CHAPTER 1

STRING PHENOMENOLOGY, SUPERSYMMETRY, AND LHC PHYSICS

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2. Introduction

The Standard Model (SM) of particle physics provides a complete description of our world, of all that we see. The basic constituents of matter are quarks and leptons, and we have a complete theory of the strong, weak, and electromagnetic forces. We also know that much is unexplained, such as what is the dark matter (DM) of the universe, and why is the universe made of matter rather than being an equal mixture of matter and antimatter, neither of which can be explained by the SM. Below I will make a longer list of questions the SM cannot answer. And there are conceptual reasons also why we expect to find new physics beyond the SM.

While a number of approaches to physics beyond the SM have been worth considering, only one so far has actually explained and predicted phenomena beyond the SM, namely the supersymmetric extension of the SM, which will be the focus of these lectures. As we will see, the supersymmetric SM has a number of successes, and as yet no unambiguous phenomenological failures. It is not yet a complete theory in the sense that we do not yet understand fully the physics of all of its parameters, but it is a complete effective theory because we can write the full effective Lagrangian of the theory. One of its important properties is that it can be a valid theory to very high energy scales or very short distances, near the Planck scale, so it provides the opportunity to connect the SM, and physics beyond the SM, to an underlying theory at short distances.

String theories have become very attractive in recent years as well. They are formulated at the Planck scale with ten or eleven dimensions and presumably unbroken supersymmetry. What is exciting about them from our point of view is that they seem to be able to accommodate the SM forces and quarks and leptons, and possibly explain how these forces and particles originate. So in these lectures we will assume that the basic theory is a string theory at the Planck scale (loosely speaking). String theory cannot be directly tested because we live in the ground state of the theory, and we do not yet know the properties of the ground state. The subfield that focus on deducing the properties of the ground state and on explaining phenomena is called "string phenomenology".

In these lectures we retain this attractive picture of a fundamental theory, presumably string theory, in 10 (or 11, the difference is not important for these lectures) small dimensions. Three dimensions inflate. Inflation ends as the energy density causing the inflation is released into a large number of massless particles, SM particles and their superpartners. The world is described by a 4D effective theory at a high scale at above 10^{16} GeV. This approach is consistent with all observed phenomena and constraints, and can address all basic questions. It is typically the approach implied by string theories. It is conservative in that a number of alternative approaches have been studied. The alternative approaches generally address fewer questions and have fewer successes. Since the supersymmetric theory is an effective theory it has parameters that have to be measured. Today there is no theory of values of quantities with dimensions of mass from first principles, so all mass parameters in any theory must be input. The parameters of the supersymmetry Lagrangian are complex, flavor-dependent masses.

Since the masses (including the trilinear couplings of the soft-breaking Lagrangian L_{soft}) are not known, supersymmetry has only limited power for studying flavor physics. Sometimes people argue that calculating fermion masses will be a convincing way to learn when a compactification is correct. But the hierarchy of masses implies that will be very hard to do. The masses are unlikely to arise entirely at the tree level, but rather depend on non-renormalizable operators and possibly on supersymmetry breaking effects. Even if the smaller masses arise at tree level in the superpotential, they are likely to have significant corrections since their numerical values are fractions of a GeV. So perhaps the large masses can be calculated, but not the smaller ones, and if the large ones have Yukawa couplings of order unity that will be common to many theories. It is of course known that huge numbers of manifolds give three families of chiral fermions.

The main thing that can be seen to test compactification and supersymmetry breaking is in fact the supersymmetry soft-breaking Lagrangian, L_{soft}. The parameters of L_{soft} are in principle measurable, though little has been known until recently about how to actually measure most of them in the general case. Part of these lectures will be about how to measure them. And they are calculable relatively easily in any compactified string theory with broken supersymmetry. They are not usually sensitive to higher order corrections unless the leading term vanishes, which can be a very important case for the gaugino masses. Progress will come from measuring L_{soft} at the weak scale, and somehow extrapolating it to the unification scale. Superpartners should be directly detected in the next few years, and once the initial excitement is past we will turn to the challenging and delightful opportunity to untangle the data and measure as much of the Lagrangian as possible.

One of the main areas that supersymmetry has addressed is Higgs physics, where it can provide an explanation of electroweak symmetry breaking. If the soft superpartner masses are set to be at the TeV scale in order to solve the hierarchy problem of the SM (that is, radiative corrections would raise the Higgs boson mass and therefore all masses to the highest scale of the theory), then this is a real explanation. For parameters that are consistent with all other data, in the minimal supersymmetric Standard Model (MSSM) the lightest Higgs boson mass should have had mass below about 100 GeV. When LEP did not find such a Higgs boson, the supersymmetric explanation apparently became somewhat fine-tuned, leading to what has been called the "little hierarchy problem". That is, one can in the MSSM have Higgs bosons with masses up to about 130 GeV but only for parameters chosen to arrange that.

There are four approaches to the little hierarchy. First, it may be an unimportant accident. While fine-tunings are not acceptable for good physics explanations, sometimes cancellations do occur. One well known cancellation is in the SM where the fermion vector couplings are proportional to $\sin^2\theta_W - 1/4$, so suppressed (since $\sin^2\theta_W \approx 0.23$) by a large factor for no deep reason. If a Higgs boson is found at the Tevatron or LHC with mass around 115 GeV this may be the explanation. Second, there may in fact be a Higgs boson lighter than the LEP limits, because the well known LEP limit is on a SM Higgs boson. The limits are lower for the MSSM analysis of the data [1], and would be even lower if complex masses were included in the analysis [2]. A number of MSSM examples are known [4] where light Higgs bosons would not have been detected at LEP. In fact, LEP has a few 2σ effects for Higgs signals, and one can find reasonable models that produce all of them [4].

Third, there is no physics reason to think the MSSM is the actual low scale effective theory. (We will define the MSSM carefully below – basically it is the theory with the SM gauge group, a superpartner for each SM particle, and two Higgs doublets.) It was entirely appropriate to study the MSSM to understand its properties and its phenomenological implications. Any low scale supersymmetric effective theory will contain the MSSM and only a few of its properties will be modified in an extended theory. Most string theories do not give a pure MSSM at low scales – often it is extended by a U(1) symmetry, in which case the fine-tuning of the MSSM goes away [5,6]. So the apparent fine tuning may just be a phenomenological clue that the real low scale theory is an extended MSSM (EMSSM).

Fourth, a number of people have used the little hierarchy as motivation to pursue alternative approaches to how the SM will be extended. These include Higgsless approaches, extra TeV sized dimensions, and more. I will not review these areas since other lectures cover them, and since most have turned out to have phenomenological problems or not to reduce fine tuning (see for example [7]), and since few have an "ultraviolet completion" (i.e. they do not connect to an underlying high scale theory and allow gauge coupling unification), and since none has emerged as a serious competitor for supersymmetry.

Further, I and increasingly many physicists feel that with LHC data coming relatively soon it is now much more important to focus on the "LHC inverse problem" than to construct additional non-supersymmetric models or try to repair those that fail [8]. The LHC inverse problem can be split into several areas:

- At a hadron collider, where only cross sections times decay branching ratios are measured, it is very hard to identify a spectrum of particles producing a signal (assuming a beyond-the-SM signal is indeed found) or measure their masses (see [9] and references therein). It is harder to construct the low scale effective Lagrangian or parts of it. A great deal of work is needed to learn how to do this in general. Some of that work could be done before the data comes, and might affect experimental analyzes and even triggers.
- Assuming a signal is found, how can we establish what the underlying physics is? Is it indeed supersymmetry? How can the spins of superpartners or other states be measured? Much more work is needed here [10].
- What we really want is to learn more about the underlying, presumably high scale, theory is. What obstacles are there to doing renormalization group running of low scale measured parameters to high scales? How can we overcome these obstacles? What features of low scale data might point to particular high scale theories (such as a class of string theories)? There will be techniques invented to make progress in doing these things, and there is great

need for people to work on these questions (there is some discussion of these issues in [11]).

Finally books on phenomenological supersymmetry are becoming available. One is already published, [12] and [13,14] will appear in early 2006. Consequently in these lectures I will focus less on some of the derivations and techniques that can be obtained from the books. Martin's Primer [15] is very useful, and I will follow it in some places. However, the books have less discussion of perspectives and of some issues such as the little hierarchy and of the LHC inverse problems that I will discuss.

Once supersymmetry is experimentally observed, supersymmetry breaking will become the central problem of particle physics. I will only discuss it from the point of view that one can write the most general effective Lagrangian at any scale. There are a number of approaches to supersymmetry breaking that have been studied, but none have yet emerged as compelling, and I will not pursue them. The books and some recent reviews can be consulted for that. These lectures are largely pedagogical and are not meant as a review. There has recently been a review of the theory and phenomenology of the soft breaking Lagrangian that has nearly 1000 references [11] so (as is already apparent) I will mainly provide references to very recent work and to work from which a topic can be traced. I apologize to authors whose work is not directly referenced here.

3. Perspectives

Supersymmetry is an idea as old as the SM, and it has not been the most fashionable way to describe the real world in recent years. It has not been taught systematically, or at schools and meetings. Consequently many students have not become familiar with supersymmetry as a practical theory, nor have they seen the arguments for its validity, or its history. Once superpartners are directly observed it will not be necessary to include these arguments, but at the present time there is some static in the messages most students get, so it is worthwhile to include some summaries of why classic supersymmetry remains the best approach. In additon, much work is needed on supersymmetry, particularly on the LHC Inverse Problem, and on supersymmetry breaking, so providing the perspectives may be useful.

It is good to recall some of the history of supersymmetry. Let us do so in the terminology of related areas.

HISTORY OF SUPERSYMMETRY

1^{st} Supersymmetry Revolution	1970-72	The idea
2^{nd} Supersymmetry Revolution	1974	Supersymmetric relativistic quantum theory
3^{rd} Supersymmetry Revolution	1975	Local supersymmetry, supergravity
4^{th} Supersymmetry Revolution	1979-83	Supersymmetry solves many problems
5^{th} Supersymmetry Revolution	2006-2009	Higgs boson and superpartners observed

Next let us consider a list of important questions that the SM does not deal with. Consequently, these can point the way beyond the SM.

3.1. *Some Physics not Described by the Standard Model*

- Gravity
- Cosmological Constant
- Dark Energy
- What is (are) the inflaton(s)?
- Strong CP problem
- Hierarchy problem
- How is the electroweak symmetry broken (EWSB)?
- Gauge coupling unification
- Matter asymmetry of the universe
- Cold dark matter
- 3 families
- Neutrino masses
- Values of quark and charged lepton masses
- $\mathrm{g}_\mu - 2$ if the current effect persists as theory and data improve

The SM *cannot* account for any of these. Any approach that claims to be making any progress (such as large extra dimension ideas) should be able to deal with some or most of these simultaneously. Where do they lead us? Supersymmetry of the form we are focusing on in these lectures is relevant to most or all of these. $g_\mu - 2$ is particularly interesting if confirmed, because it is the only one listed that cannot have an explanation at very short distances or cosmological scales.

3.2. *What Did We Learn from LEP?*

Data from LEP, combined with extensive understanding and calculations of SM predictions, implies three main results (in addition to some specific measurements such as the number of neutrinos (or more generally invisible particles into which $Z's$ decay)).

◊ Only one parameter of the SM is not measured, the Higgs boson mass m_h. The global analysis of the data demonstrates that a good fit is obtained for some values of m_h but not for others, and basically implies that $m_h \lesssim 200$ GeV. The precise value is very sensitive to details since the dependence on m_h is logarithmic, but there is an upper limit. The number could be somewhat different in a supersymmetric analysis. The implication that a real Higgs boson exists below the limit could conceivably be evaded if several kinds of new physics in the TeV region combined to fake the result, but that is both unlikely and getting harder to maintain as data gives more constraints on TeV scale phenomena. (In particular, the same limit emerges from two independent analysis of different data, the LEP global fit and the W-top quark allowed region, which makes it much harder to evade.) Thus there is strong but still indirect evidence that there is a light Higgs boson, as expected in supersymmetry if it explains EWSB.

◊ Of about twenty independent quantities accurately measured at LEP none deviate significantly from their SM expected value. Thus whatever new physics is associated with EWSB must be weakly coupled. That is automatic for supersymmetry with a conserved R-parity since then superpartners can only appear in loops, and deviations from the SM are at the level of a fraction of a per cent, unless superpartners are directly produced (which unfortunately did not happen).

◊ Beginning RGE running at LEP measured values the gauge couplings are observed to come together at somewhat above 10^{16} GeV if superpartners are included in the analysis. This implies that the underlying theory is perturbative up to the unification scale (but not that there is a desert), and that physics is simpler at high scales.

All of these point toward supersymmetry as the extension of the SM.

3.3. *The Hierarchy Problem*

The hierarchy problem is the SM problem that quantum corrections raise the Higgs boson mass up to the highest mass scale there is. It is a serious problem — as someone said, the quantum corrections are not only infinite, they are large. The high mass scales do not have to couple directly to the Higgs boson; the coupling can be through several loops, as Martin explains in some detail. All SM masses (W and Z and quarks and charged leptons) are proportional to m_h; if m_h is large so are they.

Supersymmetry was not invented or designed to solve this problem, but it did. If supersymmetry is unbroken then loops with SM particles cancel loops with their superpartners in general. For broken supersymmetry the effect is proportional to a power of the gauge couplings times the difference of the masses squared of superpartner pairs. Any proposed solution of the hierarchy problem must be insensitive to high scales, and to higher order corrections. If an approach claims to deal with the hierarchy problem it must explain how the weak and gravitational scales are determined. Later when we discuss EWSB we examine in what sense supersymmetry provides this explanation. Sometimes people make connections between the cosmological constant problem and the Higgs hierarchy problem, but they are not obviously the same because the calculation of the cosmological constant sums over all states, while the calculation of the Higgs mass only sums over states with SM gauge quantum numbers. Another way to think of the supersymmetry solution is that the Higgs doublet becomes a chiral supermultiplet so h and its superpartner have the same mass, and the fermion masses are not quadratically divergent so its superpartner mass is not divergent.

3.4. *More (Indirect) Evidence for Weak Scale Supersymmetry*

Sometimes people imagine that supersymmetry was invented to explain some of what it explains. That is not so. In fact it is the only theory in history that was not invented to explain a puzzle or phenomena. It is instructive to look at the approximate date when it was realized that supersymmetry could explain various phenomena. The theory existed even before it was realized that it solved these problems — it was not invented for any of them. For completeness we include the evidence we have already examined.

1980 — Can stabilize hierarchy of mass scales
1982 — Provides an explanation for the Higgs mechanism
1982 — Gauge Coupling Unification
1983 — Provides cold dark matter candidate
1992 — Can explain the baryon asymmetry of the universe
1982 — Heavy top quark predicted
1993 — Higgs boson must be light in general supersymmetric theory
1990 — Realization that either superpartners are light enough to find at LEP, or their effects on precision data must be very small and unlikely to be observed at LEP/SLC
1982/1995 — Starting from a high scale with a value for $\sin^2\theta_W$ of 3/8, which

arises in any theory with a unified gauge group that contains SU(5), and also in a variety of string-based theories, the value for $\sin^2\theta_W$ at the weak scale is 0.2315 and agrees very accurately with the measured value.

Some of these successes are explanations, and some are correct predictions. It is also very important that all of them are simultaneously achieved — often efforts to deal with the real world can apparently work for one effect, but cannot describe the range of phenomena we know.

There are theoretical motivations for low energy supersymmetry too. It is the last space-time symmetry not yet known to be realized in some way in nature, it adds a fermionic or quantum structure to space-time, it allows theories to be extrapolated to near the Planck scale where they can be related to gravity, local supersymmetry is supergravity which suggests a connection of the supersymmetric SM to gravity, it allows many problems in string theory and string field theory to be solved, including stabilizing the string vacuum. It is expected, though not yet demonstrated, that low energy supersymmetry is implied by string theory. In any case, improving the theory is nice but is not strong motivation for something to exist in nature, so we have emphasized the evidence that actually depends on data.

3.5. *Current Limits on Superpartner Masses*

The general limits on superpartner masses from experiment are not very strong. They are also all model dependent, sometimes a little and sometimes very much. Limits from LEP on charged superpartners are near the kinematic limits except for certain models, unless there is close degeneracy of the charged sparticle and the LSP, in which case the decay products are very soft and hard to observe. So in most cases charginos and charged sleptons have limits of about 100 GeV. Gluinos and squarks have typical limits of about 250 GeV, except that if one or two squarks are lighter the limits on them are much weaker. For stops and sbottoms the limits are about 100 GeV separately. Many published limits assume various degeneracies or relations among masses.

There are no general limits on neutralinos, though sometimes such limits are quoted. It is clear no general limits exist — suppose the LSP was pure photino. Then it could not be produced at LEP through a Z which does not couple to photinos, and suppose selectrons were very heavy so it's production via selectron exchange were very small in pair or associated production. Then no cross section at LEP is large enough to set limits.

There are no general relations between neutralino masses and chargino or gluino masses, so limits on the latter do not imply limits on neutralinos. In typical models that do assume relations between gaugino masses, such as equality at the unification scale, the limits are $M_{LSP} \gtrsim 40$ GeV, $M_{\tilde{N}_2} \gtrsim 85$ GeV. If one assumes the dark matter of the universe is all due to the LSP, and that it arises in thermal equilibrium as the universe cools, then $M_{LSP} \gtrsim$ few GeV. Superpartners get mass from both the Higgs mechanism and from supersymmetry breaking, so one would expect them to be heavier than SM particles. All SM particles would be massless without the Higgs mechanism, but superpartners would not. Many of the quark and lepton masses are small presumably because they do not get mass from Yukawa couplings of order unity in the superpotential, so one would expect naively that the normal mass scale for the Higgs mechanism was of order the Z or top masses. In models chargino and neutralino masses are often of order Z and top masses, with the colored gluino mass a few times the Z mass.

There are also no firm theoretical or indirect limits on superpartner masses. If the $g_\mu - 2$ deviation from the SM persists as the data and theory improve the first such upper limits will be deduced. If in fact supersymmetry explains all that we argue above it is explaining, particularly the EWSB, then some superpartners will be light, but this is not easily made precise. Basically, what is happening is that EWSB produces the Z mass in terms of soft-breaking masses, so if the soft-breaking masses are too large such an explanation does not make sense. The soft parameters that are most sensitive to this issue are M_3 (basically the gluino mass) and μ which strongly affects the chargino and neutralino masses. Qualitatively one therefore expects rather light gluino, chargino, and neutralino masses. If one takes this argument *seriously* one is led to expect $M_{\tilde{g}} \lesssim 500$ GeV, $M_{\tilde{N}_2}$, $M_{\tilde{C}} \lesssim 250$ GeV, and $M_{\tilde{N}_1} \lesssim 100$ GeV. These are qualitative limits, seldom saturated in models. They imply that these states should be produced in significant quantities at the Tevatron in the next few years, and of course much more at the LHC.

There are some other clues that some superpartners may be light. If the baryon number is generated at the EW phase transition then the lighter stop and charginos should be lighter than the top [16]. If the LSP is indeed the cold dark matter, and the gauginos are light as in the previous paragraph, then at least one scalar fermion is probably light enough to allow enough annihilation of relic LSPs, but there are loopholes to this argument.

Finally, it is worth commenting on one perhaps amusing point. Sometimes people who do not understand supersymmetry sometimes say it can "explain or fit anything". In fact it is the opposite. Supersymmetry is a full theory, and all that is unknown is the masses (which are matrices in flavor space) and the vacuum expectation values, exactly as for the SM. There are many conceivable phenomena that supersymmetry (not just the MSSM) could not explain, including sharp peaks in spectra at colliders, a world with no Higgs boson below about 200 GeV, a top quark lighter than the W, no gauge coupling unification, deviations from SM predictions greater than about 1% for any process with a tree-level SM contribution (including Z decay to $c\bar{c}$), leptoquarks, wide WW or ZZ resonances, excess high-P_t leptons at HERA, large violations of μ/e universality, and much more. None of these has occurred, consistent with supersymmetry, but a number of them have been reported and then gone away. Supersymmetry alone also cannot explain some real questions such as why there are three families or the $\mu - \tau$ mass ratio.

3.6. *What Might Explain What?*

It is very interesting and instructive to get an overview of what phenomena are explained by the SM, what might be explained by a supersymmetric SM (SSM), and by string theory. In the SSM case we assume that the hierarchy problem is solved by setting the soft breaking parameters with dimensions of mass to be at the TeV scale. Given that assumption, then the other results are derived. Table 1 focuses on whether questions are *addressed* (one check) and/or whether questions are *answered* (two checks). Once questions are addressed as research questions (rather than as speculative discussion) it is often not long before there is significant progress. I invite others to make similar columns for variations on these columns and for alternative theories. A few remarks comparing supersymmetry to alternative approaches might be in order. All approaches must make an assumption to explain the large hierarchy between the weak and unification (or Planck) scales. Supersymmetry does better here than other approaches since it is easy to construct supersymmetric models that connect the two scales, or "have an ultraviolet completion" in the jargon. Then supersymmetry does better with EWSB since EWSB can be derived in supersymmetry but needs a new independent assumption in alternative approaches. Supersymmetry generally does much better on gauge coupling unification since that is automatic there once the hierarchy assumption is made, while alternative approaches normally don't have gauge coupling unification and have to treat it as an accident.

