  $\lesssim$  gravitino mass** 1006.3272 (Douglas Denef 2004; Gomez-Reino, Scrucca 2006)
- **Axions** stabilized, strong CP OK, string axions OK 1004.5138
- Gluino, Multi-top searches at **LHC** (also Suruliz, Wang) 0901.336
- No flavor problems, (also Velasco-Sevilla Kersten, Kadota)
- Theory, phenomenology of  $\mu$  in M-theory 1102.0566 via Witten
- Baryogenesis, ratio of DM to baryons (also Watson, Yu) 1108.5178
- String-motivated approach to little hierarchy problem, (also Feldman) 1105.3765
- **Higgs Mass Prediction** 1112.1059 (Kumar, Lu, Zheng)

Will explain details as relevant during talk – to take Higgs results fully seriously good to know other major physics questions addressed OK in same theory

**”GENERIC”**  $\approx$  perhaps not theorem, but holds very generally – just calculate naturally without special assumptions – have to work hard to find or construct (non-generic) exceptions (if possible), and to show possible exceptions don’t have problems that exclude them

**String theory only fully predictive if results generic – not generic *means* tuning something**

**– could have nature’s theory being constrained by M/string theory framework but needing limited tuning – but hopefully not**

**Take compactifications seriously**



Next briefly compare M-**theory** derivation with **models** assuming heavy scalars – first James Wells hep-th/0302127, Nelson et al

- See many features are different – alert you to watch for them during derivations
- History very distorted, even recently

## COMPACTIFIED (STRING) M THEORY

- **Derive** solution to large hierarchy problem
- Generic solutions with **EWSB derived**
- main F term drops out of **gaugino masses** so **dynamically suppressed**
- **Trilinears**  $> M_{3/2}$  necessarily
- **$\mu$  incorporated in theory (M-theory)**
- Little hierarchy significantly reduced
- **Scalars =  $M_{3/2} \sim 50$  TeV necessarily**, scalars not very heavy
- **Glino lifetime  $\lesssim 10^{-19}$  sec**, decay in beam pipe
- **$M_h \approx 126$  GeV unavoidable**, predicted

## SPLIT SUSY (ETC) MODELS

- Assumes **no solution (possible) for large hierarchy problem**
- **EWSB assumed**, not derived
- **Gauginos suppressed by assumed R-symmetry**, suppression arbitrary
- Trilinears small, suppressed compared to scalars
- **$\mu$  not in theory** at all; guessed to be  $\mu \sim M_{3/2}$
- **No solution to little hierarchy**
- Scalars **assumed** very heavy, whatever you want, e.g.  $10^{10}$  GeV
- **Long lived gluino**, perhaps meters or more
- **Any  $M_h$  allowed**

# Now Main Derivation -- Make **assumptions**, *not closely related to Higgs sector*

- CC problem orthogonal – won't know for sure until solved
- **Our world is described by compactified M-theory on  $G_2$  manifold in fluxless sector** (can try to repeat for other corners of string theory)
- **Assume Hubble parameter  $H$  at end of inflation larger than  $M_{3/2}$**
- **Assume top quark with yukawa coupling  $\sim 1$  (work underway)**
- **Include  $\mu$  via discrete symmetry** (Witten 2002) (underway)
- **Use generic Kahler potential (Beasley, Witten, 2002)** – include volume dependence on Kahler
- **Use generic gauge kinetic function from Lukas, Morris, 2003**
- **Assume gauge group and matter content at compactification is MSSM – can repeat for any other gauge group and matter content**

**Will see that the prediction of 126 is not an accident or a planned result**

**It is here to stay in generic theory**

## GENERICALLY THESE CONDITIONS IMPLY 126 -- overview

- ❑ Compactification  $\rightarrow$  moduli  $\rightarrow M_{\text{lightest modulus}} \geq 30 \text{ TeV}$  by BBN
- ❑ ~~Susy~~ by some gaugino condensation  $\rightarrow M_{3/2} > M_{\text{lightest modulus}}$
- ❑  $CC \approx 0$ , Supergravity  $\rightarrow M_{\text{soft scalars}} > M_{\text{lightest modulus}}$
- ❑  $\mu$  doubly suppressed since need broken symmetry to remove  $\mu$  from superpotential but  $\mu \neq 0$
- ❑ REWSB conditions easy to satisfy
- ❑  $1.5 \mu \tan\beta \approx M_{3/2}$  from supergravity and EWSB
- ❑  $A \approx e^{K/2} F^\varphi K_\varphi > M_0$

## □ Moduli, gravitino constraint from BBN

In early universe, when Hubble scale  $H$  decreases, moduli begin to oscillate in their potential, and quickly dominate energy density of universe – Early universe matter dominated, a “non-thermal” history

When  $H \sim$  moduli decay width,  $\Gamma_{\text{mod}} \sim M_{\text{mod}}^3/m_{\text{pl}}^2$  then the moduli decay  $\rightarrow$  need  $M_{\text{mod}} \gtrsim 30 \text{ TeV}$  so decay occurs before nucleosynthesis – moduli decay dilutes DM, decay regenerates DM  $\rightarrow$  **wino-like LSP**

Then theorem relating lightest moduli and gravitino  $\rightarrow M_{3/2} \gtrsim 30 \text{ TeV}$  –

Then supergravity  $\rightarrow$  **scalar masses (squarks, higgs scalars)  $\gtrsim 30 \text{ TeV}$**

[Avoid BBN problem by late inflation? – Randall, Thomas 9407208-- extremely difficult – many attempts – , RandallMoroi 2000de Gouvea, Moroi, Murayama ph/9701244 – Fan, Reece, Wang 1106.6044 – Choi et al recent – **NOT GENERIC**]

## □ Generic relation of lightest moduli and gravitino masses

– basically that the gravitino is not lighter than lightest modulus –  
(assumes supersymmetry breaking is involved in stabilizing at least one moduli)

[Denef and Douglas hep-th/0411183, Gomez-Reino and Scrucca hep-th/0602246, Acharya Kane Kuflik 1006.3272]

Moduli mix with scalar goldstino, which generically has gravitino mass

Consider moduli mass matrix (but don't need to calculate it) --

Sgoldstino 2x2 piece of moduli mass matrix has mass scale  $M_{3/2}$

For pos def mass matrix smallest eigenvalue of full matrix is smaller than any eigenvalue of (diagonal) submatrices →

$$M_{\min}^2 < m_{3/2}^2 \left( 2 + \frac{|r|}{m_{pl}^2} \right)$$

→  $M_{3/2} \gtrsim M_{\text{lightest modulus}} \gtrsim 30 \text{ TeV (BBN)}$

## MODULI STABILIZATION (about 10 slides)

- All  $G_2$  moduli fields have axionic partners which have a shift symmetry in the absence of fluxes (different from heterotic or IIB) – such symmetries can only be broken by non-perturbative effects
- So in zero-flux sector only contributions to superpotential are non-perturbative, from strong dynamics (e.g. gaugino condensation or instantons) – focus on former
- In M theory the superpotential, and gauge kinetic function, in general depend on all the moduli – all moduli geometric, on equal footing
- The hidden sector gaugino condensation produces an effective potential that stabilizes all moduli



A set of Kahler potentials, consistent with  $G_2$  holonomy and known to describe some explicit examples, was given by Beasley-Witten [th/0203061](#); Acharya, Denef, Valandro [th/0502060](#), with

$$K = -3 \ln(4\pi^{1/3} V_X)$$

$$V_X = \prod_{i=1}^N s_i^{a_i}, \quad \text{with} \quad \sum_{i=1}^N a_i = 7/3$$

$$[V_X = V_7]$$

We assume we can use this. More generally the volume will be multiplied by a function with certain invariances.

Assume hidden sector gaugino condensation

$$W = \sum_{k=1}^M A_k e^{i b_k f_k}$$

gauge kinetic function

One term enough to stabilize moduli -- normally keep two terms -- enough to find solutions with good properties such as being in supergravity regime, simple enough to do most calculations semi-analytically (as well as numerically)

$b_k = 2\pi/c_k$  where  $c_k$  are dual coxeter numbers of hidden sector gauge groups ---  $A_k$  are constants of order unity, depend on threshold corrections to gauge couplings

$$b_1 = 2\pi/P, b_2 = 2\pi/Q$$

(Not “racetrack” – once moduli have any interaction they are stabilized)

The gauge kinetic functions here are integer linear combinations of all the moduli (Lukas, Morris th/0305078),

$$f_k = \sum_{i=1}^N \underline{N_i^k} z_i .$$

The microscopic constants  $a_i$ ,  $b_k$ ,  $A_k$ ,  $N_i^k$  are determined for a given  $G_2$  manifold (but not yet fully known) --they completely characterize the vacua – not dependent on moduli

For semi-analytic examples focus on the (well-motivated) case where two hidden sector gauge kinetic functions are equal (the corresponding three-cycles are in the same homology class)]

Include generic massless hidden sector chiral fermion states  $q$  with  $N_c$  colors,  $N_f$  flavors,  $N_f < N_c$  -- then (Affleck, Dine, Seiberg PRL 51(1983)1026, Seiberg hep-th/9402044, hep-th/9309335, Lebedev, Nilles, Ratz th/0603047)

$$W = A_1 e^{i \frac{2\pi}{N_c - N_f} \sum_{i=1}^N N_i^{(1)} z_i} \det(Q\tilde{Q})^{-\frac{1}{N_c - N_f}} = A_1 \phi^a e^{ib_1 f_1}$$

and define an effective meson field

$$\phi \equiv \left( \det(Q\tilde{Q}) \right)^{1/2} = \phi_0 e^{i\theta}$$

Note  
notation  
Q,q only  
here

Chiral fermions localized at pointlike conical singularities, so bulk moduli should have little effect on local physics, so assume matter Kahler potential slowly varying

$$W = A_1 \phi^a e^{ib_1 f} + A_2 e^{ib_2 f}$$

$$K = -3 \ln(4\pi^{1/3} V_X) + \phi \bar{\phi}$$

Calculate F terms  $\rightarrow F_{\text{matter}} \sim M_{3/2} M_{\text{pl}}, \quad F_{\text{mod}} \sim \alpha_{\text{gut}} M_{3/2} M_{\text{pl}}$

Meson F-terms dominate  $\rightarrow$  deS vacuum

The N=1 SUGRA scalar potential is then given by:

$$\begin{aligned}
V &= \frac{e^{\phi_0^2}}{48\pi V_X^3} [(b_1^2 A_1^2 \phi_0^{2a} e^{-2b_1 \vec{\nu} \cdot \vec{a}} + b_2^2 A_2^2 e^{-2b_2 \vec{\nu} \cdot \vec{a}} + 2b_1 b_2 A_1 A_2 \phi_0^a e^{-(b_1+b_2) \vec{\nu} \cdot \vec{a}} \cos((b_1 - b_2) \vec{N} \cdot \vec{t} + a\theta)) \\
&\times \sum_{i=1}^N a_i (\nu_i)^2 + 3(\vec{\nu} \cdot \vec{a})(b_1 A_1^2 \phi_0^{2a} e^{-2b_1 \vec{\nu} \cdot \vec{a}} + b_2 A_2^2 e^{-2b_2 \vec{\nu} \cdot \vec{a}} + (b_1 + b_2) A_1 A_2 \phi_0^a e^{-(b_1+b_2) \vec{\nu} \cdot \vec{a}} \\
&\times \cos((b_1 - b_2) \vec{N} \cdot \vec{t} + a\theta)) + 3(A_1^2 \phi_0^{2a} e^{-2b_1 \vec{\nu} \cdot \vec{a}} + A_2^2 e^{-2b_2 \vec{\nu} \cdot \vec{a}} + 2A_1 A_2 \phi_0^a e^{-(b_1+b_2) \vec{\nu} \cdot \vec{a}} \\
&\times \cos((b_1 - b_2) \vec{N} \cdot \vec{t} + a\theta)) + \frac{3}{4} \phi_0^2 (A_1^2 \phi_0^{2a} \left(\frac{a}{\phi_0^2} + 1\right)^2 e^{-2b_1 \vec{\nu} \cdot \vec{a}} + A_2^2 e^{-2b_2 \vec{\nu} \cdot \vec{a}} \\
&+ 2A_1 A_2 \phi_0^a \left(\frac{a}{\phi_0^2} + 1\right) e^{-(b_1+b_2) \vec{\nu} \cdot \vec{a}} \cos((b_1 - b_2) \vec{N} \cdot \vec{t} + a\theta)]. \tag{101}
\end{aligned}$$

