MAGNETOCONVECTION IN A LONG VERTICAL ENCLOSURE WITH WALLS WITH FINITE ELECTRICAL CONDUCTIVITY

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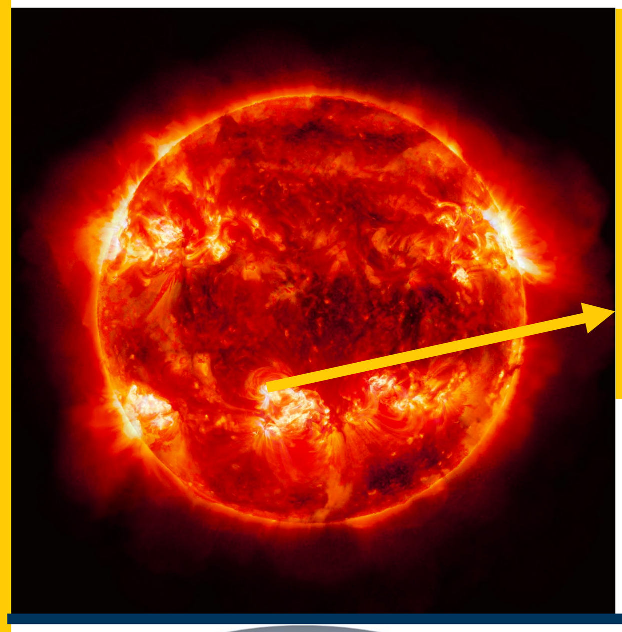
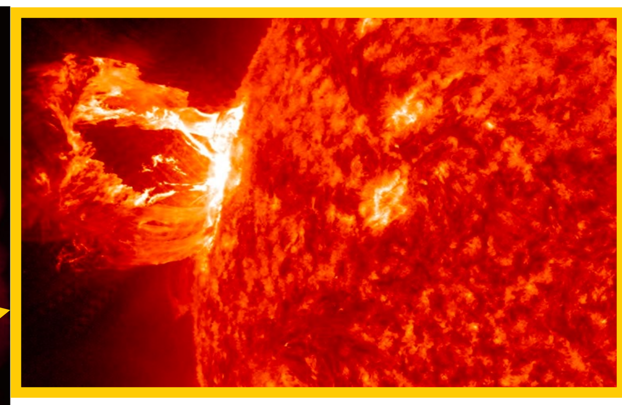
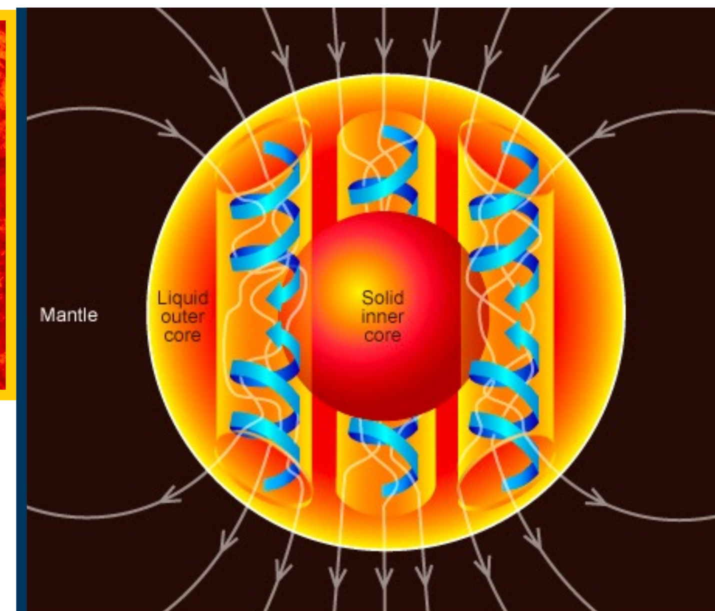
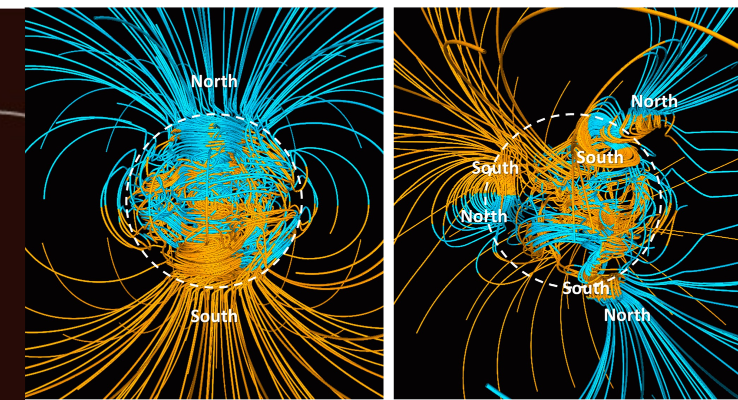
BACKGROUND