$\surd$ = question addressed, $\surd\surd$ = good answer

Table 1.

Question	Standard Model(s)	Supersymmetric SM(s), Light Superpartners	String Theory
What is matter?	$\surd$		$\surd$
What is light?	$\surd\surd$		
What interactions give our world?	$\surd$		$\surd$
Gravity?			$\surd\surd$
Stabilize weak scale hierarchy?		$\surd\surd$	
Explain weak scale hierarchy?			$\surd$
Unify gauge couplings?		$\surd\surd$	
Higgs mechanism?		$\surd$	
What is dark matter?		$\surd$	$\surd$
Baryon asymmetry?		$\surd$	
Low scale superpartners			$\surd$
How is supersymmetry broken?			$\surd$
More than one family? 3?			$\surd$
Values of quark, lepton masses?			$\surd$
Values of neutrino masses?			$\surd$
Origin of CP violation?		$\surd$	$\surd$
Inflaton?		$\surd$	$\surd$
Cosmological constant not large?			$\surd$
Dark energy?			$\surd$
What is electric charge?			$\surd$
Space-time?			$\surd$
Quantum theory?			$\surd$
Origin of universe?			$\surd$

We can comment on a few examples. The SM provides a description of matter as quarks and leptons, but it cannot tell us why they are the matter of the universe, and why they have the charges they do. String theory addresses those questions. So each has one check for that row. Given that electrons and quarks exist, then, assuming the rules for constructing a description are relativistic quantum field theory, the electrons and quarks cannot be free particles and consistent with quantum theory. Rather a vector field must exist and it must have precisely the properties of the electromagnetic field, so the existence and properties of light are fully understood in the SM assuming a relativistic quantum field theory holds. Thus two checks there. Similarly, supersymmetry does stabilize the hierarchy but it does not imply that the superpartner masses are at the TeV scale. Once it is assumed that the superpartner masses are at the TeV scale, then supersymmetry does imply a true derivation of radiative electroweak symmetry breaking over a significant part of the parameter space, and gauge coupling unification, and it provides a good dark matter candidate. And so on.

3.7. *String Phenomenology*

We want to make another distinction. String theory studies theories, which is necessary to get to understand them better. Assuming string theory is relevant, our world is the ground state of a string theory. We want to highlight the subfield that focuses on learning the properties of that ground state and on connecting it to observed phenomena. That is quite different from string theory in general. This "string phenomenology" subfield has recently become quite active, with meetings and workshops of its own. We define it by the problems it aims to solve, so string phenomenology is the subfield of physics that studies at least the following issues, each of which requires an underlying theory, and which are mostly studied by no other subfield of physics:

- ☐ Why is there no large cosmological constant?
- ☐ What is dark energy, and why is the amount what it is?
- ☐ What is the dark matter?
- ☐ What is the origin of the matter asymmetry?
- ☐ What is the inflaton?
- ☐ Why does the SM take the form it does (why quarks and leptons and their charges, why that gauge group)?
- ☐ How is the electroweak symmetry broken? How can M_Z and $\tan\beta$ be calculated from first principles?

☐ Why do the force strengths unify, and do they unify with gravity?
☐ Is there low scale supersymmetry? Why?
☐ How is supersymmetry broken and the breaking transmitted?
☐ What are the values of the parameters with dimensions of mass in the soft supersymmetry breaking Lagrangian?
☐ How many families? Why more than one?
☐ Why do the quarks and leptons have the mass values they do?
☐ Why are neutrino masses small?
☐ What is the origin of CP violation? How is the CP violation needed for the matter asymmetry related to that observed in the K and B systems?
☐ Why are flavor-changing interactions in supersymmetry small?

It is worth emphasizing that the SM itself cannot (in principle) answer any of these. Perhaps more surprising, cosmology and astronomy cannot answer any of them either – cosmology and astronomy can tell us how much dark energy and dark matter there is, and constrain its properties, but they cannot tell us what the dark matter is or what the inflaton is. Only particle physics can do that. And only data from producing and studying new particles that point to how the SM is extended can give us the information to learn the answers to these questions. We might know the theory that contains the answers, and only need to focus attention and work harder to learn them, but without the data from colliders we cannot be confident enough to carry out that program.

3.8. *Is There Collider-Scale Physics Beyond the SM?*

It is the above questions that convinces us that there is physics beyond the SM. Yet surprisingly all of these questions could conceivably have answers at very short distances and cosmological scales. Amazingly, there is not yet a single convincingly observed phenomenon that must guarantee new physics at the weak or collider scale! It is also very interesting to list what might be observed in the new future that would guarantee new physics and explanations at scales that can be directly probed experimentally:

▲ Superpartners at the Tevatron, LHC
▲ Non-SM Higgs at Tevatron, LHC
▲ Z' at Tevatron, LHC
▲ Signal from dark matter detectors
▲ Confirmation of the HEAT positron excess as a signal
▲ Signal for EDMs

▲ Signal for $B_s \to \mu\mu$ at Tevatron or LHC
▲ Signal for lepton flavor violation, e.g. $\mu \to e\gamma$, $\tau \to \mu\gamma$
▲ $A_{CP}(B \to \phi K, \eta' K) \neq \sin 2\beta$
▲ $A_{CP}(B \to s\gamma) \ncong 0$
▲ Strengthening the $g_\mu - 2$ deviation from the SM

We won't take space to go into details about these. Perhaps it is interesting that the last three, all places where signals were anticipated, have 2 - 3σ effects at present.

3.9. *Flavor Physics and String Phenomenology*

The SM, and the supersymmetric SM, can accommodate the existence of flavor physics, i.e. the existence of three families of quarks and leptons, but they do not explain or require or motivate it. If there had been one family that would have been OK. On the other hand, if we had not known about flavor physics string theory would have motivated it. Some forms of string theory give several families of massless particles after reduction to four dimensions. Thus flavor physics may be a unique and powerful way to probe string theory.

More generally, after compactification to 4D the physics is contained in the supergravity functions W, the superpotential, K, the Kahler potential, and f, the gauge kinetic function. The Kahler potential can be thought of as a metric in the space of the chiral superfields, the matter fields, and the gauge kinetic function can be thought of as a metric for the vector gauge fields. The superpotential W also contains the chiral superfields, weighted by the Yukawa coupling matrices. If W and K are not diagonal simultaneously in the matter fields, then the soft-breaking Lagrangian must contain scalar masses or trilinear couplings with off-diagonal elements connecting flavors, and these will show up in rare decays and in production of detectable cross sections at LHC for flavor-changing effects. The information from such data could teach us different physics from the information deduced from gaugino and scalar cross sections, and could be very important.

Once there are flavors, a string theory is likely to relate them by a flavor symmetry of some sort. That flavor symmetry could basically determine the masses of quarks and leptons, and the CKM matrix. It would also determine the flavor structure of the soft breaking Lagrangian, and thus rare decays [17,18].

There are several ways CP-violating effects could enter physics. It could come via these flavor interactions, in the superpotential Yukawa matrices, and in the L_{soft} trilinear coupling matrices. It could come from complex moduli F-term vevs. Data on flavor effects could distinguish. If data could favor one origin or another for CP violation it might greatly help to focus on the type of string theory that described out world. A particularly attractive situation would come from strengthening the argument that the many CPV phases of the soft lagrangian were essentially zero. One must, however, always keep in mind that some physics in addition to the SM CKM phase is required to generate the matter asymmetry.

Another flavor issue that string theory should explain is why neutrino masses are small. Even if calculating actual neutrino masses is too difficult in practice, it should be possible to provide an explanation for their size. Since some compactification may allow that and others not such studies may provide another way to point to the correct theory [19].

4. “Derivation” of the Supersymmetry Lagrangian

In order to understand the predictions and explanations of supersymmetry, particularly for the Higgs sector, we must learn the derivation of the supersymmetry Lagrangian. “Derivation” is in quotes because I will present the arguments fully but not properly derive all the steps. I will largely follow the approach of Martin, but with somewhat less detail. More detailed derivations are now available in the books, but it is worthwhile to include this derivation to understand the role of the superpotential for matter fields. This approach to a derivation is helpful from a particle point of view. An alternative derivation from a superfield point of view is more helpful for understanding the non-renormalization theorem and some general features.

Consider a massless and therefore two-component fermion, ψ whose superpartner is a complex scalar ϕ. Both have two real degrees of freedom. But in the off-shell field theory the fermion is a four-component field with four degrees of freedom, and we want supersymmetry to hold for the full field theory. So we introduce an additional complex scalar F so that there are four scalar degrees of freedom also. F is called an auxiliary field. The combined fields (ψ, ϕ, F) are called a chiral superfield or chiral supermultiplet. I will not be systematic or careful about the two-component vs. four-component notation since the context usual is clear. The Lagrangian can be taken to be

$$-L_{free} = \sum_i (\partial^\mu \phi_i^* \partial_\mu \phi_i + \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + F_i^* F_i). \qquad (1)$$

The sum is over all chiral supermulitplets in the theory. Note that the dimensions of F are $[F] = m^2$. The Euler-Lagrange equations of motion for F are $F = F^* = 0$, so on-shell we revert to only two independent degrees of freedom. One can define supersymmetry transformations that take bosonic degrees of freedom into fermionic ones; we will look briefly at them later. The supersymmetry transformations can be defined so that L_{free} is invariant. Next we write the most general set of renormalizable interactions,

$$L_{chiral} = L_{free} + L_{int} \tag{2}$$

$$L_{int} = -\frac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i + c.c. \tag{3}$$

Here W^{ij} and W^i are any functions of only the scalar fields, remarkably, and W^{ij} is symmetric. If W^{ij} or W^i depended on the fermion or auxiliary fields the associated terms would have dimension greater than four, and would therefore not be renormalizable. There can be no terms in L_{int} that depend on ϕ_i^* or ϕ_i since such terms would not transform into themselves under the supersymmetry transformations.

Now imagine supersymmetry transformations that mix fermions and bosons, $\phi \rightarrow \phi + \varepsilon\psi, \psi \rightarrow \psi + \varepsilon\phi$. We should go through these transformations in detail with indices, but one can see the basic argument simply. Here ε must be a spinor so each term behaves the same way in spin space, and we can take ε to be a constant spinor in space-time, and infinitesimal. Then the variation of the Lagrangian (which must vanish or change only by a total derivative if the theory is invariant under the supersymmetry transformation) contains two terms with four spinors:

$$\delta L_{int} = -\frac{1}{2} \frac{\delta W^{ij}}{\delta \phi_k} (\varepsilon \psi_k) \psi_i \psi_j - \frac{1}{2} \frac{\delta W^{ij}}{\delta \phi_k^*} (\varepsilon^\dagger \psi_k^\dagger) \psi_i \psi_j + c.c. \tag{4}$$

Neither term can cancel against some other term. For the first term there is a Fierz identity $(\varepsilon\psi_i)(\psi_j\psi_k) + (\varepsilon\psi_j)(\psi_k\psi_i) + (\varepsilon\psi_k)(\psi_i\psi_j) = 0$, so if and only if $\delta W^{ij}/\delta\phi_k$ is totally symmetric under interchange of i,j,k the first term vanishes identically For the second term the presence of the hermitean conjugation allows no similar identity, so it must vanish explicitly, which implies $\delta W^{ij}/\delta\phi_k^* = 0$,and thus W^{ij} cannot depend on ϕ^*! W^{ij} must be an analytic function of the complex field ϕ.

Therefore we can write

$$W^{ij} = M^{ij} + y^{ijk}\phi_k, \tag{5}$$

where M^{ij} is a symmetric matrix that will be the fermion mass matrix, and y^{ijk} can be called Yukawa couplings since it gives the strength of the coupling of boson k with fermions i, j; y^{ijk} must be totally symmetric. Then it is very convenient to define

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k \tag{6}$$

and $W^{ij} = \delta^2 W/\delta\phi_i\delta\phi_j$. W is the "superpotential", an analytic function of ϕ, and a central function of the formulation of the theory. W is fully supersymmetric and gauge invariant and Lorentz invariant, and an analytic function of ϕ (i.e. it cannot depend explicitly on ϕ^*), so it is highly constrained. It determines the most general non-gauge interactions of the chiral superfields.

A similar argument for the parts of δL_{int} which contain a spacetime derivative imply that W^i is determined in terms of W as well,

$$W^i = \frac{\delta W}{\delta\phi_i} = M^{ij}\phi_j + \frac{1}{2}y^{ijk}\phi_j\phi_k. \tag{7}$$

Because interactions are now present, the equations for F are non-trivial,

$$F_i = -W_i^*. \tag{8}$$

The scalar potential is related to the Lagrangian by $L = T - V$, so

$$V = \sum_i |F_i|^2 \tag{9}$$

This contribution is called an "F-term", and is automatically bounded from below, an important improvement.

The above analysis was appropriate for chiral superfields, which will contain the fermions and their superpartners. Now we repeat the logic for the gauge supermultiplets that contain the gauge bosons and their superpartners. Initially they are massless gauge bosons, like photons, A^a_μ, with gauge index a, and two degrees of freedom. Their superpartners are two-component spinors λ^a. But as above, off shell the fermion has four degrees of freedom, while the massive boson has three, the two transverse polarizations and a longitudinal polarization. So again it is necessary to add an auxiliary field, a real one since only one degree of freedom is needed, called D^a. Then the Lagrangian has additional pieces

$$L_{gauge} = -\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} - i\lambda^{\dagger a}\gamma^\mu D_\mu\lambda^a + \frac{1}{2}D^a D^a, \tag{10}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc}A^b_\mu A^c_\nu \tag{11}$$

and the covariant derivative is

$$D_\mu\lambda^a = \partial_\mu\lambda^a - gf^{abc}A^b_\mu\lambda^c. \tag{12}$$

Note that the notation is unfortunate, with both the covariant derivative and the new field being denoted by the standard "D". Also, I have not been careful about two component vs. four component spinors. It is crucial for gauge invariance that the same coupling g appears in the definition of the tensor F and in the covariant derivative. Lagrangians always have to contain all of the terms allowed by gauge invariance, etc., and here we can see another term to add,

$$(\phi_i^* T^a \phi_i)D^a. \tag{13}$$

There is one more term that can be added that mixes the fields, $\lambda^{\dagger a}(\psi^\dagger T^a\phi)$, and its conjugate, with an arbitrary dimensionless coefficient. Requiring the entire Lagrangian to be invariant under supersymmetry transformations determines the arbitrary coefficient and gives a resulting Lagrangian

$$\begin{aligned} L = L_{gauge} + L_{chiral} + g_a(\phi^* T^a\phi)D^a \\ -\sqrt{2}g_a[(\phi^* T^a\psi)\lambda^a + \lambda^{\dagger a}(\psi^\dagger T^a\phi)] \end{aligned} \tag{14}$$

where all derivatives in earlier forms are replaced by covariant ones. Remarkably, the requirement of supersymmetry fixed the couplings of the last terms to be gauge couplings even though they are not normal gauge interactions! The equations of motion for D^a give $D^a = -g(\phi^* T^a\phi)$, so the scalar potential is

$$V = F^{*i}F_i + \frac{1}{2}\sum_a D^a D^a = |\partial W/\partial\phi_i|^2 + \sum_a g_a^2(\phi^* T^a \phi)^2. \tag{15}$$

The sum is over $a = 1, 2, 3$ for the three gauge couplings. The two terms are called "F-terms" and "D-terms". Remarkable, the unbroken supersymmetric theory gives a scalar potential bounded from below. On the one hand that is good since unbounded potentials are a problem, but it also implies that the Higgs mechanism cannot happen for unbroken supersymmetry since the potential will be minimized at the origin. In the above,

$$L_{chiral} = D^\mu \phi_i^* D_\mu \phi_i + \bar{\psi}_i \gamma^\mu D_\mu \psi_i \tag{16}$$

$$+(\frac{1}{2}M_{ij}\psi_i\psi_j + \frac{1}{2}y^{ijk}\phi_i\psi_j\psi_k + c.c.) + F_i^* F_i.$$

This completes the derivation of the unbroken supersymmetry Lagrangian.

5. Non-Renormalization Theorem

For unbroken supersymmetry there is a very important result, called the non-renormalization theorem, that is very useful for understanding and building models to relate the theory to the real world. Because of this result the supersymmetry fields get a wave function renormalization only, so they have the familiar log running of couplings and masses, but no other renormalizations. Consequently the parameters of the superpotential W are not renormalized, in any order of perturbation theory. In particular, terms that were allowed in W by gauge invariance and Lorentz invariance are not generated by quantum corrections if they are not present at tree level, so no F-terms are generated if they are initially absent. If there is no μ –term in the superpotential (see below), none is generated. The non-renormalization theorem is difficult to prove without extensive formalism, so I just state it here. References and a pedagogical derivation are given in reference 7.

6. Toward Softly-Broken Supersymmetry with a Toy Model

Consider the Wess-Zumino model, with,

$$W = \frac{m}{2}\phi\phi + \frac{g}{6}\phi\phi\phi, \tag{17}$$

and

$$L = (\partial\phi)^2 + i\Psi^\dagger\bar{\sigma}^\mu\partial_\mu\Psi - F_\phi^* F_\phi + (\frac{1}{2}W_{\phi\phi}\Psi\Psi - W_\phi F_\phi + c.c.). \tag{18}$$

$W_\phi = -F_\phi^* = m\phi + \frac{g}{2}\phi\phi$ is the derivative of the superpotential with respect to ϕ,and $W_{\phi\phi}$ the second derivative. We put $\phi = (A + iB)/2$ and $F_\phi = (F + iG)/2$, where A, B, F, G are real scalars, and switch to four component notation. Under the supersymmetry transformations, with ε a constant spinor,

$$\delta A = \bar{\varepsilon}\gamma_5\Psi, \tag{19}$$

$$\delta B = -\bar{\varepsilon}\Psi, \tag{20}$$

$$\delta\Psi = F\varepsilon - G\gamma_5\varepsilon + \gamma^\mu\partial_\mu\gamma_5 A\varepsilon + \gamma^\mu\partial_\mu B\varepsilon, \tag{21}$$

$$\delta F = -\bar{\varepsilon}\gamma^\mu\partial_\mu\Psi, \tag{22}$$

$$\delta G = -\bar{\varepsilon}\gamma_5\gamma^\mu\partial_\mu\Psi, \tag{23}$$

the Lagrangian changes by a total derivative, so the action is invariant with the usual assumptions.

Now substitute for W_ϕ and $W_{\phi\phi}$ etc. Then the Lagrangian is

$$\begin{aligned} L = &\frac{1}{2}(\partial A)^2 + \frac{1}{2}(\partial B)^2 + \frac{i}{2}\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{1}{2}m\bar{\psi}\psi \\ &+ \frac{g}{\sqrt{2}}A\bar{\psi}\psi - \frac{ig}{\sqrt{2}}B\bar{\psi}\gamma_5\psi - \frac{1}{2}(F^2 + G^2) \\ &- \frac{m}{2}(2AF - 2BG) - \frac{g}{2\sqrt{2}}(F(A^2 - B^2) - 2GAB). \end{aligned} \tag{24}$$

The equations of motion for F, G are

$$F = -mA - \frac{g}{2\sqrt{2}}(A^2 - B^2), G = mB + \frac{g}{\sqrt{2}}AB. \tag{25}$$

Substituting these gives interaction vertices $\frac{mg}{2\sqrt{2}}A(A^2 - B^2)$.

With one coupling strength g and one mass m the full Lagrangian is supersymmetric. (Note that without supersymmetry there can be four different masses and four different couplings, so there are six relations predicted by supersymmetry which only allows one mass and one coupling.) But when supersymmetry is broken we expect the masses to differ. Suppose we allow four different masses, m_A, m_B, m_ψ, and m_g, where the last is the mass that is needed in some terms to give each term dimension four, so it multiplies g. It's clear how to rewrite the Lagrangian with these separate masses. There are four three-particle vertices, $A\bar{\psi}\psi$, $A^3, AB^2, B\bar{\psi}\psi$. Now if we write the expression for a tadpole graph,

$$\langle 0\,|L|\,A\rangle =$$

$$\frac{g}{\sqrt{2}}\left\{4m_\psi\int\frac{d^4p}{p^2-m_\psi^2}-m_g\int\frac{d^4p}{p^2-m_B^2}-3m_g\int\frac{d^4p}{p^2-m_A^2}\right\} \tag{26}$$

we see that in general this has a quadratic divergence, which cancels in the supersymmetry limit as expected. The fermion loop gives a minus sign, the factor of 4 in the first term arises from $Tr(\gamma^\mu p_\mu + m_\psi) = Tr m_\psi = 4m_\psi$, and the 3 in the last from the A^3. But — and here is the important point — the divergence still cancels if $m_A \neq m_B \neq m_g$, but not if $m_\psi \neq m_g$. Thus extra contributions to boson masses do not reintroduce quadratic divergences — they are called "soft" supersymmetry breaking. But extra contributions to fermion masses do lead to quadratic divergences, "hard" supersymmetry breaking. This result is true to all orders in perturbation theory, though this pedagogical argument does not show it. Some of the results are obvious since couplings proportional to masses will not introduce quadratic divergences, but it is still helpful to see the supersymmetry structure. After the supersymmetry breaking there are three masses and one coupling, so there are still four tests that the theory is a broken supersymmetric one.

