Political Conflicts in the Shadow of Violence:

A Preface to a Theory on the Nature and the Timing of Political Stabilizations in Crisis Situations

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Abstract

I develop a theory of political conflict in the shadow of violence in light of various domestic and international crisis situations ranging from transitional, revolutionary, and anti-colonial conflicts to internationalization of such conflicts. This theory is particularly concerned with the nature and the timing of political stabilization processes with respect to the military capabilities, instantaneous economic opportunity costs, costs associated with the use of violence, and values of winning of any two groups in conflict. In solving for a Perfect Bayesian Nash equilibrium of an incomplete information, continuous type and time, and infinite horizon war-of-attrition model with two outside options of attack and concession, I derive the following preliminary results of interest. First, my conjecture that the timing of concession and the concomitant time of stabilization increase with respect to the military capability of a group while the timing of attack and the concomitant time of stabilization decrease with respect to the military capability is a Perfect Bayesian equilibrium. Second, the decision concerning the use or the non-use of violence depends on the relative cost of using violence. The relative cost of violence is the ratio of the cost associated with the use of violence to the value of obtaining the outcome of interest. For example, the higher the benefit of obtaining political power, the more likely the use of violence, and the higher the cost associated with the use of violence, the less likely the use. This, in turn, suggests that the use of violence should be more common in non-democracies than in democracies because the benefit associated with political power is generally high in non-democracies. In addition, the second result also suggests that the use of violence should be less common in countries with high levels of economic development since there are more assets and infrastructures that can be destroyed by the use of violence. Third, the higher the level of economic development, which, in turn, means higher instantaneous economic opportunity costs for all, the sooner the political stabilization by either violent or non-violent means. More importantly, the decision concerning the timing of a group’s action depends on the instantaneous economic opportunity cost of its opponent but not on its own cost. Fourth, the higher the value of obtaining the outcome of interest, the later the political stabilization by non-violent means and the sooner the termination of the crisis situation by violent means will take place. Fifth, the costlier the use of violence, the later the political stabilization by non-violent means for groups with low military capability, and the sooner the stabilization by non-violent means for groups with mid-level military capability.

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I. Conflicts of Interest and the Question of Violence

The presence of the option of using violence to end conflict situations makes various significant domestic and international political crisis situations different from other non-political crisis situations in general and from economic conflict situations in particular. Transitional, revolutionary, anti-colonial, ethnic and territorial conflicts unfortunately often occur in the grim shadow of violence in contrast to other economic or social conflict situations where the use of violence is often too costly or is simply prohibited by various enforcement mechanisms against its use.¹ In conflict situations where the use of violence is an option, why and when do parties to the conflict use violent means instead of non-violent ones to end the crises? In this paper, I develop a theory of political stabilizations in the shadow of violence to answer the questions concerning the timing and the nature of political stabilization processes inspired by various transitional, revolutionary, anti-colonial and other domestic and international conflict situations in general.

¹ It is certainly true that we also see the use of violence in social conflict situations such as marital crises and in economic conflict situations such as strikes and wage bargaining. However, in a “civilized” society where the use of violence in the aforementioned crisis situations is strictly prohibited by law that is enforced by the state, the use of violence becomes virtually a non-option. It seems that this is one of many possible reasons why economists who study strike and wage bargaining situations do not consider the option of using violence to end the labor-management conflicts.
The literature on political transitions, in particular on transitions to democracy, has produced important empirical generalizations and theoretical conjectures concerning the relationship between economic development and political transitions. One of the less disputed stylized facts is that democracy is more likely in economically developed countries. Another little disputed stylized fact is that political transitions of all sorts are more likely in economic crisis situations. However, little consensus exists on the precise reason why we observe such empirical regularities.

There are four prominent theoretical conjectures concerning the aforementioned empirical regularities. The first conjecture is a sociological one in the sense that it emphasizes the important role played by the emergence of a middle class during economic development and how this emergence of a new class, in turn, generates the demand for democracy. The second conjecture is a psychological one in the sense that it emphasizes the importance of the newly generated overarching value consensus in demand of democracy during the time of economic development while this value consensus, in turn, is subjected to increasing specialization and differentiation of social

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2 There is continuing debate on the empirical evidence concerning the relationship between economic development and the likelihood of democracy. See Jackman (1973), Burkhart and Lewis-Beck (1994), Londregan & Poole (1990, 1996), and Epstein, Bates, Goldstone, Kristensen and O’Halloran (2003) for the supporting evidence of the positive relationship between economic development and the likelihood of democracy. See Arat (1988), Gonick and Rosh (1988), Przeworski and Lemongi (1997), and Przeworski, Alvarez, Cheibub and Lemongi (2000) for the contrary evidence. More specifically, Przeworski and Lemongi point out that economic development is only a necessary but not a sufficient condition for maintaining and consolidating an existing democratic system. Barbara Geddes (1999) interprets the findings by Przeworski et al. as “a revisionist confirmation” of the positive relationship in the sense that a democratic system is less prone to a breakdown beyond a certain level of economic development. I would argue that there is enough evidence concerning the positive relationship although no one has a completely satisfactory explanation for why this is so.


structures. The third conjecture is a political one in which the emergence of factions within the old regime and the pacts among transitional elites are the primary causes of the transition to democracy. Also, another important variation of this political conjecture emphasizes the importance of pre-transitional regime types in determining the possibility of transitions and the likelihood of consolidation. It is important to note that, in all three previous conjectures, economic change is either explicitly or implicitly the primary mover of other relevant factors that bring about political transitions. Finally, the fourth conjecture emphasizes the role of civil society in bringing about political transitions since civil society “generates the norms by which its members play the political game and holds governments accountable.” This conjecture, centered on state-society relations, however, remains unsettled because little agreement exists as to the definition and the measurement of its associational life.

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5 See Almond and Verba (1963) and Inglehart (1997).
7 See Geddes (1999) and van de Walle (2001).
8 Gibson, 201. See Hyden (1980), Zartman (1995) and Diamond (1999) for more on civil societies and the call for “taking the state out” by some prominent Africanists.
9 Huntington (1991) provides a good synthesized account of various theoretical conjectures and empirical regularities in addition to highlighting the importance of other historically contingent factors. More specifically, he questions the existence of universal socioeconomic, cultural, and political factors in explaining the recent global wave of political transitions. Although he does not dismiss the importance of certain socio-economic conditions or the importance of “political crafting,” (Di Palma’s term in Di Palma, 1990) Huntington tends to credit the relevance of such factors only under specific historical circumstances. Accordingly, this view gives a fair amount of consideration to the importance of factors that are external to a regime’s internal level of development and its transitional politics. In the case of the third wave, some of those historically contingent factors are the collapse of the USSR, the spread of Christianity, the expansion of the EU, and the global promotion of democracy by the US government. In addition, this perspective spearheaded by Huntington also gives special emphasis to the importance of the contagion effect in explaining the global wave-like occurrence of democratic transitions (Huntington, 1991; Beissinger 2005). Hence, according to this view, one can only explain why democratization occurs in specific historical circumstances given certain favorable structural conditions and transitional politics among political agents.
Probably because of the relatively peaceful nature of recent “third wave” transitions to democracy, none of the aforementioned empirical and theoretical works consider the problem of violence and its impact on the transitional dynamics. In my model, I explicitly consider the option of using violence in transitional conflicts and derive the conditions that are conducive to violent or negotiated transitions. As will be seen, the high level of economic development is linked to non-violent political transitions because the cost associated with use of violence is high in a developed economy. The cost associated with the use of violence is high simply because there are more assets and infrastructures that can be destroyed during civil war in a more economically developed country than in a less developed one.