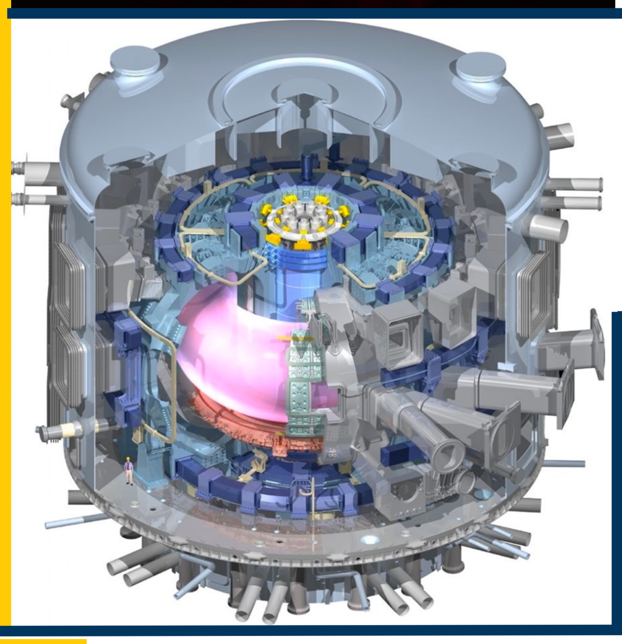


Star atmospheres





Planetary dynamo



ITER
(International Thermonuclear Experimental Reactor)

Liquid metal batteries (LMB)

PHYSICAL MODEL

GOVERNING EQUATIONS AND NONDIMENSIONAL PARAMETERS

$$\nabla \cdot \boldsymbol{u} = 0,$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \sqrt{\frac{1}{Gr}}(\nabla^2 \boldsymbol{u} + Ha^2 \boldsymbol{j} \times \boldsymbol{b}) - T\boldsymbol{e}_g$$

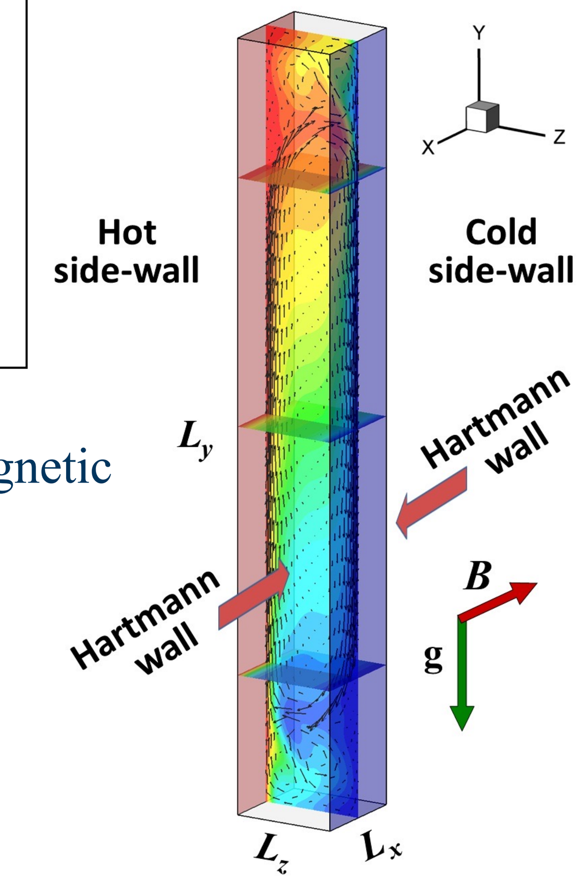
$$\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T = \sqrt{\frac{1}{Gr Pr^2}}\nabla^2 T$$

$$\boldsymbol{j} = -\nabla \phi + \boldsymbol{u} \times \boldsymbol{b}$$

$$\nabla^2 \phi = \nabla \cdot (\boldsymbol{u} \times \boldsymbol{b})$$

$$Gr \equiv \frac{g\beta \Delta T L^3}{\nu^2}, \quad Pr \equiv \frac{\nu}{\alpha}, \quad Ha \equiv BL\sqrt{\frac{\sigma}{\rho\nu}}$$