To understand the general structure of supersymmetry breaking better, recall how symmetry breaking works in the SM. It is not possible to break the $SU(2)\times U(1)$ symmetry from within the SM. So a new sector, the Higgs sector is needed. Interactions are assumed in the Higgs sector that give a potential with a minimum away from the origin, so the Higgs field gets a vev which breaks the symmetry. To generate mass for W, Z, q, l an interaction is needed to transmit the breaking to the "visible" particles W, Z, q, l. For fermions this interaction is $L_{fermion} = g_e \bar{e}_L e_R h + cc \rightarrow g_e \mathrm{v}\bar{e}e$ after h gets a vev for the fermions, and we can identify $m_e = g_e \mathrm{v}$. Similarly, for the gauge bosons the Lagrangian term $(D^\mu h)(D_\mu h) \rightarrow g^2 hhW^\mu W_\mu \rightarrow g^2\mathrm{v}^2 W^\mu W_\mu$ giving W, Z masses. The fundamental symmetry breaking is spontaneous (the Lagrangian remains symmetric, but the ground state solution no longer has the symmetry because h gets a vev), but the effective Lagrangian appears to have explicit breaking.

The situation is very similar for supersymmetry. It is not possible to break supersymmetry in the "visible" sector, i.e. the sector containing the superpartners of the SM particles. A separate sector is needed where supersymmetry is broken. Originally it was called the "hidden" sector, but that is not a good name since it need not be really hidden. Then there must be some interaction(s) to transmit the breaking to the visible sector. Since the particles of both sectors interact gravitationally, gravity can always transmit the breaking. Other interactions may as well. We will have to find out how the breaking is transmitted from data on the superpartners, their masses and decays and phases and flavor rotations. Different ways of transmitting the breaking give different patterns of the soft parameters that we discuss below. A significant and interesting complication is that the effects of the supersymmetry breaking are mixed up with effects of the transmission, so it will be a nice challenge to untangle them. All the effects of the supersymmetry breaking and of the way it is transmitted, for any theory, are embedded in the soft-breaking Lagrangian that we turn to studying. In addition, clues to the underlying theory are also embedded in the soft-breaking Lagrangian.

7. The Soft-Breaking Lagrangian

The (essentially) general form of L_{soft} is [20]

$$\begin{aligned} L_{soft} = &\frac{1}{2}(M_\lambda \lambda^a \lambda^a + c.c.) + m_{ij}^2 \phi_j^* \phi_i \qquad (27) \\ &+ (\frac{1}{2} b_{ij} \phi_i \phi_j + \frac{1}{6} a_{ijk} \phi_i \phi_j \phi_k + c.c.) \end{aligned}$$

This obviously breaks supersymmetry since only scalars and gauginos get mass, not their superpartners. It is soft as in our example above because it can be proved to not introduce any quadratic divergences. Models for supersymmetry breaking, however they originate, in string theory or supergravity or dynamically, all lead to this form. We will write it for the SSM shortly.

If all fields carry gauge quantum numbers there are terms that could be added to this without generating quadratic divergences, such as $\phi_i^* \phi_j \phi_k$, but such terms seldom arise in models so they are usually ignored [61]. If such terms are truly absent once measurements are analyzed, their absence may be a clue to how supersymmetry is broken and transmitted.

8. The Minimal Supersymmetric Standard Model

To write the supersymmetric SM we first take all of the quarks and leptons and put them in chiral superfields with superpartners. Putting them in chiral supermultiplets, with new spin-zero superpartners, is necessary because of parity violation, which requires the fermions to be chiral. Similarly, the self-adjoint gauge bosons have to be in vector supermultiplets and those have spin -1/2 superpartners.For each set of quantum numbers, such as up quarks or electrons, the scalar, fermion, and auxiliary fields (ϕ, ψ, F) form a supermultiplet in the same sense as (n, p) form a strong isospin doublet or (ν_e, e) form an electroweak doublet. All superpartners are denoted with a tilde, and there is a superpartner for each spin state of each fermion — that is important since the SM treats fermions of different chirality differently. The gauge bosons are put in vector superfields with their fermionic superpartners. Since W is analytic in the scalar fields, we cannot include the complex conjugate of the scalar field as in the SM to give mass to the down quarks, so there must be two Higgs doublets (or more) in supersymmetry, and each has its superpartners. The requirement that the trace anomalies vanish so that the theories stay renormalizable, $TR(Y^3) = TR(T_{3L}^2 Y) = 0$, also implies the existence of the same two Higgs doublets. (The relevance of anomalies may seem unclear since we are only writing an effective theory, while anomaly conditions only need to be satisfied for the full theory. But if the anomaly conditions are not satisfied it may introduce a sensitivity to higher scales that the effective theory should not have.)

We proceed by first constructing the superpotential so we can calculate the F-terms, and then writing the Lagrangian, following equation 6 and summing over all the particles. The most general superpotential, if we don't extend the SM and don't include RH neutrinos, is

$$W = \bar{u} Y_u Q H_u - \bar{d} Y_d Q H_d - \bar{e} Y_e L H_d + \mu H_u H_d. \tag{28}$$

All the fields are chiral superfields. The bars over u, d, e are in the sense of Martin's notation, specifying the conjugate fields. The signs are conventional so that masses later are positive. Indices are suppressed — for example, the fourth and first terms are

$$\mu (H_u)_\alpha (H_d)_\beta \varepsilon_{\alpha\beta} \text{ and } \bar{u}_{ai} (Y_u)_{ij} Q^a_{j\alpha} (H_u)_\beta \varepsilon_{\alpha\beta}. \tag{29}$$

The Yukawa couplings Y_u etc. are dimensionless 3×3 family matrices that determine the masses of quarks and leptons, and the angles and phase of the CKM matrix after H_u^0 and H_d^0 get vevs. They also contribute to the squark-quark-higgsino couplings etc. since the fields in W are superfields containing all the components. This is the most general superpotential for the SSM if we assume baryon and lepton number are conserved (we'll return to this question). To see the structure more explicitly we can use the approximations

$$Y_u \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Y_t \end{pmatrix}, \quad Y_d \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Y_b \end{pmatrix}, \quad Y_e \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Y_\tau \end{pmatrix}, \tag{30}$$

which gives

$$\begin{aligned} W = {} & Y_t(\bar{t}tH_u^0 - \bar{t}bH_u^+) - Y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) \\ & -Y_\tau(\bar{\tau}\nu_\tau H_d^- - \bar{\tau}\tau H_d^0) + \mu(H_u^+H_d^- - H_u^0 H_d^0) \end{aligned} \tag{31}$$

There are also other interactions from W such as vertices $H_u^0 t_R^* t_L$, $\tilde{H}_u^0 t_R^* \tilde{t}_L$, $\tilde{H}_u^0 \tilde{t}_R t_L$, etc., all with the same strength Y_t. All of them are measurable, and it will be an important check of supersymmetry to confirm they are all present with the same strength. All are dimensionless, so supersymmetry-breaking will only lead to small radiative corrections to these coupling strengths. In general one goes from one to another of these by changing any pair of particles into superpartners (except when Higgs fields that can get vevs are involved).

Before we turn to writing the full soft-breaking Lagrangian, we first look at two significant issues that depend on how supersymmetry is embedded in a more basic theory.

9. The μ Opportunity

The term $\mu H_u H_d$ in the superpotential leads to a term in the Lagrangian

$$L = \ldots\ldots.. + \mu(\tilde{H}_u^+ \tilde{H}_d^- - \tilde{H}_u^0 \tilde{H}_d^0) + \ldots\ldots. \tag{32}$$

which gives mass terms for higgsinos in the chargino and neutralino mass matrices, so μ enters there. This term also contributes to the scalar Higgs potential from the F-terms,

$$V = \ldots\ldots. \left|\mu\right|^2 \left(\left|H_u^0\right|^2 + \left|H_d^0\right|^2 + \ldots\right) + \ldots\ldots.. \tag{33}$$

so these terms affect the Higgs mass, and F-terms also give contributions to the Lagrangian that affect the squark and slepton mass matrices,

$$L = ...\mu^*(\widetilde{\bar{u}} Y_u \tilde{u} H_d^{0*} + ...) + ... \tag{34}$$

Thus phenomenologically μ must be of order the weak scale to maintain the solutions of the hierarchy problem, gauge coupling unification, and radiative electroweak symmetry breaking. The naive scale for any term in the superpotential is one above where the supersymmetry is broken, e.g. the string scale or unification scale, and since μ occurs in W one would naively expect μ to be of order that scale, far above the weak scale. In the past that has been called the "μ problem". But actually it is a clue to the correct theory and is an opportunity to learn what form the underlying theory must take. For example, in a string theory we expect all the mass terms to vanish since the SM particles are the massless modes of the theory, so in a string theory μ, which is a mass term, would naturally vanish. That could be a clue that the underlying theory is indeed a string theory. In the following we will view $\mu = 0$ as a "string boundary condition". Older approaches added symmetries to require $\mu = 0$. Note that because of the non-renormalization theorem once μ is set to zero in W it is not generated by loop corrections at any scale above that for which supersymmetry is broken.

We also know phenomenologically that the μ contribution to the chargino and neutralino masses and the Higgs mass cannot vanish, or some of them would be so light they would have been observed, so we know that somehow a piece that plays the same role as μ is generated. We will call it μ_{eff}, but whenever there is no misunderstanding possible we will drop the subscript and just write μ for μ_{eff}. Different ways of generating μ_{eff} give different relations to the other soft-breaking parameters, a different phase for μ_{eff}, a characteristic size for μ_{eff}, etc. Once it is measured we will have more clues to the underlying theory. Any top-down approach must generate μ_{eff} and its phase correctly.

There are very important points here. If supersymmetry is viewed as an effective low scale theory, then μ is a "problem", and can only be treated phenomenologically, as a parameter. But if SUSY is viewed as originating in a high scale theory their μ is a part of the theory that arises differently than Lsoft does, yet is of the same order as Lsoft. Understanding μ (including its phase) is potentially capable of providing important insights into string theory. We will not understand the supersymmetric SM until we understand how μ arises, and we can calculate μ from the underlying theory.

10. R-Parity Conservation

The μ opportunity looks like the μ problem if one views supersymmetry as an effective low energy theory without seeing it as embedded in a more fundamental high scale theory. Similarly, if we view supersymmetry as only a low energy effective theory there is another complication that arises. There are additional terms that one could write in W that are analytic, gauge invariant, and Lorentz invariant, but violate baryon and/or lepton number conservation. No such terms are allowed in the SM, which accidentally conserves B and L to all orders in perturbation theory, though it does not conserve them non-perturbatively. These terms are

$$W_R = \lambda_{ijk} L_i L_j e_k + \lambda'_{ijk} L_i Q_j \bar{d}_k + \lambda''_{ijk} \bar{u}_i \bar{d}_j d_k. \tag{35}$$

The couplings $\lambda, \lambda', \lambda''$ are matrices in family space. Combining the second and third one can get very rapid proton decay, so one or both of them must be required to be absent. That is not the way one wants to have a theory behave. Rather, B and L conservation consistent with observation should arise naturally from the symmetries of the theory. Most, but not all, theorists expect that an underlying symmetry will be present in the broader case to forbid all of the terms in W_R.

There are two approaches to dealing with W_R. We can add a symmetry to the effective low energy theory, called R-parity or a variation called matter parity, which we assume will arise from a string theory or extended gauge group. R-parity is multiplicatively conserved,

$$R = (-1)^{3(B-L)+2S} \tag{36}$$

where S is the spin. Then SM particles and Higgs fields are even, superpartners odd. This is a discrete Z_2 symmetry. Such symmetries that treat superpartners differently from SM particles and therefore do not commute with supersymmetry are called R-symmetries. Equivalently, one can use "matter parity",

$$P_m = (-1)^{3(B-L)}. \tag{37}$$

A term in W is only allowed if $P_m = +1$. Gauge fields and Higgs are assigned $P_m = +1$, and quark and lepton supermultiplets $P_m = -1$. P_m

commutes with supersymmetry and forbids W_R. Matter parity could be an exact symmetry, and such symmetries do arise in string theory. If R-parity or matter parity holds there are major phenomenological consequences,

- At colliders, or in loops, superpartners are produced in pairs.
- Each superpartner decays into one other superpartner (or an odd number).
- The lightest superpartner (LSP) is stable. That determines supersymmetry collider signatures, and makes the LSP a good candidate for the dark matter of the universe.

The second approach is very different, and does not have any of the above phenomenological consequences. One arbitrarily sets λ' or $\lambda'' = 0$ so there are no observable violations of baryon number or lepton number conservation. Other terms are allowed and one sets limits on them when their effects are not observed, term by term. In the MSSM itself R-parity must be broken explicitly if it is broken at all. If it were broken spontaneously by a sneutrino vev there would be a Goldstone boson associated with the spontaneous breaking of lepton number (called a Majaron), and some excluded Z decays would have been observed.

We will not pursue this ad hoc approach, because we do not like arbitrarily setting some terms to zero, and we do not like giving up the LSP as cold dark matter if we are not forced to. Further, large classes of theories conserve R-parity or matter parity [21]. Recently this class has been generalized [22]. Often theories have a gauged $U(1)_{B-L}$ symmetry that is broken by scalar vevs and leaves P_m automatically conserved. String theories often conserve R-parity or P_m. Often theories conserve R-parity at the minimum of the Higgs potential. Baryogenisis via leptogenesis probably requires R-parity conservation because the usual $B + L$ violation plus L violation would allow the needed asymmetries to be erased. The lepton number needed for ν seesaw masses violates L by two units and does not violate R-parity conservation. In general, when supersymmetry is viewed as embedded in a more fundamental theory, R-parity conservation is very likely and easily justified. Ultimately, of course, experiment will decide, but we will assume R-parity conservation in the rest of these lectures.

11. Definition of MSSM

At this stage we can define the effective low energy supersymmetry theory, which we call the MSSM, as the theory with the SM gauge group and particles, and the superpartners of the SM particles, and conserved R-parity, and two Higgs doublets. Perhaps it would be better to include right handed neutrinos and their superpartners as well, but for simplicity we will do that later. For the MSSM, "Minimal" refers to the particle and gauge group content.

12. The MSSM Soft-Breaking Lagrangian

We can now write the general soft-breaking Lagrangian for the MSSM,

$$\begin{aligned}
-L_{soft} = {} & \tfrac{1}{2}(M_3\tilde{g}\tilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + c.c.) \\
& +\widetilde{Q}^\dagger m_Q^2 \widetilde{Q} + \widetilde{\bar{u}}^\dagger m_{\bar{u}}^2 \widetilde{\bar{u}} + \widetilde{\bar{d}}^\dagger m_d^2 \widetilde{\bar{d}} + \widetilde{L}^\dagger m_L^2 \widetilde{L} + \widetilde{\bar{e}}^\dagger m_{\bar{e}}^2 \widetilde{\bar{e}} \\
& +(\widetilde{\bar{u}}^\dagger a_u \widetilde{Q} H_u - \widetilde{\bar{d}}^\dagger a_d \widetilde{Q} H_d - \widetilde{\bar{e}}^\dagger a_e \widetilde{L} H_d + c.c.) \\
& +m_{H_u}^2 H_u^* H_u + m_{H_d}^2 H_d^{2*} + (bH_uH_d + c.c.). \qquad (38)
\end{aligned}$$

For clarity a number of the indices are suppressed. $M_{1,2,3}$ are the complex bino, wino, and gluino masses, e.g. $M_3 = |M_3|\, e^{i\phi_3}$, etc. In the second line m_Q^2, etc, are squark and slepton hermitean 3×3 mass matrices in family space. The $a_{u,d,e}$ are complex 3×3 family matrices, usually called trilinear couplings. b is sometimes written as $B\mu$ or as m_3^2 or as m_{12}^2. Additional parameters come from the gravitino complex mass and from $\mu_{eff} = \mu e^{i\phi_\mu}$; we will usually risk writing the magnitude of μ_{eff} as just μ assuming the context will distinguish this from the original μ of the superpotential. This may seem to involve a lot of parameters, but all the physical parameters are observable from direct production and study of superpartners and their effects. The absence of observation of superpartners and their effects already gives us useful information about some of the parameters. It is important to understand that all of these parameters are masses or flavor rotation angles or phases or Higgs vevs, just as for the SM. If we had no measurements of the quark and lepton masses and interactions there would be even more parameters for the SM than here. At present, no parameter with dimensions of mass can be calculated from basic principles, so this is the minimum possible number of parameters.

With this Lagrangian we can do general, useful, reliable phenomenology, as we will see. For example, in the SM we did not know the top quark mass until it was measured. Nevertheless, for any chosen value of the top mass we could calculate its production cross section at any collider, all of its decay BR, its contribution to radiative corrections, etc. Similarly, for the superpartners we can calculate expected signals, study any candidate signal and evaluate whether it is consistent with the theory and with other constraints or data, and so on. A possible signal might have too small or large a cross section to be consistent with any set of parameters, or decay BR that could not occur here. Many examples can be given. We can also study whether superpartners can be studied at any proposed future facility. Further, most processes depend on only a few of the parameters — we will see several examples of this in the following.

Now let us count the parameters of the broken supersymmetric theory relative to the SM. There are no new gauge or Yukawa couplings, and still only one strong CP angle $\bar{\vartheta}$, so that is already rather economical. Then

- m_Q^2, etc are 5 3×3 hermitean matrices → 9 real parameters each → 45
- $a_{u,d,e}$ are 3 3×3 complex matrices → 18 real parameters each → 54
- $M_{1,2,3}$, μ, b are complex → 10
- $m_{H_{u,d}}^2$ are real by hermiticity → 2

giving a total of 111 parameters. As for the CKM quark matrix it is possible to redefine some fields and absorb some parameters. Baryon and lepton number are conserved, and there are two U(1) symmetries that one can see by looking at the Lagrangian. One arises because if μ and b are zero there is a symmetry where $H_{u,d} \to e^{i\alpha} H_{u,d}$ and the combinations $L\bar{e}, Q\bar{u}, Q\bar{d} \to e^{-i\alpha} L\bar{e}, Q\bar{u}, Q\bar{d}$. For example, one can take $Q \to e^{-i\alpha} Q$, $L \to e^{-i\alpha} L$, and $\bar{e}, \bar{u}, \bar{d}$ invariant. Such a symmetry is called a Peccei-Quinn symmetry if it holds for $\mu = 0$ but is broken when $\mu \neq 0$. The other arises because if $M_i, a_i, b = 0$ there is a continuous R-symmetry, e.g. the Higgs fields can have charge 2, the other matter fields charge 0, and the superpotential charge 2. Symmetries are called R-symmetries whenever members of a supermultiplet are treated differently.

With these four symmetries, four parameters can be absorbed. Also, the SM has two parameters in the Higgs potential, $\mu^2 \phi^2 + \lambda \phi^4$, so to count the number beyond the SM we subtract those 2. Then there are 111-4-2=105 new parameters. The SM itself has 3 gauge couplings, 9 quark and charged lepton masses, 4 CKM angles, 2 Higgs potential parameters, and one strong CP phase → 19. So there are 124 parameters altogether. When massive neutrinos are included one has RH ν masses, and the angles of the flavor rotation matrix (which has 3 real angles and 3 phases for the ν case since the Majorana nature of the neutrinos prevents absorbing two of the phases). In the following we will discuss how to measure many of the parameters. All are measurable in principle. Once they are measured they can be used to test any theory. In practice, as always historically, some measurements will be needed to formulate the underlying theory (e.g. to learn how supersymmetry is broken and to compactify) and others will then test approaches to doing that.

Only 32 of these parameters are masses of mass eigenstates! There are four neutralinos, two charginos, four Higgs sector masses, three LH sneutrinos, six each of charged sleptons, up squarks, and down squarks, and the gluino. We will examine the connections between the soft masses and the mass eigenstates below. Of the 32 masses, only the gluino occurs directly in L_{soft} — the rest are all related in complicated ways to L_{soft}! One could add the gravitino with its complex mass to the list of parameters. Even the gluino mass gets significant corrections that depend on squark masses.

Some phases, those of the gaugino masses (ϕ_1, ϕ_2, ϕ_3), of μ (ϕ_μ) and of $b(\phi_b)$, and the overall phases of the soft trilinears $(\phi_{At}, \phi_{Ab}, \phi_{Ae})$ are flavor-independent. They are significantly constrained by EDMs. The remaining (flavor-dependent) phases are from the off-diagonal entries of the trilinears and sfermion m^2 matrices. The counting of parameters and phases is presented in more detail in [11].

Some of the ways these parameters contribute is to determining the breaking of the EW symmetry and therefore to the Higgs potential, and the masses and cross sections and decays of Higgs bosons, to the relic density and annihilation and scattering of the LSP, to flavor changing transitions because the rotations that diagonalize the fermion masses will not in general diagonalize the squarks and sleptons, to baryogenesis (which cannot be explained with only the CKM phase), to superpartner masses and signatures at colliders, rare decays with superpartner loops (e.g. $b \to s + \gamma$), electric dipole and magnetic dipole moments, and more.

