Bargaining Rigidities and the Rationality of War

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Abstract

Theories of crisis bargaining implausibly assume that negotiations can take place at any time without any cost. Yet when rigidities such as the cost of changing the status quo or signaling costs, for example, are taken into account, negotiations can become too costly to revise frequently: agreements are “sticky” and extend for discrete periods of time. Non-continuous bargains leave the rising state with an opportunity loss, defined as the gains it would obtain if negotiations were updated continuously so as to ensure a perfect fit between the distribution of power and the distribution of benefits. Quick changes in relative power in a dyad and rigidities in the bargaining process can lead the rising actor to anticipate a large opportunity loss between two consecutive bargains, in which case there is no deal that both actors prefer to fighting. Ex ante negotiations can break down into war, even with complete information.

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1. Introduction

Recent research on the causes of war has focused on problems of private information and incentives to misrepresent to explain what prevents leaders from reaching *ex ante* bargains that would avoid the costs and risks of war. This literature concludes that, if both parties in a dyad have complete information about the other’s capabilities and resolve, war is never rational, as there is always a bargain that at least one of the parties prefers to fighting (Fearon 1995, Powell 1999). Actors therefore settle on a bargain that achieves the same division of benefits as the war would have, without incurring its costs and risks: “*with complete information, bargaining never breaks down in war*” (Powell, 1999).

This conclusion relies, however, on an assumption that is largely unstated in most existing theories and empirical studies of bargaining, namely that agreements can be updated at any time without any cost. The costs of reaching and enforcing an agreement are usually understood as negligible, such that the addition of a variable to a model of crisis bargaining would not be justified. Authors tend to assume a Coasian world in which adaptation would be frictionless. Yet, agreements are not and often cannot be re-negotiated continuously and uninterruptedly at every possible time $t_i$. Rather, agreements are “sticky” and extend for discrete periods of time. Negotiations and the necessary concessions they imply are difficult, costly and take time and changing the status quo is inherently costly. As a consequence, agreements tend to exhibit some degree of rigidity, such that they frequently remain in force even though the initial conditions that led to their adoption have dramatically changed. Bargaining rigidities can impede adjustments of the status quo and possibly cause war.

In fact, the mere anticipation of such discrepancies can increase the difficulty to reach agreements. Thus, a largely unaddressed puzzle is why certain deals are ever rejected, even though they constitute an improvement over the status quo. Rational, utility-maximizing leaders should always accept deals that increase their party’s gains – even if only marginally.

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2 “[...] If both states are satisfied, neither can credibly threaten to use force to revise the status quo and the status quo goes unchanged. If one of the states is dissatisfied, the satisfied state offers the dissatisfied state control over an amount equivalent in value to its payoff to fighting. The dissatisfied state accepts this offer, and the status quo is peacefully revised in its favor”, in Powell 1999.
Agreements can be “sticky”, however, such that accepting the present accord might impair the ability to obtain further concessions. The 2000 Camp David talks between Israel and Palestine illustrate this point: there is little doubt that the concessions offered by Ehud Barak would have improved the Palestinian situation over the status quo. Yet the offers were rejected by Yasser Arafat. An explanation that does not require us to abandon the assumption of perfect rationality is that the expected inability to re-negotiate was an important aspect of the bargaining setup. In other words, Arafat understood that accepting the present deal implied giving up further demands for a certain period of time.\(^3\) Thus, his decision can be rational if the long-term benefits of striking a better deal tomorrow exceed the short-term benefits of receiving limited advantages today. Anticipated rigidities explain the reluctance to accept certain deals, even though they are utility-increasing in the short-term. Negotiations and the concessions and changes they imply can be so difficult and costly to reach – both at the domestic and the international levels – that both parties implicitly understand that agreements cannot be re-negotiated at will. Only the scarcity of re-negotiation opportunities, and their cost, can explain why parties to a negotiation have minimum levels of expectations that need to be met. If accepting today’s deal implies giving up tomorrow’s, the parties may not be willing to accept any deal that only marginally increases their utility. Concessions can be judged insufficient in view of the time needed to reach a new agreement. Thus, rigidities not only explain why bargaining is infrequent, which by itself is conducive to war, but also why reaching an agreement can be so difficult.

Moreover, there exist instances in which decisions need to be taken so quickly that there is no time for negotiation. A decisive offensive military weapon, such as nuclear weapons, commensurately increases the urgency of decisions and hence the opportunity cost of time to a point where reaching an agreement is impossible. When time takes such an importance, it is easy to understand that bargaining rigidities decrease the likelihood of reaching an agreement. In other instances, time does not matter as much as cost. In many cases where some form of

\(^3\) In fact, Barak even put as a condition that Palestine would give up all of its demands in the future.
nationalism is at stake, there are issues that are difficult to address in the bargaining process. Even more important is the inherent cost of changing the status quo, as illustrated by the Israeli withdrawal from Gaza. In sum, bargaining is usually not as fluid as the literature assumes. It may even become so rigid that war can become rational for both parties even with complete information.

The issue of bargaining rigidities is most salient when relative power in the dyad is expected to evolve rapidly, such that striking a deal now makes the rising party worse off than striking it later. Thus, Arafat might have expected his bargaining power to increase (e.g., because of the population growth differential), such that he could anticipate a better deal in the future. In that sense, accepting the deal at time $t_1$ can be sub-optimal if it prevents the party to reach a significantly better deal at time $t_2$. Rigidities prevent states from updating their agreements as often as they would otherwise and, as a consequence, reduce the set of peaceful settlements that states can rationally agree upon. Under certain circumstances, this set is empty, in which case fighting is a rational choice. Hence, war can be rational on grounds other than incomplete information. Relaxing the assumption that states can bargain without any restriction or cost leads us to qualify the conclusion reached by Fearon and Powell that “*with complete information, bargaining never breaks down in war*” (Powell, 1999).

More precisely, I show here that bargaining rigidities associated with fast changes in relative power in a dyad can lead actors to be unable to locate a peaceful settlement, even under conditions of complete information, and therefore that bargaining rigidities should be taken more seriously than a simple stochastic term. Just as including transaction costs in a market of pure and perfect competition yields equilibrium outcomes that are suboptimal (Anderlini and Felli 2001), bargaining rigidities explain why engaging into a costly conflict can be a rational choice *ex ante*. To demonstrate it, I rely on a simple bargaining game with

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4 “[...] If both states are satisfied, neither can credibly threaten to use force to revise the status quo and the status quo goes unchanged. If one of the states is dissatisfied, the satisfied state offers the dissatisfied state control over an amount equivalent in value to its payoff to fighting. The dissatisfied state accepts this offer, and the status quo is peacefully revised in its favor”, in Powell 1999.
complete information\textsuperscript{5}, in which negotiations are costly. An outside option allows the players to engage into a costly conflict at any time. I show that there exist values of bargaining costs and growth differentials such that it would be Pareto-efficient for both parties to reach an agreement, and yet the unique set of subgame-perfect equilibria of the game is always for the parties to fight. In other words, under a set of specific conditions, war can be rational with complete information – leading to the conclusion that incomplete information is a sufficient but not necessary condition of war.