In addition to the rich empirical and non-formal works on political transitions, there recently has been a plethora of theoretical conjectures derived from elegant formal models. One of the most widely discussed works is Acemoglu and Robinson’s theory of political transitions (Acemoglu and Robinson, 2000 and 2001). They essentially view transition to democracy as a commitment device by which the rich ruling elites pacify the disenfranchised poor who challenge the elites with the threat of revolution. Another work of importance is by Carles Boix (2003) who develops a model of political transitions with the particular emphasis on the impact of economic inequality and capital mobility in determining the possibility of transitions and consolidations. The theories by Acemoglu and Robinson and by Boix all converge on the view that democracy is more likely and stable in equal societies and less likely and unstable in highly unequal societies since they all reason that inequality breeds the demand for redistribution by the poor. Yet both
models fail to consider the key question concerning the timing of transitions and, more generally, the timing of political stabilizations during transitional crises.

As will be seen, I derive the timing of transitions and, more generally, the timing of political stabilizations during transitional crises from my model. The key factor that determines the timing of political transitions in my model is the instantaneous economic opportunity cost of prolonging a crisis situation. It turns out that the higher the level of economic development, which, in turn, means higher instantaneous economic costs for all groups in conflict, the sooner the political transitions or non-transitions by either violent or non-violent means. The instantaneous economic opportunity cost of prolonging is high in a more developed country than in a less developed one because the degree to which a general disorder of a country damages its economy is higher in the former than the latter. More importantly, the decision concerning the timing of a group’s use of violence or a group’s unilateral concession turns out to depend on the instantaneous economic opportunity cost of the other group but not on its own. In addition, I derive the relationship between the timing of political transitions with respect to the value of prevailing and obtaining political power in transitional conflicts to the retribution cost associated with losing and to the cost associated with the use of violence.

Many theorists of conflict resolution have also developed various formal theoretical conjectures concerning the democratic transition process. In particular, there are interesting and insightful models of transitional politics that deal with the problem of violence. As suggested before, although most “third-wave” transitions to democracy were less violent, or perhaps non-violent, and were negotiated outcomes, there is a

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resurgent interest among not only the theorists of conflict but also among the scholars in comparative politics in the role of violence and of sustained mobilization by the opposition in bringing about political transitions.\textsuperscript{11} This clearly is the reasonable next step toward generalizing the lessons from the third wave transitions and moving toward a general theory of political transitions. In addition, there are models that claim to deal with the question concerning the timing of transition.\textsuperscript{12} However, for some reason, very few of these models explicitly explain the overall transitional dynamics in a satisfactory manner in a concomitantly evolving politico-economic structure with the additional option of using violence; and very few explain completely the timing of transitions with respect to key parameters of interest to social scientists such as the economic cost, the cost of using violence and the military capability. My model and its results are intended to fill such theoretical gaps in the literature and make improvements by complementing various existing formal and non-formal theories of political transitions.\textsuperscript{13}

The literature on revolutions is equally as rich and diverse in terms of its research agenda and methodology as the literature on political transitions. To a student of political conflict, it is puzzling to see why these two literatures do not synergize the lessons from their respective findings to build a general theory of political change. It seems that, to an extent, the study of revolutions can be seen as a subset of the general literature on


\textsuperscript{12} For example, see Swaminathan (1999).

\textsuperscript{13} In addition to the aforementioned works of importance, Bueno De Mesquita, Morrow, Siverson and Smith (2002, 2003) develop a general theory of political survival of leaders. Their model of political coalitions (i.e. "selectorate model") provides a unique and important insight into the dynamics of political survival with respect to the shifting size of the winning coalition and that of the selectorate. While intended for regime types of all kinds, their model seems to be particularly useful in understanding the transitional dynamics of autocratic or personalist regimes such as Iraq during Saddam Hussein’s rule.
political transitions despite the fact that transitions literature heavily focused on negotiated and less violent transitions, especially during the late 1980s and early 1990s.

Keeping in mind the possible conceptual overlaps between transitions and revolutions literature, I conduct a cursory overview of the literature on revolutions. Based on the historical narratives of classical scholars of “great revolutions” and accumulated empirical findings on revolutions in general, social scientists have developed three prominent theoretical conjectures concerning political violence and revolutions over time. The first conjecture closely mirrors the combined first and third theoretical conjectures of transitions literature and focuses on the conflicts among various social groups or key political agents who compete over conflicting objectives. Many aforementioned recent formal theoretical conjectures on political transitions are essentially analyzing the conflict of interest among key social groups. The second conjecture closely mirrors the second theoretical conjecture of transitions literature and emphasizes the importance of psychological conditions such as “relative deprivation” (Gurr, 1970) and the synchronization of existing values of a given society with new and challenging, or perhaps deviant, values (Johnson, 1966). The third conjecture closely mirrors the variation of the third conjecture of transitions literature that emphasizes the pre-transitional regime types. This conjecture is mostly ascribed to Skocpol (1979) and focuses on the structural variations of states to explain the origins of revolutions while

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14 Here, I am referring to great thinkers of the past such as Tocqueville, Weber, Burke, Marx and Engels, etc.

15 Here, I closely follow the literature review by Skocpol (1979) and by Goldstone (1994, 2001).

16 The proponents of this conjecture are usually known as political conflict theorists, and the works of Marx, Lenin, Mao, Lukacs, Gramsci, Althusser and Charles Tilly (1973, 1974) fall within this camp.
taking into account conflicts among various social groups and the impact of mass mobilization.

All of these three conjectures are primarily concerned with the origins of crisis situations that bring about revolutions. More specifically, these three conjectures and many other recent works on revolutions try to answer why and how revolutions occur. More recently, as a critique of the third theoretical conjecture on revolutions concerning its “simple state- and class-based conception of revolutions,” a new conjecture that emphasizes the process underlying revolutions has been developed by scholars of “contentious politics.” However, very few of these works are explicitly focused on the questions concerning the timing of the use of violence and the timing of political stabilization in revolutionary crisis situations. Moreover, very few, if none, focus on the question of timing of revolutions in a concomitantly and continuously evolving politico-economic structure. I believe my model of political stabilizations with some additional future work can reasonably forecast the timing and the nature of political stabilization once a crisis situation occurs despite a general skepticism by the scholars about developing a model that can predict future revolutions.

Both the literatures on political transitions and revolutions tend to only focus on successful cases of transitions and revolutions. This not only causes the selection bias problem but also causes a general myopia in theory development. I view the problem of transitions as a broader problem of political stabilization where both transitions and non-

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18 See McAdam, Tarrow, and Tilly (1997) and Tarrow (1998).
19 In particular, see Kuran (1991, 1995).
transitions are possible outcomes. From this encompassing view, the high level of economic development not only can bring about a quick and non-violent political stabilization but also can bring about an immediate and violent termination of a crisis situation. More specifically, if the ruling elites are united and have strong military capability, then the high level of economic development brings about immediate and successful crackdown of the opposition\textsuperscript{21} in a crisis situation while, if the ruling elites are divided and have weak military capability, the high level of economic development brings about the concession by the ruling elite in favor of the transition.\textsuperscript{22} In addition, from this general perspective, as will be seen, political stabilizations are less likely to be violent in a democracy and are more likely to be violent in a non-democracy since the benefit associated with the control of government tends to be higher in a non-democracy than in a democracy.

As will be seen in the later parts of this paper, especially where I discuss the model in depth, my model of political stabilizations aims to be applied beyond the specific contexts of transitional and revolutionary conflicts. It is a general model of political stabilizations in the shadow of violence and its aim is to improve our understanding of the questions concerning the timing and the nature of conflict processes in wide varieties of conflicts involving deadlocks including both international and domestic armed conflicts (e.g. both conventional and civil wars and the Cold War between the US and the USSR\textsuperscript{23}), anti-colonial conflicts and crisis bargaining\textsuperscript{24} in general.

\textsuperscript{21} A good recent example for this case is the quick and decisive action by the Chinese Communist Party during Tiananman crisis in 1989. Another good example is the Kwangju Massacre of 1980 by Chun Do Hwan and his junta in South Korea.

\textsuperscript{22} Most “negotiated third wave” transitions to democracy are good recent examples.
In addition, this model can be of use in determining the impact of third party intervention in favor of one of the two groups in conflict. As I will demonstrate in the later part of this paper, the model can provide important insights to how the intervention can or cannot change the timing and the nature of political stabilizations.

