Nonsymmetric Entanglement of Atomic Ensembles

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The entanglement of multiatom quantum states is considered. In order to cancel noise due to inhomogeneous light-atom coupling, the concept of matched multiatom observables is proposed. As a means to eliminate an important form of decoherence this idea should be of broad relevance for quantum information processing with atomic ensembles. The general approach is illustrated on the example of rotation angle measurement, and it is shown that the multiatom states that were thought to be only weakly entangled can exhibit near-maximum entanglement.

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Progress in quantum communication and information processing depends on practical schemes for the massive entanglement of quantum systems. Advances based on the entanglement of atomic ensembles are undergoing rapid development and involve independent approaches toward the entanglement of continuous light-atom variables (see, e.g., Ref. [1], and references therein) and discrete variables [2,3]. Recent progress suggests that in the short term major advances in quantum information processing with atomic ensembles are likely.

What makes quantum information processing with atomic ensembles very attractive is the relative simplicity of the experimental interface. As opposed to the case of single atoms interacting with single photons, the use of atomic ensembles does not require the technology of strong coupling cavity QED, and a free space interaction between light and atomic ensemble is sufficient for massive entanglement. In the latter case the conditions for a sufficiently strong interaction reduce to a high on-resonance optical density of the atomic sample, a condition which is relatively easy to achieve experimentally. Therefore, the field of atomic ensemble entanglement shows enormous promise as a practical way to implement quantum information processing.

On the other hand, a well-known difficulty of the field has been how to realize the required symmetric character of the atom-photon interaction in the presence of inhomogeneities in the local atom-field coupling. Inhomogeneity of this kind is a potentially fatal problem for applications of atomic ensembles to quantum information processing [4].

The apparent problem is rooted in a widely accepted notion that the multiatom states have to exhibit a high degree of symmetry in order to be strongly entangled. Implicit in these arguments has been the fact that conventional entanglement measures, such as the spin squeezing parameter, are themselves symmetric [5–9]. To the best of our knowledge, in all of the atomic ensemble entanglement work such symmetric entanglement measures have been used [2,10–27]. The symmetry of entanglement measures is directly related to the fact that additive N atom observables of the form $\hat{A} = \sum_{k=1}^{N} \hat{a}_k$, which are symmetric with respect to permutation $\hat{a}_k \leftrightarrow \hat{a}_l$ of any two single particle operators satisfying identical commutations relations, were thought to provide a proper connection to the experimental situation. Even for modest degrees of asymmetry of multiatom quantum states, the fact that $N \gg 1$ imposes severe limitations on the degree of entanglement of such symmetric observables [4].

There is, however, no *a priori* reason that the multiatom observables, and consequently, entanglement measures, should exhibit this kind of symmetry. We would like to argue in favor of a more operational approach, in which the measurement procedure itself essentially defines the entanglement measure. In atomic ensemble entanglement experiments the multiatom observables are typically nonsymmetric, because the atoms are distributed in space and couple to the local value of the electric field mode [2,14,16,20,25]. Instead of symmetric operators like $\hat{A}$, a related nonsymmetric observable $\tilde{A} = \sum_{k=1}^{N} g_k \hat{a}_k$ should be considered, where the g_k are a set of numbers whose deviation from unity is associated with the inhomogeneity. We will show that there is no asymmetry induced decoherence in the entanglement of such matched observables.

Our conjecture is that for any ensembles-based quantum information protocol, the asymmetry of the atomic ensemble cancels out and near-maximum entanglement is observed, provided every step of the ensemble-based interaction *is matched* to each other. In a sense the classical noise associated with the observable asymmetry, or equivalently the atom-field coupling inhomogeneity, can be explicitly accounted for and subtracted out.