13. Connecting High and Low Scales

Two of the most important successes of supersymmetry depend on connecting the unification and EW scales. We will not study this topic in detail here since Martin covers it thoroughly, as do the recent and coming books [13,14] but we will look at the aspects we need, particularly for the Higgs sector. The connection is through the logarithmic renormalization and running of masses and couplings, with RGEs. In general we imagine the underlying theory to be formulated at a high energy scale, while we need to connect with experiment at the EW scale. We can imagine running the theory down (top-down) or running an effective Lagrangian determined by data up (bottom-up). It is necessary to calculate for all the parameters of the superpotential and of the soft-breaking Lagrangian. The RGEs are known for gauge couplings and for the superpotential couplings to three loops, and to two loops for other parameters, for the MSSM and its RHν extension. We will only look at one-loop results since we are mainly focusing on pedagogical features. An interesting issue is that calculations must be done with regularization and renormalization procedures that do not break supersymmetry, and that is not straightforward. How to do that is not a solved problem in general, but it is understood through two loops and more loops in particular cases, so in practice there is no problem.

Since our ability to formulate a deeper theory will depend on deducing from data the form of the theory at the unification scale, learning how to convert EW data first into an effective theory at the weak scale, and then into an effective theory at the unification scale, is in a sense the major challenge for particle physics in the coming years. There are of course ambiguities in running to the higher scales. Understanding the uniqueness of the resulting high scale theory, and how to resolve ambiguities as well as possible, is very important.

For the Higgs sector we need to examine the running of several of the soft masses, whose RGEs follow. The quantity t is $\ln(Q/Q_0)$, where Q is the scale and Q_0 a reference scale.

$$16\pi^2 dM^2_{H_u}/dt \approx 3X_t - 6g_2^2\left|M_2\right|^2 - \frac{6}{5}g_1^2\left|M_1\right|^2 \tag{39}$$

$$16\pi^2 dM^2_{H_d}/d \approx 3X_b + X_\tau - 6g_2^2\left|M_2\right|^2 - \frac{6}{5}\left|M_1\right|^2 \tag{40}$$

where

$$X_{t,b} \approx 2\left|Y_{t,b}\right|^2 (M^2_{H_{u,d}} + m^2_{Q_3} + m^2_{\bar{u}_3,\bar{d}_3}) + 2\left|a_{t,b}\right|^2 \tag{41}$$

Note that $X_{t,b}$ are positive so $M^2_{H_{u,d}}$ decrease as they evolve toward the EW scale from a high scale, and unless $\tan\beta$ is very large, X_t is larger than X_b. We also need to look at just the leading behavior of the squark running,

$$16\pi^2 dM^2_{Q_3}/dt = X_t + X_b + ... \tag{42}$$

$$16\pi^2 dM^2_{\bar{u}_3}/dt = 2X_t + ... \tag{43}$$

$$16\pi^2 dM^2_{\bar{d}_3}/dt = 2X_b + ... \tag{44}$$

Think back to the SM, where the coefficient (usually called μ^2 there but remember that μ is not the same as our μ) of ϕ^2 in the Higgs potential must be negative to lead to spontaneous symmetry breaking with the minimum of the potential away from the origin. Here $M^2_{H_u}$ plays the role, effectively, of the SM μ^2. We see that because of the large X_t the right hand side of the equation for $M^2_{H_u}$ is indeed the largest, and not only does $M^2_{H_u}$ decrease as it runs but the other quantities run slower so they do not get vevs at the same time. Thus the theory naturally can lead to a derivation of the Higgs mechanism! This is extremely important. The theory could easily have had a form where no Higgs vev formed, or where a Higgs vev could only form if some squark also got a vev, which would violate charge and color conservation. The precise conditions for REWSB are somewhat more subtle in supersymmetry — $M^2_{H_u}$ does not actually need to be negative, just smaller than $M^2_{H_d}$, as we will see next.

14. Radiative Electroweak Symmetry Breaking (REWSB)

The Higgs sector is the natural domain of supersymmetry. The Higgs mechanism [23] occurs as the scale decreases from the more symmetric high scale, with vacuum expectation values becoming non-zero somewhat above the EW scale. As we will see, the Higgs mechanism is intricately tied up with supersymmetry and with supersymmetry breaking — there is no Higgs mechanism unless supersymmetry is broken. This should be contrasted with the other big issue of flavor physics, the origin of the number of families and the differences between the flavor and mass eigenstates, which is already in the structure of the theory at the unification scale. Supersymmetry accommodates the flavor issues, and allows data to constrain them, but supersymmetry can explain the Higgs physics with string boundary conditions (we'll be more precise about that later). The form the Higgs physics takes will point the way to how the SM is extended.

Once we have the superpotential and L_{soft} we can calculate the scalar potential that determines the Higgs physics — that is very different from the SM case where one adds the scalar potential in by hand. The result is, for the electrically neutral fields, where, as shown on the right, the three lines come respectively from F, D, and soft terms. Note that the F and D terms are positive definite.

$$V = |\mu_{eff}|^2 \left(|H_u|^2 + |H_d|^2\right) \qquad (F) \qquad (45)$$

$$+\frac{1}{8}(g_1^2 + g_2^2)(|H_u|^2 - |H_d|^2)^2 \qquad (D) \qquad (46)$$

$$+m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 - (bH_uH_d + c.c.). \qquad (soft) \qquad (47)$$

From now on again we will just write μ for μ_{eff}. Now we want to minimize this. If it has a minimum away from the origin vevs will be generated. If we had included the charged scalars we could use gauge invariance to rotate away any vev for (say) H_u^+. Then we would find that the minimization condition $\partial V/\partial H_d^- = 0$ implied that $\langle H_d^- \rangle = 0$, so at the minimum electromagnetism is an unbroken symmetry. The only complex term in V is b. We can redefine the phases of H_u, H_d to absorb the b phase, so we can take b as real and positive. Then by inspection we will have a minimum when the term with b subtracts the most it can, so $\langle H_u \rangle \langle H_d \rangle$ will be real and positive. Since H_u, H_d have hypercharge $\pm\frac{1}{2}$, we can use a hypercharge gauge transformation to take the two vevs separately real and positive. Therefore at the tree level CP is conserved in the Higgs sector and we can choose the mass eigenstates to have definite CP.

Writing $\partial V/\partial H_u = \partial V/\partial H_d = 0$ one finds that the condition for a minimum away from the origin is

$$b^2 > (|\mu|^2 + M_{H_u}^2)(|\mu|^2 + M_{H_d}^2). \qquad (48)$$

So $M_{H_u}^2 < 0$ helps to generate EWSB but is not necessary. There is no EWSB if b is too small, or if $|\mu|^2$ is too large. For a valid theory we must also have the potential bounded from below, which was automatic for the unbroken theory but is not when the soft terms are included. The quartic piece in V guarantees V is bounded from below except along the so-called D-flat direction $\langle H_u \rangle = \langle H_d \rangle$, so we need the quadratic terms positive along that direction, which implies

$$2b < 2\left|\mu\right|^2 + M_{H_u}^2 + M_{H_d}^2. \tag{49}$$

Remarkably, the two conditions cannot be satisfied if $M_{H_u}^2 = M_{H_d}^2$, so the fact that $M_{H_u}^2$ runs more rapidly than $M_{H_d}^2$ is essential. They also cannot be satisfied if $M_{H_u}^2 = M_{H_d}^2 = 0$, i.e. if supersymmetry is unbroken!

We write $\langle H_{u,d}\rangle = \mathrm{v}_{u,d}$. Requiring the Z mass be correct gives

$$\mathrm{v_u^2 + v_d^2 = v^2 = \frac{2M_Z^2}{g_1^2 + g_2^2}} \approx (174\mathrm{GeV})^2 \tag{50}$$

and it is convenient to write

$$\tan\beta = \mathrm{v_u/v_d}. \tag{51}$$

Then $\mathrm{v}_u = \mathrm{v}\sin\beta$, $\mathrm{v}_d = \mathrm{v}\cos\beta$, and with our conventions $0 < \beta < \pi/2$.

With these definitions the minimization conditions can be written

$$\left|\mu\right|^2 + M_{H_d}^2 = b\tan\beta - \frac{1}{2}M_Z^2\cos 2\beta \tag{52}$$

$$\left|\mu\right|^2 + M_{H_u}^2 = b\cot\beta + \frac{1}{2}M_Z^2\cos 2\beta.$$

These satisfy the EWSB conditions. They can be used (say) to eliminate b and $\left|\mu\right|^2$ in terms of $\tan\beta$ and M_Z^2. Note the phase of μ is not determined. These two equations have a special status because they are the only two equations of the entire theory that relate a measured quantity (M_Z^2) to soft parameters. Ideally we would have a high scale theory from which $\mu, m_{H_u}^2$, and $M_{H_d}^2$, and b at low scale would be calculable. Then M_Z, $tan\beta$ would be calculated from equations 52, and compared with experiment. But not yet. If the soft parameters are too large, these equations would require very precise cancellations to keep the Z mass correct.

We have two Higgs fields, each an SU(2) doublet of complex fields, so 8 real scalars. Three of them are Nambu-Goldstone bosons that are eaten by $W^{\pm}, Z$ to become the longitudinal states of the vector bosons, just as in the SM, so 5 remain as physical particles. They are usually classified as 3 neutral ones, h, H, A, and a charged pair, $H^{\pm}$. The mass matrix is calculated from V with $M_{ij}^2 = \frac{1}{2}\partial^2 V/\partial\phi_i\partial\phi_j$ where $\phi_{i,j}$ run over the 8 real scalars. Then the eigenvalue equation $\det\left|\lambda - M_{ij}^2\right| = 0$ determines the mass eigenstates. This splits into block diagonal 2×2 factors. The factors for the charged states and the neutral one in the basis $(ImH_u,\ ImH_d)$ each have one zero eigenvalue, the Nambu-Goldstone bosons. The two CP even neutrals can mix, with mixing matrix

$$\begin{pmatrix} h \\ H \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} ReH_u - \mathrm{v_u} \\ ReH_d - \mathrm{v_d} \end{pmatrix}. \tag{53}$$

The resulting tree level masses are

$$m^2_{h,H} = \frac{m^2_A + M^2_Z}{2} \mp \frac{1}{2}\sqrt{(m^2_A + M^2_Z)^2 - 4m^2_A M^2_Z \cos^2 2\beta}, \tag{54}$$

$$m^2_A = 2b/\sin 2\beta, \tag{55}$$

$$m^2_{H^\pm} = m^2_A + M^2_{W^\pm}. \tag{56}$$

From eq. 54, one can see that if $m^2_A \to 0$ then $m^2_h \to 0$, and if m^2_A gets large then $m^2_h \to 0$, so m^2_h has a maximum. A little algebra shows the maximum is

$$m_h^{tree} \le |\cos 2\beta|\, M_Z, \tag{57}$$

where we have emphasized that this maximum does not include radiative corrections. This important result leads to the strongest quantitative test of the existence of supersymmetry, that there must exist a light Higgs boson. If the gauge theory is extended to larger gauge groups there are additional contributions to the tree level mass, but they are bounded too.

There are also significant radiative corrections [24]. The Higgs potential has contributions to the h^4 term from loops involving top quarks and top squarks. These are not small because the top Yukawa coupling is of order unity and the top-Higgs coupling is proportional to the top mass. To include the effect one has to calculate the contribution to the Higgs potential, reminimize, and recalculate the mass matrix eigenvalues. The result is

$$m^2_h \lesssim \cos^2 2\beta M^2_Z + \frac{3\alpha_2}{2\pi}\frac{m^4_t}{m^2_W}\ln\frac{\tilde{m}^2_t}{M^2_Z} \approx M^2_Z(1 + \frac{1}{4}\ln\frac{\tilde{m}^2_t}{M^2_Z}) \tag{58}$$

where the last equality uses $|\cos 2\beta| = 1$, which is true for $\tan\beta \gtrsim 4$. The contributions from two loops have mainly been calculated and are small but not negligible. This result shows that if $m_h \approx 115$ GeV, it is necessary that the tree level term give essentially the full M_Z contribution, so $\cos^2 2\beta \approx 1$.

If $\tan\beta$ is large the REWSB situation is more complicated. Then the top and bottom Yukawa couplings are approximately equal, so from the RGEs [equations 39-41] we see that $M^2_{H_u}, M^2_{H_d}$ run together, and both can go negative, or the conditions [equations 48,49] may not be satisfied. The EWSB conditions can be rewritten [using equation 55] so one condition is that

$$2m_A^2 \approx M^2_{H_d} - M^2_{H_u} - M^2_Z. \tag{59}$$

Experimentally, $m_A^2 \gtrsim M_Z^2$ (or A would have been observed at LEP or the Tevatron), so the EWSB condition is that $M^2_{H_u}$ must be smaller than $M^2_{H_d}$ by an amount somewhat larger than M_Z^2. That allows a narrow window, and preferably the theory would not have to be finely adjusted to allow the REWSB to occur. Also, in this situation the other condition can be written

$$b \approx \frac{M^2_{H_d} - M^2_{H_u}}{\tan\beta} \sim \frac{M_Z^2}{\tan\beta} \ll M_Z^2 \tag{60}$$

when $\tan\beta$ is large, and this is a clear fine tuning [25] since the natural scale for b is of order the typical soft term, presumably of order or somewhat larger than M_Z^2. So REWSB is possible with large $\tan\beta$ but it is necessary to explain why this apparent fine tuning occurs. The actual effects of increasing $\tan\beta$ are complicated. The b and τ Yukawas get larger, so the top and stop and $m^2_{H_{u,d}}$ RGEs change. $m^2_{H_{u,d}}$ get driven more negative, but the larger Yukawas decrease the stop masses, which makes $m^2_{H_u}$ less negative, etc.

If $\tan\beta$ is large, theories with $M^2_{H_u}$ and $M^2_{H_d}$ split are then favored. That could occur in the unification scale formulation of the theory. One possible way to get a splitting even if $M^2_{H_u}, M^2_{H_d}$ start degenerate is via D-terms from extending the gauge theory[26]. D-terms arise whenever a U(1) symmetry is broken. Under certain circumstances their magnitude may be of order the weak scale even though the U(1) symmetry is broken at a high scale, and they can contribute if the superpartners are charged under that U(1) symmetry. If one looks at SO(10) breaking to SU(5)×U(1) and the breaking of this U(1), the soft masses are

$$\begin{aligned} m_Q^2 = m_{\bar{e}}^2 = m_{\bar{u}}^2 &= m_{10}^2 + m_D^2 \\ m_L^2 = m_{\bar{d}}^2 &= m_5^2 - 3m_D^2 \\ m_{H_{d,u}}^2 &= m_{10}^2 \pm 2m_D^2. \end{aligned}$$

The main point for us is that $m_{H_u}^2$ and $m_{H_d}^2$ are split. The splitting affects the other masses, so in principle m_D^2 is accessible experimentally if sufficiently many scalar masses can be measured.

Note that because b is in L_{soft} it is not protected by a non-renormalization theorem. So to have b small at the weak scale does not mean it is small at the unification scale. Its RGE is

$$16\pi^2 db/dt = b(3Y_t^2 - 3g_2^2 + ...) + \mu(6a_t Y_t + 6g_2^2 M_2 + ...)$$

so if it starts out at zero it is regenerated from the second term, or alternatively cancellations can make it small at the weak scale. Such cancellations would look accidental or fine tuned if one did not know the high scale theory, but the appropriate way to view them would be as a clue to the high scale theory. Similarly, large $\tan\beta$ would presumably mean that one vev is approximately zero at tree level and a small value is generated for it by radiative corrections. No theory is currently known that does that, but if an appropriate symmetry can be found that does it will be a clue to the high scale theory.

Before we leave Higgs physics we will derive one Feynman rule to illustrate how that works. From above we write

$$H_d = \mathrm{v}\cos\beta + \frac{1}{\sqrt{2}}(-\mathrm{h}\sin\alpha + \mathrm{H}\cos\alpha + \mathrm{iA}\sin\beta) \tag{61}$$

$$H_u = \mathrm{v}\sin\beta + \frac{1}{\sqrt{2}}(\mathrm{h}\cos\alpha + \mathrm{H}\sin\alpha + \mathrm{iA}\cos\beta).$$

Then from the covariant derivative term there is the Lagrangian contribution

$$\frac{g_2^2}{\cos^2\theta_W}(|H_u|^2 + |H_d|^2)Z^\mu Z_\mu \tag{62}$$

so substituting this gives the hZZ vertex

$$\frac{g_2^2 \mathrm{v}}{2\cos^2\theta_W} Z^\mu Z_\mu h(\sin\beta\cos\alpha - \cos\beta\sin\alpha) = \frac{g_2 M_Z}{\cos\theta_W}\sin(\beta-\alpha) Z^\mu Z_\mu h \tag{63}$$

Similar manipulations give the couplings

$$\begin{array}{cccc} & h & H & A \\ \bar{t}t,\ \bar{c}c,\ \bar{u}u & \cos\alpha/\sin\beta & \sin\alpha/\cos\beta & \cot\beta \\ \bar{b}b,\ \bar{\tau}\tau... & \text{-}\sin\alpha/\cos\beta & \cos\alpha/\sin\beta & \tan\beta \\ WW, ZZ & \sin(\beta-\alpha) & \cos(\beta-\alpha) & 0 \\ ZA & \cos(\beta-\alpha) & \sin(\beta-\alpha) & 0 \end{array} \tag{64}$$

The ZAh and ZHA vertices are non-zero, while the Zhh and ZHH vertices vanish; there is no tree level $ZW^\pm H^\mp$ vertex.

Finally, we note that in the supersymmetric limit where the soft parameters become zero one has

$$V = |\mu|^2 \left(|H_u|^2 + |H_d|^2\right) + \frac{g_1^2 + g_2^2}{2}\left(|H_u|^2 - |H_d|^2\right) \tag{65}$$

so the minimum is at $\mu = 0$, $H_u = H_d$; the latter implies $\tan\beta$ =1.

15. Yukawa Couplings, tan β, and Theoretical and Experimental Constraints on tan β

It's important to understand how $\tan\beta$ originates, and what is known about it. At high scales the Higgs fields do not have vevs, so $\tan\beta$ does not exist. The superpotential contains information about the quark and lepton masses through the Yukawa couplings. As the universe cools, at the EW phase transition vevs become non-zero and one can define $\tan\beta = \mathrm{v}_u/\mathrm{v}_d$. Then quark and lepton masses become non-zero, $m_{q,l} = Y_{q,l}\mathrm{v}_{u,d}$.

There are two values for $\tan\beta$ that are in a sense natural. As pointed out just above, the supersymmetric limit corresponds to $\tan\beta$ =1. Typically in string theories some Yukawa couplings are of order gauge couplings, and others of order zero. The large couplings for each family are interpreted as the top, bottom, and tau couplings. If $Y_t \approx Y_b$ then $\tan\beta \sim m_t/m_b$. Numerically this is of order 35, but a number of effects could make it rather larger or smaller, e.g. the values of m_t and m_b change considerably with scale, and with RGE running so $m_t(M_Z)/m_b(M_Z) \sim 50$. Finally $\tan\beta$ is determined at the minimum of the Higgs potential, and can be driven smaller.

There are theoretical limits on $\tan\beta$ arising from the requirement that the theory stay perturbative at high scales (remember, the evidence that the entire theory stays perturbative is both the gauge coupling unification and the radiative EWSB). Requiring that $Y_t = g_2 m_t/\sqrt{2}M_W \sin\beta$ not diverge puts a lower limit on $\sin\beta$ which corresponds to $\tan\beta \gtrsim 1.2$ when done in the complete theory, and similarly $Y_b = g_2 m_b/\sqrt{2}M_W \cos\beta$ leads to $\tan\beta \lesssim 60$. This upper limit is probably reduced by REWSB.

There are no measurements of $\tan\beta$, and as I emphasize below it is not possible to measure $\tan\beta$ at a hadron collider in general. Perhaps we will be lucky and find ourselves in a part of parameter space where such a measurement is possible, or more likely, a combination of information from (say) $g_\mu - 2$ and superpartner masses will lead to at least useful constraints on $\tan\beta$. LEP experimental groups have claimed lower limits on $\tan\beta$ from the absence of superpartner signals, but those are quite model dependent and do not hold if phases are taken into account. Similarly, there is a real lower limit on $\tan\beta$ in the MSSM from the absence of a Higgs boson below 115 GeV, as explained above and in Section 22. That limit is about 4 if phases are not included, but much lower when they are.

16. In What Sense Does Supersymmetry Explain EWSB?

Understanding the mechanism of EWSB, and its implications, is still the central problem of particle physics. Does supersymmetry indeed explain it? If so, the explanation depends on broken supersymmetry, and we have seen that in the absence of supersymmetry breaking the EW symmetry is not broken. That's OK. An explanation in terms of supersymmetry moves us a step closer to the primary theory. Historically we have learned to go a step at a time, steadily moving toward more basic understanding. If we think of supersymmetry as an effective theory at the weak scale only, then we would expect the sense in which it explains EWSB to be different from that we would find if we think of low energy supersymmetry as the low energy formulation of a high scale theory. That is, top-motivated bottom-up is different from bottom-up. It should be emphasized that one could have supersymmetry breaking without EWSB, but not EWSB without supersymmetry breaking.