I first briefly review the relevant literature on the rationality of war (Fearon 1995 and Powell 1999). Second, I define bargaining rigidities and identify factors that can lead completely informed actors to be unable to re-negotiate agreements as frequently as they would need to. Third, I develop a simplified model of bargaining rigidities in a dyad and show that relaxing the assumption of costless bargaining leads us to qualify the conclusion reached by Fearon and Powell. The game theory aspect helps to highlight the essence of the problem as opposed to its empirical content. Thus, the stylized model does not try to describe nor does it postulate an actual behavior. Rather, it aims at shedding light upon the existing literature and at understanding how authors have relied on assumptions, which, if relaxed, can lead to different conclusions. Finally, I assess the importance of these results for questions such as the effect of growth differentials on the likelihood of war; the legal form that states adopt to formalize their agreements; as well as the broad question of the rationality of war. This discussion will also lead us to understand how the present model is actually a special example of the commitment problem.

\textsuperscript{5} Obviously, bargaining rigidities are most salient when information is incomplete and the assumption of complete information might therefore – and rightfully so – appear highly restrictive. Yet, it seems appropriate here for two reasons: first, it makes comparing the present result with the existing literature easier; second, and most important, it allows me to derive a robust finding, which has the greatest generalizability to real-world cases.
2. The Rationality of War

The vast literature on the causes of war since at least Thucydides has long eluded the issue of the rationality of war as a way to solve disputes. Over the past 15 years, however, this question has started to receive a rigorous and cumulative treatment, with Fearon (1995) being the clearest specification of the puzzle: “war is costly and risky, so rational states should have incentives to locate negotiated settlements that all would prefer to the gamble of war”. Fearon “attempts to provide a clear statement of what a rationalist explanation for war is and to characterize the full set of rationalist explanations that are both theoretically coherent and empirically plausible” (Fearon 1995). This literature, extended among others by Powell (1999), largely originates in the economics literature on the rationality of strikes (see for example Fernandez and Glazer 1991). It focuses on the inefficiency of war as a way to solve disputes, and more precisely tries to explain “what prevents leaders from reaching \(ex \ ante\) bargains that would avoid the costs and risks of fighting” (Fearon 1995). The puzzle stems from the fact that, when actors are rational and information is complete, there is always a bargain that at least one side prefers to war and war is therefore never rational \(ex \ post\). Fearon and others thus convincingly expose the deductive flaws in existing realist and rationalist explanations for war.

To solve the puzzle, authors have relied on two main arguments: first, it is argued that informational problems explain misperception (Jervis 1976) and miscalculation (Waltz 1979, Fearon 1995, Powell 1999): states possess private information on their resolve and capabilities and have incentives to misrepresent this information in order to gain additional bargaining power. Miscalculation follows from such misrepresentation, and can lead both actors in a dyad to rationally expect a positive utility from the use of force.

A second type of argument relies on the idea of commitment problems (Fearon 1995, Powell 2004). Commitment problems arise when parties to a negotiation cannot credibly commit to uphold the agreement, because of incentives to renege on it (Fearon 1998). In this regard, Powell (2004) suggests that “rapid shifts in relative bargaining power can lead to
bargaining breakdowns even if there is complete information”. However, he also suggests that “the shifts in the distribution of military power due to differential rates of economic growth are empirically too small to account for war through this mechanism”.

The rigorous specification of these theories, as well as the shift of units of analysis from a systemic perspective to a dyadic perspective have improved our understanding of the micro-foundations of conflict and have had a major impact on the way we analyze interstate conflicts. I intend here to build directly upon what has been developed by specifying even more rigorously under what circumstances we can expect these theories to hold, and when we can legitimately doubt their applicability. More precisely, I look at how modifying the assumption of no bargaining rigidities yields different results and can provide an explanation for the fact that war can be rational even under conditions of complete information.

While the issue of bargaining rigidities has remained largely unaddressed in political science, it has received some limited treatment in the economics literature under the concept of “transaction costs”. Transaction costs are traditionally understood as the time spent ‘preparing’ for the Coasian negotiation. When transaction costs are included in the market of pure and perfect competition, the market can fail to settle at a Pareto-efficient equilibrium and the Coase theorem hence does not hold anymore (Anderlini and Felli, 2001). The present paper looks at the impact of similar costs in situations of crisis bargaining and shows that they can also have a disruptive effect on states’ ability to reach a peaceful equilibrium.

3. Bargaining Rigidities
Agreements need to be updated frequently to adapt to constant changes in international political and economic structures. Existing theories of crisis bargaining have implicitly assumed that such adjustments are free and easy. Yet resources are scarce both at the domestic and the international level, and change is costly as a result. The lengthy rounds of the World Trade Organization, the sporadic North-South Korean talks or the difficult Israeli-

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6 The Coase theorem states that “whenever property rights are allocated, we should observe only outcomes which are constrained efficient in the sense that all potential gains from trade (net of transaction costs) are exploited.” (Anderlini and Felli, 2001).
Palestinian negotiations are all examples drawn from the real world that agreements are not and often cannot be re-negotiated continuously and uninterruptedly at every possible time $t_i$. Rather, agreements are sticky and extend for discrete periods of time of the kind $t_1$ to $t_{1+n}$, where $n \geq 1$. Rigidities inherent in the bargaining process, which are caused by domestic, institutional, signaling or implementation costs, prevent actors from bargaining in the same way as they would in the absence of these costs.

I define bargaining rigidities as costs or restrictions imposed on a state’s ability to engage into negotiations with another state in the same way as it would without these restrictions, such that negotiations cannot take place continuously and uninterruptedly at every possible time $t_i$. The causes of bargaining rigidities can be grouped under two main categories: first, the process of bargaining itself is costly. Second, the concessions that are necessarily entailed by negotiations are costly for the leader as well as the country as a whole.

\textit{a. Costly Bargaining}

i. Bargaining takes time

Complete information is not necessarily a given: it needs to be reached. Both at the interstate and the domestic levels, signals must be sent, beliefs changed and actors convinced. Agreements are typically hard to reach and negotiations take time. Time can in turn generate a cost for either player. Thus, if the challenger has a strong interest in achieving an agreement quickly (i.e., if the per period loss that it incurs as a consequence of not revising the agreement is high), the amount of time needed to reach a deal can be decisive in its willingness to continue bargaining. If, for example, the fate of co-nationals or an important source of economic revenue is at stake in the short term, it is easy to understand that bargaining time entails a major opportunity cost. Most obvious is the case of a country threatened by nuclear weapons: the speed of such weapon and the amount of destruction it causes generate a cost such that very little time, if any, can be devoted to negotiations. It becomes clear here that it is bargaining rigidities, in other words the inability to reach an
agreement instantly, that make the situation escalate. Time is essential in any military situation and bargaining can often become a luxury that leaders cannot take the risk to afford.