While the notion of matched observables is quite general, for the sake of clarity we will illustrate it on a particular atomic ensemble scheme: the sensitivity of rotation angle measurement. First let us recall the general ideas involved. The simplest kind of rotation measurement involves preparing an input quantum state of an N particle system, passing it through an interferometer where a phase rotation is encoded, and then measuring an output observable sensitive to the phase rotation. The action of the quantum interferometer (photon or massive particle) can quite generally be understood in terms of the

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rotation of a pseudoangular momentum operator $\hat{\mathbf{F}}$, comprised of an ensemble of N spin 1/2 systems $\hat{\mathbf{F}} = \sum_{k=1}^{N} \hat{\mathbf{f}}^k$ [28]. The sensitivity of the rotation measurement is evidently conditioned on the quantum fluctuations of the input state of $\hat{\mathbf{F}}$. For a coherent spin state (CSS), a nonentangled state, the rotation sensitivity is given by the so-called standard quantum limit $\delta\phi \sim 1/\sqrt{N}$. If instead we prepare a spin squeezed state (SSS), a massively entangled state of N particles, the phase sensitivity improves and can approach the Heisenberg limit $\delta\phi \sim 1/N$ [28]. This example rather clearly illustrates the practical consequences of entanglement in quantum measurements. The crucial issue, then, is how to prepare the entangled SSS? One proposal involves allowing the spin ensemble to interact for a certain time with an auxiliary ensemble of n spin 1/2 particles $\hat{\mathbf{s}}^i$, $i = 1, \ldots, n$, prior to entering the interferometer [12]. Two essential new elements of this scheme, in addition to the action of the interferometer itself, should be noted. The interaction between the two spin ensembles is designed to prepare a SSS, and for this purpose a quantum nondemolition (QND) interaction is ideal. Moreover, an external measurement on the auxiliary spin is required $\hat{\mathbf{S}} = \sum_{i=1}^{n} \hat{\mathbf{s}}^i$, in order to condition the input state to the interferometer. We shall also see later that entanglement of the atomic spin ensemble by means of a QND interaction with an ancilliary ensemble is not essential. One can also exploit entanglement between the two ensembles when the atomic ensemble is in a separable state. However, we will first concentrate on the former case.

The discussion hides an important practical problem, however, the difficulty of implementing the desired QND interaction. It is at this point that the issue of nonsymmetric observables arises in practice. Analysis of the example of rotation angle measurement demonstrates how random noise associated with the asymmetry feeds into the measurement preventing a sensitivity approaching the Heisenberg limit. Consider two ensembles of spin 1/2 systems, the *atoms* $\hat{\mathbf{f}}^i$, $i = 1, \ldots, N$ which are passed through the interferometer, and the *photons* $\hat{\mathbf{s}}^i$, $i = 1, \ldots, n$ which act as the auxiliary state preparation device. While at this stage we use the names atoms and photons merely as labels, we have in mind practical quantum optical systems where the angular momenta do correspond to a collection of atoms and a single mode probe light field, respectively. The total angular momenta are represented by $\hat{\mathbf{S}} = \sum_{i=1}^{n} \hat{\mathbf{s}}^i$ and $\hat{\mathbf{F}} = \sum_{k=1}^{N} \hat{\mathbf{f}}^k$. We assume that initially the states of both of the ensembles are uncorrelated CSS, in which the average values of $\hat{\mathbf{S}}$ and $\hat{\mathbf{F}}$ are directed along the x axis.

The interaction between the atoms and photons is designed in order to entangle the ensemble of atomic spins and is given by the QND interaction

$$\hat{H} = \hbar\Omega \sum_{i=1,k=1}^{n,N} g_k \hat{s}_z^i \hat{f}_z^k = \hbar\Omega \hat{S}_z \tilde{F}_z, \tag{1}$$

where Ω is a frequency, the g_k are dimensionless coupling weights, and $\tilde{F}_z = \sum_{k=1}^{N} g_k \hat{f}_z^k$ is a nonsymmetric atomic operator unless all of the g_k are equal. In a realistic experimental scenario coupling weights of varying magnitude arise when an interaction of this kind is created using an off-resonant interaction between a single mode light field and a collection of atoms. Then, the distribution of coupling weights g_k maps out the variation of mode intensity seen by the spatially distributed atoms $k = 1, \ldots, N$.