It may clarify the issues to first ask what needs explanation. Recall that in the SM one writes the Higgs potential as $V = \mu^2\phi^2 + \lambda\phi^4$. Then one must derive that $\mu^2 < 0, |\mu^2| \lesssim (TeV)^2, \lambda > 0, \lambda \gtrsim g^2$ in order to explain the breaking of the electroweak SU(2)xU(1) symmetry. We can explicitly list:

1. Why are there Higgs scalar fields, i.e. scalars that carry SU(2)×U(1) quantum numbers, at all?
2. Why does the Higgs field get a non-zero vev?
3. Why is the vev of order the EW scale instead of a high scale?
4. Why does the Higgs interact differently with different particles, in particularly different fermions?

Let us consider these questions.

At least scalars are naturally present in supersymmetric theories, and generally carry EW quantum numbers, whereas in the SM scalars do not otherwise occur. If we connect to a high scale theory, some (most) explicitly have SM-like Higgs fields, e.g. in the E_6 representation of heterotic string theories. Basically as long as we view supersymmetry as embedded in a high scale theory we will typically have Higgs scalars present, though not in all possible cases. That in turn can point to the correct high scale theory.

We have seen that the RGE running naturally does explain the origin of the Higgs vev if the soft-breaking terms and μ_{eff} are of order the weak scale, and if one Yukawa coupling is of order the gauge couplings. If we view the theory as a low energy effective theory we have seen that we do not know why μ in the superpotential is zero, but if we view the theory as embedded in a string theory then it is natural to have $\mu = 0$ in the superpotential. We referred to this as string boundary conditions. Then how μ_{eff} is generated points toward the correct high scale theory. If μ_{eff} is of order the weak scale then it is appropriate to explain the Higgs mechanism *and* gauge coupling unification. Similarly, the mechanism of supersymmetry breaking has to give soft masses of order the weak scale if supersymmetry explains (or, as some prefer to say, predicts) gauge coupling unification.

In a string theory, for example, we expect some Yukawa couplings to be of order the gauge couplings. We identify one of those with the top quark. Then the running of $M^2_{H_u}$ is fast and it is driven negative, or decreases sufficiently, to imply the non-zero Higgs vev. The relevant soft-breaking terms, particularly $M^2_{H_u}$ and $M^2_{H_d}$ must be of order the weak scale. The theory accommodates different couplings for all the fermions. It does not explain the numerical values of the masses, but allows them to be different — that is non-trivial.

So a complete explanation requires thinking of supersymmetry as embedded in a deeper theory such as string theory (so scalar fields exist in the theory, and $\mu \approx 0$, and the top Yukawa is of order 1), and requires that the soft terms are of order the weak scale after supersymmetry is broken. If we only think of supersymmetry as a low energy effective theory not all of these elements are present, so the explanation is possible but incomplete. It is not circular to impose soft-breaking parameters of order the weak scale to explain the EWSB since one is using supersymmetry breaking to explain EW breaking, which is important progress — that is how physics has increased understanding for centuries.

Perhaps it is amusing to note that two families are needed to have both a heavy fermion so the EW symmetry is broken, and light fermions that make up the actual world we are part of. No reasons are yet known why a third family is needed — it is clear that CP violation could have arisen from soft phases with two families, and does not require the three family SM.

Now that we have developed some foundations we turn to applications in several areas.

17. Current and Forthcoming Higgs Physics

There are two important pieces of information about Higgs physics that both independently suggest it will not be too long before a confirmed discovery. But of course it is such an important question that solid data is needed.

The first is the upper limit on m_h from the global analysis of precision LEP (or LEP + SLC +Tevatron) data [27]. Basically the result is that there are a number of independent measurements of SM observables, and every parameter needed to calculate at the observed level of precision is measured except m_h. So one can do a global fit to the data and determine the range of values of m_h for which the fit is acceptable. The result is that at 95% C.L. m_h should be below about 200 GeV. The precise value does not matter for us, and because the data really determines $\ln m_h$ the sensitivity is exponential so it moves around with small changes in input. What is important is that there is an upper limit. The best fit is for a central value of order 100 GeV, but the minimum is fairly broad. The analysis is done for a SM Higgs but is very similar for a supersymmetric Higgs over most of the parameter space.

In physics an upper limit does not always imply there is something below the upper limit. Here the true limit is on a contribution to the amplitude, and maybe it can be faked by other kinds of contributions that mimic it. But such contributions behave differently in other settings, so they can be separated. An entirely independent analysis using the relation between the measured values of M_W and M_{top} in the SM or MSSM also implies a light Higgs exists with mass less than about 200 GeV, and centered near 100 GeV. Since alternative explanations enter differently here, this strengthens the case that the need for a Higgs is not being faked by other new effects in the TeV region.

The second new information is a possible signal from LEP [28] in its closing weeks for a Higgs boson with m_h=115 GeV. Fortunately, its properties are nearly optimal for confirmation at the Tevatron, since its mass is predicted, and cross section and branching ratio to $b\bar{b}$ are large. Less is required to confirm a signal in a predicted mass bin than to find a signal of unknown mass, so only a few fb^{-1} of integrated luminosity will be required if the LEP signal is indeed correct. If funding and the collider and the detectors all work as planned confirming evidence for h could come in 2007. In fact, LEP has reported evidence for several 2σ efforts with Higgs bosons lighter than the LEP limits. Any or all of the effects could be real in the MSSM when full account is taken of allowed parameter ranges and phases [4].

Suppose the LEP h is indeed real. What have we learned [29]? Most importantly, of course, that a point-like, fundamental Higgs boson exists. It is point-like because its production cross section is not suppressed by structure effects. It is a new kind of matter, different from the century old matter particles and gauge bosons. It completes the SM, and points to how to extend the SM. It confirms the Higgs mechanism, since it is produced with the non-gauge-invariant ZZh vertex, which must originate in the gauge-invariant $ZZhh$ vertex with one h having a vev.

The LEP signal, assuming it is correct, can only provide us a limited amount of information since it only supplies two numbers, m_h and $\sigma \times BR$. The full Higgs potential depends on at least 7 parameters [2], so none of them can be explicitly measured. Because the potential depends on the stop loops, it depends on the hermitean stop mass matrix (equation 69 below).

Since the elements are complex, in general the loop contributions to the Higgs potential will be complex, so the potential will have to be reminimized taking into account the possibility of a relative phase between the Higgs vevs. One can write

$$H_d = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathrm{v_d} + \mathrm{h_d} + \mathrm{ia_d} \\ h_d^- \end{pmatrix}, \qquad H_u = \frac{e^{i\theta}}{\sqrt{2}} \begin{pmatrix} h_u^+ \\ \mathrm{v_u} + \mathrm{h_u} + \mathrm{ia_u} \end{pmatrix}. \tag{66}$$

At the minimum of the potential it turns out that θ cannot be set to zero or absorbed by redefinitions. The resulting θ is a function of the phase of μ, ϕ_μ, and of the phase(s) in a_t (and of course of other parameters). Thus m_h and $\sigma_h \times BR(b\bar{b})$ are functions of the magnitudes of μ and a_t, m_Q^2, m_u^2, b, $\tan\beta$, and the physical phase(s) $\phi_\mu + \phi_{a_t}$ at least. Since some of these are matrices they can involve more than one parameter. Also, if $\tan\beta$ is large there will be important sbottom loops, and chargino and neutralino loops can contribute. So only in special cases can data about the Higgs sector be inverted to measure $\tan\beta$ and the soft parameters, and only then if there are at least 7 observables.

If θ is significant then even and odd CP states mix and there are 3 mixed neutral states which could all show up in the $b\bar{b}$ or $\gamma\gamma$ spectrum, and those spectra could show different amounts of the three mass eigenstates. Both cross section and BR for the lightest state can be different from the SM and from the CP conserving supersymmetry case.

One can check that the phase can be very important. For example, if a Higgs is observed at LEP and the Tevatron one can ask what region of parameter space is consistent with a given mass and $\sigma_h \times BR(b\bar{b})$. The answer is significantly different, for example for $\tan\beta$, if the phase is included. Or if no Higgs is observed one can ask what region of parameters is excluded. If the phase is included the actual limit on m_h is about 10% lower than the published limits from LEP, below 100 GeV. Similarly, lower values of $\tan\beta$ are allowed if phases are included than those reported by LEP experimenters. It should be emphasized that this Higgs sector phase is from the normal MSSM Lagrangian, not a new phase. It enters the Higgs sector through the stop loops.

18. The Stop Mass Matrix

Arranging the stop mass terms from the Lagrangian in the form

$$\begin{pmatrix} \tilde{t}_L^* & \tilde{t}_R^* \end{pmatrix} m_{\tilde{t}}^2 \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix},$$

the resulting Hermitean stop mass matrix is

$$m_{\tilde{t}}^2 = \begin{pmatrix} m_{Q_3}^2 + m_t^2 + \Delta_u & \mathrm{v}(a_t \sin\beta - \mu Y_t \cos\beta) \\ & m_{\bar{u}_3}^2 + m_t^2 + \Delta_{\bar{u}} \end{pmatrix}. \tag{67}$$

The $\Delta's$ are D-terms, from the $(\phi^* T \phi)^2$ piece of the Lagrangian — $\Delta_u = (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)\cos 2\beta M_Z^2$, $\Delta_{\bar{u}} = \frac{2}{3}\sin^2\theta_W \cos 2\beta M_Z^2$. These EW D-terms are proportional to the T_3 and hypercharge charges. The pieces proportional to $\sin^2\theta_W$ come from the breaking of the U(1) symmetry and vanish if $\sin^2\theta_W \to 0$. The m_t^2 comes from the F-terms in the scalar potential, $Y_t^2 H_u^{0*} H_u^0 \tilde{t}_L^* \tilde{t}_L$ and a similar term for $\tilde{t}_R$, when the Higgs get vevs. F-terms in V also give the term -$\mu Y_t \tilde{t}^* \tilde{t} H_d^{0*}$ which gives the second term in the 12 position when H_d^0 gets a vev. The soft term $a_u \tilde{t}^* \tilde{Q}_3 H_u^0$ gives the first 12 term when the Higgs gets a vev. Similar mass matrices are written for all the squarks and sleptons. For the lighter ones the Yukawas and possibly the trilinears are small, and the fermion masses are small, so only the diagonal elements are probably large. Each of the elements above is a 3×3 matrix, so $m_{\tilde{t}}^2$ is a 6×6 matrix. a_t and μ and even v are in general complex. This is relevant to the previous section, and to rare decays with superpartners in loops.

19. What Can be Measured in the Higgs Sector?

Assuming the LEP signal is indeed valid and is confirmed at the Tevatron, what can we eventually learn? I will focus on the Tevatron and LHC since they will be our only direct sources of Higgs information in the next decade. The Tevatron can use the WWh, ZZH channels. In addition once m_h is known the inclusive channel, with about a pb cross section, can be used. If the total cross section at the Tevatron for Higgs production is 1.5 pb, and each detector gets 10 fb^{-1} of integrated luminosity, the total number of Higgs bosons produced is about 30,000 in a known mass bin. At some level it will be possible to get information about $g_{WWh} g_{b\bar{b}h}$ and $g_{ZZh} g_{b\bar{b}h}$ from σxBR for the WWh and ZZh channels, so their ratio tests whether h couples to gauge bosons proportional to mass. Once m_h is known it will be possible to see or get useful limits on $h \to \tau\bar{\tau}$ in both inclusive production and associated production with a W, and test if the coupling to fermions is proportional to mass. A similar test comes from not seeing $h \to \mu\bar{\mu}$ (or seeing a few events of this mode since it should occur a bit below the 10^{-3} level). The inclusive production is dominantly via a top loop so it measures g_{tth} indirectly, and this is complicated since superpartner loops contribute as well as SM ones. It may be possible to see the $t\bar{t}h$ final state directly [30]. Since $BR(\gamma\gamma)$ is at the 10^{-3} level an observation or useful limit will be possible here if the resolution is good enough. All of these can give very important tests of what the Higgs sector is telling us. Whether the Tevatron will generate sufficient luminosity to probe the Higgs sector is unclear.

It is also interesting to ask if data can distinguish a SM Higgs from a supersymmetric one, though most likely there will be signals of superpartners as well as a Higgs signal so there will not be any doubt. If $\tan\beta$ is large the ratio of $b\bar{b}$ to $\tau\bar{\tau}$ is sensitive to supersymmetric-QCD effects and can vary considerably from its tree level value [31]. The ratio of top to bottom couplings is sensitive to ways in which the supersymmetric Higgs sector varies from the SM one. If $\tan\beta$ is large and m_A is less than about 150 GeV it is possible A can be observed at the Tevatron. Altogether, the Tevatron may be a powerful Higgs factory if it takes full advantage of its opportunities. It is still unlikely that there will be enough independent measurements at the Tevatron to invert the equations relating the soft parameters and $\tan\beta$ to observables. The lighter stop mass eigenstate $\tilde{t}_1$ may be observable at the Tevatron, and provide another observable for the Higgs sector.

At LHC it is very hard to learn much about the lightest Higgs h if its mass is of order 115 GeV. It will most likely be observed in the inclusive production and decay to $\gamma\gamma$, but observation in the $\gamma\gamma$ mode does not tell us much about the Higgs physics once the Higgs boson has been discovered, which will have occurred if indeed $m_h \approx 115$ GeV. The $\gamma\gamma$ mode does not demonstrate the Higgs mechanism is operating since it occurs for any scalar boson. The SM does have a definite prediction for BR($\gamma\gamma$) from the top and W loops, and superpartner loops can be comparable, so a measurement would be very interesting. Note that one cannot assume the $\gamma\gamma$ BR is known.

Maybe it will be possible to detect the $\tau\bar{\tau}$ mode at LHC using WW fusion to produce h and tagging the quarks [32]. This mode also confirms the non-gauge-invariant WWh vertex. Considerable additional information about the Higgs sector may come from observing the heavier Higgs masses and $\sigma \times BR$, and the heavier stop. Since $A \to \gamma\gamma$ but not to ZZ, WW it may be possible to see A if it is not above the $t\bar{t}$ threshold. Decays of the heavy Higgs to $\tau's$ are enhanced if $\tan\beta$ is large. Note that one cannot assume only SM decays of h in analysis since channels such as h$\to LSP + LSP$ are potentially open and can have large BR since they are not suppressed by factors such as m_b^2/M_W^2. The combined data from the Tevatron and LHC may provide enough observables to invert the Higgs sector, at least under certain reasonable and checkable assumptions. Two aspects of (particularly LHC) Higgs physics are seldom studied. Superpartners may have decays into h and indeed may be large sources of h. And heavy Higgs states will decay into superpartners if kinematically allowed, changing BR, and providing superpartners. A recent brief summary of LHC Higgs sector physics with references is [33].

20. Upper Limit on Lightest Higgs Mass Regardless of Soft Terms, Gravitino Mass

Since it is not well known, and not entirely obvious, we recall here a general upper limit on the mass of the *lightest* Higgs boson. We assume only that the masses of Z, W arise from the Higgs mechanism, and that the theory stays perturbative up to of order the scale where gauge couplings unify. One might expect that if all soft terms (such as $m^2_{H_u}, m^2_{H_d}$), or vevs of scalars that could mix, were very large, then the masses of the Higgs mass eigenstates would be too.

In fact there is an upper limit of order twice M_Z on the lightest Higgs mass eigenstates m_h [34,35]. The precise value of the upper limit depends on whether Higgs triplets are present, whether other gauge groups are present, and so on, at the 10-15% level, but the limit is always present. This is the most firm test of general supersymmetric theories — not just the MSSM. On the other hand, that means that the existence of a light Higgs does not imply any upper limit on soft mass parameters.

21. Charginos

The lightest superpartners are likely to be the neutralinos and charginos, possibly the lighter stop, and the gluino. Their mass matrices have entries from the higgsino-gaugino mixing once the SU(2)×U(1) symmetry is broken, so the mass eigenstates are mixtures of the symmetry eigenstates. When phases are neglected these matrices are described in detail in many places so I will not repeat that here. However, it is worth looking at the most general case including phases for several instructive reasons [36]. The chargino mass matrix follows from the L_{soft}, in the wino-higgsino basis:

$$M_{\widetilde{C}} = \begin{pmatrix} M_2 e^{i\phi_2} & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu e^{i\phi_\mu} \end{pmatrix}. \tag{68}$$

The situation is actually more subtle — this is a submatrix of the actual chargino mass matrix, but this contains all the information — and the reader should see Martin or earlier reviews for details. Also, the off-diagonal element can be complex too since it arises from the last term in eq.14 when the Higgs gets a vev, and the vev can be complex as explained above; I will just keep the phases of M_2 and μ here. The masses of the mass eigenstates are the eigenvalues of this matrix. To diagonalize it one forms the hermitean matrix $M^\dagger M$. The easiest way to see the main points are to write the sums and products of the mass eigenstates,

$$M^2_{\widetilde{C}_1} + M^2_{\widetilde{C}_2} = Tr M^\dagger_{\widetilde{C}} M_{\widetilde{C}} = M_2^2 + \mu^2 + 2M_W^2, \tag{69}$$

$$\begin{aligned} M^2_{\widetilde{C}_1} M^2_{\widetilde{C}_2} &= \det M^\dagger_{\widetilde{C}} M_{\widetilde{C}} \\ &= M_2^2\mu^2 + 2M_W^4 \sin^2 2\beta - 2M_W^2 M_2 \mu \sin 2\beta \cos(\phi_2 + \phi_\mu) \end{aligned} \tag{70}$$

Experiments measure the masses of the mass eigenstates. One thing to note is that the masses depend on the phases ϕ_2 and ϕ_μ, even though there is no CP violation associated with the masses. Often it is implicitly assumed that phases can only be measured by observing CP-violating effects, but we see that is not so. The combination $\phi_2 + \phi_\mu$ is a physical phase, invariant under any reparameterization of phases, as much a basic parameter as $\tan\beta$ or any soft mass.

If one wants to measure the soft masses, μ, $\tan\beta$, $\phi_2+\phi_\mu$ *it is necessary to invert such equations.* This is obvious, but its implications are often ignored. Since there are fewer observables than parameters to measure, additional observables are needed. One can measure the production cross sections of the mass eigenstates. But then additional parameters enter since exchanges of sneutrinos (at an electron collider) or squarks (at a hadron collider) contribute. One can decide to neglect the additional contributions, but then one is not really doing a measurement. If one "measures" $\tan\beta$ from the above equations by setting the phase to zero, as is usually done, the result is different from that which would be obtained if the phase were not zero. When the phases are present the phenomenology, and any deduced results, can be quite different. We saw that for the Higgs sector above. It is studied for the chargino sector in ref. [37]. Similar arguments apply for the neutralino mass matrix.

One implication of this analysis is that $\tan\beta$ is not in general measurable at a hadron collider — there are simply not enough observables [36]. One can count them, and the equations never converge. Depending on what can be measured, by combining observables from the chargino and neutralino sectors, and the Higgs sector, it may be possible to invert the equations. This is a very strong argument[36] for a lepton collider with a polarized beam, where enough observables do exist if one is above the threshold for lighter charginos and neutralinos, because measurements with different beam polarizations (not possible at a hadron collider) double the number of observables, and measurements with different beam energies (not possible at a hadron collider) double them again. The precise counting has to be done carefully, and quadratic (and other) ambiguities and experimental errors mean that one must do a thorough simulation[38] to be sure of what is needed, but there appear to be sufficient observables to measure the relevant parameters. The issue of observing the fundamental parameters of L_{soft} is of course broader, as discussed in Section 17. There are 33 masses in the MSSM including the gravitino, but 107 new parameters in L_{soft} (including the gravitino). The rest are flavor rotation angles and phases. Many can be measured by combining data from a linear electron collider above the threshold for a few superpartners and hadron colliders. It is also necessary to include flavor changing rare decays to measure the off-diagonal elements of the sfermion mass matrices and the trilinear couplings. If a linear collider is not available it will be necessary to invent new approaches to learn about the underlying theory from the data.