The existence of bargaining rigidities is, in fact, implicitly recognized in most democratic constitutions where crisis situations threatening the security of the country usually call for extraordinary measures in which the executive branch is given exceptional power. Crises demand quick responses for which single leaders as opposed to deliberative bodies are better suited.

ii. Bargaining costs
Agreements need to be negotiated, and not everything can be delegated to outsiders such as ambassadors. Thus, agreements need to be ratified by the parliament in most democracies. Yet, policy analysis and bargaining within the states’ various institutions takes time and has a cost. “Decisions are not a free good. Governmental resources for policy analysis and decision-making are costly and in short supply. Individuals and organizations seek to conserve those resources for the most urgent and pressing matters. […] Efficiency dictates considerable policy continuity.” (Chayes and Chayes 1993). Existing models, however, “include no scarcity of the type that makes holding another vote or implementing a new policy expensive” (Lupia and McCubbins 2005). Yet, parliaments and executive committees, among other, cannot always meet because of temporal and energy constraints. Constant re-negotiations are costly and sometimes impossible. Again, the existence of procedures such as the fast track procedure for trade agreements in the United States illustrates the importance of bargaining rigidities and the need to address them.

iii. Institutional cost
International institutions aim at facilitating the bargaining process and thus at reducing bargaining rigidities. As Keohane (1984) points out, they reduce bargaining costs by instituting clear rules of negotiation, procedures and reputation costs among other. However, an important cause of their efficiency is precisely that they are costly to create, and therefore
costly to change. They are typically structures with a certain degree of rigidity. Institutions’ structures, procedures and formal rules thus prevent states from changing them without incurring a certain cost. Institutions, by locking states’ interactions into a certain pattern of behavior that might no longer be optimal, impose rigidities on states’ negotiations, as the institution first needs to be removed or amended to accommodate the new needs. Institutions might prevent states from bargaining in the same way as they would without the institution and therefore generate a bargaining rigidity.

Obviously, this hypothesis would not remain unchallenged by realist scholars. Many of them would probably argue that institutions have no influence on states’ behavior when security is at stake. Such critic misses one important point however: institutions might precisely be one of the reasons that lead states to reach that point where security becomes central. In other words, by imposing bargaining rigidities on states and therefore preventing them from updating their agreement as often as they would want to, institutions can lead states to postpone their agreement, until postponing is no longer possible and the states reach a point where security becomes a central issue and an agreement must be reached to avoid war. By imposing bargaining rigidities, ill-suited institutions can thus exacerbate the tension between two states and the importance of security issues in their relations.

Finally, institutions are often not well adapted to fast changes in the status quo and might therefore become a burden. Yet, institutions are costly to discard. The very cost of creating institutions forces their members to be careful when putting them aside, as reviving them can be costly or even impossible. Thus, for example, ignoring the recommendations of the United Nations is usually not taken lightly, as it decreases the legitimacy of the institution and therefore puts its very existence at risk.

\[ b. \text{Costly Concessions} \]

Bargaining necessarily implies concessions from at least one of the parties. There exists an inherent cost of making such concessions.
i. Implementation costs
Changes that have been agreed upon need to be implemented, but changing the status quo is inherently costly. The Israeli withdrawal from the Gaza strip is an obvious example of a costly change in the status quo. More generally, movements of population (in case of a negotiation over a border), adaptation of economic activities (e.g. bilateral trade negotiations), changes in domestic law are examples of tangible costly changes. Finally, implementing the respect of the agreement is also costly (e.g., monitoring the German reparations following World War I).

ii. Signaling issues
States are traditionally not limited in their relations to a single country. Rather, they typically interact with numerous other states, and their behavior towards one of them affects the perception of the others of this country’s strategies, resolve, and capabilities. Therefore, even though conceding to the other party's demands might be rational if the analysis is limited to the dyad, its cost vis-à-vis third parties needs to be taken into account. Immediately giving in to demands sends the signal that bargaining is not costly and third parties, which do not necessarily have complete information, will therefore be more likely to ask for concessions.

As a result, making bargaining less easy and hence more costly (maybe to the point of fighting) can actually be voluntarily used as a strategy to deter other countries from making demands. The level of analysis is fundamental here: a behavior that could seem irrational at the dyadic level because it yields a lower payoff than an agreement is actually rational at a more global level: taking a strong stance in a negotiation by increasing negotiation costs (i.e., “tying your own hands”), even if it is not credible to the other party who is completely informed, achieves two essential results: first, it builds a reputation of being “tough”, and can therefore give more bargaining leverage in present and future negotiations with third parties. Second, it prevents revealing your type and reservation value: giving up too fast in a negotiation gives clues to other states about your resolve and capabilities. Making this
information “public” implies less bargaining power in future negotiations, and has therefore a cost. Tying one’s hands now by imposing a bargaining cost on negotiations can be efficient in the long term. Artificially increasing bargaining costs can thus be a rational strategy in the context of an \( n > 2 \) players’ game. An extreme illustration of this is the case of Chechnya: while the cost of the war with Chechnya is probably higher than the benefits \( \text{per se} \) for Russia, the fact that it sends a strong signal to other nations that might be tempted to make the same kind of demands for independence makes it rational in an \( n \)-player strategic context.

In sum, complete information in a dyad does not necessarily imply that there are no rigidities in the bargaining process. Several causes of rigidities have been put forward, and the next section demonstrates formally how they can have a major influence on states’ ability to reach an agreement\(^7\).

### 4. The model

I model bargaining rigidities as a fee that both actors must pay to enter negotiations. The cost of a single negotiation can be so high that actors prefer to fight immediately rather than try to reach an agreement. Even more likely to affect the outcome are cumulated costs of re-negotiations. Fast changes in the underlying bargaining power call for frequent re-negotiations of agreements, and hence repeated “fees”. Given that constant re-negotiation is infinitely costly under these conditions, agreements must be given a duration that is a function of the level of cost. High costs, however, can decrease the frequency of re-negotiations to a point where war becomes a rational choice. In essence, bargaining costs force actors to space their negotiations out and therefore impede adjustment of the status quo to a point where war can become rational.

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\(^7\) It is important to note, however, that I do not pretend to represent a theory that could be readily used for policy analysis. In fact, a world where actors have complete information about each other is highly implausible in most real-life contexts. Nevertheless, it allows us to see the role that the assumption of no bargaining rigidities plays in previous models, holding other factors constant and as such intends to contribute to the cumulativeness of the study of international conflict.
a. **Setup**

I consider a bargaining game over a known and fixed quantity \( Q \) between two players indexed by \( i \in \{ D, S \} \). The game follows the logic of a simple alternating offers bargaining game with complete information and potentially infinitely many rounds of alternating offers, in which the players discount the future at a strictly positive rate (Rubinstein, 1982), to which I add an outside option (Powell, 1999). In all odd rounds, player \( S \) has the chance to make offers, and player \( D \) to respond. The roles are reversed in all even periods. The players’ payoff consist of their share of the surplus, minus any cost paid, appropriately discounted.