Imposed Steady Uniform Magnetic Field



Walls with finite electrical conductivity – thin wall assumption

$$C_w = \frac{\sigma_w \tau_w}{\sigma L}, \qquad \tau_w \ll L \;\Rightarrow\; \frac{\partial \phi}{\partial n}\Big|_{wall} = C_w \nabla_\perp^2 \phi\Big|_{wall}, \qquad 0 \le C_w < \infty$$

$* \text{ For } C_w = \infty, \qquad \phi = const.$

NUMERICAL MODEL

- Second-order nearly fully finite difference scheme on an arbitrary non-uniform structured grid with collocated grid arrangement [1,2]
- Time discretization is semi-implicit and based on the Adams-Bashforth/Backward-Differentiation method of the second order

1. Tensor – Product – Thomas Solver (TPT) [3]
   - Modification of the classical tensor-product technique
   - Thomas algorithm in one direction
   - Matrix multiplication in other two directions
   - Using Intel MKL deeply optimized DGEMM in order to do matrix multiplication (15% faster)
2. 2D Cyclic reduction
   - Fast Fourier Transform in one direction (requires uniform grid and conventional boundary condition

Efficiency of TPT

- Arbitrary non-uniform grids with efficient near-wall clustering
- Electric potential boundary condition on conducting walls is implemented directly instead of using outer iterations