22. Neutralinos

In a basis $\Psi^0 = (\widetilde{B}, \widetilde{W}_3, \widetilde{H}_d, \widetilde{H}_u)$ terms in the Lagrangian can be rearranged into -$\frac{1}{2}(\Psi^0)^T M_{\tilde{N}} \Psi^0$ with the symmetric

$$M_{\tilde{N}} = \begin{pmatrix} M_1 e^{i\phi_1} & 0 & -\frac{g_1}{\sqrt{2}} H_d^{0*} & \frac{g_1}{\sqrt{2}} H_u^{0*} \\ 0 & M_2 e^{i\phi_2} & \frac{g_2}{\sqrt{2}} H_d^{0*} & -\frac{g_2}{\sqrt{2}} H_u^{0*} \\ & & 0 & -\mu e^{i\phi_\mu} \\ & & & 0 \end{pmatrix} .$$

Although the elements are complex, this matrix can still be diagonalized by a unitary transformation. Its form in a basis $\Psi' = (\tilde{\gamma}, \widetilde{Z}, \tilde{h}_s, \tilde{h}_a)$ is sometimes useful:

$$M_{\tilde{N}} = \begin{pmatrix} M_1 s_W^2 + M_2 c_W^2 & (M_1 - M_2) s_W c_W & 0 & 0 \\ & M_1 s_W^2 + M_2 c_W^2 & M_Z & 0 \\ & & \mu \sin 2\beta & -\mu \cos 2\beta \\ & & & -\mu \sin 2\beta \end{pmatrix} .$$

If $M_1 \approx M_2$ and/or if $\tan\beta$ is large (so $\sin 2\beta \approx 0$) this takes a simple form.

The lightest neutralino is the lightest eigenvalue of this, and may be the LSP. Its properties then determine the relic density of cold dark matter (if the LSP is indeed the lightest neutralino). It also largely determines the collider signatures for supersymmetry. It will be a linear combination of the basis states,

$$\widetilde{N}_1 = \alpha \widetilde{B} + \beta \widetilde{W}_3 + \gamma \widetilde{H}_d + \delta \widetilde{H}_u$$

with $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Learning the content of the LSP will be crucial for determining its relic density and whether it is the dark matter.

An interesting limit that is at least pedagogically instructive arises if we take $M_1 \approx M_2$ (at the EW scale) and $\tan\beta \approx 1$, and $\mu < M_Z$. Then $\widetilde{N}_1 \approx \tilde{h}$, where $\tilde{h} = \tilde{h}_d \sin\beta + \tilde{h}_u \cos\beta$, and $M_{\tilde{N}_1} \approx \mu$. $\widetilde{N}_2 \approx \tilde{\gamma}$, with $M_{\widetilde{N}_2} \approx M_2$, $\alpha \approx -\beta \approx -45°$ so $\cos(\alpha - \beta)$ and $\cos 2\beta \to 0$. At tree level the $Z\tilde{\gamma}\tilde{h}$, $h\tilde{\gamma}\tilde{h}$, and $Z\tilde{h}\tilde{h}$ vertices vanish, and the dominant decay of the second neutralino is $\widetilde{N}_2 \to \widetilde{N}_1 + \gamma$. $M_{\tilde{C}_1} \gtrsim M_{\widetilde{N}_2}$.

23. Effects of Phases

The effects of phases have been considered much less than the masses. As we saw above for charginos and the Higgs sector they affect not only CP-violating observables but essentially all observables. They can have significant impacts in a variety of places [36,39,2], including $g_\mu - 2$, electric dipole moments (EDMs), CP violation in the K and B systems, the baryon asymmetry of the universe, cold dark matter, superpartner production cross sections and branching ratios, and rare decays. We do not have space to give a complete treatment, but only to make some points about the importance of the observations and what they might teach us about physics beyond the SM in general; while we focus to some extent on the phases because they are usually not discussed, our concern is relating them to the entire L_{soft}.

There are some experiments that suggest some of the phases are small, mainly the neutron and electron EDMs. On the other hand, we know that the baryon asymmetry cannot be explained by the quark CKM phase, so some other phase(s) are large, and the soft phases are good candidates. It has been argued that very large phases are needed if baryogenesis occurs at the EW phase transition [40]; see also [41]. Further, there is no known symmetry or basic argument that the soft phases in general should be small. If the outcome of studying how to measure them was to demonstrate that some were large that could be very important because both compactification and supersymmetry breaking would have to give such large phases. The phase structure of the effective soft Lagrangian at the weak scale and at the unification scale are rather closely related, so it may be easier to deduce information about the high scale phases from data than about high scale parameters in general. If the outcome of studying how to measure the phases was to demonstrate that the phases were small that would tell us different but very important results about the high scale theory. It would also greatly simplify analyzing weak scale physics, but that is not sufficient reason to assume the phases are small.

24. $g_\mu - 2$

In early 2001 it was reported that the anomalous magnetic moment of the muon was larger than the SM prediction by a significant amount. Even if the effect disappears, it is worth considering $g-2$ experiments, because in a supersymmetric world the entire anomalous moment of any fermion vanishes if the supersymmetry is unbroken, so magnetic moments are expected to be very sensitive to the presence of low energy supersymmetry, and particularly of broken supersymmetry. The analysis can be done in a very general and model independent manner [42], and illustrates nicely how one can say a great deal with supersymmetry even though it seems to have a number of parameters. So it is also pedagogically interesting. There are only two supersymmetric contributions, a chargino-sneutrino loop and a smuon-neutralino loop. One can see that starting from the complete theory, with no assumptions beyond working in the MSSM, there are only 11 parameters that can play a role out of the original set of over 100,

$$|M_2|\,,|M_1|\,,|\mu|\,,|A_\mu|\,,m_{\tilde{\mu}_L},m_{\tilde{\mu}_R},m_{\tilde{\nu}},\tan\beta,\phi_2+\phi_\mu,\phi_1+\phi_\mu,\phi_A+\phi_\mu.$$

In the general case all 11 of them can be important, and the experimental result will give a complicated constraint among them. But if we ask about putting an upper limit on superpartner masses, which would be of great interest, we can say more. For larger masses one can see that the chargino-sneutrino diagram dominates, and in addition that it is proportional to $\tan\beta$; The $\tan\beta$ factor arises from the needed chirality flip on a chargino line. Thus only the magnitudes of M_2, and μ, $\tan\beta$, $m_{\tilde{\nu}}$ and the phase enter in this limit. If we illustrate the result by assuming a common superpartner mass $\tilde{m}$ (just for pedagogical reasons, not in the actual calculations), we find that

$$a_\mu^{susy}/a_\mu^{SM} \approx \left(\frac{100\ GeV}{\tilde{m}}\right)^2 \tan\beta\cos(\phi_2+\phi_\mu). \tag{71}$$

Further, to put upper limits on the masses we can take the phase to be zero since it turns out to enter only in the above form under these assumptions (for the general case see [43]). And if we express results in terms of the lighter chargino mass rather than M_2 and μ we can eliminate one parameter; for a given chargino mass there will be ranges of M_2 and μ. So we are down to three parameters, with no uncontrolled approximations or assumptions. If the effect persists there will be significant upper limits on the superpartner masses. Note the relevant physical phase here is $\phi_2 + \phi_\mu$. If superpartner masses are measured at colliders, the sensitivity of $g_2 - 2$ to $\tan\beta$ may turn out to be the best way to constrain $\tan\beta$, which (as discussed above) is very difficult to measure at hadron colliders.

It is interesting to consider the supersymmetry limit so the supersymmetric SM contribution vanishes. In that limit the two lighter neutralino masses vanish, and their contribution cancels the photon contribution, the two heavier neutralino masses become M_Z and their contribution cancels that of the Z, and the two charginos have M_W and cancel the W contribution. Since the chargino has a sign opposite to that of the W in the supersymmetric limit but the same sign for the broken supersymmetry physical situation it is important to check that indeed the piece proportional to $\tan\beta$ does change sign as needed.

25. Electric Dipole Moments

In the SM electric dipole moments are unobservably small, of order $10^{-33}e$ cm. That is basically because they are intrinsically CP-violating quantities, and for CP violation to occur in the SM it is necessary for all three families to affect the quantity in question. Otherwise one could rotate the CKM matrix in such a way that the phase did not occur in the elements that contributed. So it must be at least a two-loop suppression. There must also be a factor of the electron or quark mass because of a chirality flip, with the scale being of order M_W. In addition there is a GIM suppression. Interpreting results will be complicated because the neutron, and any nuclear EDM, can have a contribution from strong CP violation, while the electron can only feel effects from EW interactions.

Naively, the EDM is the imaginary part of a magnetic moment operator, and the real part is the magnetic moment. So EDMs can arise from the same diagrams as $g_\mu - 2$, but for the electron and for quarks (in neutrons). It is more complicated in reality because the part of the amplitude that has an imaginary part may not give the dominant contribution to the magnetic moment. It has been known for a long time that if the soft phases were of order unity and if all contributions were independent, then the supersymmetry contributions to EDMs are too large by a factor of order 50. However, over a significant part of parameter space various contributions can cancel. Some of that cancellation is generic, e.g. between chargino and neutralino in the electron EDM because of the relative minus sign in eq.32.

The smallness of EDMs may be telling us that the soft phases are small. Then we need to find out why they are small. Or it may be telling us that cancellations do occur. Cancellations look fine tuned from the point of view of the low energy theory, but small phases look fine tuned too. Relations among soft parameters in the high scale theory will look fine-tuned in the low scale theory if we do not know the origin of those relations. If $\tan\beta$ is very large cancellations become unlikely since the chargino contribution will dominate the eEDM just as it does for $g_\mu - 2$, but if $\tan\beta$ is of order 4-5 the situation has to be studied carefully.

26. Measuring Phases at Hadron Colliders

Phases, as well as soft masses, can affect distributions at colliders. We briefly illustrate that here for an oversimplified model [39]. Consider gluino production at a hadron collider. The Lagrangian contains a term

$$M_3 e^{i\phi_3} \lambda_{\tilde{g}} \lambda_{\tilde{g}} + c.c. \tag{72}$$

It is convenient to redefine the fields so the phase is shifted from the masses to the vertex, so one can write $\psi_{\tilde{g}} = e^{i\phi_3/2}\lambda_{\tilde{g}}$. Then writing the Lagrangian in terms of ψ the vertices $q\tilde{q}\tilde{g}$ get factors $e^{\pm i\phi_3/2}$. The production cross sections for gluinos, for example from $q + \bar{q} \to \tilde{g} + \tilde{g}$ by squark exchange, have factors $e^{+i\phi_3/2}$ at one vertex and $e^{-i\phi_3/2}$ at the other, so they do not depend on the phase. That is clear from general principles since ϕ_3 is not by itself a physical, reparameterization-invariant phase. But gluinos always decay, and for example in the decay $\tilde{g} \to q + \bar{q} + \tilde{B}$ mediated by squarks there is a factor of $e^{i\phi_3/2}$ at the $q\tilde{q}\tilde{g}$ vertex and a factor $e^{i\phi_1/2}$ at the $q\tilde{q}\tilde{B}$ vertex, so the rate depends on the physical relative phase $\phi_3 - \phi_1$. In general it is more complicated with all the relative phases of the neutralino mass matrix entering. In this simple example the experimental distribution in Bino energy is

$$d\sigma/dx \sim m_{\tilde{g}}^4 \left(\frac{1}{\tilde{m}_L^4} + \frac{1}{\tilde{m}_R^4} \right) \times \tag{73}$$

$$[x - 4x^2/3 - 2y^2/3 + y(1 - 2x + y^2)\cos(\phi_3 - \phi_1)]$$

where $x = E_{\tilde{B}}/m_{\tilde{g}}$ and $y = m_{\tilde{B}}/m_{\tilde{g}}$. Other distributions are also affected. If one tries to obtain information from gluino decay distributions without taking phases into account the answers will be misleading if the phases are not small. The same result is of course true for many superpartner decays. It is important to realize that the same phases are appearing here as appear for example in studying ε and ε' in the kaon system or in $b \to s + \gamma$. At LHC it will be possible to study CPV by measuring the charges of light quark jets with a resultant large improvement in statistics.

27. LSP Cold Dark Matter

If it is stable, the LSP is a good candidate for the cold dark matter of the universe. Historically, it is worth noting that this was noticed before we knew that non-baryonic dark matter was needed to understand large scale structure. It is a prediction of supersymmetry. We discussed above why we expected R-parity or a similar symmetry to hold, with the stability of the LSP as one of its consequences. Then the basic argument is simple. As the universe cools, soon after the EW phase transition all particles have decayed except photons, $e^{\pm}, u^{\pm}, d^{\pm}$, neutrinos, and LSPs. The quarks form baryons, which join with electrons to make atoms. The relic density of all but LSPs is known to be $\Omega_{SM} < 0.05$, while $\Omega_{matter} \approx 0.3$. The LSPs annihilate as the universe cools, with a typical annihilation cross section $\sigma_{ann} \sim \rho_{LSP} G_F^2 E^2$, and in the early universe $E \sim T$. The expansion rate is governed by the Hubble parameter $H \sim T^2/M_{Pl}$. The LSPs freeze out and stop annihilating when their mean free collision path is of order the horizon, so $\sigma_{ann} \sim H$. This gives a density $\rho_{LSP} \sim 1/M_{Pl} G_F^2 \sim 10^{-9}$ GeV^3. At freeze-out $T \sim 1$ GeV, and ρ_γ is of order the entropy $S \sim T^3 \sim 1$ GeV^3, so $\rho_{LSP}/\rho_\gamma \sim 10^{-9}$, similar to the density of baryons. Thus $\Omega_{LSP} \sim (M_{LSP}/M_{proton})\Omega_{baryon}$. Quantitative calculations in many models confirm this. Numerically $\Omega_{matter} \approx 0.3$ is a natural result.

But the actual calculations of the relic density depend on several soft parameters such as masses of sleptons and gauginos, and also on $\tan\beta$ and on soft phases. In the absence of measurements or a theory that can convincingly determine all of these, we cannot in fact say more than that qualitatively the LSP is a good candidate, *even if WIMPs are apparently discovered.* Since we have argued above that in practice it is unlikely that $\tan\beta$ will be measured accurately at hadron colliders (though we may be lucky with $g_\mu - 2$ plus hadron colliders), it may be difficult to compute Ω_{matter} accurately even after LSPs are detected. It should be emphasized that detection of LSPs is not sufficient to argue they are actually providing the cold dark matter [44,45] — LSPs could be detected in direct experiments scattering off nuclei, and in space based searches, and at colliders even if

$\Omega_{LSP} \lesssim 0.05$. Alternatively, they could make up the CDM even though they were not detected in direct and space based experiments.

Further, in recent years it has come to be understood that LSPs may be produced dominantly by a number of processes that are not in thermal equilibrium rather than the equilibrium process described above (for some references see [45]). In addition, the presence of dark energy can affect the evolution of the Hubble parameter before freeze-out, and thus affect the relic density even if the LSP is in thermal equilibrium (e.g. see [46]). In that case the relic density is not so simply connected to the LSP nature. It is still true that supersymmetry provides a very good dark matter candidate, and it is not an accident that it can give $\Omega_{matter} \approx 0.3$, but the full argument can be more complex.

28. Comments on Relating CP Violation and String Theory; Could the CKM Phase be Small?

Where does CP violation originate? Can the pattern of CP phenomena give us important clues to formulating and testing string theory? Very little work has been done about the fundamental origins of CP violation. In 1985 Strominger and Witten discussed how to define CP transformations in string theory, and in 1993 Dine, Leigh, and McIntyre argued that CP was a gauge symmetry in string theory, for both strong and EW CP violation. As a gauge symmetry it could not be broken explicitly, perturbatively. or non-perturbatively. More recently Bailin et al, Dent, Geidt, and Lebedev have discussed aspects of this question. Little thought has been given to CP violation in D-brane worlds, Type IIB theories with SM particles as Type I open strings, and so on.

From the point of view of connecting to the observable world, however the CP violation originates it will appear as phases in either the Yukawa couplings in the superpotential, or as phases in L_{soft}. Any theory for CP violation will produce characteristic patterns of such phases. So if we could measure those phases perhaps we would have rather direct information about such questions as moduli dependence of Yukawas, supersymmetry breaking and transmission, vevs of moduli and the dilaton, and the compactification manifold.

If one begins with a string theory including proposed solutions to how to compactify, and to break supersymmetry, the connection to the observable world is first made by writing down the Kahler potential, gauge kinetic function, and superpotential, $W = Y_{\alpha\beta\gamma}\phi_\alpha\phi_\beta\phi_\gamma$. Then L_{soft} is calculated for the assumed approach to supersymmetry breaking, etc. The trilinear terms, for example, are linear combinations of the Yukawas and derivatives of the Yukawas with respect to moduli fields. So if the Yukawas have large phases it seems likely the trilinear terms will also have large phases. On the other hand, phases could enter the trilinears through the Kahler potential even if they were not present in the Yukawas. Recalling that the quark CKM phase is unable to provide the CP violation needed for the baryon asymmetry, it is interesting to consider the possibility that all CP violation originates in the soft phases. It is possible to describe CP violation in the kaon and B systems with only soft phases [47].

Phenomenologically there are a number of ways that soft phases could be shown to be large. One is observing an eEDM. The nEDM is not so simple to interpret since it could arise from strong CP violation, but perhaps the relative size of the nEDM and HgEDM could show the effect of soft phases. The Higgs sector could show phase effects, as could superpartner masses, production cross sections, and decay BR. The size of $K_L \to \pi^0 \nu\bar{\nu}$ could deviate from the SM prediction because of phases. If b-quark and Kaon phenomenology give a unique result for the CKM phase with small errors it will probably be correct that is the right interpretation, but until we also understand the soft phases and baryogenesis we cannot be confident.

29. Phases (and Flavor Structure) of L_{soft}

The soft-breaking Lagrangian has, as we have seen, many phases, and interesting and potentially important flavor structure. Few top-down models, e.g. string based models, have studied or even looked at the phase and flavor structure. There is and will be much more data on these topics, and there should be much more theoretical analysis of them. Following the framework of Ibanez, Munoz, and Rigolin[48], we have looked at how the phases emerge in some D-brane models [49].

If one embeds the MSSM on one brane, usually the gaugino masses M_i all have the same phase, and using the freedom from a U(1) symmetry one can rotate that phase away. An interesting structure emerges if one embeds the SM gauge groups on two intersecting branes. We studied the simplest case with SU(2) on one brane, and SU(3)×U(1) on the other. While we did not try to derive such a structure from an actual compactification, it is known that explicit compactifications of intersecting branes exist, and that open strings connecting D-branes intersecting at non-vanishing angles lead to theories with chiral fermions, so it is plausible that such a model can exist. We follow Ibanez et al in assuming the supersymmetry breaking occurs in a hidden sector, and is transmitted by complex F-term moduli vevs to the superpartners. Then this model gives for soft terms

$$M_1 = M_3 = -A_t \sim e^{i\alpha_1}, \qquad M_2 \sim e^{-i\alpha_2}, \tag{74}$$

and all the other soft terms are real. One important lesson is that such a theory has only 9 parameters — the many parameters of L_{soft} have been reduced by the theory down to this number. They are

$$\alpha_2 - \alpha_1, m_{3/2}, \tan\beta, |\mu|, |A_t|, \phi_\mu, X_1, X_2, X_3. \tag{75}$$

Here only the relative phase of the moduli vevs enters, $m_{3/2}$ is the gravitino mass and sets the overall mass scale, and the X_i are measures of the relative importance of different moduli. The X_i could be measured, in which case they would tell us about the structure of the theory, and/or they could be computed in a good theory. Measuring the string-based parameters here would teach us about formulating and testing string theory. Any theory will have relations among the soft parameters so the actual number of parameters is far smaller than the full number of L_{soft}. This number could be reduced further by some testable assumptions. Also, not all of them will contribute in any given process, as we have seen. The resulting theory can be used to simultaneously study collider physics and LSP cold dark matter as is usual, and also CP violation. An extended version of the model [50] can also address flavor issues.

In this model one can illustrate how results of the low energy theory can appear fine-tuned and somewhat arbitrary because they are not apparently due to a symmetry when they originate in dynamics that occur at the high scale and are hidden at the low energy scale. If the gluino-gluon box diagram indeed explains direct CP violation in the kaon system, then one needs a certain phase relation to hold,

$$\arg(\phi_{A_{sd}} M_3^*) \approx 10^{-2}, \tag{76}$$

which seems fine-tuned. But as we saw in eq. 74, M_3 and the elements of A have the same phase in this D-brane based theory, and so the quantity in eq. 76 is zero at the high scale. Since the phases of M_3 and of A run differently, a small phase is generated at the low scale. While we are not arguing this is the actual explanation for ε'_K, it does nicely illustrate how such phases could be related by an underlying theory yet not follow from any low energy symmetry.

30. Direct Evidence for Superpartners? — at the Tevatron?

So far all the evidence for low energy supersymmetry is indirect. Although the evidence is strong, it could in principle be a series of coincidences. More indirect evidence could come soon from improved $g_\mu - 2$, other rare decays, b-factories, proton decay, CDM detection. But finally it will be necessary to directly observe superpartners, and to show they are indeed superpartners. That could first happen at the Tevatron collider, or at LHC as early as 2008.

Accepting that supersymmetry explains EWSB, we expect the gluinos, neutralinos, and charginos to be rather light. The lighter stop may be light as well. Sleptons may also be light though there is somewhat less motivation for that. We can list a number of channels and look at the signatures for each of them. Almost all cases require a very good understanding of the SM events that resemble the possible signals, both in magnitude (given the detector efficiencies) and the distributions. Missing transverse energy will be denoted by $\not{E}_T$. It is reasonable to expect the Tevatron to have an integrated luminosity of 2 fb^{-1} per detector by sometime in 2006, and over 5 fb^{-1} by sometime in 2008. Until we know the ordering of the superpartner masses we have to consider a number of alternative decays of $\widetilde{N}_2$, $\widetilde{C}_1$, $\tilde{t}_1$, $\tilde{g}$, etc. [51]. Consider various channels. Since the Tevatron still has two-three years before signals are very likely from LHC, we orient the discussion some what that way. Further, the Tevatron will provide information somewhat complementary to the LHC if the light superpartners indeed exist since at the Tevatron initial partons are mainly quarks (rather than mainly gluons as at LHC), and SM backgrounds are different. Most of the analysis is relevant to any hadron collider.