If the atom and photon spin ensembles interact for a time τ under Eq. (1), the atomic spins evolve according to $\hat{f}_x^k(\tau) = \cos(g_k\chi\hat{S}_z)\hat{f}_x^k - \sin(g_k\chi\hat{S}_z)\hat{f}_y^k$, $\hat{f}_y^k(\tau) = \sin(g_k\chi\hat{S}_z)\hat{f}_x^k + \cos(g_k\chi\hat{S}_z)\hat{f}_y^k$, $\hat{f}_z^k(\tau) = \hat{f}_z^k$, where $\chi = \Omega\tau$. The individual spins have a dispersion of frequencies associated with the distribution of weights, while the collective spin $\hat{\mathbf{S}}$ satisfies

$$\begin{pmatrix} \hat{S}_x \\ \hat{S}_y \\ \hat{S}_z \end{pmatrix}(\tau) = \begin{pmatrix} \cos(\chi\tilde{F}_z) & -\sin(\chi\tilde{F}_z) & 0 \\ \sin(\chi\tilde{F}_z) & \cos(\chi\tilde{F}_z) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{S}_x \\ \hat{S}_y \\ \hat{S}_z \end{pmatrix}. \tag{2}$$

The atoms are then passed into the interferometer where a phase rotation ϕ is imposed by means of a rotation about the y axis, so that $\hat{f}_{a,\mathrm{out}}^k = e^{i\phi\hat{F}_y(\tau)}\hat{f}_a^k(\tau)e^{-i\phi\hat{F}_y(\tau)}$, $a = x, y, z$. To determine the phase ϕ we could measure the z component of total angular momentum at the output $\hat{F}_{z,\mathrm{out}} = \hat{F}_z\cos\phi - \sin\phi\sum_{k=1}^{N}[\cos(g_k\chi\hat{S}_z)\hat{f}_x^k - \sin(g_k\chi\hat{S}_z)\hat{f}_y^k]$. In fact we would really like to measure $\hat{F}_{z,\mathrm{out}} - \hat{F}_z(\tau) = \hat{F}_{z,\mathrm{out}} - \hat{F}_z$, the change in angular momentum due to the interferometer, canceling any superfluous noise carried in the SSS input. The QND interaction enables us to obtain information about $\hat{F}_z$, since $\hat{S}_y(\tau) \approx \chi\tilde{F}_z\hat{S}_x + \hat{S}_y$, for $\chi \triangle F_z = \chi\sqrt{N/4} \ll 1$. Hence, using $\langle\hat{S}_x\rangle_{\mathrm{CSS}} = N/2$, it follows that the operator $2\hat{S}_y(\tau)/(n\chi) \approx \tilde{F}_z(2\hat{S}_x/n) + 2\hat{S}_y/(n\chi)$ is linear in $\tilde{F}_z$ with a coefficient of unity (the coherent state is an eigenstate of $\hat{S}_x$, and thus it can be replaced by the c number $n/2$). This operator is as close to $\hat{F}_z$ as we can get by measuring a component of the auxiliary ensemble. Note that the observable which appears is the nonsymmetric partner $\tilde{F}_z$ rather than the (symmetric) angular momentum $\hat{F}_z$, and thus classical noise associated with the distribution of weights g_k feeds into measurements of the observable $\hat{F}_z' \equiv \hat{F}_{z,\mathrm{out}} - 2\hat{S}_y(\tau)/(n\chi)$.