Condition for deS minimum is

$$3 - \frac{8}{Q-P} - \frac{28}{P \ln\left(\frac{A_1 Q}{A_2 P}\right)} < 0.$$

So setting this to zero allows eliminating  $P \ln\left(\frac{A_1 Q}{A_2 P}\right)$ . Then meson vev is

$$\phi_0^2 \approx -\frac{1}{8} + \frac{1}{Q-P} + \frac{1}{4} \sqrt{1 - \frac{2}{Q-P}} + \frac{2}{Q-P} \sqrt{1 - \frac{2}{Q-P}}.$$

So from square root see  $Q-P > 2$ . For  $Q-P > 3$  the supergravity validity may fail  $\rightarrow$  take  $Q-P=3$  for solutions. Then moduli vevs are

$$s_i = \frac{14Q}{\pi N(3(Q-P) - 8)}.$$

Can show  $M_{3/2}$  dependence on  $Q$ ,  $N$  (number of moduli) enters only through  $V_7$  - (For  $N_F$  chiral fermion flavors take  $Q-P > 3N_F$ )

# □ From Planck scale to 50 TeV “dimensional transmutation”

Scale of gaugino condensation  $\Lambda \approx M_{\text{pl}} \exp(-8\pi^2 / 3Qg^2) \approx \exp(2\pi \text{Im}f / 3Q)$

where  $\text{Im}f = \sum N_i s_i$

Q is rank of condensing gauge group

With  $Q-P=3$ ,  $\text{Im}f=14Q/\pi \rightarrow \Lambda \approx M_{\text{pl}} e^{-28/3} \approx 2 \times 10^{14}$  GeV, so

**$\Lambda \approx 10^{-4} M_{\text{pl}} \approx \text{scale at which supersymmetry broken}$**

Then  $W \sim \Lambda^3 \sim 10^{-12} M_{\text{pl}} \sim 2 \times 10^6$  GeV =  $2 \times 10^3$  TeV. Also expect inverse volume factor  $1/V_7$  from  $e^{K/2}$  so

$$\mathbf{M_{3/2} \approx e^{K/2} W \sim 50 \text{ TeV}}$$

Note  $\text{Im}f/Q$  not explicitly dependent on  $Q$  – still dependent because of  $V_7$  and  $P_{\text{eff}}$ , but weakly – so  $\Lambda$  rather well determined



## More details on gravitino mass – semi-analytic example

$$m_{3/2} \equiv m_p^{-2} e^{\frac{K}{2m_p^2}} |W|$$

Q,P ranks of typical gauge groups from 3-cycle singularities, Q=6,7,8,9 –  
moduli vevs  $\sim 3Q \sim 1/\alpha_{\text{GUT}}$  -- put CC=0 to solve for  $\text{PIn}(\ )=P_{\text{eff}}$

$$m_{3/2} = m_{\text{pl}} \frac{\alpha_{\text{GUT}}^{7/2}}{\sqrt{\pi}} \frac{|Q-P|}{Q} e^{-\frac{P_{\text{eff}}}{Q-P}}$$

→  $m_{3/2} \approx 50 \text{ TeV}$

$$(e^{-20} \approx 10^{-9}, P_{\text{eff}} = \frac{14(3(Q-P)-2)}{3(3(Q-P)-2\sqrt{6(Q-P)})} \sim 60 \text{ when } Q - P = 3)$$

$$M_{\text{GUT}} = M_{11} \alpha_{\text{Gut}}^{1/3}$$

## □ DE SITTER VACUUM, GAUGINO MASSES SUPRESSED

-- With only compactification moduli one gets AdS extrema – minima, maxima, saddle points (no go theorems, Maldacena and Nunez...) – some break susy, some preserve it

-- For M theory, positive F terms from chiral fermion condensates automatically present, cancel for CC and give deS minima – “uplift”

-- also, in M theory case the deS minima come from susy preserving extremum if ignore meson F terms, so the minima is near a susy preserving point in field space where gaugino masses would vanish

-- so SM gaugino masses are doubly suppressed – vanish at susy preserving point, and get no contribution from large F terms of mesons

$$M_{1/2} \sim K_{mn} F_m \partial_n \mathbf{f}_{SM}$$

-- can't calculate suppression precisely, estimate  $\sim$  scalars/50

-- general situation not known – gauginos suppressed in heterotic, IIB?

(nightmare scenario?)

## □ Including $\mu$ parameter in string theory ( $W = \mu H_u H_d + \dots$ so $\mu \sim 10^{16}$ GeV)

- Normally  $\mu$  and  $\tan\beta$  treated as parameters, constrained to get EWSB
- **Ultimately want to derive them from first principles**
- If  $\mu$  in  $W$  then it should be of order string scale
- **Need symmetry to set  $\mu=0$**
- Witten, hep-ph/0201018 – found discrete symmetry for  $G_2$  compactification, closely connected to doublet-triplet splitting problem, proton lifetime, R-parity
- **Unbroken discrete symmetry so  $\mu \equiv 0$**  – when moduli are stabilized the effects generally not invariant so in M-theory with moduli stabilized the symmetry is broken
- **$\mu$  proportional to  $M_{3/2}$**  since  $\mu \rightarrow 0$  if susy unbroken
- **Also  $\mu$  proportional to moduli vev** since  $\mu \rightarrow 0$  if moduli not stabilized
- **Stabilization led to moduli vev/ $M_{pl} \lesssim 0.1$**
- **So finally expect  $\mu < 0.1 M_{3/2}$**
- discrete symmetry anomalous,  $Z_9$  ok – sub group unbroken  $\rightarrow$  Rparity

arXiv:1102.0556, Acharya, Kane,  
Kuflik, Lu

## □ WHY IS $M_H$ LIGHT? -- QUICK SUMMARY

-- Recall no EWSB at high scale, generated by RGE running

High scale, compactified M theory, orbifold and conical singularities → gauge and chiral matter → gaugino and meson condensates, F-terms, supersymmetry-breaking, moduli stabilization, deS vacuum

Typical gauge groups → gaugino condensation  $\sim 10^{-4-5} M_{\text{planck}}$ , cubed in superpotential, so  $M_{3/2} \sim 50 \text{ TeV}$  (top down)

$M_{3/2} >$  smallest eigenvalue of moduli mass matrix  $\gtrsim 30 \text{ TeV}$ , from BBN

**Calculate** soft-breaking Lagrangian: scalars, trilinears, b -- ALL  $\sim M_{3/2}$

$\mu$  superpotential term zero from discrete symmetry – broken by moduli stabilization, so  $\mu_{\text{eff}} \sim (\text{moduli vev}/M_{\text{pl}}) M_{3/2} < \text{few TeV}$

At high scale Higgs sector soft terms  $\sim M_{3/2}$ , no EWSB

Then  $M_{H_u}^2$  runs down, satisfies EWSB conditions (REWSB)

**Now go through details**

## Higgs sector

In supersymmetric theory two higgs doublets present for anomaly cancellation – by “Higgs mass” mean mass of lightest CP-even neutral scalar in Higgs sector

Precise value depends on all the soft-breaking parameters including  $B, \mu$

**Why 125 GeV? – no simple formula, must do RGE running, relate terms, smallest eigenvalue of matrix**

Higgs potential at any scale – calculated at compactification scale, no parameters, then do RGE running to other scales

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (b H_u^0 H_d^0 + \text{c.c.}) + \text{D terms}$$

→ Higgs mass matrix  $\begin{pmatrix} m_{H_u}^2 + \mu^2 & -b \\ -b & m_{H_d}^2 + \mu^2 \end{pmatrix}$

Need negative eigenvalue for EWSB

$\tan\beta = v_u/v_d$  only meaningful after EWSB, doesn't exist at high scales  
– parameter before, now calculate it approximately

# Renormalization Group Equations

$$8\pi^2 \frac{dm_{H_u}^2}{dt} = 3|\lambda_t|^2 (m_{H_u}^2 + m_Q^2 + m_T^2 + |A_{top}|^2)$$

$$8\pi^2 \frac{dm_T^2}{dt} = 2|\lambda_t|^2 (m_{H_u}^2 + m_Q^2 + m_T^2 + |A_{top}|^2)$$

$$8\pi^2 \frac{dm_Q^2}{dt} = |\lambda_t|^2 (m_{H_u}^2 + m_Q^2 + m_T^2 + |A_{top}|^2)$$

$$8\pi^2 \frac{dA_{top}}{dt} = 6\lambda_t^2 A_{top}$$

$$8\pi^2 \frac{dB}{dt} = 3\lambda_t^2 A_{top}$$

## EWSB, $\mu$ , $\tan\beta$ , naturalness

Usual EWSB conditions [so higgs potential minimum away from origin]:

$$M_Z^2 = -2\mu^2 + 2(M_{Hd}^2 - M_{Hu}^2 \tan^2\beta)/\tan^2\beta = -2\mu^2 + 2M_{Hd}^2/\tan^2\beta - 2M_{Hu}^2$$

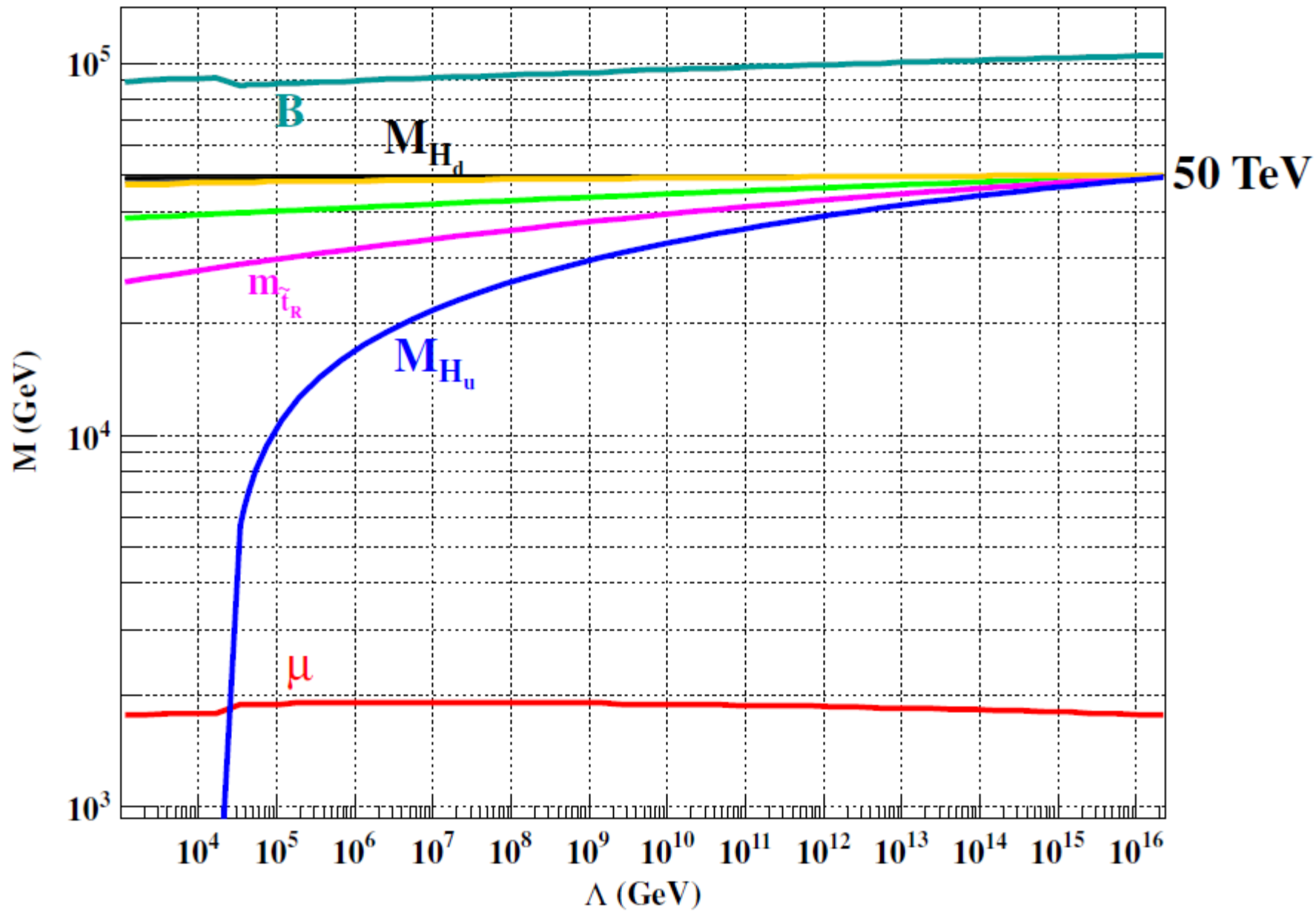
$$2B\mu = \sin 2\beta (M_{Hu}^2 + M_{Hd}^2 + 2\mu^2)$$