I assume that actors’ preferences are such that both bargainers prefer more than less and sooner than later. Both players know the other’s preferences and are aware of each other’s rationality. The size of the surplus to be split between the players is normalized to one and any offer made in period \( t \) by player \( i \in \{ D, S \} \) is denoted by \( x_i \in [0, 1] \). This denotes \( D \)’s share of the pie. The discount factor of player \( i \in \{ D, S \} \) is denoted by \( \delta_i \in [0, 1) \). The growth rate of player \( i \in \{ D, S \} \) at time \( t \) is assumed to be exogenously given and is denoted by \( \Delta_i \), and the growth differential between \( S \) and \( D \) at time \( t \) is denoted by \( \beta_t = |\Delta_t^S - \Delta_t^D| \). At any time, actors can impose a settlement (henceforth understood as “war”), winning the entire flow of benefits with probabilities \( p_t \) and \( (1 - p_t) \) respectively for actors \( D \) and \( S \). The probability of winning \( p_t \) can in fact be conveniently understood as the distribution of power (Powell 1999). \( p(t) \), henceforth written \( p_t \), is a non-decreasing, continuous function defined on \([0, 1]\).\(^8\) For convenience, power is exogenously given as a function of time. Finally, the status quo is denoted by \( q \). That is, at time \( t_0 \), \( D \) and \( S \) have per period payoffs of \( (q) \) and \( (1-q) \). All of this is common knowledge.

\(^8\) See Fearon (1996) for example where \( p \) is endogenously determined
Let us also assume that $D$ is dissatisfied. Following Powell (1999), let $U_D(t_1)$ denote $D$’s payoff to go to war at time $t_1$. $S$’ payoff can be expressed similarly.

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(1) \quad U_D(t_1) = p_{t_1} \cdot \left[ \sum_{i=1}^{\infty} \delta_i^D (1 - c_i^D) \right] + (1 - p_{t_1}) \cdot \left[ \sum_{i=1}^{\infty} - c_i^D \delta_i^D \right],
\]

$D$’s per period payoff to going to war simplifies to $p - c_h^D$, while $S$’ payoff simplifies to $1 - p - c_h^S$, with $c_i^D$ representing the cost of war for player $i \in \{D, S\}$ at time $t_i$. $S$ therefore offers $p - c_h^D$, which is the minimum payoff that $D$ is willing to accept. $D$ accepts and the new status quo settles at $p - c_h^D$. In other words, the agreement reached is the reflection of the distribution of power $p$ minus the cost of war.

\[\text{Figure 1: Game Tree of the Initial Bargaining Model}\]

\[\text{a.}\]

\[\text{Introducing bargaining costs}\]

Let us assume for the time being that $\Delta^S_i = \Delta^D_i$, so that there is no growth differential and hence no need to re-negotiate. I model bargaining rigidities as a fee that actors must pay upon entering negotiations. Actors thus need to decide whether they are willing to pay the fee (“enter”) or not (“not enter”). If one of them chooses not to enter, the other player has only a choice between attacking (“attack”) and maintaining the status quo (“not attack”). If the other

\[\text{b.}\]

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player enters, then the game follows the offer and counter-offer pattern discussed in the previous section (see Figure 1).

Clearly, if one of the two players does not pay his bargaining fee, $D$ remains dissatisfied and will attack as soon as possible. Knowing that, $S$ chooses to attack in the first node.

**Proposition 1:** If one of the players does not pay his bargaining fee and hence does not enter the negotiation, war starts.

Intuitively, we see that if the cost of negotiating is too high, at least one of the parties will not enter. Then, given that $D$’s payoff to go to war is higher than its payoff to the status quo $q$ (i.e., $D$ is dissatisfied), war will follow (see Proposition 1).
More generally, given a bargaining fee $b$, the minimum $D$ will be willing to accept is $p_i - c_i^D + b$, while the maximum $S$ is willing to give is $p_i + c_i^S - b$. The reason is the following: $D$’s payoff to go to war is $p_i - c_i^D$ (Powell, 1999), while his payoff to negotiating is $x_i - b$, that is the size of the offer discounted by the bargaining fee it had to pay to enter the negotiation. Since $D$ will not rationally accept any offer that yields a payoff smaller than his payoff to go to war, the smallest offer $D$ will rationally accept is $x = p_i - c_i^D + b$. Similarly, $S$ can only offer up to $p_i + c_i^S$ discounted by the bargaining fee, that is $p_i + c_i^S - b$.

It is thus obvious that increases in $b$, the cost of bargaining, result in a smaller set of possible bargains between $D$ and $S$. $b$ can even be so high as to prevent actors from being able to locate a peaceful settlement and war can therefore become rational. More precisely, there is no deal that both actors prefer to war if $b > \frac{c_i^D + c_i^S}{2}$. Intuitively, war is rational if the total cost of bargaining is higher than the total cost of war.

**Proposition 2:** There are values of $b$, with $0 < b < 1$, for which neither player pays his fee, and war begins.

Overall, however, the cost of a single negotiation is seldom high enough to deter parties from negotiating or reaching an agreement. More plausible is that the cumulated cost of bargaining fees paid over multiple re-negotiations can be sufficient to prevent actors from locating a peaceful agreement. We therefore want to look at situations where frequent re-negotiations are necessary. Shifts in the underlying power structure, caused by differences in rates of growth, impose such frequent re-negotiations.

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12 For simplicity, and without loss of generality, the bargaining fee for $S$ and $D$ is assumed to be equal
c. Growth differential and the need to re-negotiate

Let us go back to the original setup and assume now that there is a growth differential and, without loss of generality, that it takes the form \( \Delta_t^s < \Delta_t^D \). As a result, \( D \)'s power increases faster than \( \Delta_t^S \) and \( D \) becomes increasingly dissatisfied. Over time and as \( D \)'s power increases, we therefore expect to see a series of deals that update the original agreement. Each time, the agreement should be re-evaluated so that the “distribution of benefits” corresponds to the “distribution of power” (Powell 1999).

Since the agreement always settles at the point where a marginal increase of \( D \)'s power increases its payoff to go to war to a level where war becomes rational, an equilibrium reached at time \( t_1 \) is immediately obsolete, given that \( \Delta_t^s < \Delta_t^D \); at \( t_2 \), \( D \)'s power has increased again, rendering this actor no longer satisfied with the minimum payoff \( p_{t_1} - c_i^{p} \) it was willing to accept at time \( t_1 \). A new agreement is therefore necessary. Permanent equilibrium necessitates constant re-negotiation of agreements. As long as negotiations are costless and always possible, \( S \) and \( D \) will always bargain according to Powell’s model (1999), and war will therefore never be rational.