- $\widetilde{\mathbf{N}}_\mathbf{1} + \widetilde{\mathbf{N}}_\mathbf{1}$ can be tagged with a monojet, or a photon, but then the rate is much smaller.

This channel is very hard to tag at a hadron collider.

- $\widetilde{\mathbf{N}}_\mathbf{1} + \widetilde{\mathbf{N}}_\mathbf{2,3}$

These channels can be produced through an s-channel Z or a t-channel squark exchange. The signatures depend considerably on the character of $\widetilde{N}_2$, $\widetilde{N}_3$. $\widetilde{N}_1$ escapes. If $\widetilde{N}_2$ has a large coupling to $\widetilde{N}_1 + Z$ (for real or virtual Z) then the $\widetilde{N}_1$ will escape and the Z will decay to e or μ pairs each 3% of the time, so the event will have missing energy and a prompt lepton pair.

There will also be tau pairs and jet pairs, but those are somewhat harder to identify. Or, perhaps $\widetilde{N}_2$ is mainly photino and $\widetilde{N}_1$ mainly higgsino, in which case there is a large BR for $\widetilde{N}_2 \to \widetilde{N}_1 + \gamma$ and the signature of $\widetilde{N}_2$ is one prompt γ and missing energy. The production cross section can depend significantly on the wave functions of $\widetilde{N}_1, \widetilde{N}_2$. If the cross section is small for $\widetilde{N}_1 + \widetilde{N}_2$ it is likely to be larger for $\widetilde{N}_1 + \widetilde{N}_3$. Most cross sections for lighter channels will be larger than about 50 fb, which corresponds to 200 events (not including BR) for an integrated luminosity of 2 fb^{-1} per detector.

- $\mathbf{\widetilde{N}_1 + \widetilde{C}_1}$

These states are produced through s-channel $W^{\pm}$ or t-channel squarks. The $\widetilde{N}_1$ escapes, so the signature comes from the $\widetilde{C}_1$ decay, which depends on the relative sizes of masses, but is most often $\widetilde{C}_1 \to l^{\pm} + \not{E}_T$. This is the signature if sleptons are lighter than charginos ($\widetilde{C}_1 \to \tilde{l}^{\pm} + \nu$, followed by $\tilde{l}^{\pm} \to l^{\pm} + \widetilde{N}_1$), or if sneutrinos are lighter than charginos by a similar chain, or by a three-body decay ($\widetilde{C}_1 \to \widetilde{N}_1 + virtual\ W$, $W \to l^{\pm} + \nu$). But it is not guaranteed — for example if stops are lighter than charginos the dominant decay could be $\widetilde{C}_1 \to \tilde{t} + b$. In the case where the lepton dominates the event signature is then $l^{\pm} + \not{E}_T$, so it is necessary to find an excess in this channel. Compared to the SM sources of such events the supersymmetry ones will have no prompt hadronic jets, and different distributions for the lepton energy and for the missing transverse energy.

- $\mathbf{\widetilde{N}_2 + \widetilde{C}_1}$

If $\widetilde{N}_2$ decays via a Z to $\widetilde{N}_1 + l^{+} + l^{-}$ and $\widetilde{C}_1$ decays to $\widetilde{N}_1 + l^{\pm}$, this channel gives the well-known "tri-lepton" signature, three charged leptons, $\not{E}_T$, and no prompt jets, which may be relatively easy to separate from SM background. But it may be that $\widetilde{N}_2 \to \widetilde{N}_1 + \gamma$, so the signature may be $l^{\pm} + \gamma + \not{E}_T$.

- $\mathbf{\tilde{l}^{+} + \tilde{l}^{-}}$

Sleptons may be light enough to be produced in pairs. Depending on masses, they could decay via $\tilde{l}^{\pm} \to l^{\pm} + \widetilde{N}_1$, $\widetilde{C}_1 + \nu$, $W + \tilde{\nu}$. If $\widetilde{N}_1$ is mainly higgsino decays to it are suppressed by lepton mass factors, so $\tilde{l}^{\pm} \to l^{\pm} + \widetilde{N}_2$ may dominate, followed by $\widetilde{N}_2 \to \widetilde{N}_1 + \gamma$ [52].

- **Gluinos**

Gluinos can be produced via several channels, $\tilde{g} + \tilde{g}$, $\tilde{g} + \widetilde{C}_1, \tilde{g} + \widetilde{N_1}$, etc. If supersymmetry indeed explains EWSB it would be surprising if the gluino were heavier than about 500 GeV, as argued above. Then the total cross section for its production should be large enough to observe it at the Tevatron. If all its decays are three-body, e.g. $\tilde{g} \to \tilde{q} + \bar{q}$ followed by $\tilde{q} \to q + \widetilde{C}_1$, etc, then the signature has energetic jets, $\not{E}_T$, and sometimes charged leptons. There are two channels that are particularly interesting and not unlikely to occur — if $t + \tilde{t}$ or $b + \tilde{b}$ are lighter than $\tilde{g}$ then they will dominate because they are two-body. The signatures can then be quite different, with mostly b and c jets, and smaller multiplicity. In general gluino decays may include $t + \bar{t}$ in various ways, so searches based on tops will be important[53,54].

Gluinos and neutralinos are normally Majorana particles. Therefore they can decay either as particle or antiparticle. If, for example, a decay path $\tilde{g} \to \bar{t}(\to W^- \bar{b}) + \tilde{t}$ occurs, with $W^- \to e^- \nu$, there is an equal probability for $\tilde{g} \to e^+ + \ldots$. Then a pair of gluinos can with equal probability give same-sign or opposite sign dileptons! The same result holds for any way of tagging the electric charge — we just focus on leptons since their charge is easiest to identify. In particular, same-sign b's or top's may be a very good signature for supersymmetry. The same result holds for neutralinos. The SM allows no way to get prompt isolated same-sign leptons (or b's or t's), so any observation of such events is a signal beyond the SM, and very likely a strong indication of supersymmetry. Reference [10] is a recent analysis of how robust the same-sign signal is, and what might be most important if the same-sign signal is suppressed.

- **Stops**

Stops can be rather light, so they should be looked for very seriously. They can be pair-produced via gluons, with a cross section that is about 1/8 of the top pair cross section. It is smaller because of a p-wave threshold suppression for scalars, and a factor of 4 suppression for the number of spin states. They could also be produced in top decays if they were lighter than $m_t - M_{\tilde{N}_1}$, and in gluino decays if they are lighter than $m_{\tilde{g}} - m_t$, which is not at all unlikely. Their obvious decay is $\tilde{t} \to \widetilde{C} + b$, which will indeed dominate if $m_{\tilde{t}} > m_{\widetilde{C}}$. If this relation does not hold, it may still dominate as a virtual decay, followed by $\widetilde{C}$ real or virtual decay (say to $W + \widetilde{N}_1$), in which case the final state is 4-body after W decays, and suppressed by 4-body phase space. That may allow the one-loop decay $\tilde{t} \to c + \widetilde{N}_1$ to dominate stop decay. As an example of how various signatures may arise, if the mass ordering is $t > \widetilde{C}_1 > \tilde{t} > \widetilde{N}_1$ and $t > \tilde{t} + \widetilde{N}_1$, then a produced $t\bar{t}$ pair will sometimes (depending on the relative branching ratio, which depends on the mass values) have one top decay to $W + b$ and the other to $c + \widetilde{N}_1$, giving a $W + 2$ jets signature, with the jets detectable by b or charm tagging, and thus an excess of such events. Such a mass ordering is well motivated if the matter asymmetry is from electroweak baryogenesis.

- An event was reported by the CDF collaboration from Tevatron Run

1, $p\bar{p} \to ee\gamma\gamma\not{E}_T$, that is interesting for several reasons, both as a possible signal and to illustrate some pedagogical issues. That such an event might be an early signal of supersymmetry was suggested in 1986. It can arise [52,55] if a selectron pair is produced, and if the LSP is higgsino-like, in which case the decay of the selectron to $e + \widetilde{N}_1$ is suppressed by a factor of m_e. Then $\tilde{e} \to e + \widetilde{N}_2$ dominates, followed by $\widetilde{N}_2 \to \widetilde{N}_1 + \gamma$. The only way to get such an event in the SM is production of $WW\gamma\gamma$ with both $W \to e + \nu$, with an overall probability of order 10^{-6} for such an event in Run 1. Other checks on kinematics, cross section for selectrons, etc., allow a supersymmetry interpretation, and the resulting values of masses do not imply any that must have been found at LEP or as other observable channels at the Tevatron, though over some of the parameter space some associated signal could have been seen. Because of the needed branching ratios there would be no trilepton signal since $\widetilde{N}_2$ decays mainly into a photon instead of l^+l^-, and the decay of $\widetilde{N}_3$ would be dominated by $\tilde{\nu}\nu$. There are many consistency conditions that must be checked if such an interpretation is allowed, and a number of them could have failed but did not. Indeed, a related interpretation that had the decay of the selectron to electron plus very light gravitino is excluded by the absence of a signal at LEP for events with two photons and large missing energy. N_2 and/or N_3 decays are often invisible, e.g. to $\nu\tilde{\nu}$ followed by $\tilde{\nu} \to \nu + LSP$[64]. If this event were a signal additional ones would soon occur in Run 2; since such events have not been seen this event was probably not new physics.

For a complete treatment of any process one should list all the related channels, and combine those that can lead to similar signatures. The total sample may be dominated by one channel but have significant contributions from others, etc. It should also be emphasized that the so-called "backgrounds" are not junk backgrounds that cannot be calculated, but from SM events whose rates and distributions can be completely understood. Determining these background rates is essential to identifying a signal and to identifying new physics, and requires powerful tools in the form of simulation programs, which in turn require some expertise to use correctly. The total production cross section for all neutralino and chargino channels at the Tevatron collider is expected to be between 0.1 and 10 pb, depending on how light the superpartners are, so even in the worst case there should be several hundred events in the two detectors. If the cross sections are on the low side it will require combining inclusive signatures to demonstrate new physics has been observed.

Although it might look easy to interpret any non-standard signal or excess as supersymmetry, in fact a little thought shows it is very difficult. As illustrated in the above examples, a given signature implies an ordering of superpartner masses, which implies a number of cross section and decay branching ratios. All must be right. All the couplings in the Lagrangian are determined, so there is little freedom once the masses are fixed by the kinematics of the candidate events. To prove a possible signal is indeed consistent with supersymmetry one has also to check that relations among couplings are indeed satisfied. Such checks will be easy at lepton colliders, but harder at hadron colliders, so we do not focus on them here. There can of course be alternative interpretations of any new physics, but in all cases it will be possible to show the supersymmetry one is preferred (if it is indeed correct) — that is a challenge we would love to have .

31. Extensions of the MSSM

I want to emphasize that it may be very important to not restrict analysis of data by over constraining the MSSM with additional assumptions. I have focused on the MSSM for pedagogical simplicity, but nature could define simplicity differently. Surely the neutrino sector must be added, and that affects RGE's for the sectors we have examined. There is good motivation for extra U(1) symmetries, which may lead to extra D-terms and to extra neutralinos that mix to affect the neutralino mass eigenstates behavior and the CDM physics, and change the Higgs sector. There will be Planck-scale suppressed operators that may be crucial for flavor physics and for understanding the fermion masses and for precise calculations of gauge coupling unification. There may be extra scalars related to inflation, and axions, which affect cosmology and CDM physics. By using the MSSM without assuming relations among parameters many of these affects can be allowed for, while if parameters are related by ad hoc assumptions the extensions could only appear if inconsistencies appeared in the analysis — that is hard to see because of the initially large experimental uncertainties. For example, extra D-terms shift various scalar masses and separate $M_{H_u}^2$ and $M_{H_d}^2$, so assuming all the scalars masses are degenerate does not allow the D-term contributions to appear.

32. The LHC Inverse Problem

The initial goal of the Tevatron and LHC is to discover a signal of physics beyond the SM. Then the goal switches to figuring out what has been discovered, and to learning what deeper underlying theory the data points to.

Instead of addressing these latter goals, most of the work on collider phenomenology to date has been done in the “forward” direction, studying the map from parameter or model space into the space of observable collider signatures. The signals for a specific model are studied in great detail, with the goal of seeing how well the parameters of the model can be measured or constrained. Often, many of the signals are tailor-made to the model at hand and aren’t effective for other models, particularly not for the general case. To make the studies more tractable, they are usually performed within simplified models with very few parameters—in the context of SUSY, for instance, these have been carried out with mSUGRA, gauge mediated and anomaly mediated SUSY breaking. The hope is that if enough models are simulated in the forward direction, we will gain familiarity with the associated signals and will be able to spot and interpret them if they arise at the LHC.

If we think about what will happen when data comes we see the situation will be one we are less prepared for. First one has to decide what has been discovered in order to have a framework at all. Here we will of course assume that is supersymmetry. If it were something else, much of what we discuss would still be relevant. Then we have to deduce from the data which superpartners have been detected, their masses and properties. That is difficult because at the hadron collider what is measured are cross sections times branching ratios for a few channels, plus some kinematical distributions. Typically a given set of hadron collider signatures can be produced by several or even many different sets of MSSM parameters and spectra (“Degeneracies”[56]). Even if one could solve the degeneracy problem one still wants to deduce more completely the entire spectrum and the soft-breaking Lagrangian at the weak scale[11]. That will not satisfy us either – we want to learn the Lagrangian at the scale where gauge couplings unify and the theory is presumably simpler[9]. And then we want to learn the underlying 10D string theory[57].

We’ll discuss these issues in terms of LHC for simplicity, even though the discoveries could be made first at the Tevatron, and even if they are first made at LHC the Tevatron with its reduced backgrounds and simpler environment may contribute crucially to untangling the physics implications. There are several parts to the LHC Inverse Problem.

LHC-1A: Is it SUSY?
LHC-1B: Learning the spectrum and MSSM parameters – solving

the degeneracy problem.
LHC-1C: Can we figure out the high scale Lagrangian? What are the obstacles?
LHC-1D: What is the 10D string theory?

Of course we could wait until there is a discovery to begin to think about these questions, but actually we can work on all of them before the discovery. There is a lot of useful thinking and analysis that can be done now, and unfortunately has been neglected until now. It is likely that being better prepared will lead to interactions with experimenters and to asking questions of the data that might otherwise be neglected, and perhaps to progress that might not otherwise occur. Better understanding of such issues could lead to modified triggers and cuts. Here we will briefly consider some of these issues. Little systematic work has been done on these questions, and here we will just mention some recent work and activities from which one can begin studies, for each of these areas.

What are the most important things we want to learn? First, of course, that there is a signal for physics beyond the SM. Second, that it is indeed superpartners (or whatever it is). Probably third is learning about the LSP because if it exists and is stable it is some or all of the dark matter of the universe. Of course everyone will assume the dark matter is the LSP, but for such an important question it will be crucial to actually calculate its relic density and confirm it is providing about a quarter of $\Omega = 1$[45]. Data from colliders will complement that from direct and indirect dark matter detectors to allow such calculations.

33. Is it SUSY?

Once a signal is found by the experimenters, we will all be eager to interpret it. How could we establish it is indeed superpartners that are observed? Of course supersymmetry has a number of well known signatures, particularly an excess of events with large missing transverse energy. Another is same-sign dileptons, which occur in supersymmetry because the gauginos are Majorana particles and can decay as themselves or their antiparticles. This signature is a strong indication if it is present, since no SM process will produce it (assuming decays within jets are cut). Any given signature can be approximately faked by some other new physics, of course, so it will be necessary to find two or more to draw definitive conclusions. A recent study of such issues is ref. [10]. It examines the conditions under which the main supersymmetry signatures should hold, what other signatures typically accompany them, and in various special circumstances where one or another signature is missing it examines what replaces the missing signature.

Traditionally, it has been hoped that supersymmetry could be confirmed by measuring the spins of superpartner candidates, and checking that their interactions had the same gauge couplings as the particles. On the time scale of interest to us that will have to be done at hadron colliders, and is difficult, since isolating any particular superpartners is unlikely. Ref. [10] pointed out that using the unique relation between mass and cross section and spin one could replace the spin measurement by a cross section measurement if one had an experimental way to fix a mass scale. This was done independently in reference [58] and [59], which also emphasized that there could be ambiguities since one was really measuring the mass differences between the produced particles and the lightest one into which they decay. In practice that means one will have to make measurements of two mass scales and associated cross sections. Because supersymmetry is a real theory, with only masses unknown, once there are signals associated with mass scales a large number of relative rates and branching ratios are fixed and it will not be difficult to make a very convincing case that indeed supersymmetry is being observed. The main danger here is equating supersymmetry with some particular model of masses rather than testing the general theory.

Further, decay branching ratios are normally rather different in alternative approaches. Typically for supersymmetry decays of charginos and neutralinos have large contributions from real or virtual W's and Z's, unless squarks and sleptons are quite light. Then their branching rations to quarks are usually considerably larger than those to leptons, just as for W and Z. In the alternative models typically the decays do not go through W,Z and often those branching ratios are more commonly to leptons (e.g. see Table 3 of [10]).

There are many signatures that would be inconsistent with supersymmetry as well, such as TeV wide resonances and these must all be absent.

Altogether, contrary to what is sometimes said, I don't think it will be hard to distinguish supersymmetry from other alternatives even at hadron colliders, and it will not take long to be convinced what has been discovered is supersymmetry (if it is). Probably better techniques to actually measure the spins of superpartners will also be developed and the traditional tests can eventually be done. The cross sections and decay branching ratios also depend on the couplings and it won't be too hard to confirm they are consistent with the supersymmetry predictions too.

34. The LSP

As described above, learning about the LSP, its mass and its composition – how much is bino or wino or higgsino? – is essential for doing the calculations needed to confirm the LSP from the collider is the entire dark matter. While that will be taken as obvious at the NY Times level, it actually is not. First, escaping the detector only requires that the lifetime be longer than about 10^{-8} seconds. Detection in a direct experiment is needed to confirm the LSP has a lifetime at least as long as the universe. Second, doing the calculation needed to show it does provide the relic density observed by cosmological experiments requires detailed knowledge of the LSP properties (and those of other superpartners), as well as detailed knowledge of the cosmology.

At a hadron collider, with two LSPs escaping, it will not be possible to simply deduce its mass by the usual kinematical methods. The problem is actually very difficult, and it is exacerbated by the existence in general of multiple solutions ("degeneracies") for the parameters that can produce any given set of LHC signals. In addition, knowledge of the mass alone is not enough to deduce the relic density. Data from the Tevatron collider (or eventually ILC) may be needed. Nevertheless, it is probably possible to learn what we need to learn from LHC plus direct detection and space-based experiments, again because supersymmetry is a real theory with many constraints relating all kinds of data. Higgsino LSPs lead to different branching ratios from gluino and squark decays than do wino or bino LSPs, etc. With limited information, people who studied the LHC Olympics (see below) blackbox models did well in learning the mass and nature of the LSP – see particularly the talk of Natalia Toro at the 2nd LHC Olympics. Further study is needed to ensure the relevant information is available.

35. The LHC Olympics

Many particle physicists, theorists and experimenters, want to be involved in the exciting opportunity to figure out how the SM gets extended, once data from the LHC (and the Tevatron) point the way. Even those who have worked on LHC physics already have usually done so in the context of what sort of mass reach the LHC might have rather than interpreting the physics there or the general LHC inverse problem. How might we develop the techniques and experience to learn to start with the data and interpret it? Or suppose you have a theory and you want to check it against the data. Your theory is in the form of a Lagrangian at 10^{16} GeV. How can you convert it into predictions for LHC that can be tested against what is observed there, say the number of events with two opposite sign leptons and two or more energetic jets and large missing transverse energy? Several people decided a good approach was to simulate one or more models that might come from the LHC and post the "data". Then anyone can take up the challenge to interpret it, learn what spectrum it came from, and what underlying theory. The data is available in several forms. What will happen in the real world is that experimenters will give talks and post papers analyzing data and reporting signals inclusively, such as the number of various kinds of events, and kinematical distributions. The Olympics data is posted in such a form, just as it will be at the first conference where the real data will be reported. If one wants additional distributions or rates in the real world one has to ask the experimenters, who provide them, and the Olympics are similar. The Olympics data is also posted the way it will come from the detector and anyone can plot their own graphs and calculate their own rates. One can simply google LHC Olympics and click to get to the web sites to participate. Guides and Primers to lower the barrier to participation are there too and are being improved.