$M_{Hu}^2$  runs to be small,  $M_{Hd}^2$  and  $B$  don't run much,  $\mu$  suppressed,  
 $\sin 2\beta \approx 2/\tan\beta$

**If no  $\mu$  from superpotential, and visible sector Kahler metric and Higgs bilinear coefficient independent of meson field, and if  $F_{\text{mod}} \ll F_\phi$  then  $B$  (high scale)  $\approx 2M_{3/2}$  – recall  $\mu < 0.1M_{3/2}$**

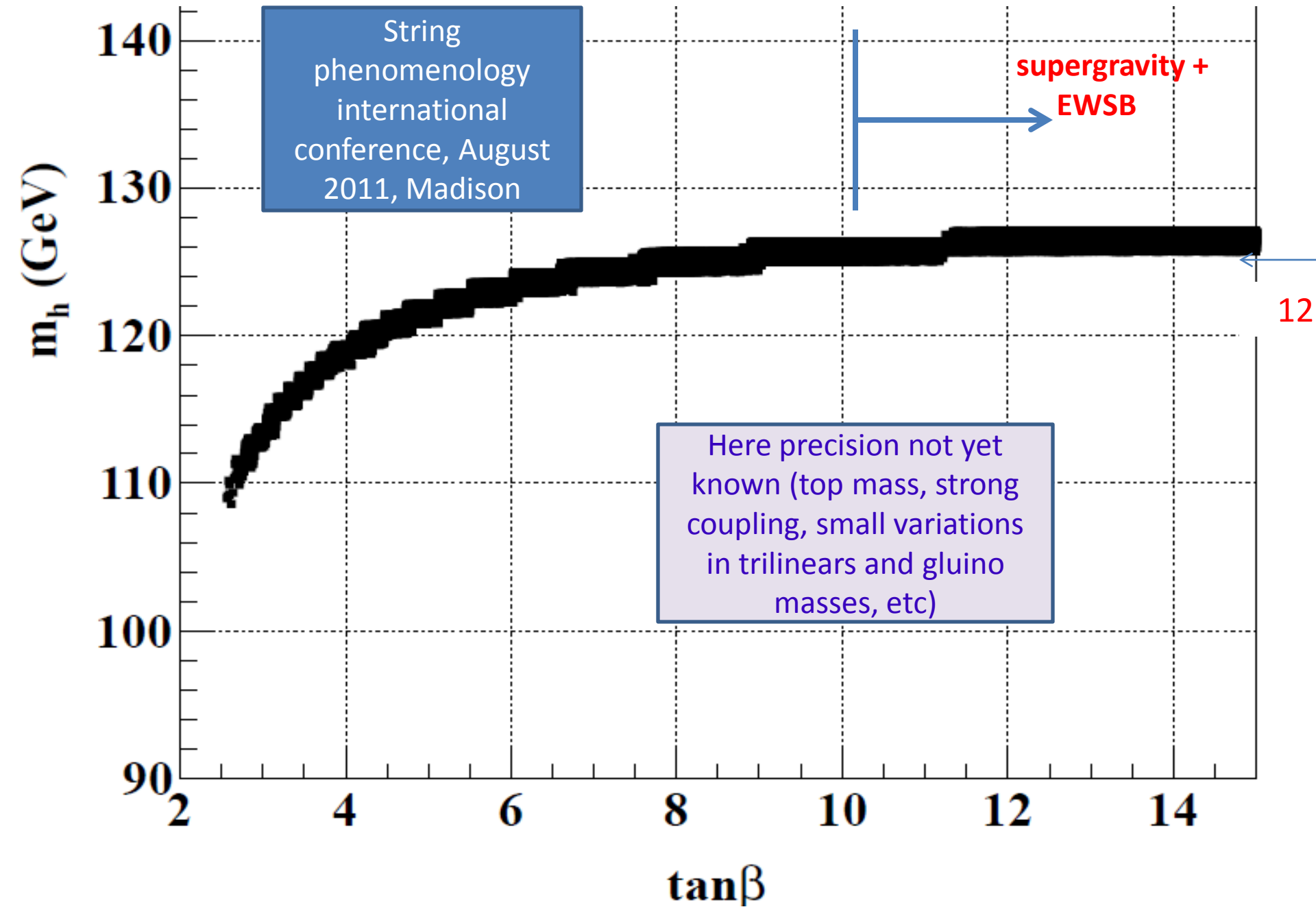
$$\rightarrow \tan\beta \approx M_{Hd}^2/B\mu \approx M_{3/2}^2/B\mu \rightarrow \tan\beta \approx M_{3/2}/2\mu (\sim 15)$$