What happens, however, if negotiations are not free? In other words, what are the consequences of introducing a fee for entering into negotiations, as was the case in the first model? If actors want to update their agreement continuously, they now need to pay an infinite number of fees and the total cost (a deadweight loss) hence becomes infinite for any \( 0 < b \). Constant re-negotiation thus being impossible, agreements must have a duration and hence a frequency. The maximum frequency of re-negotiation can easily be calculated on the basis of the maximum number of re-negotiations possible before at least one of the state has a higher utility going to war, which itself depends on the cost of negotiating \( b \) and the “margin of negotiation” that both actors get: \( (p + c_i^S) - (p - c_i^D) = (c_i^S + c_i^D) \), that is, the total cost of war. The total cost of negotiations \( (bn) \) must be smaller than this margin:

\[
(2b \times n) \leq (c_i^S + c_i^D) \quad \Rightarrow \quad n \leq \frac{(c_i^S + c_i^D)}{2b}
\]
$n$ represents the maximum number of negotiations that can take place over a time interval.

**Proposition 3:** Given a time interval $[t_1, t_2]$ and a level of costs $b$, there exists a maximum number of re-negotiations and hence a maximum frequency of re-negotiation if war is to be avoided.

**d. Relative loss**

We now want to know whether this number of re-negotiations will be sufficient to avoid war. The answer depends on how frequently agreements need to be updated, and hence on the growth differential in the dyad. This section shows that a large growth differential forces both actors to re-negotiate frequently if they want to avoid war.

Since re-negotiations are discrete and not continuous, $D$ anticipates at time $t_n$ a relative loss because the agreement will not be re-negotiated until $t_{n+1}$, although $D$’s power will have increased in the meantime. In other words, $D$ anticipates a relative loss whose size is a function of the growth rate differential and of the time interval between the present negotiation and the future one: $D$ has a relative loss equal to the area of $L_4$ on figure 3.

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**Figure 3: D’s gains and relative losses from successive bargains**

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13 Remember that both $D$ and $S$ anticipate a change in relative power at a rate $\beta'$. 
Non-continuous bargains leave the rising state with a relative loss, which is a function of the
time elapsed between the two bargains and of the growth differential, $\beta$.

The size of the growth differential and the time interval between two negotiations (which
depends on the number of renegotiations, itself a function of the cost of negotiation $b$: see
proposition 3) determine the size of $L_1$ and hence how much $D$ will be asking for to
compensate for the anticipated relative loss ($L_1$) between $t_1$ and $t_2$ that arises from a
discrepancy between the distribution of power (the curve) and the distribution of benefits (the
agreement line $y = p_{t_i} - c_{t_i}^D$). The higher $D$’s rate of growth, the faster the differential
between distribution of power and distribution of benefits will increase and therefore, the
faster $D$ will grow dissatisfied. Hence, agreements must either take place more frequently to
satisfy $D$ and prevent it to go to war, or include the whole area of $L_1$ that $D$ expects to obtain
over the $[t_1,t_2]$ period if agreements where updated continuously. As will become clear,
however, the larger $L_1$ is, the less attractive a bargain is to $S$ and therefore the more likely is
war.

$D$ could win more if the agreements were re-negotiated at every time $t_i$. However, since the
cost $b$ of negotiating imposes a duration to agreements (see proposition 3) and therefore re-
renegotiation is not possible between $t_1$ and $t_2$ (i.e., this is the highest possible re-negotiation
frequency), $D$ will want at time $t_1$ a deal that compensates for this expected loss (based on his
expected increase in relative power): at time $t_1$, $D$ will ask for what it expects it should be able
to get between $t_1$ and $t_2$ (because of its increasing relative power), that is $L_1$, in addition to
($p_{t_i} - c_{t_i}^D + b$). To avoid war in the presence of bargaining costs, $S$ and $D$ need to reach an
agreement such that, at no time between the present negotiation ($t_1$) and the next one ($t_2$) will
$D$’s payoff to go to war be higher than its payoff to maintaining the status quo. Formally, $S$
needs to grant $D$ at least the following:

$$
x_{s_i} = \sum_{i=0}^{t_i} \delta^i q^{rad} + p \left[ \sum_{i=1}^{\infty} \delta^i (1 - c_{t_i}^D) \right] + (1 - p) \left[ \sum_{i=1}^{\infty} - c_{t_i}^D \delta^i \right] + \int_{t_i}^{t_2} p_{t_i} (\beta_t - 1) \, dt
$$

19
To simplify, let us assume that the growth curve is linear and monotonic, and let us normalize the time interval \([t_1-t_2]\) to one. The above equation can then be simplified to 
\[
x_{t_1} = \left( \beta p_{t_1} - c_{t_1}^D + b \right).
\]
Therefore, to avoid war between \(t_1\) and \(t_2\), \(D\) will ask for \(\beta p_{t_1} - c_{t_1}^D + b\), where \(\beta\) represents the growth differential. To put it differently, \(D\) needs to anticipate the future and therefore makes a demand that is not supported by its present level of power.

e. The No-Agreement Zone

While \(D\) demands \(\beta p_{t_1} - c_{t_1}^D + b\), \(S\) is willing to give up to \(p_{t_1} + c_{t_1}^S - b\). But where \(p_{t_1} + c_{t_1}^S - b\) now lies is unclear. To determine whether there are agreements that both parties can prefer to war, we need to determine the sign of 
\[
\left( \beta p_{t_1} - c_{t_1}^D + b \right) - \left( p_{t_1} + c_{t_1}^S - b \right).
\]
War is only rational for both sides if there is no agreement that both prefer to war and therefore if the above inequality is strictly positive. Simplifying leaves us with 
\[
\beta > 1 + \frac{c_{t_1}^D + c_{t_1}^S - 2b}{p}.
\]
It is now clear that the larger \(\beta\) (the growth differential), the more attractive war is.

![Figure 4: The No-agreement Zone](image)

If \(p + c_{t_1}^S - b\) lies at the left of \(\beta p_{t_1} - c_{t_1}^D + b\) (see Fig. 4 above), then there is no deal that both players prefer to war.

In other words, if the growth rate of one of the actors is very large and agreements cannot be re-negotiated often enough (i.e. \(\beta\) is too large), then there is no agreement that both actors would prefer to war and war is rational.
Proposition 4: Given a time interval \([t_1, t_2]\) and a level of costs \(b\), there exists a minimum number of re-negotiations and hence a minimum frequency of re-negotiation if war is to be avoided.

Theorem: Consider the game with bargaining fees described above. The game has a subgame perfect equilibrium (SPE) in which agreement is reached in finite time if and only if \(\Delta_{i_n}^I\) for \(i \in \{D, S\}\) and for any time \(n \in [t_1, t_2]\) satisfies:

\[
(A4) \quad 1 + \frac{c_{i_n}^D + c_{i_n}^S - 2b}{p_{i_n}} + \Delta_{i_n}^S \geq \Delta_{i_n}^D \quad \text{and} \quad 1 + \frac{c_{i_n}^D + c_{i_n}^S - 2b}{p_{i_n}} + \Delta_{i_n}^S \geq \Delta_{i_n}^D
\]

It has been made clear that the growth of the power differential defines a minimum frequency at which agreements need to be re-negotiated if war is to be avoided. Agreements should be re-negotiated at least often enough so that \(D\) is always satisfied. It has also been shown that bargaining costs impose a maximum frequency of re-negotiations. There are, however, values of the growth differential and the bargaining fee \(b\) that make these maximum and minimum frequencies of renegotiation incompatible, such that war is rational.