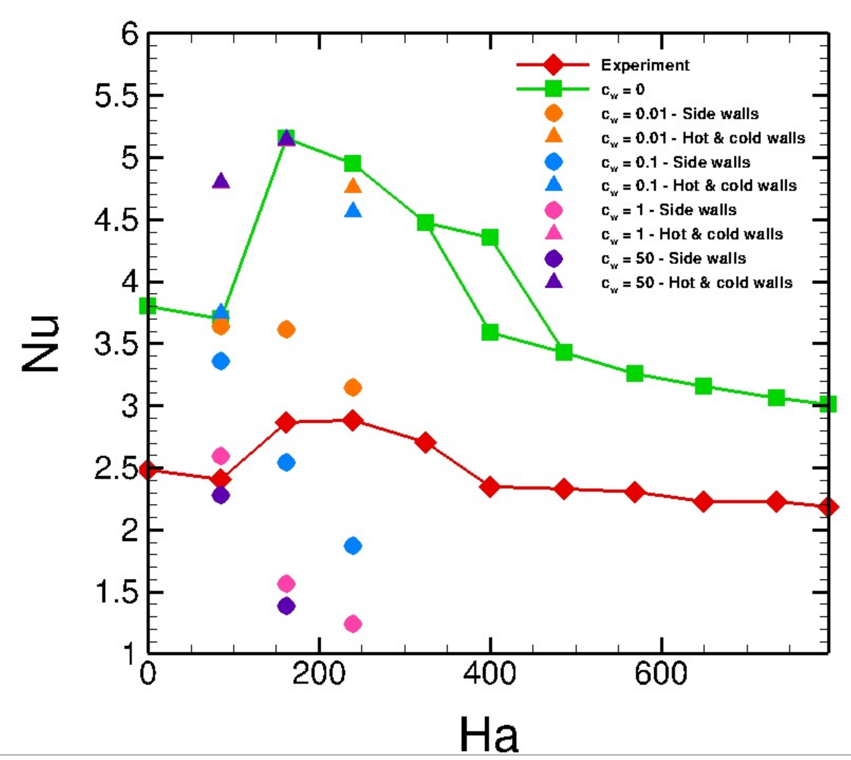
RESULTS

Control parameters

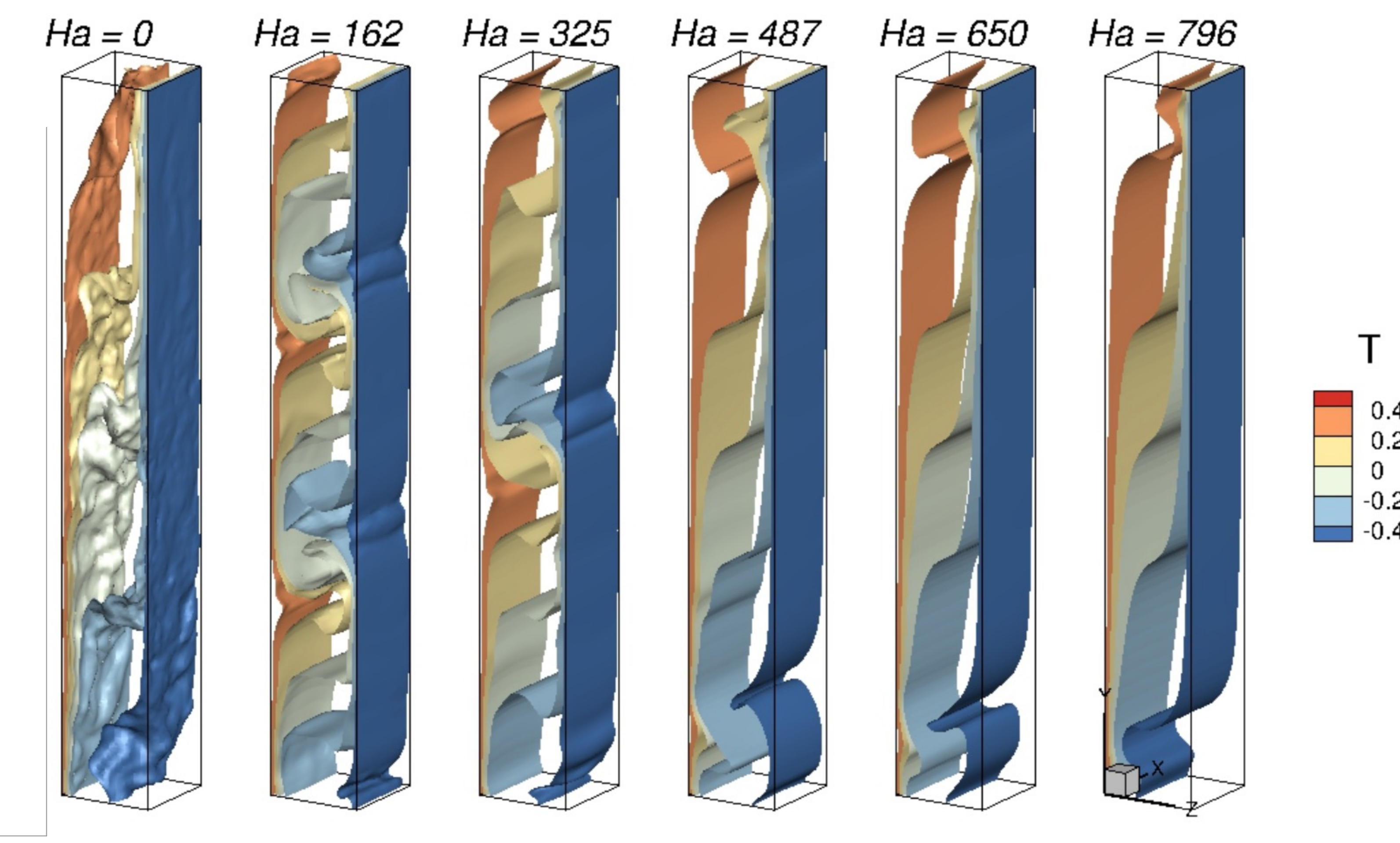
Gr	10^7
Pr	0.025
Ha	0 – 796
L(x,y,z)	1, 7.5, 1
C_w	0, 0.01, 0.1, 1, 50

Geometry and mesh

N_x	64 / 128 / 96
N_y	480 / 480 / 720
N_z	64 / 64 / 96



Iso-surfaces of temperature field T



CONCLUSIONS & FUTURE WORK

- ✓ Development and verification a new code combining TPT with a highly conservative scheme
- ✓ Direct solution of electrical potential in domains with thin walls of finite electric conductivity
- ✓ In agreement with experiments [4], an imposed magnetic field enhances heat transfer (peak of *Nu* at *Ha* about 200)
- ✓ Multiple flow states with hysteresis at Ha=400

Future work: Exploration of the effect of wall conductivity on the flow structure and convective heat transfer

AKNOWLEDGMENTS & REFERENCES

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[1] D. Krasnov, O. Zikanov, T. Boeck. Comparative study of finite difference approaches to simulation of magnetohydrodynamic turbulence at low magnetic Reynolds number. Comp. Fluids, vol. 50 (2011), pp. 46–59.
[2] O. Zikanov, Y. Listratov, V. G. Sviridov. Natural convection in horizontal pipe flow with strong transverse magnetic field. J. Fluid Mech., vol. 720 (2013), pp. 486–516.
[3] D. Krasnov, A. Akhtari, O. Zikanov, J. Schumacher. Tensor-product-Thomas elliptic solver for liquid-metal magnetohydrodynamics. J. Comp. Physics, Submitted.
[4] G. Authié, T. Tagawa, R. Moreau. Buoyant flow in long vertical enclosures in the presence of a strong horizontal magnetic field. Part 2. Finite enclosures. European Journal of Mechanics(2003), B/Fluids, 22(3), 203–220.