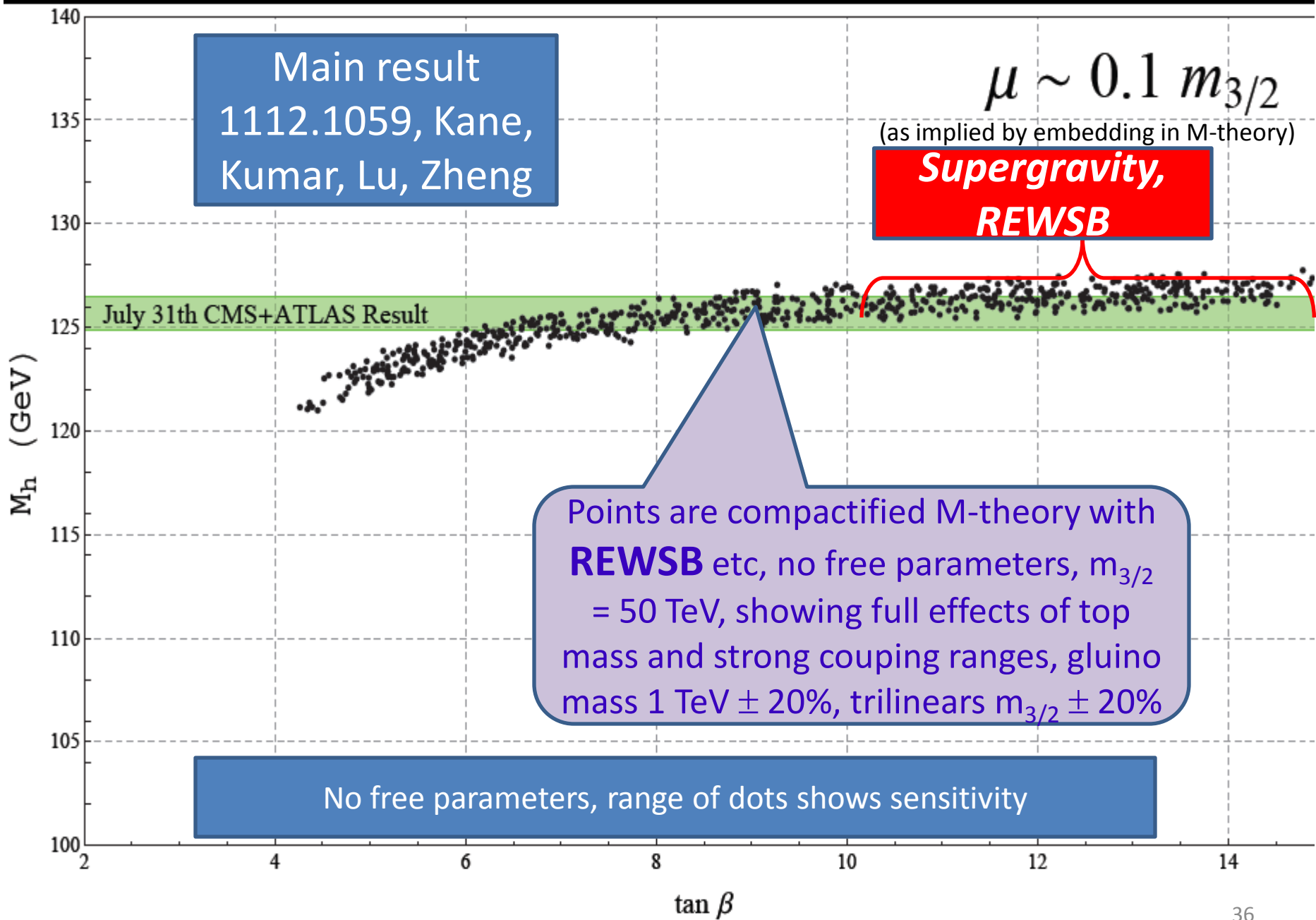


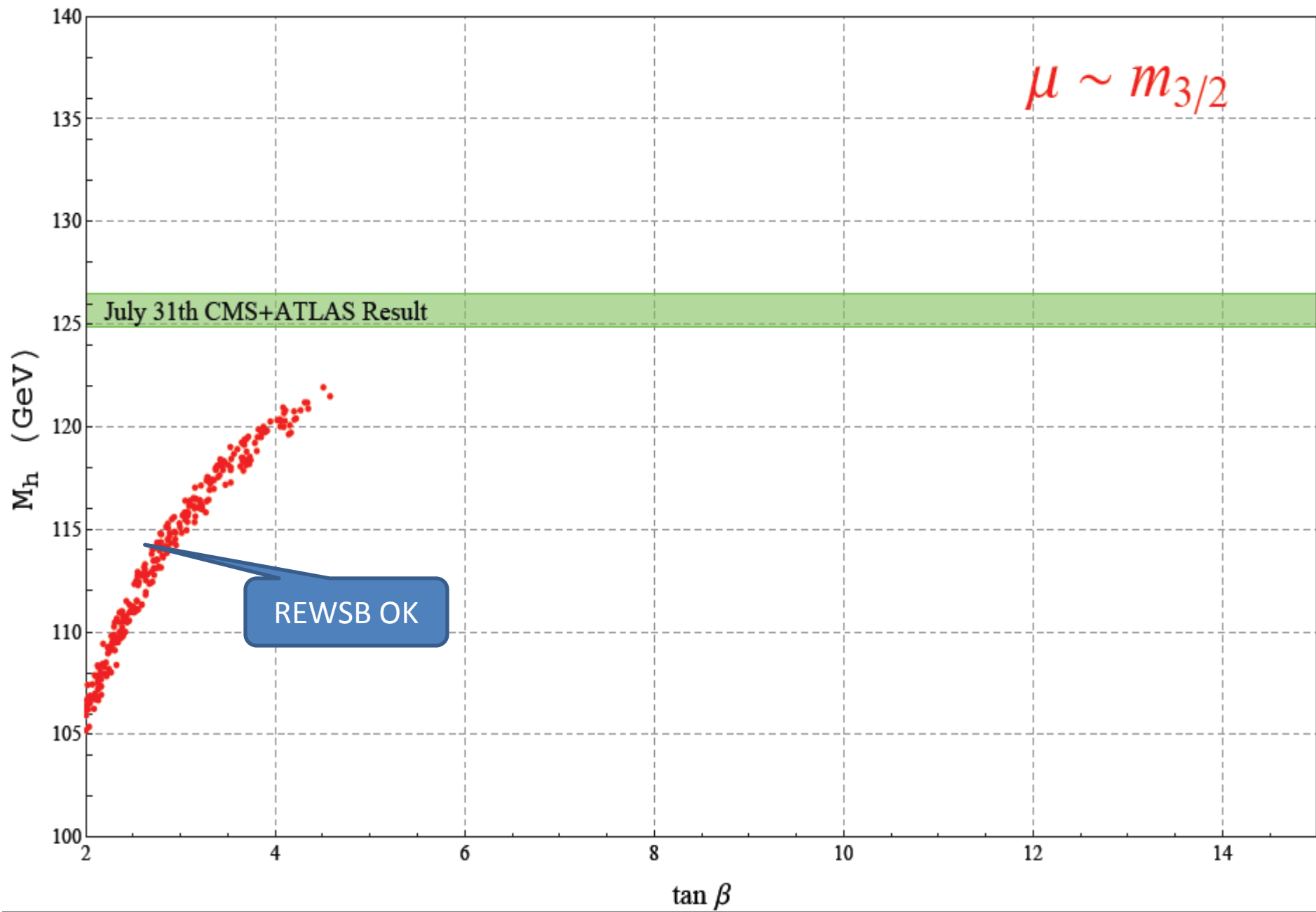


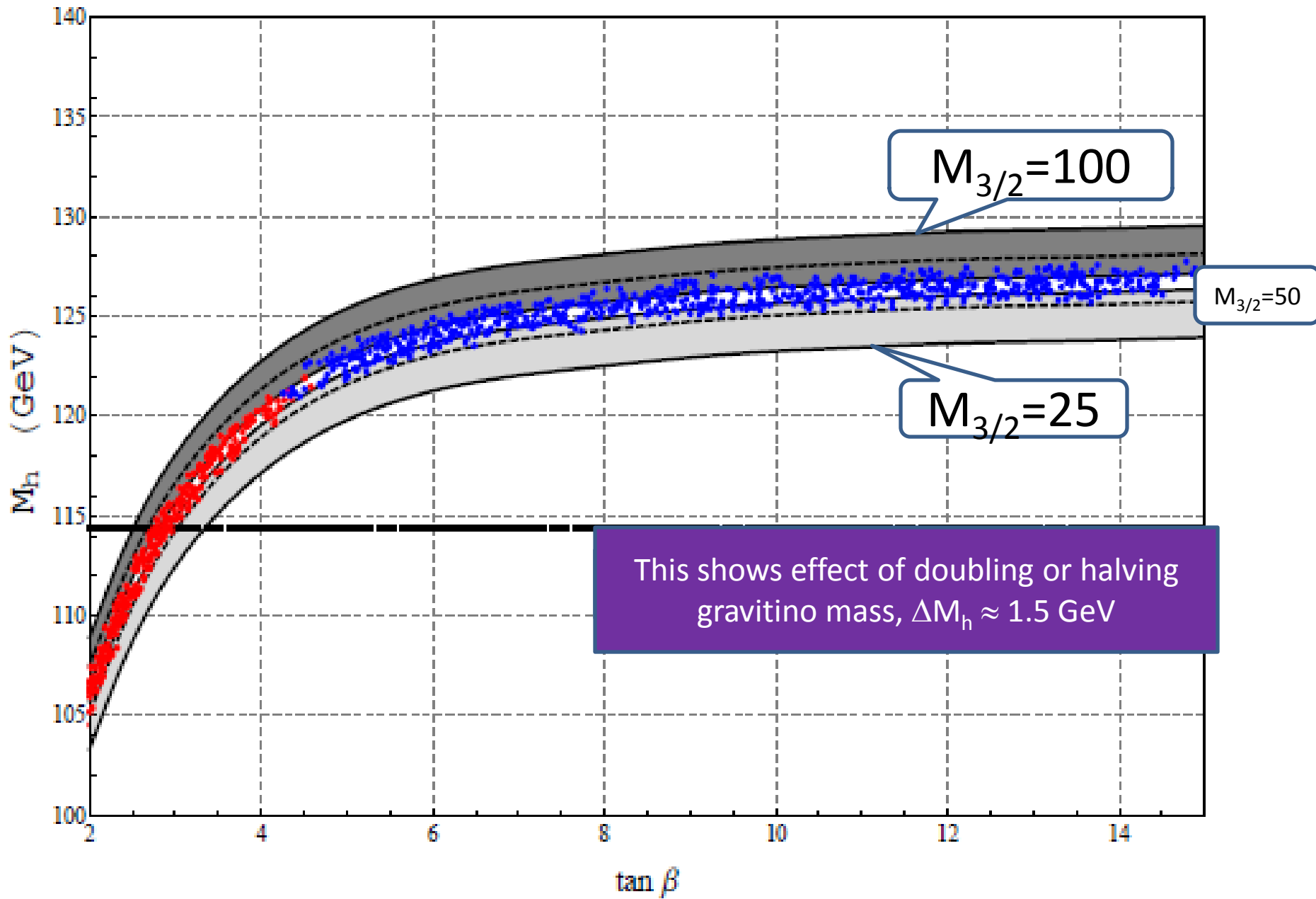
## THEORY AT HIGH SCALE, TECHNICAL DETAILS OF COMPUTING $M_H$

- Write theory at scale  $\sim 10^{16}$  GeV, fix soft-breaking Lagrangian parameters by theory – no free parameters
- Run down, maintain REWSB
- Use “match-and-run” and also SOFTSUSY and Spheno, compare – match at  $(M_{\text{stop1}}M_{\text{stop2}})^{1/2}$  – two-loop RGEs – expect public software to work since scalars not too large
- Main sources of imprecision for given  $M_{3/2}$  are  $M_{\text{top}}$  (1 GeV uncertainty in  $M_{\text{top}}$  gives 0.8 GeV in  $M_h$ ),  $\alpha_{\text{strong}}$ , theoretical gluino mass (allow 600 GeV to 1.2 TeV), trilinear couplings (allow  $0.8-1.5M_0$ )









## Is h SM-like?

Theory -- all scalar terms in the soft-breaking Lagrangian predicted to be of order gravitino mass,  $\gtrsim 30$  TeV so “decoupling” limit

Still supersymmetric Higgs sector of course, but H, A, H $^\pm$  also about equal to the gravitino mass  $\gtrsim 30$  TeV, h light and SM-like

**h is the lightest eigenvalue of the supersymmetric higgs mass matrix, in the decoupling limit  $\rightarrow$  BR are SM-like**

Typically chargino and neutralino loops give few per cent deviations

$$(\sigma \times \text{BR summed})_{\text{data}} / (\sigma \times \text{BR summed})_{\text{SM}} = 1.11 \pm 0.16$$

[but watch  $\gamma\gamma$ , etc, channels]

**We assumed MSSM is gauge group and matter content at compactification – must calculate one gauge group and matter content at a time because of RGE running etc**

- Can find models extending MSSM that give  $M_h$  same value as MSSM
  - Some U(1) extensions with no extra matter do not change mass value or BR
    - SO(10) with  $RH\nu$ , no other extra matter gives 126
    - MSSM plus U(1) plus singlet charged under U(1) does not generically give 126
    - **We have no examples with  $M_h = 126$  and increased  $\gamma\gamma$  width larger than  $\sim 10\%$**

**→ probably strong prediction that BR( $\gamma\gamma$ ), ZZ, WW, bb,  $\tau\tau$  have SM value,**



# LITTLE HIERARCHY PROBLEM – NEW APPROACH

Running of  $M^2_{Hu}$  in string/M theory [arXiv:1105.3765 Feldman, GK, Kuflik, Lu]

Compactified M theory  $\rightarrow A_0 > M_0$

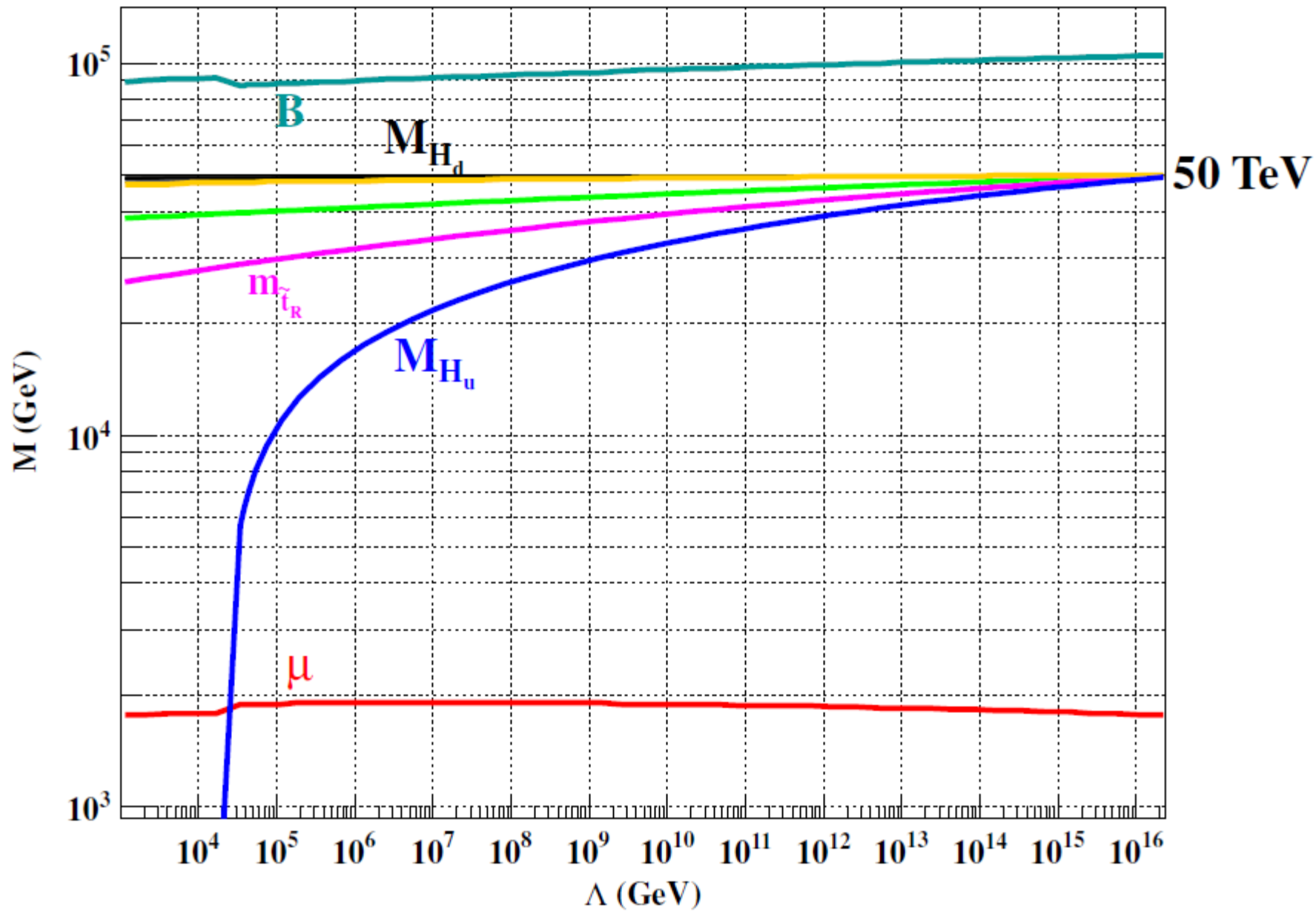
$$M^2_{Hu}(t) \approx f_M(t) M^2_0 - f_A(t) A^2_0$$

$$A_0 > M_0 = M_{3/2} \approx 50 \text{ TeV}$$

$f_M, f_A$  calculated from SM inputs, both about 0.12-0.13

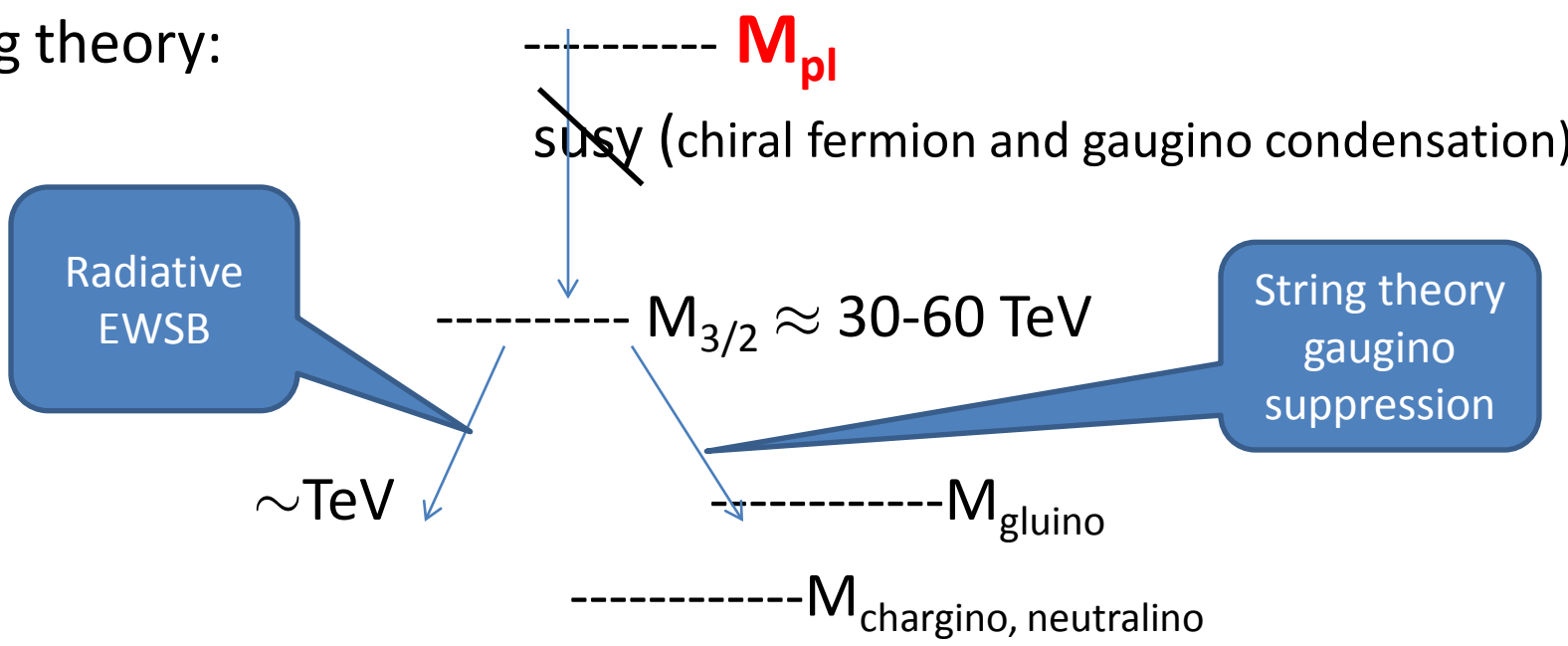
So stringy prediction is a decrease  $\sim 50$  in  $M^2_{Hu}$  – **if trilinears not large get order of magnitude less decrease in  $M^2_{Hu}$**