In sum, the only way to avoid war when the growth differential is high is to negotiate frequently. When this impossible because of high bargaining costs, war is rational.

5. Consequences of the rigidity problem

I have shown that, when relative power changes quickly between two actors, bargaining costs can make war a rational solution even to completely informed actors. This finding leads us to derive conclusions about the effect of power shifts on war. It also allows us to sketch hypotheses about the effect of growth differential on the legal form that we should expect agreements to take. Finally, I evaluate the idea that the bargaining rigidities problem can be resolved if there are no commitment problems.
a. Power shifts and war

The link between changes in relative power and the likelihood of war has been the subject of numerous theories, of which two are particularly relevant to the present discussion:

The first body of literature, which originates in the so-called power-transition theory (Gilpin 1981, Kugler and Organski 1989), sees power transition as the cause of major wars. According to the theory, wars are most likely to occur at the transition period, that is, when the challenger’s level of power roughly equals that of the status quo hegemon. The idea can be traced back to Thucydides’ argument that “what made the Peloponnesian War inevitable was the growth of Athenian power and the fear that this caused in Sparta”: power transitions occur when one state rising in power surpasses others that are relatively declining. These theories therefore look at the phenomenon of changing power differentials between states that arises from uneven rates of growth. They focus mainly on the rational timing of war (on preventive war, see Levy 1987). However, the evidence to support the power transition theory is rather weak (Houweling and Siccama 1988). More important, the reasons why the transition period should be more war-prone than other stages remain somewhat unclear, and recent work has challenged this theory. Thus, it has been argued that what matters is actually not so much the rate of growth of the challenger, as “predictable, long run changes in relative capabilities as opposed to transitions” (Morrow and Kim 1992), in other words the anticipated growth differential. Further, Powell (1999), shows that the increased likelihood of war does not come from the transition or shifts in power per se, but rather that the disparity between the distribution of power and the distribution of benefits (as a result of growth differentials) in a dyad causes one state to be willing to use force if that disparity is not matched by agreements to adapt the status quo.

Adopting the same logic as Powell and introducing bargaining rigidities, I show that changes in relative power do affect the likelihood of war under complete information.

14 It should be noted that the power-transition literature is not unanimous about when wars are more likely: some argue that the period preceding the transition is more war-prone (Organski 1958), while other argue that the period following it is more likely to witness a major conflagration. These debates have only minor relevance to the present topic and I refer the reader to extensive existing debates on the issue for further information.
although not for the reasons put forward by the power transition theory. In an environment of bargaining rigidities, fast rates of growth differential increase the size of the anticipated opportunity loss faced by the rising power, and hence raise the attractiveness of war as an alternative to bargaining.

Second, the balance of power literature follows a similar logic. In a balance of power environment, the most war-prone configuration is one where an actor’s relative power becomes large and therefore necessitates a preemptive military intervention to stop its growth (Waltz 1979). Conversely, this school expects war to be least likely “when equality is greatest among the poles” (Mearsheimer 1990). Powell’s criticism and hence the present article applies here just in the same way as for the power transition theory: there is no compelling reason to believe that power imbalances are more war-prone than power balance. Moreover, this literature fails to address the major question: why do states go to war rather than reach an agreement that would achieve the same division of benefits as war would have, without incurring its costs and risks?

Overall, the present findings corroborate the ones of Powell (2004): in addition to the disparity between the distribution of power and benefits, the disparity in growth rates per time unit also impacts the likelihood of war. All things equal, I expect a positive relationship between on the one hand the rate of growth differential (power or wealth growth) and the rigidity of the bargaining process, and on the other hand the likelihood of conflict in a dyad. Contrary to Powell (1999) statement that “faster shifts [are not] more dangerous than slower shifts”, but in conformity with Powell (2004), the pace of the growth differential actually affects the likelihood of war in the presence of bargaining rigidities.

Finally, what rational strategies can we expect in an environment of bargaining rigidities? If rigidities are too high, then war will take place at some point over the $[t_1, t_2]$ interval. The question now becomes What is the best strategy for actors, given that war will occur? In other words, how do rigidities affect the timing of war? The answer is not as simple as may seem at first sight. On the one hand, $D$ – the rising power – has fewer chances to win now than later in
the interval, when its relative power will have increased. At the same time, fighting now would allow it to enjoy a higher expected payoff longer. The reverse reasoning is true for $S$. In mathematical terms, $D$ needs to compare

$$U_D(t_i) = p_{t_i} - c_D^{t_i} \quad \text{and} \quad U_D(t_w) = \int_{t_i}^{t_w} (q_t) \, dt + p_w - c_w^D,$$

where $w$ is the time of the war and $w > 1$. If $D$ expects a high increase in its relative power, fighting later would give him more chances to win, and waiting would hence seem to be the rational strategy. However, any major increase in relative power in the presence of bargaining rigidities is also associated with a proportionately high relative loss (Fig. 3). The opposite is true for $S$. As a result, both players should be indifferent between fighting now or later in the $[t_1, t_2]$ interval.

b. **Hard vs. Soft law**

If the rigidity of agreements affects the likelihood of war when there are large growth disparities, then we should expect agreements to take different forms according to the level of growth differentials. Intuitively, we expect agreements that are easier to change when growth differentials are high and more rigid agreements when more stability is anticipated.

The past decade has seen an increase in the number of publications linking political processes to the type and forms of legal international agreements. More precisely, this literature has addressed the question of Why international actors sometimes rely on legally binding and inflexible legal instruments to legalize their agreements, and why they at other times prefer loose, imprecise and more flexible ones? To address this question of the “degrees and forms of legalization” (Abbott and Snidal, 2000), scholars have used a terminology based on the “softness” or “hardness” of the form of legal tool chosen\textsuperscript{15}. Their taxonomy is essentially based on three dimensions, namely the levels of “obligation”, “precision” and “delegation” of the law (Abbott et al., 2000). Thus for example, gentlemen agreements are

\textsuperscript{15} For a full presentation of soft vs. hard law, see Abbott and Snidal, 2000
categorized as soft law, while legally binding treaties enforced by an independent institution (e.g. the E.U. pact of stability) are obviously considered as hard law. More generally, a law can be categorized as soft when the three dimensions are low (i.e., low precision, delegation and obligation), while it can be categorized as hard when “all three […] are high” (Abbott and al., 2000): “Highly legalized institutions are those in which rules are obligatory on parties through links to the established rules and principles of international law, in which rules are precise […], and in which authority to interpret and apply the rules has been delegated to third parties acting under the constraint of rules” (Abbott and al., 2000). Thus, where soft law provides flexibility and low negotiation and implementation costs, hard law provides clarity and reliable commitment.