36. Degeneracies

Of course, the most pressing question at the LHC will be to figure out whether there is any evidence for physics beyond the standard model, and then most broadly what theoretical framework best describes the new physics—for instance is it SUSY, or strong dynamics, or something else, as discussed above? But there is also a major aspect of the inverse problem purely within the context of low-energy SUSY. The parameter space of the model is so huge, and gives rise to such a large range of possible signals, that we can already ask, even assuming we have a signal we are confident is low-energy SUSY, whether we can even roughly determine the correct region of parameter space from LHC data. This has now been studied systematically in reference [56], which we summarize briefly in this section.

In this context, the "parameter space" is clear—fixing for instance the minimal field content of the supersymmetric standard model, we can vary the 105 soft parameters of the theory. In practice, it is useful to consider a smaller subset that still captures much of the variety of physics to be expected. Many of the 105 soft parameters are relevant only to flavor physics and do not have much effect on collider physics. The relevant ones consist of three gaugino masses, the Higgsino mass parameter μ, degenerate soft masses for the first two generations of scalar fields (in order to avoid large flavor violations), and separate soft terms for the third generation scalars. If we also include $\tan\beta$ this gives a total of 15 parameters. Some trilinears can be included.

The "signature space" is also easily defined. After imposing the appropriate cuts, we associate a number with every LHC observable. For instance, any number count of events of a particular kind is a direction in signature space. Any kinematic histogram can be divided up into deciles, and the boundaries of each decile is a direction in signature space. In this way, we quickly get a huge dimensionality for signature space. Since signature space has a much higher dimensionality than parameter space, naively one would think that the inverse map from signature to parameter space would be unique. Actually, this is incorrect, largely because the signatures tend to be highly correlated with each other, so the effective dimensionality of signature space is much smaller. In fact, we can imagine dividing signature space up into bins, with size determined by statistical fluctuations and experimental errors. Even scanning over all possible MSSM parameters, the number of bins in signature space—the number of experimentally distinguishable outcomes at the LHC—is not enormously large.

An obvious strategy for studying the inverse map is to simply simulate the MSSM in all regions of its parameter space. For instance, we can imagine taking each soft mass parameter in 100 GeV increments between, say, 100 GeV and 1 TeV. Even fixing $\tan\beta$ and taking our simplified 14 relevant soft parameters at the LHC, this is a total of 10^{14} models. In practice it is impossible to simulate so many models—simulating the first year of LHC data for a single SUSY model with fast detector simulation takes about 1 CPU hour. This is why so many studies are performed in models with a much smaller dimension parameter space, like the 5 dimensional parameter space of mSUGRA even though such studies may not be relevant. The net result is that there are a large number of "degeneracies", distinct sets of MSSM parameters that give indistinguishable LHC signatures. The degeneracies are effectively islands in parameter space that map into a single region of signature space.

The degeneracies have simple physical interpretations. As is common in hadron collider phenomenology, the cleanest handle on new physics often comes from looking at leptons. In SUSY models where sleptons are not copiously produced in a long SUSY cascade decay chain, the leptons dominantly come from Ws and Zs produced in electroweak-ino cascade decays. In this case, we find large ambiguities in the spectrum of the remaining electroweak superpartners. Two models can have identical LHC signals by being "flippers" where electroweak-ino mass eigenvalues are fixed but with different eigenstates, "sliders" where the electroweak-ino spectrum is moved up or down keeping mass differences fixed, and "squeezers" where the information of some of the electroweak-inos is hidden because the mass splittings are small enough that the leptons in the decay products are too soft to be seen. Of course, the changes in the electroweak-ino sector are accompanied by suitable changes in the colored superparticle spectrum to match rates and other kinematical distributions between the models. The number of degeneracies for a given model arising in this way is of order 10 to 100. When sleptons are forced to be present in the cascade decays, there are more leptons in final states and the degeneracies nearly disappear, although there may still be an ambiguity in swapping the left-and right-handed sparticle spectrum. But in the general case, our estimate of the number of degeneracies is as interesting as it could have been—the number is not one (i.e. there certainly are degeneracies) but nor is it 100. Therefore while the existence of degeneracies represents a challenge, it is one that can likely be overcome by devising clever independent new observables to eliminate them. This could allow us to determine essentially all important aspects of SUSY physics with LHC data.

Broadly defined, LHC signatures are anything that can be measured with an ATLAS- or CMS-like detector. To simulate 10 fb-1 of LHC data, reference [56] used PYTHIA to generate parton level interactions and hadron showering and piped PYTHIA output to a modified version of the CDF fast detector simulator PGS written by John Conway. This modified version was developed by Steve Mrenna and approximates an ATLAS- or CMS-like detector. PGS yields reasonable efficiencies and fake rates and includes the effect of energy smearing. A simplified output from PGS was used for our study, namely a list of objects in each event labeled by their identity (photon, electron, muon, hadronic tau, jet, b-tagged jet, missing E_T) and their four-vector. Leptonic objects are also labeled by their charges. Using this information, one can construct almost any LHC signature imaginable. Standard model background was ignored, but if one cannot interpret a signal without backgrounds it will only be harder with them.

While standard model background was not included, nor initial state radiation or multiple interactions, cuts and triggers were selected in a way that is aware of the challenges from these issues. The complete list of the signatures used is given in appendix B of [56]. There are two different types of signatures considered, counting signatures and kinematic histograms.

Counting signatures give the number of events that pass a certain set of criteria. Because getting an accurate measurement of superpartner rates is very difficult at the LHC, only one signature that counts the total number of SUSY events that pass the above cuts was used; all other counting signatures are given as ratios.

In the regions of parameter space where the sleptons are not produced in long cascade decay chains, it is found that there is very little handle on the slepton masses, or on degeneracies in the electroweak-ino sector. The typical number of degeneracies for any given model is of order 10 - 100. These degeneracies have simple interpretations. For instance, we can have "flippers" where (N_2, N_1) are either $(\tilde{B}, \tilde{W})$ or $(\tilde{W}, \tilde{B})$, with accompanying changes in the rest of the spectrum to match LHC signatures.

With sleptons produced on-shell in cascade decays as is the case in many of the well-studied mSUGRA models, but perhaps not in nature, the situation is better, but there may still be possible degeneracies involving left/right swaps. The study of the inverse problem reconciles two orthogonal views one often hears about what the LHC can determine about weak-scale physics. There is the school of thought that says that the LHC is only a discovery machine, but that any more precise determination of the underlying physics must await the construction of a linear collider. Another school of thought holds that not only can the LHC discover new physics, it can also determine model parameters to few percent accuracy! The inverse map shows the sense in which both of these pictures can be correct. There can indeed be a relatively large number of different models compatible with LHC data, partially justifying the first view, but each of these is a small island in parameter space, partially justifying the second view. The number of degeneracies is as interesting as it could have been, however. The number is not millions or thousands; it is just large enough to represent a non-trivial challenge and just small enough to spur us to think of clever new signatures to resolve the small number of ambiguities in SUSY. And it is easy to determine whether a new set of signatures is effective in enlarging signature space—we simply add the signature and compute the new average number of degeneracies.

Obviously, the number of degeneracies will be smaller if a more restrictive parameter space is chosen. Again, this can be looked at by repeating our analysis for the restricted models. If a simple model reproduces LHC data, it is less likely to have a degenerate pair within its own model space, and despite the fact that it may have degenerate pairs in an enlarged parameter space, it would clearly be preferred over other generic points. It is quite interesting that $\tan\beta$ is difficult to measure in general, and it is challenging to find signatures that are sensitive to $\tan\beta$ alone. Perhaps when information about the superpartner spectrum is known from the LHC, $\tan\beta$ could be determined from other information that is very sensitive to $\tan\beta$, such as the higgs sector, $g_\mu - 2$ and $B_s \to \mu^+\mu^-$ constraints. Our approach to the inverse problem was carried out here in the context of low-energy SUSY, but the same ideas can be applied to any theory of physics beyond the standard model, including theories of extra dimensions with KK parity and of little Higgs models with T-parity. Indeed, SUSY is likely a more challenging example due to its large parameter space. Furthermore, our entire discussion has been at the electroweak scale in four dimensions. Once the 4D effective lagrangian is determined at the weak scale, we can begin probing the underlying higher scale or higher dimensional physics.

37. Concluding Remarks

These lectures have emphasized how to construct a supersymmetric description of nature at the weak scale based on forthcoming data from colliders, rare decays, static properties, cold dark matter studies, and more, and how to connect that to a unification scale description, so that we can eventually learn a complete effective Lagrangian near the Planck scale. If we also understand string theory (and we do not distinguish here between string theory and M-theory) well enough, possibly we will be able to bridge the gap to the Planck scale in 10 dimensions and formulate a fundamental theory. If so a number of features of the effective theory will be able to test ideas about the fundamental theory. The most important consequences of the experimental discovery of supersymmetry will be threefold: we will understand the natural world better; we will know we are on the right track to make more progress; and we will be opening a window to see physics at the Planck scale, which makes immensely more likely that we will be able to formulate and test a fundamental theory at the Planck scale.

Sometimes I am asked “what is left to compute” by students or postdocs looking for interesting projects, and interested workers in related areas. Much is indeed already known about supersymmetry from over two decades of work by a number of good people. But in fact we have just gotten to the stage where the most important problems can be addressed!. Little is known about how to relate data to the parameters of L_{soft} in practice, little is known about the flavor and CPV properties of L_{soft} and how to compute them theoretically or extract them from data uniquely, and little is known about how to relate data at the weak scale to an effective Lagrangian at the string scale. There is much to understand and to compute. These issues will be the main focus of supersymmetry research once superpartners are being directly studied. Much thinking and analysis is needed before we can understand the implications of the hoped-for signals.

There are several practical features that should be emphasized. Unless we are missing important basic ideas, a Higgs boson and superpartners could be produced at the Tevatron collider. Supersymmetry signals have two escaping LSP's, so they are never dramatic or obvious or easy to interpret. They will appear as excesses in several channels, where channels are labeled by numbers of leptons and jets, and missing transverse energy. Once superpartners are found, the entire challenge to experimenters is to measure the parameters of L_{soft}, which has been the main subject of these lectures. The relations of the parameters of L_{soft} to data is complicated, and it is easy to get the wrong answers if care is not taken. Although there seem to be a large number of parameters, any given measurement depends only on a few, and most parameters enter in a number of places, so combining information from several analysis will greatly facilitate progress, and may be essential. Interpreting the data and learning its implications will be challenging, and it is a challenge we are eager to have.

Acknowledgments

These lectures have benefited greatly from my collaborations and discussions with Mike Brhlik, Dan Chung, Lisa Everett, Steve King, Lian-Tao Wang, Brent Nelson, Ting Wang, Henery Frisch, Steve Mrenna, Nima Arkani-Hamed, Jesse Thaler and Piyush Kumar.

References

1. Search for CP Violating Neutral Higgs Bosons In The MSSM at LEP, LEP Collaboration (Philip Bechtle for the Collaboration), hep-ex/0602046
2. G.L.Kane and L.-T. Wang, Phys. Lett. B488(2000)383, hep-ph/0003198; M.Carena, J.Ellis, A.Pilaftsis, and C.Wagner, Phys. Lett. B495(2000)155, hep-ph/0009212; T.Ibrahim and P.Nath, Phys. Rev. D63(2001)035009, hep-ph/0008237
3. *Perspectives on Supersymmetry*, G.L.Kane (ed.), Singapore, World Scientific (1998)
4. Theoretical Implications of the LEP Higgs Search, G.L. Kane, Ting T. Wang, Brent D. Nelson, and Lian-Tao Wang, Phys.Rev.D71:035006,2005 hep-ph/0407001

5. The Minimal U(1)' Extension of the MSSM, Durmus A. Demir, Gordon L. Kane, and Ting T. Wang, Phys.Rev.D72:015012,2005, Hep-Ph/0503290
6. Consistency of LEP0Event Excesses With An $H \to AA$ Decay Scenario And Low-Fine-Tuning NMSSM Models, R. Dermisek and J.F. Gunion, hep-ph/0510322
7. See for example Implications for New Physics from Fine-Tuning Arguments. II. Litttle Higgs Models, J.A. Casas, J.R. Espinosa, and I. Hidalgo, JHEP 0503:038,2005, hep-ph/0502066
8. Supersymmetry and the LHC Inverse Problem, N. Arkani-Hamed, G.L. Kane, J. Thaler, and L.-T. Wang, hep-ph/0512190
9. How Can We Go From Hadron Collider Data Toward the Underlying Theory that Extends the Standard Model? After the Champagne... , G.L. Kane, Czech.J.Phys.55:B197-B204,2005, hep-ph/0504257
10. Is It Susy?, A. Datta, G.L. Kane, and M. Toharia, hep-ph/0510204
11. The Supersymmetric Soft-Breaking Lagrangian: Theory and Applications, D.J.H.Chung, L.Everett, G.L.Kane, S.F.King, J. Lykken, and L.-T.Wang, in preparation.
12. Theory and Phenomenology of Sparticles: An Account of Four-Dimensional N=1 Supersymmetry in High Energy Physics, M. Drees, R. Godbole, and P. Roy, 2004, 555pp. Hackensack, Usa: World Scientific (2004) 555pp
13. Supersymmetry, P. Binetruy, Oxford University Press, 2006
14. Weak Scale Supersymmetry: From Superfields to Scattering Events (Hardcover), H. Baer and X. Tata, Cambridge University Press, 2006
15. A Supersymmetry Primer, S.P.Martin, hep-ph/9709356
16. Electroweak Baryogenesis and Dark Matter in the NMSSM, A. Menon, D.E. Morrissey, and C.E.M. Wagner, Phys.Rev.D70:035005, 2004, hep-ph/0404184
17. Kahler Corrections and Softly Broken Family Symmetries, S.F. King, Iain N.R Peddie, G.G. Ross, L. Velasco-Sevilla, and O. Vives, JHEP 0507;049, 2005 hep-ph/0407012
18. Flavor Symmetries and Kahler Operators, J.R. Espinosa and A. Ibarra, JHEP 0408:010, 2004 hep-ph/0405095
19. Massive Neutrinoes and (Heterotic) String Theory, J. Giedt, G.L. Kane, P. Langacker, and B.D. Nelson, Phys.Rev.D71:115013, 2005 hep-th/0502032
20. S.Dimopoulos and H.Georgi, Nucl. Phys. B193(1981)150; L.Girardello and M.Grisaru, Nucl. Phys. B194(1982)65
21. L.Ibanez and G.G.Ross, Nucl. Phys. B368(1992)3; S.P.Martin, Phys. Rev. D54(1996)2340; hep-ph/9602349
22. What is the Discrete Gauge Symmetry of the MSSM?, H.K. Dreiner, C. Luhn, and M. Thormeier, hep-ph/0512163
23. L.Ibanez and G.G.Ross, Phys. Lett. B110(1982)215; L. Alvarez-Gaume, M.Claudsen, and M.Wise, Nucl. Phys. B221(1983)495; K.Inoue, A.Kakuto, H.Komatsu, and S.Takeshita, Prog. Theor. Phys. 68(1982)927
24. See H.Haber in *Perspectives on Supersymmetry,* G.L.Kane (ed.), Singapore, World Scientific (1993)
25. A. Nelson and L. Randall, Phys. Lett. B316 (1993) 516; hep-ph/9308277
26. For some review and references see Martin, [1].

27. LEP Electroweak Working Group, LEPEWWG/
28. LEP Higgs Working Group for Higgs boson searches hep-ex/0107029; ALEPH-2001-066; ALEPH Collaboration hep-ph/0201014
29. G.L.Kane, S.F.King, and L.-T. Wang, Phys. Rev. D64(2001)095013; hep-ph/0010312
30. J.Goldstein, C.Hill, J.Incandela, S.Parke, D.Rainwater, D.Stuart, Phys. Rev. Lett. 86(2001)1694; L.Reina, S.Dawson, and D.Wackeroth, hep-ph/0109066; W.Beenakker,S.Dittmaier, M.Kramer, B.Plumper, M.Spira, P.Zerwas, hep-ph/0107081
31. The literature can be traced from M.Carena, H.Haber, H.Logan, S.Mrenna, hep-ph/0106116
32. D.Rainwater, D.Zeppenfeld, K.Hagiwara, Phys. Rev. D59(1999)014037; hep-ph/9808468
33. K.S. Cranmer, hep-ph/0512154
34. Calculable Upper Limit on the MAss of the Lightest Higgs Boson in any Perturbatively Valid Supersymmetric Theory, G.L. Kane, C.F. Kolda, and J.D. Wells, Phys.Rev.Lett.70:2686-2689, 1993 hep-ph/9210242
35. Upper Bounds on the Lightest Higgs Boson Mass in General Supersymmetry Standard Models, J.R. Espinosa, and M. Quiros, Phys.Lett.B302:51-58, 1993 hep-ph/9212305
36. M.Brhlik and G.L.Kane, Phys. Lett. B437(1998)331; hep-ph/9803391
37. N.Ghodbane, S.Katsanevas, I.Laktineh, and J.Rosiek, hep-ph/0012031
38. S.Y.Choi, J.Kalinowski, G.Moortgat-Pick, P.M.Zerwas, hep-ph/0202039
39. S.Mrenna, G.L.Kane, and L.-T.Wang, Phys. Lett. B483(2000)175; hep-ph/9910477
40. J.Cline, M.Joyce, and K.Kainulainen, hep-ph/0110031
41. M.Carena, J.M.Morena, M.Quiros, M.Seco, and C.Wagner, Nucl. Phys. B599(2001) 158; hep-ph/0011055
42. L. Everett, G.L.Kane, S.Rigolin, and L.-T. Wang, Phys. Rev. Lett 86(2001)3484; hep-ph/0102145
43. T.Ibrahim and P.Nath, hep-ph/9908443
44. M.Brhlik, D.Chung, and G.L.Kane, Int. J. Mod. Phys. D10(2001)367; hep-ph/0005158
45. What is the Cosmological Significance of a Discovery of Wimps at Colliders or in Direct Experiments?, J.L. Bourjaily and G.L. Kane hep-ph/0501262
46. Dark Matter Relic Abundance and Scalar - Tensor Dark Energy, R. Catena, N. Fornengo, A. Masiero, M. Pietroni, and F. Rosati, Phys.Rev.D70:063519, 2004 astro-ph/0403614
47. M.Brhlik, L.Everett, G.L.Kane, S.F.King, and O.Lebedev, Phys. Rev. Lett. 84(3041)2000; hep-ph/9909480
48. L. Ibanez, C. Munoz, and S. Rigolin, hep-ph/9812397; Nucl. Phys. B553(1999)43.
49. M. Brhlik, L. Everett, G.L. Kane, and J. Lykken, Phys. Rev. Lett. 83(1999)2124, hep-ph/9905215; Phys. Rev. D62(2000)035005, hep-ph/9908326
50. L. Everett, G.L. Kane, S. King, S. Rigolin, and L.-T. Wang, hep-ph/0202100;

See also S. Abel, S. Khalil, and O. Lebedev, hep-ph/0103031 and references therein

51. See chapters by J.-F. Grivaz; M. Carena, R.L. Culbertson, S. Eno, H.J. Frisch, and S. Mrenna; J.F. Gunion and H. Haber in ref.2.
52. See chapter by G.L. Kane in ref.2.
53. Do About Half the Top Quarks at FNAL Come from Gluino Decays?, G.L. Kane and S. Mrenna, Phys.Rev.Lett.77:3502-3505, 1996 hep-ph/9605351
54. Searching for a Light Stop at the Tevatron, G. Mahlon and G.L. Kane, Phys.Rev.D55:2779-2791, 1997 hep-ph/9609210
55. S. Ambrosanio, G.L. Kane, G. Kribs, S. Martin, and S. Mrenna, Phys. Rev. Lett. 76(1996)3498
56. Supersymmetry and the LHC Inverse Problem, N. Arkani-Hamed, G.L. Kane, J. Thaler, and L.-T. Wang hep-ph/0512190
57. From LHC Data to the 10D String Theory?, G.L. Kane, P. Kumar, and J. Shao, in preparation
58. Top Partners at the LHC: Spin and Mass Measurement, P. Meade and M. Reece, Jan 2006, hep-ph/0601124
59. Top Partners in Little Higgs Theories with T-Parity, H.-C. Cheng, I. Low, and L.-T. Wang, Oct 2005, hep-ph/0510225
60. S.P.Martin and P.Ramond, Phys.Rev. D51(1995)6515; hep-ph/9501244; G.Kribs, Nucl. Phys. B535(1998)41; hep-ph/9803259; K.Dienes, Physics Reports 287 (1997) 447
61. The literature can be traced from I.Jack and D.R.T.Jones, Phys. Lett. B457(1999)101; hep-ph/9903365; J.L.Diaz-Cruz, Proceedings Merida 1999 "Particles and Fields", 299.
62. G.L. Kane, Proceedings of Supersymmetry 1997, ed. M. Cvetic and P. Langacker.
63. Recognizing Superpartners at LEP, G.L. Kane and G. Mahlon, Phys.Lett.B408:222-228, 1997 hep-ph/9704450
64. Low-Energy Supersymmetry with a Neutralino LSP and the CDF E E Gamma Gamma + Missing E)T) Event, S. Ambrosanio, G.L. Kane, G. Kribs, S. Martin, and S. Mrenna, Phys.Rev.D55:1372-1398, 1997 hep-ph/9607414