Greatly reduces “little hierarchy problem” – covers gap from  $M_{3/2}$  to TeV



# Naturalness? Fine-tuning? Little hierarchy?

M/String theory:



Suppose string theory gives a successful description of our string vacuum – ***Can string theory be unnatural?***

If calculated  $M_h$  directly instead of ratio to  $M_Z$ , would get larger number, e.g.  $M_Z \sim 1\text{-}2\text{ TeV}$  – this is the natural result  $\rightarrow??$

Interesting to think about how precisely Higgs vev is constrained in order to give our world

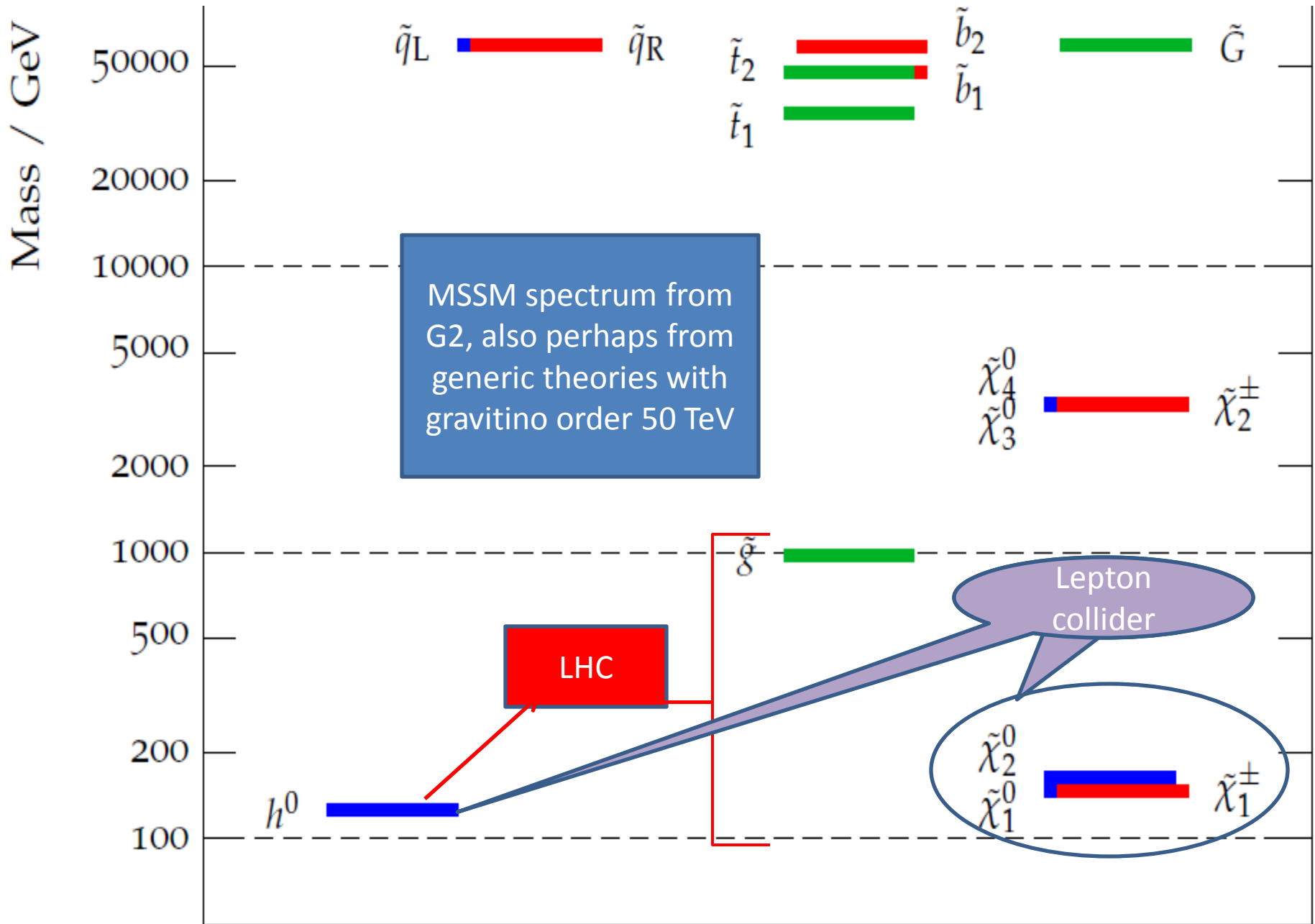
Much weaker than usual landscape issues

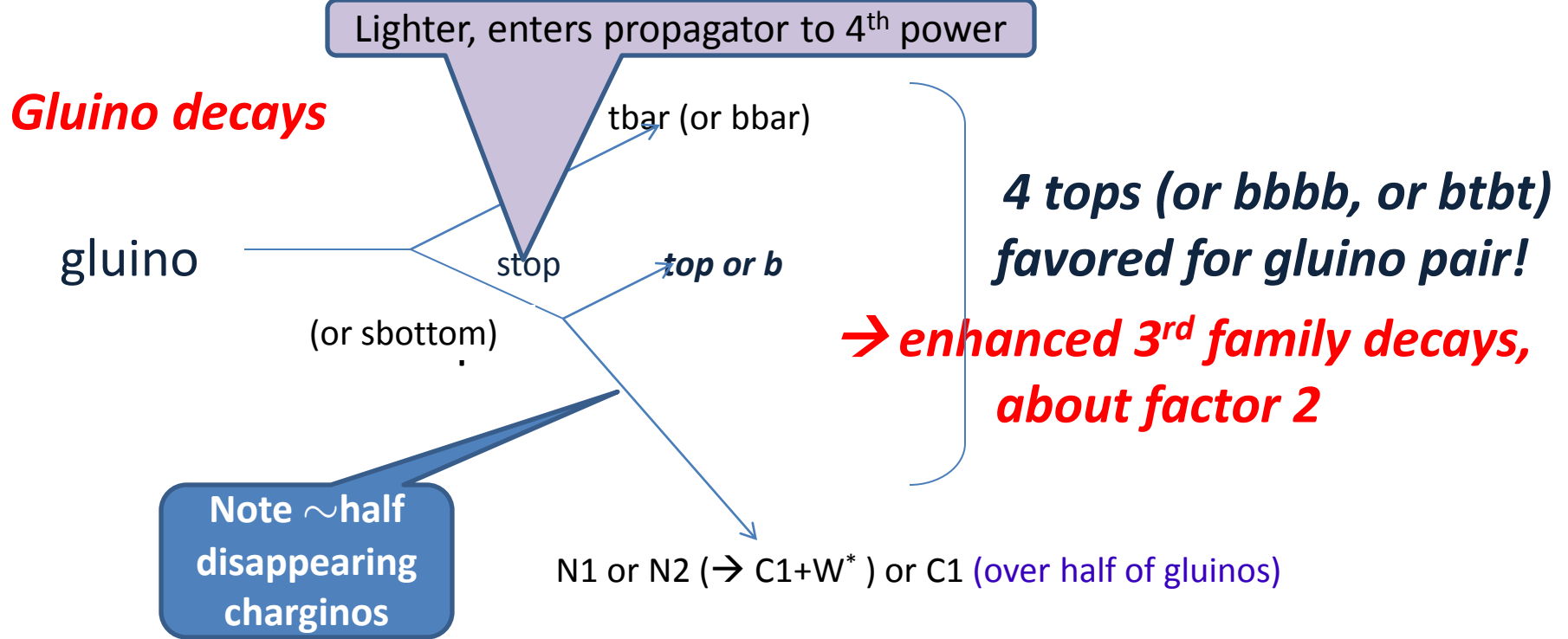
– Donoghue, Dutta, Ross, Tegmark 0903.1024 argued that the higgs vev can vary a factor of a few without any change in SM physics

## ➤ String/M theory crucial for *deriving Higgs* results!

- Must have theory with **stabilized moduli and spontaneous supersymmetry breaking** – compactified string theories
- Must have gravitino-moduli connection to get lower limit on gravitino mass
- Must derive soft terms, otherwise could choose anything – e.g. large trilinears important, but people in past guessed they were small – string theory gave prediction of large trilinears
- Must have  $\mu$  embedded in string theory
- Must exhibit string solutions with REWSB
- Must have effectively no parameters
- No R symmetry, since trilinears heavy and gauginos light

## Some LHC predictions





Glino lifetime  $\sim 10^{-19}$  sec, decays in beam pipe

**Glino decays flavor-violating**

Current limit for gluinos with enhanced 3<sup>rd</sup> family decays, very heavy scalars  $\lesssim 900$  GeV

Papers LHC14,0901.3367; LHC7, 1106.1963



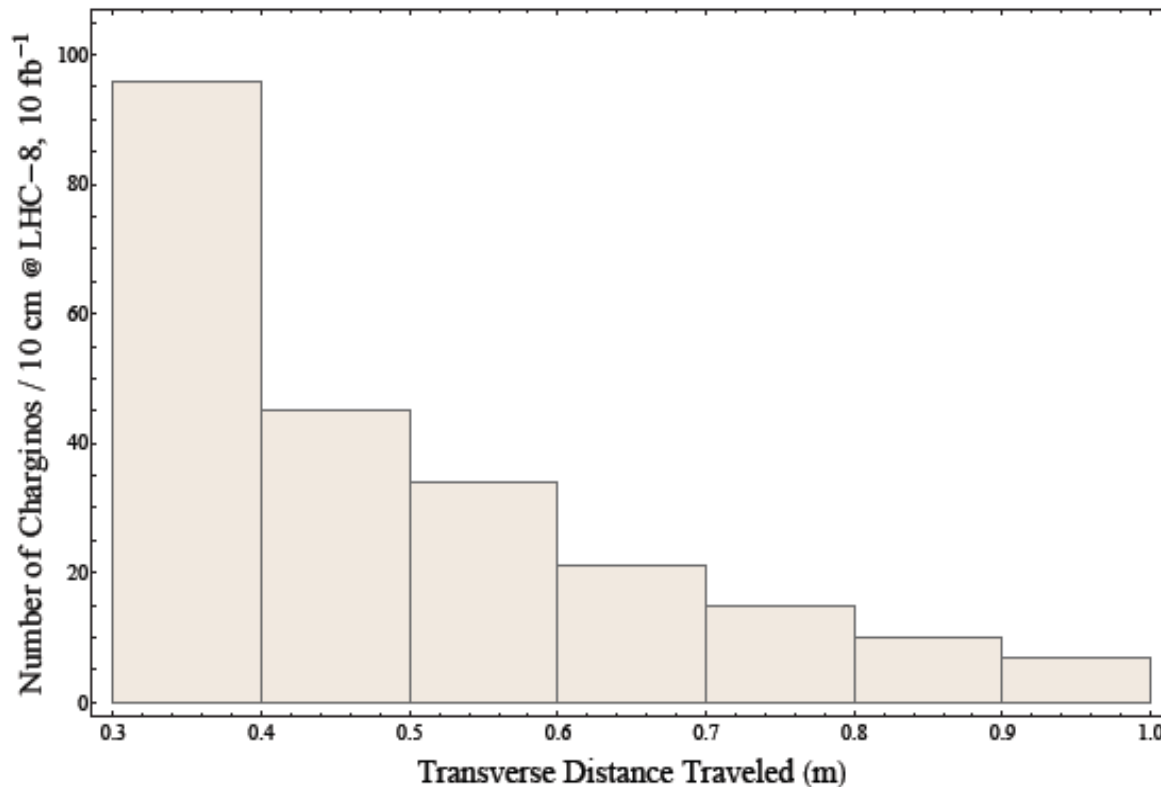
# Realistic Branching Fraction

$$\left. \begin{array}{l} m_{3/2} = 50 \text{ TeV} \\ M_{\text{gluino}} = 900 \text{ GeV} \\ M_{\text{LSP}} = 145 \text{ GeV} \end{array} \right\} \begin{array}{l} BR(\tilde{g} \rightarrow t \bar{t} \tilde{\chi}^0) \approx 0.15 \\ BR(\tilde{g} \rightarrow t \bar{b} \tilde{\chi}^- + h.c.) \approx 0.28 \\ BR(\tilde{g} \rightarrow b \bar{b} \tilde{\chi}^0) \approx 0.08 \end{array}$$