The present findings provide some elements of answer as to when we should expect to see states rely on soft vs. hard law. The underlying logic is straightforward: hard law is preferable because it defines parties’ rights and obligations clearly. However, this very advantage becomes cumbersome when the law has to be changed: changing a “hard law” entails negotiation and implementation costs, time, etc. Hard law is therefore most appropriate when revisions are not expected for a relatively large amount of time. On the contrary, soft law can be revised with much less difficulty and as such is more flexible than hard law. It is therefore well-suited to situations where revisions are frequent.

Clearly, international agreements are a reflection of a specific state of affairs in time. They typically reflect the distribution of power and existing norms and beliefs. As a consequence, agreements frequently need to be revised to adapt to changes in the international order (whether it be at the bi- or multilateral level). Therefore, states should rely on more flexible agreements and institutions (soft law) when they expect major changes in relative power and/or when rigidities in the bargaining process are strong: such changes make revision of existing agreements more frequent in order to adapt the distribution of power to the distribution of benefits. Soft law being relatively less costly to change, the overall cost of re-
negotiation is lower over time. We can therefore expect rational and fully informed actors to anticipate this and opt for soft law as a consequence.

On the contrary, states should rely on more highly legalized frameworks (i.e., more costly to create but more clear and stable) when relative levels of power are stable and/or rigidities are weak between parties to the negotiation (e.g. The European Union and the stability pact).

c. Overcoming rigidities: the importance of the commitment problem

The practical problem exposed in the present paper is How states facing rigidities such as the ones described above can avoid war? Clearly, the idea that both members of a dyad would be better off settling on an agreement *ex ante* that achieves the same division of benefits as war would have remains valid. War reduces the benefits to be distributed by the very costs that it entails. Yet the problem, as has been made clear, is that there exist values of growth differential and of bargaining rigidities such that even rational and fully informed actors may not be able to locate an agreement that would make them better off in the long-term.

While the immediate reason for this inefficient outcome is that bargaining is costly, the deeper reason is more one of commitment. Thus it can easily be shown that the problem raised by bargaining rigidities is actually an instance of a broader pattern that links failure to reach a settlement to issues of commitment. The fact that war can be rational under conditions of complete information does not imply that it will necessarily take place, even if bargaining rigidities and growth of power differentials are very high. War can still be avoided but bargaining is more difficult because issues of commitment come into play.

The logic of commitment problems, developed in more depth over the past decade (Fearon 1995, Powell 2004), goes as follows: “Bargainers are sometimes unable to commit themselves to following through on an agreement and have incentives to renege on it. These incentives may undermine the efficient outcomes. When they do, complete information bargaining breaks down in the inefficient use of power.” (Powell 2004). The relevance of this literature to the present question is the following: the main contention in the present model was that $D$
had to demand more than its power could “afford” in order to avoid war in the period between two agreements, and $S$ cannot rationally grant $D$ a sufficient amount at time $t_1$ to satisfy $D$ over the whole $[t_1, t_2]$ interval. As a result, $D$ suffers from an opportunity loss and war becomes rational. Yet the fact remains that, while $D$ is suffering from an opportunity loss, $S$ enjoys an extra-amount of benefits exactly equal to $D$’s loss. $S$ cannot, however, offer this “opportunity gain” to $D$ at time $t_1$ because it has not enjoyed it yet.

War could be avoided if $S$ could commit to give its extra-amount of benefits back to $D$ at point $t_2$. The commitment problem comes into play because $S$ needs to be credible in its offer to give its benefits back to $D$. $S$’ credible commitment depends, among other, on whether $S$ and $D$ will have a chance to interact and bargain again in the future (i.e., for example, that $S$ will not have disappeared, that is, that the game can be repeated). Unfortunately, there are reasons to believe that when bargaining is infrequent, issues of commitment might be salient, such that each process reinforces the other. As Schelling has put it (Schelling 1980: 45), “one of the purposes of piecemeal bargaining is to cultivate the necessary mutual expectations”, in other words the “shadow of the future” (Axelrod, 1984). Clearly, if bargains are not repeated frequently, cultivating expectations and mutual trust will not be facilitated and problems of commitment are likely to prevent possible agreements that would overcome the bargaining rigidities problem exposed above. Problems of commitment might thus well make bargaining harder and more rigid, which in turn make issues of commitment more salient. One solution to the commitment problem is to choose a legal form that makes reneging costlier. Thus, Snidal writes that “when a commitment is cast as hard law, the reputational effects of a violation can be generalized to all agreements subject to international law, that is, to most international agreements”. While casting an agreement in hard law decreases the salience of the commitment issues, it has the side effect of increasing the costs of re-negotiations and therefore both increases and decreases the likelihood of war. Which effect dominates remains unclear. Overall, however, I agree with Fearon that “preventive war stems from a commitment problem rather than from differential power growth per se” (Fearon, 1995).
6. Concluding remarks

I have shown that war can be rational, even under conditions of complete information, by adding the assumption that bargaining is costly and that agreements can therefore not be constantly re-evaluated (agreements are “sticky”). The rate of growth differential defines how often agreements should be re-negotiated. But re-negotiation is costly and if an actor’s power rises so fast that the cost of frequent bargaining becomes higher than the cost of war, or if rigidities prevent the actors from negotiating often enough, then war becomes rational. Thus Powell’s (1999) affirmation that “the overall probability that the declining state’s offers are rejected and that the shift ends in war is the same” independently of the speed of the shift needs to be amended to take into account bargaining rigidities and opportunity losses. Moreover, adding bargaining costs to the model gives an empirically more plausible explanation for the fact that fast shift in growth differences might provide incentives to go to war.

In conformity with Powell (1999) however, I do not find any theoretical evidence that would justify the proposition that the most dangerous phase of a shift in power occurs at a power transition where the two states are equally powerful. Similarly to Morrow and Kim (1992), I find that what matters instead is the growth of power differential. The use of power is inefficient but rational.

These results complete Fearon and Powell’s accounts of why states go to war. Just as the market price can fail to reach a Pareto optimal equilibrium when there are negotiations costs (Anderlini and Felli, 2001), bargaining costs such as the audience costs of concessions create rigidities in negotiations and can hence explain why war is rational ex ante. It helps give a more plausible explanation to how changes in relative power can affect the decision to go to war. Bargaining rigidities lead to a situation where commitment problems become decisive. Incomplete information is a sufficient but not necessary condition of the occurrence of war.
Appendix: Proofs

**Proposition 1**: If one of the players does not pay his bargaining fee and hence does not enter the negotiation, war starts.

**Proof**: The game is solved by backward induction. By definition, $D$’s payoff to the status quo is $q$, while it has been shown elsewhere (Powell 1999) that his payoff to fighting is $p_i - c^D_i$, with $p_i - c^D_i > q$. It follows that at any node following $S$’ choice, $\delta^n(p_i - c^D_i) > \delta^s q$. Since the payoff to go to war is always greater than the payoff to the status quo and $p_i - c^D_i > \delta^n(p_i - c^D_i)$, $D$ chooses to attack at his first node, yielding a payoff of $p_i - c^D_i$. Therefore, if any player chooses not to pay his bargaining fee, war ensues.