To determine the sensitivity of rotation measurements we now evaluate

$$\delta\phi = \left(\sqrt{\langle(\Delta\hat{F}_z')^2\rangle} \Big/ \left| \frac{d\langle\hat{F}_z'\rangle}{d\phi} \right| \right)_{\phi=0}. \tag{3}$$

It is sufficient to evaluate $\langle\hat{F}_z'\rangle$ correct to first order in ϕ, i.e., $\langle\hat{F}_z'\rangle \approx -(N\phi/2)\langle e^{-\xi g^2/2}\rangle = -(N\phi/2) \times (e^{-\xi(1+\Delta g^2\xi)^{-1}/2})/\sqrt{1+\Delta g^2\xi}$ for $g_k^2\chi^2 \ll 1$ and $\xi(\Delta g)^2 \ll 1$, where $\xi \equiv n\chi^2/4$. Here we have used properties of the initial coherent spin state and assumed that the weights g_k are Gaussian distributed with mean value $\langle g \rangle = 1$ and variance $(\Delta g)^2 = \langle g^2 \rangle - 1$.

The variance $\langle (\Delta \hat{F}'_z)^2 \rangle$ may be computed at $\phi = 0$, and using conservation of $\hat{F}_z$ it reduces to $\langle (\Delta \hat{F}'_z)^2 \rangle = \langle (\Delta \hat{F}_z)^2 \rangle + 4\langle \hat{S}_y^2(\tau) \rangle/(n^2\chi^2) - 2(\langle \hat{F}_z \hat{S}_y(\tau) \rangle + \text{c.c.})/(n\chi)$. The first and second members have shot noise contributions which are canceled by the third term, as designed, leaving residual quantum fluctuations. Explicitly $4\langle \hat{S}_y^2(\tau) \rangle/(n^2\chi^2) \approx [N + 1/\xi + N(\Delta g)^2]/4$, assuming $N\chi^2 \ll 1$, where the three terms represent shot noise, quantum fluctuations associated with the input entangled state, and classical noise due to the distribution of weights, respectively. Hence, canceling the atomic shot noise, we find $(\Delta \hat{F}'_z)^2 = [1 + N\xi(\Delta g)^2]/(4\xi)$, which gives a phase error of

$$\delta\phi(\xi) = \sqrt{[1 + N\xi(\Delta g)^2][1 + \xi(\Delta g)^2]}\frac{e^{\xi[1+\xi(\Delta g)^2]^{-1}/2}}{N\sqrt{\xi}}. \tag{4}$$

In the symmetric limit $\Delta g = 0$, $\langle g^2 \rangle = 1$, this reduces to $\delta\phi(\xi) = e^{\xi/2}/(N\sqrt{\xi})$. It takes a minimum value approaching the Heisenberg limit for $\xi = 1$, where $\delta\phi_{\min} = \sqrt{e}/N$ [12]. Note that $\xi = 1$ implies $n\chi^2 = 4$, while we have also assumed $N\chi^2 n^{1/2} \ll 4$ to obtain the approximate expression for the variance. The two conditions are consistent provided the number of photons greatly exceeds the number of atoms $\sqrt{n} \gg N$. In order to approach the Heisenberg limit, therefore, it is clear that we must satisfy $(\Delta g)^2 \ll \frac{1}{N}$, a criterion which is very difficult to achieve for a macroscopic sample of atoms where typically $10^6 < N < 10^9$.

The procedure we have described is based on a measurement of $\hat{F}_z$, but the QND interaction does not correlate the auxiliary spin angular momentum $\hat{S}_z$ directly to $\hat{F}_z$, rather to its nonsymmetric partner $\tilde{F}_z$. Hence the inhomogeneity noise will dominate the phase measurement error unless very strict limits on the distribution variance are satisfied. We have explicitly demonstrated the conventional wisdom that asymmetry washes out the entanglement. The important question is, can the entanglement be distilled out if suitable multiparticle measurements are made to cancel the classical noise? Our answer is in the affirmative. The crucial point is that since the form of the QND interaction correlates the photon spin to $\tilde{F}_z$, we require a multiparticle measurement which gives $\tilde{F}_z$ also at the output of the interferometer. The difference signal can then in principle cancel the coupling distribution noise. This strategy is quite clear from the operational point of view taken here, but runs counter to standard theoretical measures of entanglement based on symmetric operators, such as the spin squeezing parameter $\eta = \sqrt{N}\delta\phi$, which are implicitly sensitive to distribution noise.