II. Political Conflicts from a War-of-Attrition Perspective

Panmunjom, Korea—(UPI)—The American general and the North Korean general glared at each other across the table and the only sound was the wind howling across the barren hills outside their hut. Major General James B. Knapp, negotiator for the United Nations Command (UNC), was waiting for Major General Ri Choonsun of the Democratic People’s Republic of North Korea to propose a recess. They sat there, arms folded, for 4 ½ hours. Not a word. Finally, General Ri got up, walked out and drove away.


In this paper, I extend the standard of war of attrition game to improve our theoretical understanding of political conflicts in the shadow of violence in the substantive areas of interest such as political transitions, revolutions, anti-colonial conflicts and crisis bargaining. In addition, this extension makes technical contributions to the study of timing games in game theory by solving an incomplete information, continuous time and type, and infinite horizon war of attrition with two outside options of concession and attack. This interesting extension of the standard attrition game has not

23 This model with a minor modification can be useful in understanding possible reasons why the US chose not to confront the USSR immediately after the Second World War when the US had a nuclear monopoly. Some leaders of the free world, Winston Churchill in particular, insisted on the immediate military showdown against the USSR. Some, especially George Kennan, insisted on engaging in a non-violent economic war of attrition until the USSR economic or other forms of internal collapse. Kissinger’s account of the history of Cold War provides a convincing case for the application and the use of my model. See Kissinger (1994) for details.

24 In particular, this model can be useful in understanding the nuclear crisis bargaining between the US and North Korea. As will be seen in the later parts of this paper, this particular crisis can be analyzed as the special case of the model where the economic costs for the two states are common knowledge and where the military capability of North Korea is unclear given the fact that we do not know for sure whether North Korea have nuclear capability as it claims. Despite being a model that does not take into account all relevant factors of consideration, the model, I confidently believe, has some interesting policy implications for the US in dealing with this particular rogue regime.
been solved until now probably because of various technical difficulties in dealing with an additional outside option in difficult-to-solve supergames such as the war of attrition. More importantly, the fact that my project was has been driven by the problem of violence in crisis situations from the start allowed me to develop this extension with the additional outside option.

What is the standard war of attrition? It is also commonly known as the game of chicken. It is a two-player timing game where players can either prolong the conflict in a declining payoff situation for both or concede. The player who outlasts the other is the winner. In general, consideration of asymmetric payoff coordination games or public goods games in dynamic settings yields virtually identical wars of attrition. Most, if not all, wars of attrition games have one outside option of making a concession and have been mostly developed in theoretical biology and in economics and has been widely applied to study various Hobbesian “shrinking pie” conflict situations such as reproductive competitions in the animal kingdom (Bishop, Cannings and Maynard Smith, 1978; Riley, 1981) strikes and wage bargaining (Kennan and Wilson, 1990), fiscal and monetary stabilizations (Alesina and Drazen, 1991), the provision of public goods (Bliss and Nalebuff, 1983), firm exit decisions in declining market share (Fudenberg and Tirole, 1987), and the role of audience cost in the escalation of international conflicts (Fearon, 1994). For the wars of attrition with one outside option, the standard result is that the

25 More recently, this framework is being used to study the emergence of property rights in primitive human society (Hafer, forthcoming) and wars in general (Morrow, unpublished). Some may point out that Fearon’s model is a war-of-attrition with two outside options (Fearon, 1994). However, Fearon’s model is not a case with two outside options because, until a player reaches the horizon point in which war starts, his game is a standard war of attrition with one outside option. Explicitly adding an additional outside option at all times of the game painfully increases the mathematical complexity. More importantly, doing so significantly changes the dynamics of the standard war of attrition as will be seen in this paper.
weaker player (i.e. the player who has higher instantaneous cost of prolonging or who places a lower value on the prize) gives in before the stronger competitor.

Then, what is the value of explaining political conflict processes in the shadow of violence using the war of attrition with two outside options? To answer this question, I point out the following four common denominators of transitional, revolutionary, anti-colonial and crisis bargaining situations, all of which suggest the suitability of using the two outside options framework. Although some biologists and economists may disagree, as an amateur observer of animal and firm competition, I am curious to know why they do not also consider the option of fighting between animals or firms in competition.26

First, in most crisis situations, there is agreement on the need for political stabilizations. This is simply because the continuation of crisis situations is costly for all. However, a deadlock over which group shall be responsible for the burden of stabilization or over how the burden of stabilization should be shared prevents a timely resolution of the conflict.27 In the baseline model that I analyze in this paper, the loser of the game is responsible for the entire burden of stabilization while all players accumulate their respective, privately known, instantaneous costs of prolonging the crisis situation. I focus on the analysis of deadlock between two groups primarily because of mathematical convenience.28 In sum, the presence of the deadlock that involves seemingly costly delay

26 I am quite certain that animals actually do fight over a mate and firms do fight corporate wars in addition to engaging in wars of nerves.

27 It is also possible to think in terms of the benefit of stabilization instead of the burden. If so, the key issue is how to distribute this benefit among the parties in conflict.

28 This restriction concerning the number of players can be of concern to some readers. It is certainly true that, in some cases of transitional, revolutionary, anti-colonial and crisis bargaining situations involving deadlocks, there are more than two groups in conflict. I agree that the consideration of multiple groups makes the model more “realistic.” Yet, this also increases the mathematical complexity without a clear promise of generating new theoretical insights. In their analysis of n-player war-of-attrition games with one
strongly suggests the appropriateness of using the war-of-attrition framework, which, in turn, has been widely used to study various “shrinking pie” situations.

Second and most important in terms of using the war-of-attrition framework, there is little or no explicit and meaningful bargaining in many crisis situations until one side makes a considerable concession. Only after this major concession that brings about the turning point toward the resolution of conflict, specific negotiations over the details of settlement may occur. For example, in the case of South African transition to democracy, F. W. de Klerk’s unilateral declaration lifted the bans on the major anti-apartheid organizations and pledged the release of Nelson Mandela and other political prisoners in February 2, 1990. Similarly, in the case of South Korean transition to democracy, Roh Tae Woo’s unilateral declaration promised a direct presidential election and that pledged the release of opposition leaders, Kim Dae Jung in particular, in June 29, 1987. These are some notable examples of the aforementioned grand concessions in transitional crises.²⁹ This feature concerning the major concession corresponds to the characterization of wars of attrition in which the conflict terminates when one side unilaterally makes a concession. It is certainly true that conflicts do not simply end after the concession. Yet, it seems reasonable to consider the point of major concession as the turning point toward ending the deadlock. Wars of attrition may be less pertinent in modeling explicit negotiations over the details of stabilization. However, it is possible to imbed explicit bargaining elements to the attrition framework.

²⁹ See an earlier version of this paper for a detailed examination of the South African and the South Korean transitions to democracy from a comparative perspective (Kim, 2001; Kim, 2004).
Third, a few failed attempts at conflict resolutions usually precede a successful one. More importantly, the longer the conflict process is prolonged, the terms of conflict resolution that were unacceptable previously become acceptable. This is not a surprising common feature in various crisis situations of interest given the fact crisis situations are costly for all and the respective costs accumulate over time. As will be seen in my model, the instances where neither group makes a concession nor attacks can be construed as failed attempts at political stabilizations.

Fourth and finally, as emphasized throughout this paper, most political crisis situations occur in the shadow of violence. In transitional or in revolutionary crises, the deadlock between the ruling group and its opponent with respect to the control of government almost always occur with the possibility of military crackdown from the top or with that of violent uprising from the bottom. When either side escalates with the use of violence, the ensuing civil war or violent conflict of lesser scale can bring about an end to the deadlock. When the opposition prevails, this, in turn, results in a revolutionary outcome and the concomitant stabilization occurs in favor of the opposition. When the ruling elites prevail, this, in turn, results in return to “normalcy” by force. A similar simplification applies to anti-colonial conflicts or, more generally, international crisis situations involving deadlocks with the potential use of force. The presence of the option to use violence strongly suggests that we should consider this additional option in using the standard war-of-attrition framework.