$Q.E.D.$

**Proposition 2**: There are values of $b$, with $0 < b < 1$, for which neither player pays his fee, and war starts.

**Proof**: The first step is to show that $D$ will not accept any offer $x < (p - c^D_i + b)$. It has been demonstrated elsewhere (Fearon, 1995) that $D$ will not accept any offer that yields a payoff smaller than $(p - c^D_i)$. In the presence of bargaining costs $b$, $D$ will, therefore, only rationally accept offers that satisfy

$$(A1) \quad (x - b) \geq (p - c^D_i) \quad \Rightarrow \quad x \geq (p - c^D_i + b), \text{ with } 0 \leq x \leq 1$$

The second step is to show that there are values of $b$ such that $S$ cannot credibly commit to offer $x \geq (p - c^D_i + b)$ to $D$ at any stage following the payment of the bargaining fee. It has been demonstrated elsewhere (Fearon, 1995) that $S$ cannot rationally make any offer $x \geq p + c^S_i$. In the presence of rigidities, $S$ can, therefore, only rationally offer up to

$$(A2) \quad (x + b) \leq (p + c^S_i) \quad \Rightarrow \quad x \leq (p + c^S_i - b), \text{ with } 0 \leq x \leq 1$$
The third step is to show that there exists a \( b \) such that both (A1) and (A2) cannot be satisfied. In other words, we need to show that there is a \( b \) such that

\[
(A3) \quad (p + c_i^s - b) - (p - c_i^p + b) < 0
\]

\( b > \frac{c_i^p + c_i^s}{2} \) satisfies this condition. It follows that there exist values of \( b \) for which \( S \) cannot credibly commit to offer \( x \geq (p - c_i^p + b) \) and therefore that neither player will pay its fee.

Finally, we know from Proposition 1 that if at least one of the players does not pay his fee, the only rational choice is for the actors to attack.

\[ Q.E.D. \]

**Proposition 3:** Given a time interval \([t_1, t_2]\) and a level of costs \( b \), there exists a maximum number of re-negotiations and hence a maximum frequency of re-negotiation if war is to be avoided.

**Proof:** consider the case where \( D \) and \( S \) re-negotiate their agreement at every time \( t \), over an interval \([t_1, t_2]\). Consider a function \( D(t) \) representing \( D \)'s demands at any time \( t \in [t_1, t_2] \), with \( D(t) = p_i - c_i^p + tb \), and a function \( S(t) \) representing \( S \)'s maximum offer at any point in time, with \( S(t) = p_i + c_i^s + tb \). Negotiations take place as long as \( S \) can offer at least as much as \( D \) demands, that is for any \( t \) such that

\[
S(t) \geq D(t) \quad \Rightarrow \quad p_i + c_i^s + tb \geq p_i - c_i^p + tb
\]

\[
\Rightarrow t \leq \frac{c_i^s + c_i^p}{2b}
\]

Hence for all \( t > \frac{c_i^s + c_i^p}{2b} \), \( S \) cannot rationally satisfy \( D \)'s demands, and war ensues as a result. Hence, no matter how much is offered in each negotiation, there is a maximum number of re-negotiations before war is rational, and therefore a maximum frequency of re-negotiation.

\[ Q.E.D. \]
Proposition 4: Given a rate of growth differential $\beta > 0$ and a cost of war $c^i_t > 0$, there exist time intervals $[t_1, t_i]$ such that actors must renegotiate their agreement at a minimum frequency if war is to be avoided.

Proof: In order to be satisfied at a time $i \in [t_1, t_2]$, $D$'s payoff to not fighting must equal at least $\int_{t_1}^{t_i} \left( p_t - c^D_t + b \right) dt$ at any point in time. We know from Powell (1999) that the maximum offer $x$ that $S$ can rationally make at any point $i$ in time is $p_t + c^S_t - b$. Consider a single offer $x^S_t = p_t + c^S_t - b$, yielding for $D$ a total payoff of $\int_{t_1}^{t_i} \left( p_t + c^S_t - b \right) dt$. Given that $p(\cdot)$ is a monotonously increasing function over $[t_1, t_i]$, and that $\int_{t_1}^{t_i} \left( p_t + c^S_t - b \right) dt$ is by definition non-increasing, there necessarily exists a value of $i$ such that $\int_{t_1}^{t_i} \left( p_t + c^S_t - b \right) dt < \int_{t_1}^{t_i} \left( p_t - c^D_t + b \right) dt$, and there is, therefore, no offer $x^S_t$ that would avoid renegotiations on any interval $[t_1, t_i]$.

Q.E.D.

Theorem: Consider the game with bargaining fees described above. The game has a subgame perfect equilibrium (SPE) in which agreement is reached in finite time if and only if $\Delta^i_n$ for $i \in \{D, S\}$ and for any time $n \in [t_1, t_2]$ satisfies:

\[ 1 + \frac{c^D_t + c^S_t + b}{p_t} + \Delta^S_i \geq \Delta^D_i \quad \text{and} \quad 1 + \frac{c^D_t + c^S_t + b}{p_t} + \Delta^S_i \geq \Delta^D_i \]

Proof: The first step is to show that for any $\Delta^i_n$ that does not satisfy (A4), the game cannot have a peaceful SPE.
Consider any $\Delta^i_{t_n}$ that does not satisfy (A4), in other words $\beta > 1 + \frac{c^D_{t_n} + c^S_{t_n} - 2b}{p_{t_n}}$. From the text, we know that $D$ will not accept any offer $x^S_{t_n} < \int_{t_i}^{t_f} \left( p_{t_n} - c^D_{t_n} + b \right) dt$. Given that the growth differential is assumed to be constant, this simplifies to:

$$
\begin{align*}
x^S_{t_n} &\leq \int_{t_i}^{t_f} \left( \beta p_{t_n} - c^D_{t_n} + b \right) dt \\
\Rightarrow x^S_{t_n} &\leq \int_{t_i}^{t_f} \left[ \left( 1 + \frac{c^D_{t_n} + c^S_{t_n} - 2b}{p_{t_n}} \right) p_{t_n} - c^D_{t_n} + b \right] dt \\
x^S_{t_n} &\leq \int_{t_i}^{t_f} \left( p_{t_n} + c^S_{t_n} - b \right) dt
\end{align*}
$$

Consider now the maximum offer $S$ can make at time $t_1$, $x^S_{t_1} = \int_{t_i}^{t_f} \left( p_{t_1} + c^S_{t_1} - b \right) dt$

Since there are no offer that $S$ can rationally make that satisfy $D$, $D$ will choose not to pay its negotiation cost and therefore there will be no agreement.

The second step is to show that for any $\Delta^i_{t_n}$ that satisfies (A4), there exists a peaceful SPE.

The proof is the exact reverse from above. Given $\beta \leq 1 + \frac{c^D_{t_n} + c^S_{t_n} - 2b}{p_{t_n}}$, there must be an offer that $S$ can credibly make and that $D$ can rationally accept.

Q.E.D.
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