So **BR (third family)**  $\approx \frac{1}{2}$ ,

BR (1<sup>st</sup> + 2<sup>nd</sup> families  $\approx \frac{1}{2}$ ) per gluino

If wino-like LSP, chargino and LSP are nearly degenerate, so chargino  
 $\rightarrow$  LSP plus very soft  $\pi^+$   $\rightarrow$  **disappearing charginos in gluino**  
**decays** --  $\gamma_{CT} \approx 10$  cm



GK, Lu,  
Zheng  
1202.4448

See Moroi et  
al for pair  
production of  
disappearing  
charginos

FIG. 1: Charged Winos resulting from gluino pair production, binned as a function of transverse distance traveled from the beam line. These results correspond to  $10 \text{ fb}^{-1}$  of LHC-8 data ( $\sigma_{\tilde{g}\tilde{g}} \sim 235 \text{ fb}$ ), with  $m_{\tilde{g}} = 750 \text{ GeV}$ ,  $m_{\tilde{W}} = 150 \text{ GeV}$ . For graphical purposes, charginos traveling a transverse distance  $< 30 \text{ cm}$  are not shown.

## GENERIC PREDICTIONS from compactified string theories

- Squarks, sleptons 30-60 TeV, trilinears  $>$  scalars, no R symmetry
- **Non thermal cosmological history**
- Low scale gauge mediation not significant source of supersymmetry breaking since gravitino mass of order 50 TeV
- **$B_s \rightarrow \mu\mu$  within 1-2% of SM**
- $(g-s)_\mu$  within 5-10% of SM
- **$\tan\beta \gtrsim 15$**
- $M_h = 126 \pm 2$ , susy higgs sector decoupling so  $H, A, H^\pm > 30$  TeV
- **No invisible h decays**
- Gluino  $\sim 1$  TeV, gluino decays flavor violating, 3<sup>rd</sup> family larger
- **$EDMe \approx 10^{-30}$**
- LSP wino-like but  $\mu$  small so mixing
- **Relic density of LSPs, axions both order 1**
- $\sigma_{SI} \sim 10^{-46}$

Needed results, issues to understand our string vacuum

- **Study variations in Kahler potential, gauge kinetic function**
- Don't have general global G2 singular compact manifolds yet
- **Improve detailed understanding of hierarchy problem – little and mini hierarchies**
- Derive “R-parity” or not (*underway*)
- Derive large top Yukawa in M-theory (*underway*)
- **Calculate gaugino masses better**
- Higgs mass calculation happens to be precise because scalars heavy – error small because dependence on  $\tan\beta$  and  $M_{3/2}$  small – sharpen?
- **Study other gauge groups at compactification**
- Study other corners of string theory – soft-breaking Lagrangian,  $\mu$  (stabilization, deS?, scalars heavy?, gauginos suppressed?!)
- CC set to zero – sufficiently fine scanning ok?
- Think about meaning of M/string theory  $\leftrightarrow$  data, “generic”

## Final remarks

- *Higgs data looks like data from compactified constrained string theory with stabilized moduli should look! – 126 GeV not unnatural! – SM-like Higgs not surprising!*
- Higgs looks like a fundamental particle – normal susy h in decoupling region – not weird or fine-tuned
- *BR near SM seems unavoidable prediction*
- *String theory maturing into a useful predictive framework that relates many explanations, tests*
- M theory compactified on  $G_2$  manifold looks like a good candidate to continue to explore for describing our string vacuum – explains many phenomena, predicts some -- some features generic for other corners of string theory too
- Compactified M/string theory, squarks, sleptons 30-60 TeV
- $\mu, \tan\beta$  in theory, not free parameters – no free parameters!

# Backup slides

“if people don’t want to come to the ballpark nobody’s  
going to stop them”

Yogi Berra

- Can also divide all string theorists into those who want to understand our real world and ...



## BARYOGENESIS [GK, Shao, Watson, Yu arXiv:1108.5178]

- **Affleck-Dine** baryogenesis, flat direction in superpartner scalar space lifted by supersymmetric breaking, field oscillates – early universe dominated by A-D fields and moduli
- Baryon asymmetry  $\sim$  unity generated
- Moduli decay generates large entropy  $\sim 10^9$  which suppresses asymmetry to observed number
- Non-thermal cosmological history  $\rightarrow$  moduli decay also over-generates LSP dark matter – annihilate via Boltzmann equation to relic density
- So both baryons and dark matter from moduli decay! – can get equation for ratio, about right!

Numerical structure...

$\sim 10$

$$M_h^2 \approx M_Z^2 + (3/16\pi^2) (M_{\text{top}}^4 / v^2) [\ln (M_{\text{stop}}^2 / M_{\text{top}}^2 )$$

$$+(2 \alpha_s \ln^2 (M_{\text{stop}}^2 / M_{\text{top}}^2 ) \dots]$$

$\sim 7$

$$M_{3/2} = 50 \text{ TeV} \rightarrow M_{\text{stop}} \approx 30 \text{ TeV}$$

$$(126)^2 = (91)^2 + (87)^2$$

# Volume Modulus Inflation and the Gravitino Mass Problem

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J.P. Conlon<sup>1,2</sup>, R. Kallosh<sup>3,4</sup>, A. Linde<sup>3,4</sup> and F. Quevedo<sup>2</sup>

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<sup>2</sup>*DAMTP, Centre for Mathematical Sciences,  
Wilberforce Road, Cambridge, CB3 0WA, United Kingdom*

<sup>3</sup>*Department of Physics, Stanford University, Stanford, CA 94305, USA*

<sup>4</sup>*Yukawa Institute for Theoretical Physics, Kyoto, Japan*

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<sup>2</sup>For a recent discussion in the context of M-theory compactifications of the overshooting problem and how to avoid it, see [27].

– 5 –

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[27] B. S. Acharya, P. Kumar, K. Bobkov, G. Kane, J. Shao and S. Watson, arXiv:0804.0863

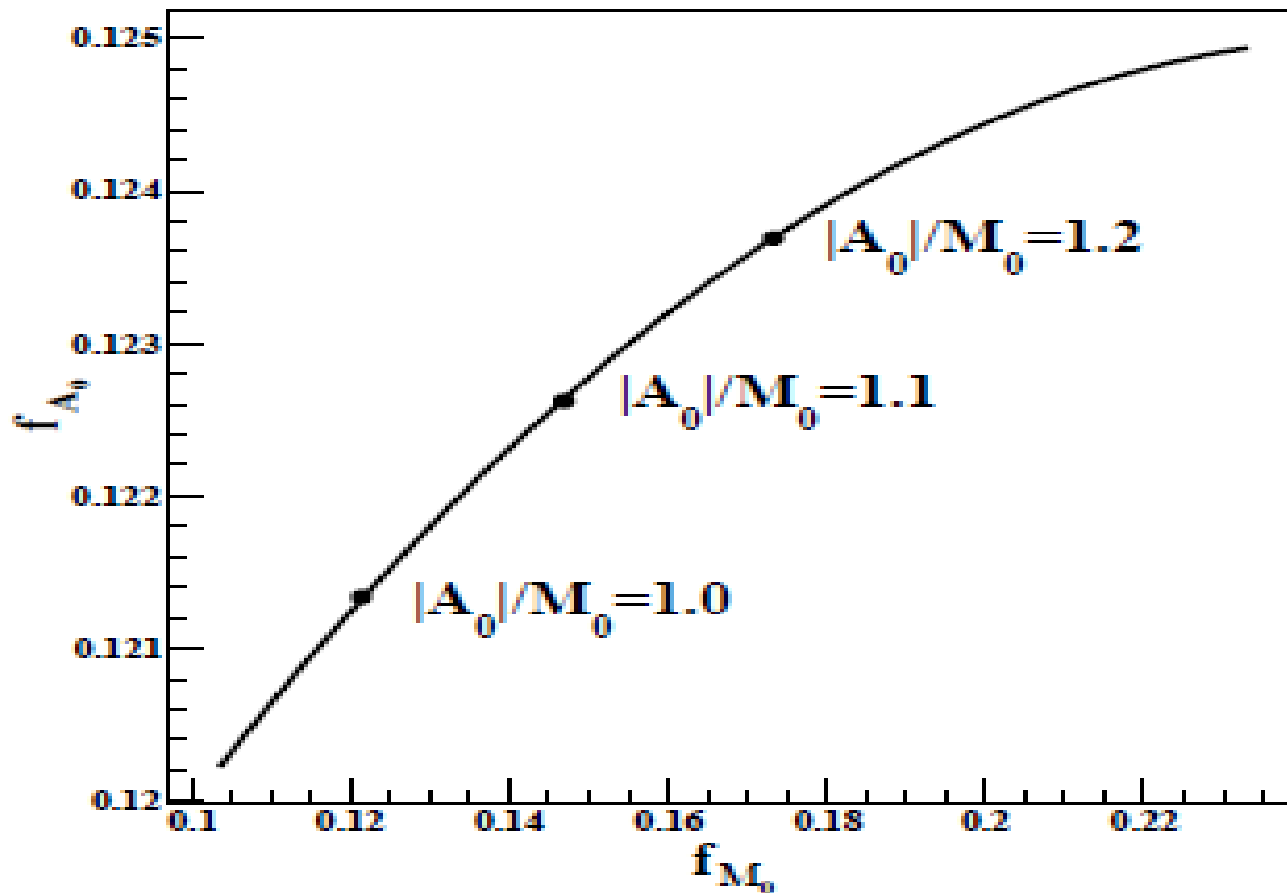
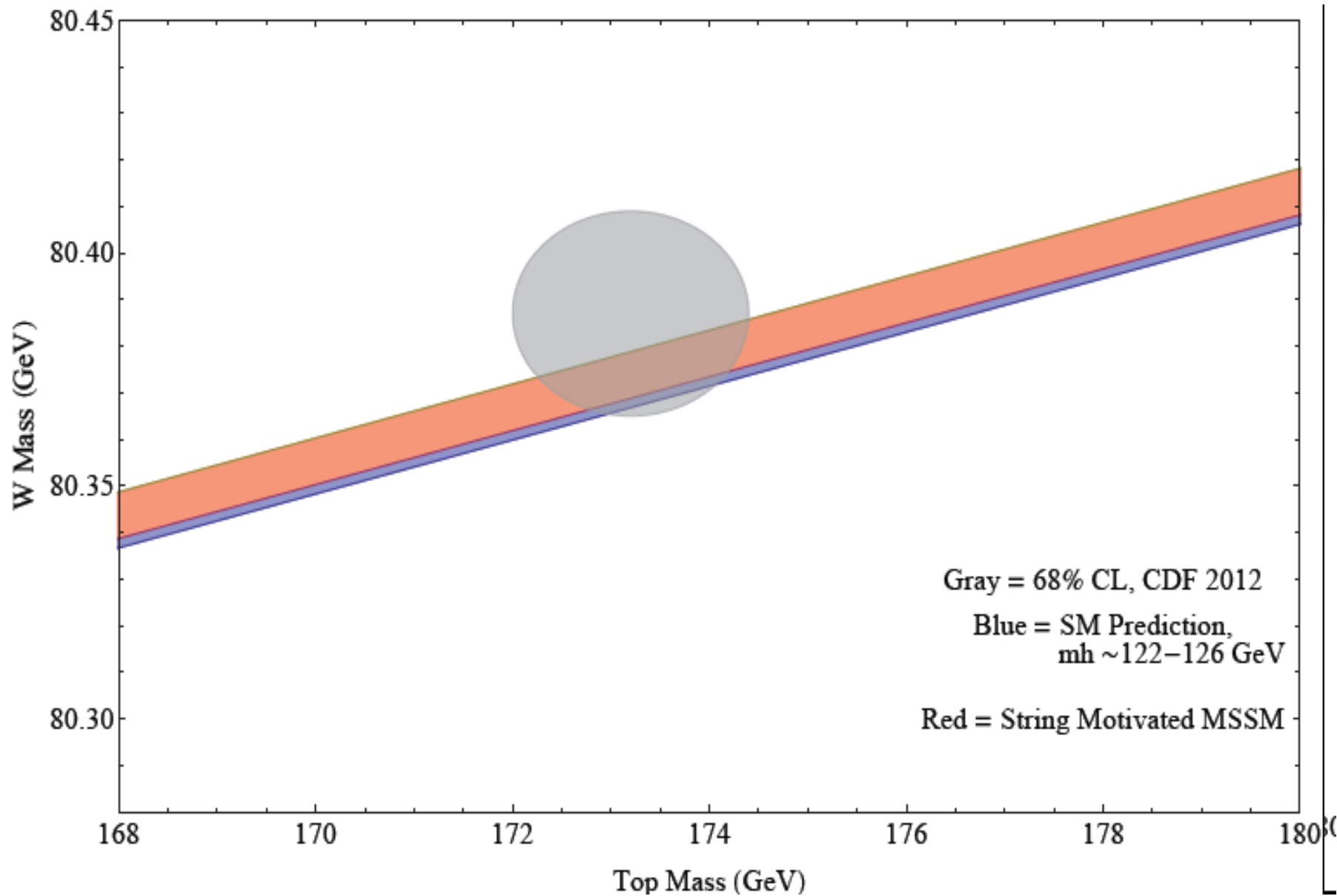
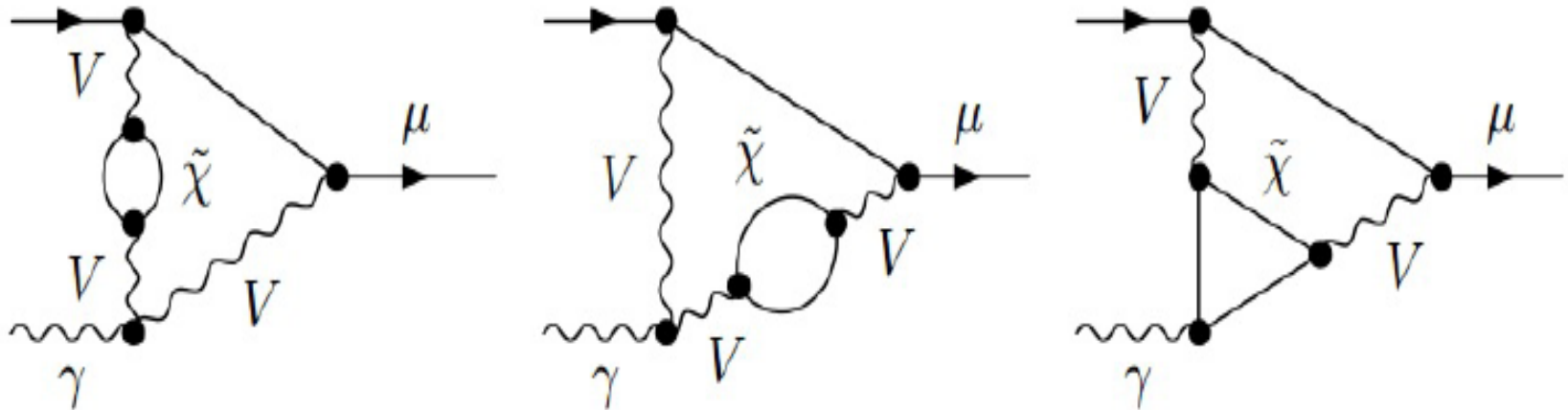