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30 Following Fearon and Laitin’s definition (2003), I also define civil war as a violent conflict between “agents of a state and organized, nonstate groups who sought either to take control of government or to use violence to change government policies.” Fearon and Laitin also put restrictions on the minimum number of casualties on each side to distinguish between civil wars and other violent conflicts of smaller scale.
In the following section, I show how political conflicts in the shadow of violence can be modeled while taking these four common features into account and analyze the baseline model with two outside options to derive theoretical propositions.

III. A Model of Political Conflict in the Shadow of Violence

This is an incomplete information, continuous type and time, and infinite horizon war of attrition with two outside options of attack and concession. For a good discussion of other variations of wars of attrition with one outside option and timing games in general, see Fudenberg and Tirole’s *Game Theory* (Fudenberg and Tirole, 1996: 117-126 and 216-219). For another good introduction to this game in an applied context in fiscal and monetary stabilization, see Drazen’s *Political Economy in Macroeconomics* (2000).

It is important to note that, given the complexity of my model with the additional outside option of attack, the standard technique in solving wars of attrition in the aforementioned books cannot be used. As will be seen, a new technique has been used to solve this game and this technique markedly differs from the existing ones.

*Basic Setup*\(^{31}\)

There are two groups, \(i\) and \(j\), and each group’s type, \(\theta\), is distributed uniformly on \([0,1]\) and is independent of the other. Each group knows its type but only knows about the distribution of the type of the other group. In substantive terms, \(\theta\) can be construed as the

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\(^{31}\) In this model, I am not concerned about the origins of various crisis situations. Hence, I simply assume that two groups are in a crisis situation from time zero. For example, in domestic crisis situations, common causes of the crises are poor economy, foreign policy failure, the demise of external patron state and domestic scandal, all of which hurt the legitimacy of the ruling group. Here, what is meant by “legitimacy” has been widely debated. See Linz and Stepan (1996) or Huntington (1991) for a brief summary of this debate.
The higher the value of $\theta$ for a group, the stronger the group. The assumption concerning the uniform distribution of type is purely for mathematical convenience and is not required for obtaining the results of interest.

A group’s decision consists of the type and the time of action. Denote the decision by $(d, s)$ where the type of action is $d$ and the time of action is $s$. Here, $d \in \{a, c, \Delta\}$ and $s \in [0, \infty]$. When $d = a$, the type of action is attack. When $d = c$, the type of action is concession. When $s = \infty$, groups don’t need to choose any specific action. In this case, we set $d = \Delta$ for convenience. The collection, $D$, of all possible decisions is called the decision space. Notice that $D = \{(d, s) : d \in \{a, c\} \land 0 \leq s < \infty \lor (d, s) = (\Delta, \infty)\}$. A group’s strategy, $(\pi, \sigma)[0,1] \to D$, is a mapping from the set $[0,1]$ of all types of the group to the decision space, $D$, of the group.

The payoffs are the following. When either group attacks, the group that has the higher $\theta$ wins and the winner’s payoff is $W - \psi - \kappa(i \land j)$. Here, $\kappa$ is the instantaneous cost of prolonging and $\kappa_i \neq \kappa_j$. In substantive terms, $\kappa$ can be construed as the instantaneous economic opportunity cost from the continuation of a crisis situation. $\psi$ is the cost associated with the use of violence, and this cost is imposed on both groups if either chooses to attack. This cost represents the destruction of assets and infrastructures as the result of a militarized conflict. The winner’s prize is $W$. $s_i$ and $s_j$ are time of action by groups $i$ and $j$. When a group

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32 More generally, $\theta$ is a group’s power. In general, a group’s power includes its military capability and other tangible and intangible factors that allow the group to prevail in a militarized conflict. For simplicity and conceptual convenience, I assume $\theta$ to be a group’s military capability.

33 Almost all crisis situations are economically costly to all groups in conflict. General disorder and mayhem that result from both domestic and international crises bring about foregone opportunities to engage in economically gainful activities among the groups in conflict. The extent to which such situations hurt a particular group varies. Accordingly, I assume that $\kappa_i \neq \kappa_j$. 
outlasts the other without war, this group wins and the winner’s payoff without war is
\[ W - \kappa(s_i \land s_j) \]. Whichever group concedes first or whichever group is the lower type when
fighting loses and the loser’s payoff is \[ L - \kappa(s_i \land s_j) \] when it concedes and \[ L - \psi - \kappa(s_i \land s_j) \]
when it fights and loses. The loser’s payoff without the war cost and without the accumulated
instantaneous cost is \[ L \]. Here, \( W - L > \psi \).

A pair of strategies, \( (\pi_i, \sigma_i), (\pi_j, \sigma_j) \), constitutes a Bayesian Nash equilibrium if
\[
E[ U_{(\pi_i, \sigma_i)}(\theta, \pi_i(\theta), \sigma_i(\theta))] = \max_{d, s \in \mathcal{D}} E[ U_{(\pi_i, \sigma_i)}(\theta, d, s)] \text{ for all } \theta \in [0,1] \quad (*)
\]
and
\[
E[ U'_{(\pi_j, \sigma_j)}(\theta, \pi_j(\theta), \sigma_j(\theta))] = \max_{d, s \in \mathcal{D}} E[ U'_{(\pi_j, \sigma_j)}(\theta, d, s)] \text{ for all } \theta \in [0,1]. \quad (**) \]
Here, \( U_{(\pi_i, \sigma_i)}(\theta, d, s) \) is the payoff for group \( i \) when \( i \) is of type \( \theta \) and chooses the decision
\( (d, s) \) and when \( j \) plays the strategy, \( (\pi_j, \sigma_j) \). \( U'_{(\pi_i, \sigma_i)}(\theta, d, s) \) is similarly defined by changing
the roles of \( i \) and \( j \).

**Equilibrium Conjecture**

Suppose that \( (\pi_i, \sigma_i), (\pi_j, \sigma_j) \) is an equilibrium that satisfies the following three
conditions:

1. There exists \( \theta^* \in [0,1] \) such that \( \pi_i(\theta) = \begin{cases} c & \text{if } \theta < \theta^* \\ a & \text{if } \theta > \theta^* \end{cases} \).
2. \( \sigma_i \) is strictly increasing on \([0, \theta^*] \) and is strictly decreasing on \((\theta^*, 1]\).
3. \( \sigma_i \) is continuous on \([0,1] - \{\theta^*\} = [0, \theta^*] \cup (\theta^*, 1]\).

The following is a schematic representation of the conjectured equilibrium.
Here, $\Phi^c(s)$ is the type that concedes at $s$ and $\Phi^a(s)$ is the type that attacks at $s$. Also, note that $\lim_{\theta \to \theta^*} \sigma(\theta) = \infty$. The next step in solving this game is to derive a set of differential equations that satisfy this equilibrium conjecture.

**Derivations**

$U_{(i,\sigma)}(i, a, x)$ is the payoff for $i$ when $i$ is of type $\theta_i$ and $i$ chooses $(a, x)$ where $x$ is $s$ in free variable form. Similarly, $U_{(i,\sigma)}(i, c, x)$ is the payoff for $i$ when $i$ is of type $\theta_i$ and $i$ chooses $(c, x)$ and $U_{(i,\sigma)}(i, \Delta, \infty)$ is the payoff for $i$ when $i$ is of type $\theta_i$ and $i$ chooses $(\Delta, \infty)$.