To determine which multiparticle measurements will achieve our goal, we consider the operator $\tilde{F}_z$ at the interferometer output. To first order in ϕ

$$\tilde{F}_{z,\text{out}} = \tilde{F}_z - \phi\tilde{F}_x(\tau). \tag{5}$$

Now we introduce a second set of auxiliary spins $\hat{\mathbf{j}}^i$, $i = 1, \ldots, n$ which are coupled with the atomic spin operators at the interferometer output in exactly the same way as spin $\hat{\mathbf{S}}$, i.e., $H_{\text{int}} = \hbar\Omega\hat{J}_z\tilde{F}_z$, where $\hat{\mathbf{J}} = \sum_{i=1}^n \hat{\mathbf{j}}^i$. In practice this ensemble represents a second pulse of light in the same spatial mode of the electromagnetic field, and we assume that it is prepared in a CSS pointing in the positive x direction, so that $\langle \hat{J}_x \rangle = n/2$. Thus after interacting for a time τ with the atoms we have to sufficient accuracy for $N\chi^2 \ll 1$ that $\hat{S}_y(\tau) = \hat{S}_y + \chi\tilde{F}_z\hat{S}_x$, and $\hat{J}_y(\tau) = \hat{J}_y + \chi\tilde{F}_{z,\text{out}}\hat{J}_x = \hat{J}_y + \chi\tilde{F}_z\hat{J}_x - \phi\tilde{F}_x(\tau)\hat{J}_x$.

Let us now consider the difference signal $\hat{A}(\phi) \equiv \hat{J}_y(\tau) - \hat{S}_y(\tau)$, so that

$$\hat{A}(\phi) = \hat{J}_y - \hat{S}_y + \chi\tilde{F}_z(\hat{J}_x - \hat{S}_x) - \phi\chi\tilde{F}_x(\tau)\hat{J}_x. \tag{6}$$

As the auxiliary spins are prepared in eigenstates of $\hat{S}_x$ and $\hat{J}_x$, respectively, the variance in $\hat{A}(\phi = 0)$ reduces to the sum of shot noise contributions from the two auxiliary spin systems $\langle [\Delta\hat{A}(\phi = 0)]^2 \rangle = \langle (\Delta\hat{J}_y)^2 \rangle + \langle (\Delta\hat{S}_y)^2 \rangle = n/2$, and the weight distribution noise has canceled out. The ensemble average of the differential change in signal at $\phi = 0$ measures $\tilde{F}_x(\tau)$ directly, i.e.,

$$\begin{aligned}\left(\frac{d}{d\phi}\langle \hat{A}(\phi) \rangle\right)_{\phi=0} &= -\chi\langle \hat{J}_x \rangle \langle \tilde{F}_x(\tau) \rangle \\ &= -\frac{\chi n N}{4}\frac{e^{-\xi[1+\xi(\Delta g)^2]^{-1}/2}}{[1 + \xi(\Delta g)^2]^{3/2}},\end{aligned} \tag{7}$$

and the corresponding phase error is given by

$$\delta\phi(\xi) = \sqrt{\frac{2}{\xi}}\frac{[1 + \xi(\Delta g)^2]^{3/2}}{N}e^{-\xi[1+\xi(\Delta g)^2]^{-1}/2}. \tag{8}$$

For $(\Delta g)^2 \ll 1$, the minimum phase error approaches the Heisenberg limit $\delta\phi_{\min} = \sqrt{2e}/N$, where the factor of $\sqrt{2}$ arises from the independent shot noise contributions of the two auxiliary spin ensembles. It is interesting to note that even when $\Delta g \to \sqrt{\langle g^2 \rangle}$, the phase error still scales as $1/N$. In other words this measurement scheme could operate close to the Heisenberg limit of phase measurement accuracy under the same conditions where a symmetric squeezing parameter predicts little or no entanglement. The measurements are appropriately *matched* to the nonsymmetric observable of interest, and the corresponding phase error yields an improved, operational measure of entanglement.