FIG. 1: The 1 loop RGE coefficients  $f_{M_0}$  and  $f_{A_0}$  at  $Q_{\text{EWSB}}$  as given in Eq.(2). The amount of cancellation in the Eq.(2) for  $m_{H_u}^2(Q_{\text{EWSB}})$  depends on  $|A_0|/M_0$ , and we show the values that minimize  $m_{H_u}^2$  at one-loop. In this figure,  $M_0$  runs from 10 TeV at the lower end of the curve to 50 TeV at the top of the curve. (See Fig. (3) for the full analysis with with 2 loop running and the threshold/radiative corrections.)

Squarks and sleptons give very small contributions to rare decay loops so  $B_s \rightarrow \mu^+ \mu^-$ ,  $\mu \rightarrow e \gamma$ , should not deviate significantly from SM --  $g_\mu - 2$  gives 5-10% increase





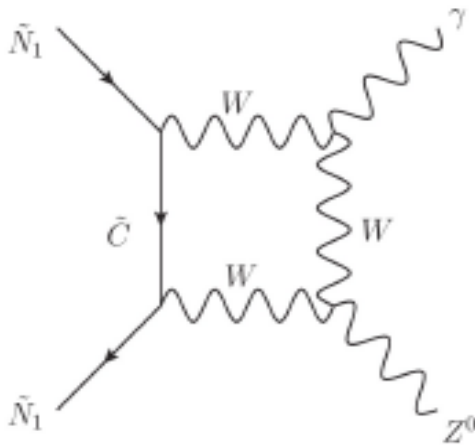
$$\delta a_\mu \sim 5 - 10 \times 10^{-10}$$

$$\delta a_\mu^{exp} = (26.1 \pm 8.0) \times 10^{-10}$$

Heinemeyer et. al  
arXiv:0405255

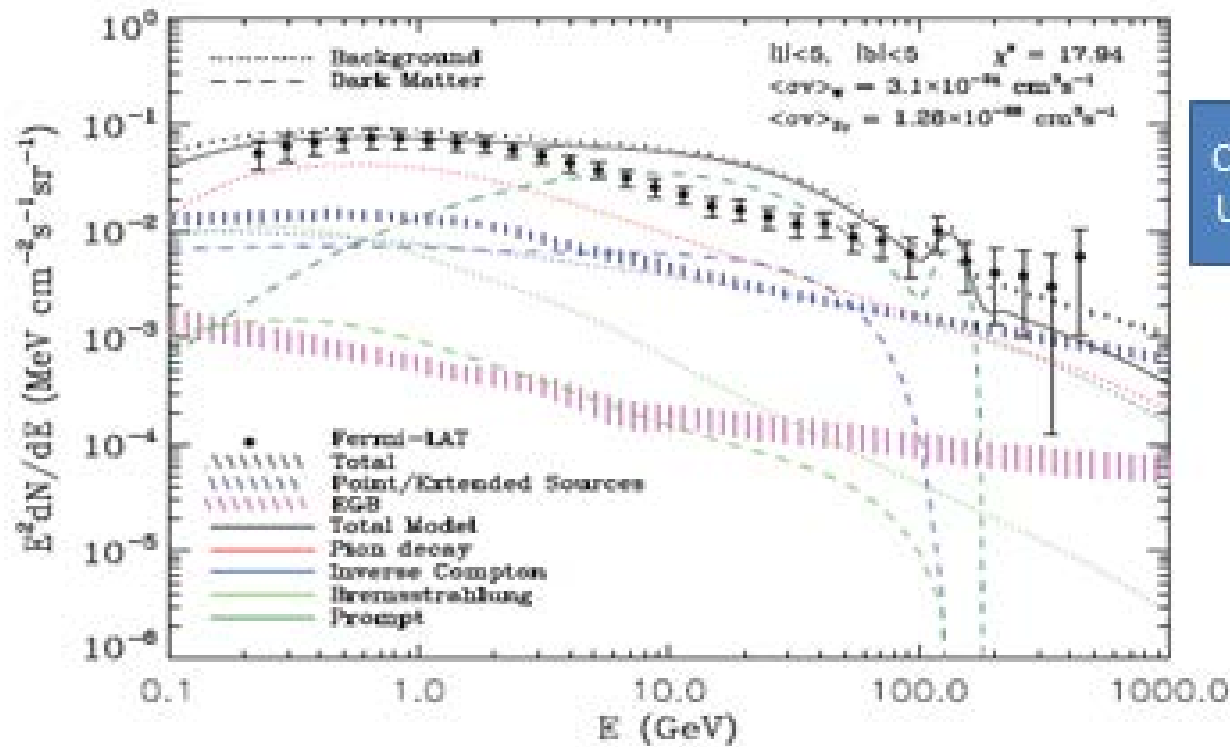
Moduli decaying before BBN – wash out all DM, baryon asymmetry, etc, before that – DM from moduli decay, needs large annihilation rate  $\rightarrow$  wino-like LSP – overclose universe for others – “non-thermal cosmological history”!

Dark matter -- 130 GeV monoenergetic gamma from DM annihilation, non-thermal cosmological history  $\rightarrow$  wino-like DM, LSP mass  $\approx$  144 GeV



$$E_\gamma = m_{LSP} \left( 1 - \frac{m_Z^2}{4m_{LSP}^2} \right)$$





Cholis, Tavakoli,  
Ullio1207.1468

FIG. 6: Wino/Axion model of [28].  $m_\chi = 145 \text{ GeV}$   
 $\langle\sigma v\rangle_{\chi\chi \rightarrow Z\gamma} = 1.26 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ ,  $\langle\sigma v\rangle_{\chi\chi}^{\text{tot}} = 3.2 \times 10^{-24} \text{ cm}^3 \text{ s}^{-1}$ .

# MODULI MASS MATRIX – RELATE MODULI AND GRAVITINO MASS

- Can write 4D scalar potential V in terms of function

$$G = K + m_{pl}^2 \ln(W\bar{W}/m_{pl}^6) \quad V = m_{pl}^2 e^{G/m_{pl}^2} (G^i G_i - 3m_{pl}^2)$$

- Then calculate scalar mass matrix (CC=0) Douglas, Denef, th/0411183

$$\begin{pmatrix} M_{i\bar{j}}^2 & M_{ij}^2 \\ M_{\bar{i}j}^2 & M_{\bar{i}\bar{j}}^2 \end{pmatrix}$$

Gomez-Reino, Scrucca, th/0602246

Acharya, Kane, Kuflik 1006.3272

$$M_{i\bar{j}}^2 = e^{G/m_{pl}^2} \left( \nabla_i G_k \nabla_{\bar{j}} G^k - R_{i\bar{j}k\bar{l}} G^k G^{\bar{l}} + G_{i\bar{j}} \right)$$

$$M_{ij}^2 = e^{G/m_{pl}^2} \left( 2\nabla_i G_j + G^k \nabla_i \nabla_j G_k \right)$$

- Look near minima of V, mass matrix positive definite – use theorem smallest eigenvalue of mass matrix is less than

$$\xi^\dagger M \xi \text{ for any unit vector } \xi. \quad (1006.3272 \text{ appendix c})$$

- Take  $\xi = (G^{\bar{j}} \quad c G^j) / \sqrt{3(1 + |c|^2)}$  as projection in scalar Goldstino direction, with c any complex number
- Get a one complex parameter set of constraints on upper bound of lowest mass moduli eigenvalue

$$\begin{aligned}
m_{\min}^2 &\leq \frac{1}{3(1+|c|^2)} (G^i \ c^\dagger G^{\bar{i}}) \begin{pmatrix} M_{i\bar{j}}^2 & M_{ij}^2 \\ M_{\bar{i}j}^2 & M_{\bar{i}\bar{j}}^2 \end{pmatrix} \begin{pmatrix} G^{\bar{j}} \\ cG^j \end{pmatrix} \\
&\leq m_{3/2}^2 \left( 2 \frac{|1-c|^2}{1+|c|^2} + \text{Re} \left\{ \frac{2c}{1+|c|^2} \frac{u}{m_{pl}^2} \right\} - \frac{r}{m_{pl}^2} \right)
\end{aligned}$$