For $\theta_i < \theta^*$,

$$E[U_{(i,\sigma)}(i, c, x)] = L + (W - L)F_j(\Phi^c(x)) - \psi(1 - F_j(\Phi^c(x))) - \kappa_j \int_0^x (F_j(\Phi^c(y)) - F_j(\Phi^c(y))) \ dy.$$
For notational convenience, $g_j^*(s) \equiv F_j^*(\Phi_j^*(s)) \equiv \Phi_j^*(s)$ and $g_j^*(s) \equiv F_j^*(\Phi_j^*(s)) \equiv \Phi_j^*(s)$.

\[
\frac{\partial}{\partial c} E[U_{(x,\sigma)}(\theta, c, x)]_{x=\sigma(\theta)} = (W - L)g_j^{\ast}(\sigma_j(\theta_j)) + \psi g_j^{\ast}(\sigma_j(\theta_j)) - \kappa_i(g_j^*(\sigma_j(\theta_j)) - g_j^*(\sigma_j(\theta_j))) = 0.
\]

\[
(W - L)g_j^{\ast}(s_j) + \psi g_j^{\ast}(s_j) - \kappa_i(g_j^*(s_j) - g_j^*(s_j)) = 0. \tag{1}
\]

For $\theta_i > \theta^*$,

\[
E[U_{(x,\sigma)}(\theta, a, x)] = L + (W - L)F_j(\theta_j) - \psi(1 - F_j(\Phi_j^*(x))) - \kappa_i \int_{0}^{\sigma_j^*} (F_j(\Phi_j^*(y)) - F_j(\Phi_j^*(y))) dy.
\]

\[
\frac{\partial}{\partial x} E[U_{(x,\sigma)}(\theta, a, x)]_{x=\sigma(\theta)} = \psi g_j^{\ast}(\sigma_j(\theta_j)) - \kappa_i(g_j^*(\sigma_j(\theta_j)) - g_j^*(\sigma_j(\theta_j))) = 0.
\]

\[
\psi g_j^{\ast}(s_j) - \kappa_i(g_j^*(s_j) - g_j^*(s_j)) = 0. \tag{2}
\]

Equations (1) and (2) are $i$ 's expected payoff maximizing response functions given the equilibrium conjecture concerning $j$. $j$ 's expected payoff maximizing response functions given the equilibrium conjecture concerning $i$ can be easily found by simply switching the roles of $i$ and $j$.

It is important to note that equations, (1) and (2), have an unusual characteristic in that the expected payoff of a group is maximized regardless of the time of action chosen by the group given the equilibrium strategy conjecture of the other group. However, as will be seen, unless both groups play the conjectured equilibrium strategies, the conjecture no longer is an equilibrium and, to be the best response functions for each other, both groups must play the conjectured equilibrium strategy functions.\textsuperscript{34} This technique is clearly different from the

\textsuperscript{34} This is an analogous technique to finding the mixed strategy equilibrium of the rock, paper, and scissor game. In the rock, paper, and scissor game, the mixed strategy equilibrium is to mix and play the game with the following probability distribution: (rock, paper, scissor: 1/3, 1/3, 1/3). Despite the fact that a player’s expected payoff is the same among choosing one of the three actions given that the other player plays this
standard technique of Fudenberg and Tirole or that of Drazen since one cannot solve for the respective unique optimal times of action for all types of $i$ given the decision by $j$ in the war of attrition with the additional outside option.

**Finding Equilibrium**

Rearranging (2), we get

$$g^e_j(s_i) = \frac{\kappa_i}{\psi} \left( g^a_j(s_i) - g^c_j(s_i) \right).$$

(3)

Substituting (3) into (1), we get

$$g^a_j(s_i) = \frac{1}{\psi} \left( \kappa_i - \frac{W - L}{\psi} \kappa_i \right) (g^a_j(s_i) - g^c_j(s_i)).$$

(4)

Subtracting (3) from (4), we get

$$g^a_j(s_i) - g^c_j(s_i) = \frac{W - L}{\psi^2} \kappa_i (g^a_j(s_i) - g^c_j(s_i)).$$

Substituting (5) into (3), we get

$$g^e_j(s_i) = \frac{\kappa_i}{\psi} e^{-\frac{W - L}{\psi^2} \kappa_i s_i}. $$

$$g^c_j(s_i) = \int_0^{s_i} \frac{\kappa_i}{\psi} e^{-\frac{W - L}{\psi^2} \kappa_i x} dx = \frac{\psi}{W - L} \left( 1 - e^{-\frac{W - L}{\psi^2} \kappa_i s_i} \right).$$

(6)

The cutoff type, $\bar{\theta}$, is

$$\bar{\theta} = g^c_j(\infty) = \frac{\psi}{W - L}. $$

(A)

mixed strategy, unless this player plays this mixed strategy, the specified mixed strategy cannot be an equilibrium strategy of the game. Note that my equilibrium decision is a pure, not mixed, equilibrium.
Since $g_j^c(s) \equiv F_j(\Phi_j^c(s)) \equiv \Phi_j^c(s)$,

$$g_j^c(s_i) = F_j(\Phi_j^c(s_i)) = \frac{\psi}{W-L} \left( 1 - e^{-\frac{W-L}{\psi \kappa_i \psi}} \right).$$

$$\Phi_j^c(s_i) = F_j^{-1} \left( \frac{\psi}{W-L} \left( 1 - e^{-\frac{W-L}{\psi \kappa_i \psi}} \right) \right) = \frac{\psi}{W-L} \left( 1 - e^{-\frac{W-L}{\psi \kappa_i \psi}} \right). \quad (B)$$

$$\sigma_j^c(\theta) = \frac{\psi^2 \ln \left( \frac{L \theta_j - W \theta_j + \psi}{\psi} \right)}{L \kappa_i - W \kappa_i}. \quad (C)$$

Substituting (6) into (5), we get

$$g_j^a(s_i) = F_j(\Phi_j^a(s_i)) = \frac{\psi}{W-L} + \left( 1 - \frac{\psi}{W-L} \right) e^{-\frac{W-L}{\psi \kappa_i \psi}}. \quad (7)$$

Since $g_j^a(s) \equiv F_j(\Phi_j^a(s)) \equiv \Phi_j^a(s)$,

$$\Phi_j^a(s_i) = F_j^{-1} \left( \frac{\psi}{W-L} + \left( 1 - \frac{\psi}{W-L} \right) e^{-\frac{W-L}{\psi \kappa_i \psi}} \right) = \frac{\psi}{W-L} + \left( 1 - \frac{\psi}{W-L} \right) e^{-\frac{W-L}{\psi \kappa_i \psi}}. \quad (D)$$

$$\sigma_j^a(\theta) = \frac{\psi^2 \ln \left( \frac{L \theta_j - W \theta_j + \psi}{L - W + \psi} \right)}{L \kappa_i - W \kappa_i}. \quad (E)$$

**Theorem 1. (A Theory of Political Conflicts in the Shadow of Violence)**

$(\pi_i, \sigma_i)$ and $(\pi_j, \sigma_j)$ is a Perfect Bayesian Nash equilibrium of this game and is defined as
Proof. See Appendix.

The conjecture that, for groups below the cutoff type, as the type of group increases, the time of concession increases while, for groups above the cutoff type, as the type of group increases, the time of attack decreases is a Perfect Bayesian Nash equilibrium. Not that it is a trivially “subgame perfect” equilibrium because the only proper subgame is the game itself. It
is also a “Bayesian” equilibrium in the sense that the game is an incomplete information game where the belief concerning each group’s type is represented by the assumption concerning the distribution of each group’s type. In other words, the Nash equilibrium of this game trivially meets the requirement for being a Perfect Bayesian equilibrium.

In sum, this theorem simply implies that groups with military capability below the cutoff type, $\overline{\theta}$, make concessions in a strictly increasing manner with respect to the time of concession while the groups with military capability above $\overline{\theta}$ attack in a strictly decreasing manner with respect to the time of attack. The cutoff type is endogenously determined by the relative cost of using violence, and this relative cost is the ratio between $\psi$ and $W - L$. Adding the option of using violence to the standard incomplete information, continuous time and type, and infinite horizon war of attrition has generated a new result that differs significantly from the standard result. In other words, instead of the standard result where the strongest type waits for the longest time, the strongest type is the first one to act and immediately uses violence.\(^{35}\)

IV. Comparative Statics

From (A) of the Finding Equilibrium section, we know that the cutoff type, $\overline{\theta}$, is a function of $\psi, W$, and $L : \overline{\theta} = \frac{\psi}{W - L}$. From this and using the facts, $W > 0$, $L < 0$, and $\psi < 0$, we derive the first proposition of this model concerning the decision to use violence.