It is possible to avoid using the second photon ensemble $\hat{\mathbf{j}}^i$, $i = 1, \ldots, n$, altogether. Instead, the first photon ensemble $\hat{\mathbf{s}}^i$, $i = 1, \ldots, n$ can be stored after the first QND interaction. After the phase rotation ϕ, the photon ensemble is coupled to the atomic spins with interaction strength $\chi_1 = -\chi$ (this can be effectively achieved by flipping the z component of either atom or photon spin before the interaction takes place). The $\hat{S}_y^f$ component of the photon ensemble serves as the

ϕ-dependent observable $\hat{B}(\phi) \equiv \hat{S}_y^f$. We find $\hat{B}(\phi) = \hat{S}_y + [\chi \tilde{F}_z - \chi \cos(\phi)\tilde{F}_z + \chi \sin(\phi)\tilde{F}_x]\hat{S}_x$. It follows that $\langle \hat{B}(\phi)\rangle = -\langle \hat{A}(\phi)\rangle$, and $\langle[\Delta\hat{B}(\phi = 0)]^2\rangle = \frac{1}{2}\langle[\Delta\hat{A}(\phi = 0)]^2\rangle$, since now the shot noise of only one photon ensemble contributes. The phase accuracy $\delta\phi(\xi)$ is improved by a factor of $\sqrt{2}$ over that given by Eq. (8). It is straightforward to check that in this situation spin squeezing parameters [3,6,8] for the atomic ensemble always exceed or equal unity at all times even for perfectly symmetric coupling. Thus the atomic state is not "spin squeezed," or entangled. Nevertheless, Heisenberg-limited performance of the atomic interferometer is achieved. Therefore, we argue that atom-photon entanglement in itself is the primary reason behind the improved accuracy of the considered QND-enhanced atomic interference measurement; the measurement-induced atomic squeezing is secondary and its use may be avoided.

Our results are of importance for the synergy of the fields of cavity QED [29] and of atomic ensembles [1]. Using a cavity of finesse F would effectively increase the strength of the ensemble-light interaction by the factor of F [12]. It has been suggested in the context of atomic ensembles that a ring cavity with running-wave cavity modes and large Gaussian waists should be used, so that all atoms see the same electric dipole mode coupling intensity [12,26,27]. On the contrary, here we put forward an argument that suggests even standing-wave cavity modes can be used to realize a QND atom-light interaction. In this case $\langle(\Delta g)^2\rangle \rightarrow \langle g\rangle^2$, but as we have shown, it is still possible to approach the Heisenberg limit. In present-day experiments various kinds of decoherence would practically limit the amount of entanglement before the Heisenberg limit is reached. With that in mind, we could say that standing-wave probe fields are just as good as running-wave ones. The existing high finesse cavities typically used in cavity QED are all of standing-wave type, and the cavity modes have a sinusoidal spatial dependence along the cavity axis [30–33].

To what extent is the matching of observables feasible in practice? Let us consider the most demanding case of the standing-wave cavity interacting with an ensemble of cold atoms. The atomic velocity would be on the order of a few cm/s, thus if the time interval between the light pulses $<10\ \mu$s, the two measurements would be very well matched. For comparison, in recent experiments that demonstrated atomic memory effects in atomic ensembles [2,25] the relevant time intervals were $<1\ \mu$s. In the case of a running-wave probe field, where the typical length of intensity variations would be a fraction of a centimeter, time intervals as long as 0.1 s would be possible.