Where  $u \equiv \frac{1}{3} G^i G^j G^k \nabla_i \nabla_j G_k$ ,  $r \equiv \frac{1}{3} R_{i\bar{j}k\bar{l}} G^i G^{\bar{j}} G^k G^{\bar{l}}$   $m_{3/2}^2 = m_{pl}^2 e^{G/m_{pl}^2}$

- $r$  is the holomorphic sectional curvature of the scalar field space, projected in the sgoldstino directions

- So 
$$\mathcal{M}_{\min}^2 = m_{3/2}^2 \left( 2 + \frac{|r|}{m_{pl}^2} \right)$$

If only scales are set by  $m_{pl}$  (which includes  $M_{3/2}$ ) then  $r/m_{pl}^2 \lesssim 1$  – e.g. in simple isotropic  $G_2$  case  $r=14/N_{\text{mod}}$

- If other scales put in by hand, bound correct but maybe not useful
- Recently Piyush showed no  $r$  dependence (1204.2795 VIB) if positive contributions to  $V$  depend on moduli only via  $V^{-n}_X$  -- holds for  $G_2$  etc

Non-thermal, matter dominated, history  $\rightarrow$  two major implications

- Upper limit on axion decay constant lifted close to compactification scale – solves long-standing string axion problem – allows axions to make up  $\sim 1$  of dark matter (Acharya Bobkov Kumar)
- Moduli decay via Planck suppressed operators to all MSSM particles -  
- get

$$n_X^X \sim \frac{\Gamma_X^2 m_{pl}^2}{m_X} \sim \frac{D_X^2 m_X^6}{m_{pl}^2 m_X}, \quad n_e \sim \frac{\Gamma_X}{\langle \sigma v \rangle} \sim \frac{D_X m_X^3}{m_{pl}^2 \langle \sigma v \rangle}$$

So DM must annihilate down to not badly overclose universe – only wino-like LSP has large enough annihilation rate to do that

$\rightarrow$  with non-thermal history predict wino-like LSP

(0804.0863; Acharya, Kane, Kumar, Lu, Zheng 1205.5789)

Gauge mediation at lower scales with small gravitino mass violates moduli lower bound – so lower scale gauge mediation will not be important source of supersymmetry breaking in string theory

- In non-canonical basis:  $m_{\bar{\alpha}\beta}^2 = m_{3/2}^2 \tilde{K}_{\bar{\alpha}\beta} - \Gamma_{\bar{\alpha}\beta}$
- Using homogeneity properties of moduli space, show  
*Acharya, Bobkov 0810.3285*

$$\Gamma_{\bar{\alpha}\beta} \propto \tilde{K}_{\bar{\alpha}\beta} + \text{higher order corrections}$$

- Corrections expected to be small if SUGRA approx. is valid
- Then, going to the canonical basis, find  $\hat{m}_{\bar{\alpha}\beta}^2 \simeq m_{3/2}^2 \delta_{\bar{\alpha}\beta}$

- Key to understanding higgs etc is to understand generic properties of moduli
- Universe post inflation, pre BBN, matter dominated
- $F_{\text{mod}} \sim \alpha_{\text{gut}} M_{3/2} M_{\text{pl}}$
- $F_{\text{matter}} \sim M_{3/2} M_{\text{pl}}$
- Get small  $M_{\text{Hu}}^2$  needs  $M_0=10 \rightarrow A_0=M_0$ ,  $M_0=30 \rightarrow A_0=1.2M_0$ ,  
 $M_0=50 \rightarrow A_0=1.5M_0$

The  $\mathcal{N} = 1$  supergravity theory obtained in four dimensions is then characterized by the following hidden sector superpotential:

$$W = m_p^3 \left( C_1 P \phi^{-(2/P)} e^{ib_1 f_1} + C_2 Q e^{ib_2 f_2} \right); \quad b_1 = \frac{2\pi}{P}, \quad b_2 = \frac{2\pi}{Q} \quad (1)$$

Here  $\phi \equiv \det(Q\tilde{Q})^{1/2} = (2Q\tilde{Q})^{1/2}$  is the effective meson field (for one pair of massless quarks) and  $P$  and  $Q$  are proportional to one loop beta function coefficients of the two gauge groups which are completely determined by the gauge group and matter representations. For concreteness we can consider the gauge group to be  $SU(Q) \times SU(P+1)$  with one vector like family of quarks charged under  $SU(P+1)$ . The normalization constants  $C_1$  and  $C_2$  are calculable, given a particular  $G_2$ -manifold.  $f_{1,2}$  are the (tree-



To “compactify”, specify a 4D superpotential  $W$  for matter and for moduli

And specify a “gauge kinetic function” that is basically a metric for gauge fields

And specify a Kahler potential for matter and for moduli, essentially a metric for scalar fields

The moduli potential is flat to all orders of perturbation theory if susy unbroken – so non-perturbative, look for susy-breaking to generate it – generically expect “gaugino condensation” of gauge fields arising from orbifold singularities in 3-cycles to give this – typical gauge groups  $SU(6)$ ,  $SU(8)$ ,  $E_6$ , etc

**Once moduli have any interaction they are stabilized**

Running is log, for such gauge groups typically get strong interactions at scales of order  $10^{14}$  GeV – this scale divided by  $M_{pl}$  enters cubed in  $W$

The metric corresponding to the Kähler potential (1) is given by

$$K_{i\bar{j}} = \frac{3a_i}{4s_i^2} \delta_{i\bar{j}}.$$

The  $\mathcal{N} = 1$  supergravity scalar potential given by

$$V = e^K (K^{i\bar{j}} F_i \bar{F}_{\bar{j}} - 3|W|^2),$$

where

$$F_i = \partial_i W + (\partial_i K) W,$$

can now be computed. The full expression for the scalar potential is given by

$$\begin{aligned} V = & \frac{1}{48\pi V_X^3} \left[ \sum_{k=1}^2 \sum_{i=1}^N a_i \nu_i^k \left( \nu_i^k b_k + 3 \right) b_k A_k^2 e^{-2b_k \vec{\nu}^k \cdot \vec{a}} + 3 \sum_{k=1}^2 A_k^2 e^{-2b_k \vec{\nu}^k \cdot \vec{a}} \right. \\ & + 2 \cos[(b_1 \vec{N}^1 - b_2 \vec{N}^2) \cdot \vec{t}] \sum_{i=1}^N a_i \prod_{k=1}^2 \nu_i^k b_k A_k e^{-b_k \vec{\nu}^k \cdot \vec{a}} \\ & \left. + 3 \cos[(b_1 \vec{N}^1 - b_2 \vec{N}^2) \cdot \vec{t}] \left( 2 + \sum_{k=1}^2 b_k \vec{\nu}^k \cdot \vec{a} \right) \prod_{j=1}^2 A_j e^{-b_j \vec{\nu}^j \cdot \vec{a}} \right] \end{aligned}$$

where we introduced a variable

$$\nu_i^k \equiv \frac{N_i^k s_i}{a_i} \quad (\text{no sum})$$

## Cosmological Constant?

Of course no solution here – OK to proceed, anticipate no issues?

**We assume CC problem is solved by other physics – orthogonal, decoupled**

Solving CC problem seems unlikely to help predict higgs mass or any other collider observable, dark matter

Not solving CC problem seems unlikely to prevent calculating  $M_h$  etc

Cannot be sure until it is solved

# Moduli Stabilization in $M$ theory

▶ Moduli vevs  $s_i \sim 3Q = \frac{1}{\alpha_{GUT}}$

▶ So, eg,  $Q=6,7,8,9$

▶  $m_{pl}^2 = Vol(X)M_{11}^2 \sim \frac{1}{\alpha_{GUT}^{7/3}} M_{11}^2$

▶  $M_{GUT} = M_{11}\alpha_{GUT}^{1/3}$

▶  $m_{3/2} = m_{pl} \frac{\alpha_{GUT}^{7/2}}{\sqrt{\pi}} \frac{|Q-P|}{Q} e^{-\frac{P_{eff}}{Q-P}}$

▶  $P_{eff} = \frac{14(3(Q-P)-2)}{3(3(Q-P)-2)\sqrt{6(Q-P)}} \sim 60$  when  $Q - P = 3$

▶ So,  $m_{3/2} \sim O(50)$  TeV. Note:  $Q - P \geq 3$ , so  $Q - P = 4$  doesn't work.

▶ So, moduli can decay before BBN.

▶ There are two INTEGER parameters  $P, Q$  which determine  $\alpha_{GUT}, M_{GUT}, M_{pl}, m_{3/2}$  all consistently.

$\alpha_{GUT} \approx 1/25$

Gaugino masses

$$M_{1/2} = m_p \frac{e^{\hat{K}/2} \hat{K}^{n\bar{m}} F_{\bar{m}} \partial_n f_{sm}}{2i \operatorname{Im} f_{sm}},$$

Scalar masses

$$m_{\bar{\alpha}\beta}^2 = (m_{3/2}^2 + V_0) \tilde{K}_{\bar{\alpha}\beta} - e^{\hat{K}} F^{\bar{m}} (\partial_{\bar{m}} \partial_n \tilde{K}_{\bar{\alpha}\beta} - \partial_{\bar{m}} \tilde{K}_{\bar{\alpha}\gamma} \tilde{K}^{\gamma\bar{\delta}} \partial_n \tilde{K}_{\bar{\delta}\beta}) F^{\bar{m}}$$

$$A'_{\alpha\beta\gamma} = \frac{\hat{W}^*}{|\hat{W}|} e^{\hat{K}/2} F^{\bar{m}} [\hat{K}_{\bar{m}} Y'_{\alpha\beta\gamma} + \partial_{\bar{m}} Y'_{\alpha\beta\gamma} - (\tilde{K}^{\delta\bar{\rho}} \partial_n \tilde{K}_{\bar{\rho}\alpha} Y'_{\alpha\beta\gamma} + \alpha \leftrightarrow \gamma + \alpha \leftrightarrow \beta)]$$

$$(m_{\alpha}^2) = (m_{3/2}^2) \left[ 1 - \frac{(139 + 396\sqrt{3})^2}{524593216} \frac{1}{4\pi} \sum_i \{ l^2 \psi_{ii}^{\alpha} \sin^2(2\pi\theta_i^{\alpha}) + l^2 \psi_i^{\alpha} \sin(4\pi\theta_i^{\alpha}) - 2l \psi_i^{\alpha} \sin(2\pi\theta_i^{\alpha}) \} \right]$$

$$\approx (m_{3/2}^2) \left[ 1 - \frac{0.0013}{4\pi} \sum_i \{ l^2 \psi_{ii}^{\alpha} \sin^2(2\pi\theta_i^{\alpha}) + l^2 \psi_i^{\alpha} \sin(4\pi\theta_i^{\alpha}) - 2l \psi_i^{\alpha} \sin(2\pi\theta_i^{\alpha}) \} \right]$$

$$\approx m_{3/2}^2.$$

(291)

$$\begin{aligned}
\mu &= \left( \frac{\hat{W}^*}{|\hat{W}|} e^{\hat{K}/2} \mu' + m_{3/2} Z - e^{\hat{K}/2} F^{\bar{m}} \partial_{\bar{m}} Z \right) (\tilde{K}_{H_u} \tilde{K}_{H_d})^{-1/2} \\
B\mu &= (\tilde{K}_{H_u} \tilde{K}_{H_d})^{-1/2} \left\{ \frac{\hat{W}^*}{|\hat{W}|} e^{\hat{K}/2} \mu' (e^{\hat{K}/2} F^m [\hat{K}_m + \partial_m \ln \mu'] - m_{3/2}) + (2m_{3/2}^2 + V_0) Z - m_{3/2} F^{\bar{m}} \partial_{\bar{m}} Z \right. \\
&\quad \left. + m_{3/2} F^m [\partial_m Z - Z \partial_m \log(\tilde{K}_{H_u} \tilde{K}_{H_d})] - F^{\bar{m}} F^{\bar{n}} [\partial_{\bar{m}} \partial_{\bar{n}} Z - \partial_{\bar{m}} Z \partial_{\bar{n}} \log(\tilde{K}_{H_u} \tilde{K}_{H_d})] \right\} \quad (310)
\end{aligned}$$