\(^{35}\) With some effort, one can prove that there is no equilibrium with an additional outside option of using violence where the stronger types wait longer than the weaker types before using violence. Obviously, there are other possible equilibria of this game and I have found them. However, all share this essential feature.
**Proposition 1.** The higher the relative cost of using violence, \( \frac{\psi}{W - L} \), the less likely the use of violence.

**Proof.** See Appendix.

The decision concerning the use or non-use of violence depends on the relative cost of using violence. This relative cost is the function of the cost associated with the use of violence, \( \psi \), the payoff from winning, \( W \), and that from losing, \( L \). Note that the instantaneous economic opportunity cost, \( \kappa \), does not influence the decision concerning the use of violence. The higher the \( \psi \), the higher the value for cutoff type, \( \theta \), and the lower the type space for attack. This simply means that when the use of violence becomes more costly, groups are less likely to use violence. The bigger the \( W \), the lower the \( \theta \), and the higher the type space for attack. In other words, when the size of the outcome of interest increases (i.e., the higher the value of obtaining political power), groups are more likely to use violence. The bigger the \( L \), the higher the \( \theta \), and the lower the type space for attack. In other words, when losing becomes more costly, groups are less likely to use violence.

From this theoretical proposition, we can derive the following to empirical implications. First, non-violent political stabilizations are more likely in countries that have high levels of economic development. This is because the high level of economic development entails the high cost associated with the use of violence. Second, violent political stabilizations are more likely in non-democracies and non-violent stabilizations are more likely in democracies. This is because the value of obtaining political power is generally higher in a non-democracy than in a democracy.
Now, let’s turn to the question of timing. Using the facts, \( W - L > \psi \), \( W > 0 \), \( L < 0 \), \( \psi < 0 \), \( \kappa_{i,j} > 0 \) and \( \theta : [0,1] \), we conduct comparative statics on the time of attack, \( \sigma^A \), and the time of concession, \( \sigma^c \), with respect to \( \kappa, W, L, \) and \( \psi \). Rewriting (C) and (E) from _Finding Equilibrium_, we get

\[
\sigma^c_j(\theta_j, W, L, \psi, \kappa_i) = \frac{\psi^2 \ln \left( \frac{\psi}{\psi - (W - L)\theta_j} \right)}{(W - L)\kappa_i} \quad \text{if} \quad 0 \leq \theta_j < \frac{\psi}{W - L}, \quad (*C)
\]

and

\[
\sigma^a_j(\theta_j, W, L, \psi, \kappa_i) = \frac{\psi^2 \ln \left( \frac{W - L - \psi}{(W - L)\theta_j - \psi} \right)}{(W - L)\kappa_i} \quad \text{if} \quad \frac{\psi}{W - L} < \theta_j \leq 1. \quad (*E).
\]

**Proposition 2.** Both the time of attack, \( \sigma^c_j \), and the time of concession, \( \sigma^c_j \), strictly decrease with respect to the other group’s increasing instantaneous cost of prolonging the conflict, \( \kappa_i \).

**Proof.** See Appendix.

As the instantaneous economic opportunity cost of prolonging the conflict by the other group increases, both the time of attack of a group, if the type of the group is below the cutoff type, and the time of concession, if the type of the group is above the cutoff, decrease. This is a counterintuitive result in the sense that a group’s time of action depends on the other group’s instantaneous cost of prolonging instead of its own. The reason why we get this counterintuitive result is because, in solving the game, we look for a group’s decision that makes the other group indifferent and this, in turn, makes the
group’s decision a function of the other group’s instantaneous cost. The instantaneous economic opportunity cost of a group, to an extent, represents the resolve of the group with respect to continuing this race to the bottom, and thus the other group takes action sooner as the group’s instantaneous cost rises. In substantive terms, the higher the economic opportunity cost of a group (i.e., the continuation of a crisis hurts this group more than the other), the sooner the time of concession by the other group and the concomitant stabilization, if the other group has low military capability, and also the sooner the time of attack by the other group and the concomitant termination of the crisis situation, if the other group has high military capability.

If we only focus on the symmetric equilibrium of this game, we can conclude that, when the instantaneous economic opportunity cost of prolonging the conflict increases, the group that is stronger than the cutoff type attack sooner and the group that is weaker than the cutoff concede sooner. Hence, we can safely derive an empirical implication that the higher the level of economic development, the higher the level of instantaneous economic opportunity cost of a crisis situation, and the sooner the timing of political stabilization by either violent or non-violent means.

**Proposition 3.** The time of attack, $\sigma^a$, strictly decreases, and the time of concession, $\sigma^c$, increases with respect to the increasing size of the winner's prize, $W$.

**Proof.** See Appendix.

This simply means that, when the value of obtaining the outcome of interest increases, groups that are stronger (i.e., have higher military capability) than the cutoff type attacks sooner and groups that are weaker (i.e., have lower military capability) than
the cutoff concedes later. This, in turn, implies that the violent political stabilization will take place sooner and the non-violent stabilization later in non-democracies where the benefit associated with having the control over government tends to be higher than in democracies.

**Corollary 3.1.** The time of attack, $\sigma^a$, strictly increases, and the time of concession, $\sigma^c$, decreases with respect to the increasing size of the loser’s retribution payoff, $L$.

In other words, when losing becomes more costly, groups that are stronger than the cutoff type attack later and groups that are weaker than the cutoff type concede sooner.

**Proposition 4.** The time of attack, $\sigma^a$, strictly increases with respect to the increasing cost associated with use violence, $\psi$, while the time of concession, $\sigma^c$, increases for the low types, and decreases for the medium types with respect to the increasing $\psi$.

*Proof.* See Appendix.

In other words, the higher the level of economic development (i.e., the higher the cost associated with the use of violence), the later the time of attack by groups that are above the cutoff and the later the political stabilization by violent means. The higher the level of economic development, the sooner the time of concession by a group that has low military capability (i.e., less than $\bar{\theta}^{36}$), and the later the time of concession by a group that has mid-level military capability (i.e., higher than $\bar{\theta}$ but still lower than $\bar{\theta}$). It

$\frac{\bar{\theta}}{2} < \bar{\theta} < \bar{\theta}$. 

36
is important to keep in mind that groups that concede are all below the cutoff type and are, therefore, weak in general. When considering the time of concession, I am comparing relative military capabilities among groups that are below the cutoff type.

After the appendix, a Matlab code is attached so that readers can generate graphs of interest using different parameter values.

V. Conclusion

In this paper, I developed and analyzed a model of political conflict in the shadow of violence using incomplete information, continuous time and type, and infinite horizon war of attrition with two outside options of conceding and attacking. The equilibrium conjecture that groups make concessions according to their military capabilities in a cascading manner if the groups are below the cutoff type and that groups attack according to their military capabilities also in a cascading manner if the groups are above the cutoff is proved to be a Perfect Bayesian Nash equilibrium of this game. More specifically, in this equilibrium, the time of concession and the concomitant time of non-violent political stabilization by a group’s unilateral concession increases with respect to the group’s military capability. The time of attack and the concomitant time of violent political stabilization decreases with respect to the group’s military capability. Adding the option of attack changes the dynamics of wars of attrition significantly in that militarily stronger groups act early and use violence.

The decision concerning the use of violence is determined by the cost associated with use of violence, the significance of the winner’s prize, and the retribution payoff of the loser. More specifically, the greater the costs of using violence, the less likely the
groups are to use violence. The greater the values of prevailing in the conflict, the more likely the groups are to use violence. The greater the penalties for succumbing to the other group, the less likely the groups are to use violence. In the context of transitional conflicts, these results imply that non-violent political stabilizations are more likely in countries with high levels of economic development and that political stabilizations through the use of violence are more likely in non-democracies.