Our results should have broad relevance for situations involving multiatom entangled states. By presenting a strategy to eliminate the noise due to asymmetry of atomic states, we have significantly simplified the path towards use of atomic ensembles for quantum information processing. Quantum metrology applications such as atomic clocks, gravimeters, and atomic electric dipole moment searches would also benefit from enhancement in precision that entangled atomic states can provide. Beyond the field of atomic ensembles, we hope that this work opens a discussion of multiparticle entanglement outside the symmetric entangled states that have been considered so far.

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[1] A. Kuzmich and E. S. Polzik, in *Quantum Information with Continuous Variables*, edited by S. L. Braunstein and A. K. Pati (Kluwer, Dordrecht, 2003).
[2] A. Kuzmich *et al.*, Nature (London) **423**, 731 (2003).
[3] L.-M. Duan *et al.*, Nature (London) **414**, 413 (2001).
[4] C. P. Sun *et al.*, Phys. Rev. A **67**, 063815 (2003).
[5] B. Yurke, Phys. Rev. Lett. **56**, 1515 (1986).
[6] M. Kitagawa and M. Ueda, Phys. Rev. Lett. **67**, 1852 (1991); Phys. Rev. A **47**, 5138 (1993).
[7] D. J. Wineland *et al.*, Phys. Rev. A **46**, R6797 (1992).
[8] D. J. Wineland *et al.*, Phys. Rev. A **50**, 67 (1994).
[9] A. Sørensen *et al.*, Nature (London) **409**, 63 (2001).
[10] G. F. Grom and A. M. Kuzmich, JETP Lett. **61**, 900 (1995).
[11] A. Kuzmich *et al.*, Phys. Rev. Lett. **79**, 4782 (1997).
[12] A. Kuzmich *et al.*, Europhys. Lett. **42**, 481 (1998).
[13] K. Mølmer, Eur. Phys. J. D **5**, 301 (1999).
[14] J. Hald *et al.*, Phys. Rev. Lett. **83**, 1319 (1999).
[15] A. Kuzmich *et al.*, Phys. Rev. A **60**, 2346 (1999).
[16] A. Kuzmich *et al.*, Phys. Rev. Lett. **85**, 1594 (2000).
[17] M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000).
[18] A. Kuzmich and E. S. Polzik, Phys. Rev. Lett. **85**, 5639 (2000).
[19] L.-M. Duan *et al.*, Phys. Rev. Lett. **85**, 5643 (2000).
[20] B. Julsgaard *et al.*, Nature (London) **413**, 400 (2001).
[21] S. D. Jenkins and T. A. B. Kennedy, Phys. Rev. A **66**, 043621 (2002).
[22] I. Bouchoule and K. Mølmer, Phys. Rev. A **65**, 041803 (2002).
[23] A. Di Lisi and K. Mølmer, Phys. Rev. A **66**, 052303 (2002).
[24] L.-M. Duan, Phys. Rev. Lett. **88**, 170402 (2002).
[25] C. H. van der Wal *et al.*, Science **301**, 196 (2003).
[26] A. S. Sørensen and K. Mølmer, Phys. Rev. A **66**, 022314 (2002).
[27] A. S. Sørensen and K. Mølmer, Phys. Rev. Lett. **90**, 127903 (2003).
[28] B. Yurke *et al.*, Phys. Rev. A **33**, 4033 (1986).
[29] H. J. Kimble, Phys. Scr. **76**, 127 (1998).
[30] W. P. Smith *et al.*, Phys. Rev. Lett. **89**, 133601 (2002).
[31] J. A. Sauer *et al.*, quant-ph/0309052.
[32] J. McKeever *et al.*, Phys. Rev. Lett. **90**, 133602 (2003).
[33] P. W. H. Pinkse *et al.*, Nature (London) **404**, 365 (2000).