The timing of attack and the concomitant time of political stabilization by the use of violence are dependent upon the cost of using violence, the significance of the winner’s prize, the retribution payoff of the loser, and the instantaneous economic opportunity cost. More specifically, it has been shown that the greater the cost of using violence, the later the stabilization through the use of violence. The greater the value of prevailing, the sooner the stabilization by the use of violence will take place. The greater the penalty for losing, the later the stabilization by the use of violence. Finally, the greater the instantaneous economic opportunity cost of prolonging the conflict, the sooner the stabilization by the use of violence.

The timing of concession and the concomitant time of non-violent political stabilization are also dependent upon the cost of using violence, the significance of the winner’s prize, the retribution payoff of the loser, and the instantaneous economic opportunity cost. When a group has low military capability, this group’s time of concession and the concomitant stabilization with respect to the increasing cost of using violence will occur later. When a group has mid-level military capability, this group’s time of concession and the concomitant stabilization with respect to the increasing cost of using violence will occur sooner. The greater the value of prevailing, the later the
stabilization by a group’s concession will take place. The greater the penalty for losing, the sooner the stabilization by a group’s concession will take place. Finally, the greater the instantaneous economic opportunity cost of prolonging the conflict, the sooner the stabilization by a group’s concession will take place.

In the context of transitional conflicts, there are three important implications with respect to the timing of political stabilizations. First, the higher the level of economic development, which, in turn, means the higher instantaneous economic opportunity costs for all groups in conflict, the sooner the political stabilizations (i.e., both transitions and non-transitions) by either violent or non-violent means. More importantly, the decision concerning the timing of a group’s use of violence or a group’s unilateral concession depends on the instantaneous opportunity cost of the other group but not on its own. Second, the higher the level of economic development, which, in turn, means the higher cost associated with the use of violence, the later the timing of the use of violence if a group has high military capability, the sooner the timing of concession if a group has low military capability, and the later the timing of concession if a group has mid-level military capability. Third, violent political stabilizations will take place sooner and non-violent stabilizations later in non-democracies where the benefit associated with having the control over government tends to be higher than in democracies.

Using my model of political conflicts, the impact of a third party intervention in a deadlocked crisis situation between two groups can be easily analyzed. A third party military intervention in favor of a group increases this group’s military capability. A third party economic intervention in favor of a group decreases this group’s instantaneous economic cost. A combined military and economic intervention in favor of a group
improves this group’s military capability and reduces its economic cost. Using the comparative statics results above, readers can easily deduce the impact of third party interventions on the timing and the nature of political stabilization processes.

As the title of this paper foreshadowed, this model is a small, and yet significant, part of my dissertation project. I will go on to refine and tailor the theorem and the four propositions to be empirically testable, and I will test them using event history analysis. Some possible theoretical extensions of interest are adding the use of violence as an inside option\textsuperscript{37} and making the war outcome probabilistic instead of deterministic. These theoretical extensions are almost complete. Detailed case studies of both domestic and international crisis situations are in progress and will be available in the near future. Three cases of particular interest are South Korean and South African transitions to democracy from a comparative perspective, Franco-Vietnamese anti-colonial conflict, and the US-North Korean nuclear bargaining. Additional future research agendas include computational extensions of the formal model, modeling the process of mass mobilization in transitional conflicts, and the application of the model to study the history of Cold War.

\textsuperscript{37} See Muthoo (1999) for a detailed explanation of this term.
APPENDIX

Proof of Theorem 1.

We show (*) from Basic Setup by considering the following three cases separately:

\[ \theta_i < \frac{\psi}{W-L}, \quad \theta_i > \frac{\psi}{W-L} \quad \text{and} \quad \theta_i = \frac{\psi}{W-L}. \]

Case 1: \( \theta_i < \frac{\psi}{W-L} \).

Let \( h_\epsilon(s) = E[U(\theta, c, s)] \).

Then, \( h_\epsilon(s) = L + (W - L)g_j^\epsilon(s) - \psi(1 - g_j^a(s)) - \kappa_i \int_0^s (g_j^a(x) - g_j^\epsilon(x))dx \),

where \( g_j^\epsilon \) and \( g_j^a \) are respectively defined as (6) and (7) in Finding Equilibrium.

Therefore, \( \frac{d}{ds} h_\epsilon(s) = (W - L) \frac{d}{ds} g_j^\epsilon(s) + \psi \frac{d}{ds} g_j^a(s) - \kappa_i (g_j^a(s) - g_j^\epsilon(s)) \).

By (3) and (4) from Finding Equilibrium,

\[ \frac{d}{ds} h_\epsilon(s) = 0. \quad (\ast 1) \]

Let \( h_a(s) = E[U(\theta, a, s)] \).

Then, \( h_a(s) = L + (W - L)(\theta_i \lor g_j^\epsilon(s)) - \psi(1 - g_j^\epsilon(s)) - \kappa_i \int_0^s (g_j^a(x) - g_j^\epsilon(x))dx \).  \( (\ast 2) \)

Therefore, by (3),

\[ \frac{d}{ds} h_a(s) = \psi \frac{d}{ds} g_j^\epsilon(s) - \kappa_i (g_j^a(s) - g_j^\epsilon(s)) = 0 \text{ if } s < \sigma_j(\theta_i). \quad (\ast 3) \]

By (\ast 2), (3), and (5) in Finding Equilibrium,
\[
\frac{d}{ds} h_a(s) = (W - L + \psi) \frac{d}{ds} g_j^*(s) - \kappa_i (g_j^*(s) - g_j^c(s)) = \frac{(W - L)\kappa_i e^{\frac{W - L}{\psi} s}}{\psi} \quad \text{if } s > \sigma_j(\theta). \tag{4}
\]

By (3) and (4),

\[ h_a(s) \leq \lim_{x \to \infty} h_a(x) \quad \text{for all } s \geq 0. \tag{5} \]

Since \( E[U^i_{\sigma, \sigma_j}(\theta_i, \Delta, \infty)] = \lim_{x \to \infty} h_c(x) = \lim_{x \to \infty} h_a(x), \tag{6} \)

(*1) and (5) lead to (*).

Case 2: \( \theta_i > \frac{\psi}{W - L} \).

Let \( h_c(s) = E[U^i_{\sigma, \sigma_j}(\theta_i, c, s)] \).

Then, \[ h_c(s) = \begin{cases} 
L + (W - L)g_j^c(s) - \psi(1 - g_j^a(s)) - \kappa_i \int_0^s (g_j^a(x) - g_j^c(x))dx & \text{if } s \leq \sigma_j(\theta_i) \\
L + (W - L)g_j^c(s) + \theta_i - g_j^a(s) - \psi(1 - g_j^a(s)) - \kappa_i \int_0^s (g_j^a(x) - g_j^c(x))dx & \text{if } s > \sigma_j(\theta_i) 
\end{cases} \]

Therefore, by (3) and (4),

\[ \frac{d}{ds} h_c(s) = (W - L) \frac{d}{ds} g_j^c(s) + \psi \frac{d}{ds} g_j^a(s) - \kappa_i (g_j^a(s) - g_j^c(s)) = 0 \quad \text{if } s < \sigma_j(\theta_i). \]

By (3), (4), and (5),

\[ \frac{d}{ds} h_c(s) = (W - L) \frac{d}{ds} g_j^c(s) + (\psi - W + L) \frac{d}{ds} g_j^a(s) - \kappa_i (g_j^a(s) - g_j^c(s)) > 0 \quad \text{if } s > \sigma_j(\theta_i). \]

Hence, \( h_c(s) \leq \lim_{x \to \infty} h_c(x) \) for all \( s \geq 0 \). \tag{7}

Let \( h_a(s) = E[U^i_{\sigma, \sigma_j}(\theta_i, a, s)] \).

Then, \[ h_a(s) = L + (W - L)\theta_i - \psi(1 - g_j^c(s)) - \kappa_i \int_0^s (g_j^a(x) - g_j^c(x))dx. \]
By (3),
\[
\frac{d}{ds} h_u(s) = \psi \frac{d}{ds} g_j(s) - \kappa_i \left( g_j(s) - g_j^c(s) \right) = 0 \quad \text{for all } s > 0. \quad (**8)
\]

By (**6), (**7), and (**8), (*) holds.

Case 3: \( \theta_i = \frac{\psi}{W - L} \).

Let \( h_c(s) = E[U_{(i,j)}(\theta_i, c, s)] \).

Then, \( h_c(s) = L + (W - L)g_j^c(s) - \psi \left( 1 - g_j^a(s) \right) - \kappa_i \int_0^s (g_j^a(x) - g_j^c(x)) dx \).

By (3) and (4),
\[
\frac{d}{ds} h_c(s) = 0. \quad (**9)
\]

Let \( h_a(s) = E[U_{(i,j)}(\theta_i, a, s)] \).

Then, \( h_a(s) = L + (W - L)\theta_i - \psi \left( 1 - g_j^a(s) \right) - \kappa_i \int_0^s (g_j^a(x) - g_j^c(x)) dx \).

By (3),
\[
\frac{d}{ds} h_a(s) = 0. \quad (**10)
\]

by (**9) and (**10),
\[
E[U_{(i,j)}(\theta_i, c, s)] \text{ is the same for all } (d, s) \in D.
\]

Hence, (*) holds.

Note that the expected payoff for either making concession or attacking immediately is not higher than playing the equilibrium decisions as can be seen from (1) and (2) in Derivations.
Proof of Proposition 1.

Using the facts, $W > 0$, $L < 0$, and $\psi < 0$,

$$\frac{\partial \bar{\theta}}{\partial \alpha} > 0 \text{ where } \alpha = \frac{\psi}{W - L},$$

$$\frac{\partial \bar{\theta}}{\partial \psi} = \frac{1}{W - L} > 0,$$

$$\frac{\partial \bar{\theta}}{\partial W} = -\frac{\psi}{(W - L)^2} < 0,$$

$$\frac{\partial \bar{\theta}}{\partial L} = \frac{\psi}{(W - L)^2} > 0.$$

Proof of Proposition 2.

Here, $W - L > \psi$, $W > 0$, $L < 0$, $\psi < 0$, $\kappa_{i,j} > 0$ and $\theta : [0,1]$.

Also, $\sigma_j^c(\theta_j,W,L,\psi,\kappa) = \frac{\psi^2 \ln \left( \frac{\psi}{\psi - (W - L)\theta_j} \right)}{(W - L)\kappa_i}$ if $0 \leq \theta_j < \frac{\psi}{W - L}$,

$$\sigma_j^a(\theta_j,W,L,\psi,\kappa) = \frac{\psi^2 \ln \left( \frac{W - L - \psi}{(W - L)\theta_j - \psi} \right)}{(W - L)\kappa_i} \text{ if } \frac{\psi}{W - L} < \theta_j \leq 1.$$

Note that these givens are used in the proofs of propositions 3 and 4 and the proof of corollary 3.1.

$$\frac{\partial \sigma_j^a}{\partial \kappa_i} = -\frac{\psi^2}{(W - L)\kappa_i^2} \ln \left( \frac{(W - L) - \psi}{(W - L)\theta_j - \psi} \right) < 0,$$

$$\frac{\partial \sigma_j^c}{\partial \kappa_i} = \frac{\psi^2}{(W - L)\kappa_i^2} \ln \left( \frac{\psi - (W - L)\theta_j}{\psi} \right) < 0.$$
Proof of Proposition 3.

\[
\frac{\partial \sigma^\varphi_j}{\partial W} = \frac{\psi^2}{(W - L)^2 \kappa_i} \left[ -\left(1 - \theta_j\right)\psi \left(\frac{W - L - \psi}{(W - L)\theta_j - \psi}\right) + \ln\left(\frac{W - L - \psi}{(W - L)\theta_j - \psi}\right) \right] < 0.
\]

\[
\frac{\partial \sigma^\varphi_i}{\partial W}(\theta_j, W, L, \psi, \kappa_i) = \frac{\psi^2}{(W - L)^2 \kappa_i} \left[ \frac{(W - L)\theta_j}{\psi - (W - L)\theta_j} - \ln\left(\frac{\psi}{\psi - (W - L)\theta_j}\right) \right].
\]

\[
\frac{\partial \sigma^\varphi_i}{\partial W}(0, W, L, \psi, \kappa_i) = 0. \quad (\clubsuit)
\]

\[
\frac{\partial^2 \sigma^\varphi_i}{\partial W^2}(\theta_j, W, L, \psi, \kappa_i) = \frac{\psi^2}{(W - L)^2 \kappa_i} \left[ \frac{(W - L)^2 \theta_j}{(\psi - (W - L)\theta_j)^2} \right] > 0. \quad (\heartsuit)
\]

By (\heartsuit) and (\clubsuit), \(\frac{\partial \sigma^\varphi_i}{\partial W} > 0\).

Proof of Corollary 3.1.

\[
\frac{\partial \sigma^\varphi_j}{\partial W} = \frac{\psi^2}{(W - L)^2 \kappa_i} \left[ \frac{(1 - \theta_j)\psi}{(W - L - \psi)(W - L)\theta_j - \psi} + \ln\left(\frac{W - L - \psi}{(W - L)\theta_j - \psi}\right) \right] > 0.
\]

\[
\frac{\partial \sigma^\varphi_i}{\partial L}(\theta_j, W, L, \psi, \kappa_i) = \frac{\psi^2}{(W - L)^2 \kappa_i} \left[ -\frac{(W - L)\theta_j}{\psi - (W - L)\theta_j} + \ln\left(\frac{\psi}{\psi - (W - L)\theta_j}\right) \right].
\]

\[
\frac{\partial \sigma^\varphi_i}{\partial L}(0, W, L, \psi, \kappa_i) = 0. \quad (\ast \heartsuit)
\]
\[
\frac{\partial^2 \sigma^c_i}{\partial \theta_j \partial \psi} (\theta, W, L, \psi, \kappa_i) = \frac{1}{(W - L) \kappa_i (\psi - (W - L) \theta_j) } \left[ \psi - 2(W - L) \theta_j \right] > 0 \text{ iff } \theta_j < \frac{\psi}{2(W - L)} = \frac{\bar{\theta}}{2} .
\]

\[
\frac{\partial^2 \sigma^c_i}{\partial \theta_j \partial \psi} (\theta, W, L, \psi, \kappa_i) = \frac{1}{(W - L) \kappa_i (\psi - (W - L) \theta_j) } \left[ \psi - 2(W - L) \theta_j \right] < 0 \text{ iff } \theta_j > \frac{\psi}{2(W - L)} = \frac{\bar{\theta}}{2} .
\]

There exists \( \bar{\theta} \) such that \( \frac{\bar{\theta}}{2} < \bar{\theta} < \bar{\theta} \).

\[
\frac{\partial \sigma^c_i}{\partial \psi} \geq 0 \text{ iff } \theta \leq \bar{\theta} \text{ and } \frac{\partial \sigma^c_i}{\partial \psi} < 0 \text{ iff } \theta > \bar{\theta} .
\]
The following is the Matlab code for generating visual aids. Simply, copy, paste and enjoy.

Please use Matlab versions 7.0 or higher.

```matlab
W = 5;
L = -1;
p = 1;
k = 1;

theta_upperstar = p / (W - L);
M = 6 / ((W - L) * k / p / p);
s = [0:M/1000:M];
a = exp(-(W - L) / p / p * k * s);
plot(theta_upperstar*(1-a),s,theta_upperstar+(1-theta_upperstar)*a,s);
```
Reference


Skocpol, Theda. 1979. *States and Social Revolutions*. Cambridge, UK: Cambridge University Press.


