



Eudoxus' simultaneous risings and settings

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Abstract

The article provides a reconstruction of Eudoxus' approach to simultaneous risings and settings in his two works dedicated to the issue: the *Phaenomena* and the *Enoptron*. This reconstruction is based on the analysis of Eudoxus' fragments transmitted by Hipparchus. These fragments are difficult and problematic, but a close analysis and a comparison with the corresponding passages in Aratus suggests a possible solution.

Since the sun's motion across the sky marks the passing of time, during the day the Greeks and the Romans could reckon time by means of sundials. During the night, time could be measured by tracking celestial bodies in the sky which could be connected to the sun's motion. These are the 12 zodiacal constellations that lie along the ecliptic, which is the path that the sun completes in one year in its apparent eastward motion. Each night six zodiacal constellations rise, dividing the night into roughly six equal parts of ca. two seasonal hours each¹. Therefore, people who see a certain zodiacal constellation rising in the east and also know in which zodiacal constellation the sun is when the observation is performed (the latter information could be easily found out

¹ 'Seasonal hours' are obtained by dividing daytime and night time into 12 equal parts each, so that the duration of a seasonal hour changes during the year and depends on latitude. These are the hours usually used by the Greeks in their civil life. However, astronomers, starting from ca. 300 BCE (the first attestation is in *PHibeh* 1.27, dating to that period) also used 'equinoctial hours', which are obtained by dividing the night-and-day combination into 24 parts of equal length.

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in the *parapegmata*²), will also know the seasonal hour of the night, or how many seasonal hours remain before sunrise.³

This system is behind the section of Aratus' *Phaenomena* generally known as *Simultaneous Risings and Settings* (*Phaen.* 559–732). In it, however, Aratus does not speak of seasonal hours and the time it takes for each zodiacal constellations to rise; rather, he simply considers the rising of successive zodiacal constellations as a way to determine when the sunrise will occur. More specifically, he lists the 12 zodiacal constellations (starting with the Crab) and for each of them he says which other fixed constellations either rise or set when this zodiacal constellation is *beginning to rise*. This means that the rest of the zodiacal constellation is not visible yet; what counts is just the first stars of it that are rising above the horizon. This is functional to Aratus' goal in this section of the poem. As the poet says at the opening of *Simultaneous Risings and Settings*, the goal is to be able to determine when the sunrise will occur (*Phaen.* 559–568). However, as the zodiacal constellation occupied by the sun in a given month rises, no other constellation can be seen due to the sun's light. Hence the best solution is to pay attention to the constellations which rise and set *just before* a specific zodiacal constellation is rising, focusing thus on the very beginning of the rising of the latter. In this way, by knowing the constellations which are rising and setting, and thereby already visible when a specific zodiacal constellation is beginning to rise, Aratus' readers can know when the sun will rise—once they know in which zodiacal constellation the sun is in a specific month.

While Aratus focuses on the zodiacal constellation which is beginning to rise, in his *Exegesis of the Phaenomena of Eudoxus and Aratus* Hipparchus looks at the zodiacal sign rising, that is, from the beginning to the end of its rising. For an astronomer like Hipparchus, who worked with a celestial globe, it was indeed possible to see the synchronous risings and settings of the other constellations for the entire timespan in which a zodiacal sign was rising, since the light of the sun in this case did not come into question. Consequently, when discussing this section of Aratus' *Phaenomena*, Hipparchus focuses on the constellation/sign that comes before the one discussed by Aratus, and in paraphrasing Aratus he automatically makes the switch.

As for Eudoxus, Hipparchus says (2.1.26) that he also “assumes the beginnings of the zodiacal signs at the point of their rising (ὑποτίθεται τὰς ἀρχὰς τῶν ζῳδίων ἐπὶ τῆς ἀνατολῆς)”.⁴ The fact that Eudoxus too focused on the moment in which

² The *parapegmata* are calendars sometimes organized by the position of the sun in the zodiacal constellations (or signs) and listing the rising and setting of other fixed constellations as well as weather predictions. We have examples of *parapegmata* in manuscript form (as e.g. *P.Hibeh* 1.27, dated to the early third century BCE, or the *parapegma* at the end of Gemini's *Introduction*) and stone *parapegmata* displayed in public spaces which kept track of cyclical celestial phenomena through movable pages (e.g. the two *parapegmata* from Miletus, dating to the second century BCE). Cf. Lehoux 2007, 12–19.

³ With zodiacal signs, which are a division of the ecliptic into equal arcs of 30 degrees each, and equinoctial hours, things are more precise. Indeed, six zodiacal signs (i.e. 180 degrees on the ecliptic) rise every night. Throughout this paper I will always distinguish between zodiacal signs and zodiacal constellations, since this distinction is at the core of the different approaches of Aratus and Eudoxus (who use zodiacal constellations) and Hipparchus (who uses zodiacal signs).

⁴ The sequence ὑποτίθεται ... τὰς ἀρχὰς τῶν ζῳδίων ἐπὶ τῆς ἀνατολῆς exactly corresponds to what Hipparchus says of Aratus in 2.2.2 ὑποθέμενος τοίνυν ὁ Ἄρατος ἐπὶ τῆς ἀνατολῆς τὴν ἀρχὴν τοῦ

the zodiacal constellation is beginning to rise is for Hipparchus a further proof that indeed Aratus has followed Eudoxus.

However, when we look at the fragments of Eudoxus, we clearly see some discrepancy with this statement of Hipparchus. The few scholars (Boll, Martin, Dekker) who noted the difference in describing the simultaneous risings and settings between Aratus (who focuses on the zodiacal constellation which is beginning to rise; I will call this 'Aratus' system') and Hipparchus (who focuses on the zodiacal constellation/sign which is rising; I will call this 'Hipparchus' system') all concluded that the system adopted by Eudoxus was that of Hipparchus.⁵

Things are, however, more complicated, as Eudoxus' fragments are quite difficult to interpret, mostly due to the fact that we have only Hipparchus' testimony for Eudoxus' work on simultaneous risings and settings and in most cases Hipparchus paraphrases Eudoxus—quoting him verbatim only rarely. Needless to say, when dealing with texts preserved through paraphrases the amount of uncertainty is great. In addition, Hipparchus is often biased in reporting what he reads in Aratus, as a comparison with the *Phaenomena* shows.⁶ We cannot exclude a similar level of 'imprecise' reporting in the case of Eudoxus. Last but not least, Eudoxus wrote two works on the topic (at least according to Hipparchus, who is the only source for this information): the *Phaenomena*, which is the text used by Aratus, and the *Enoptron* or *Mirror*.⁷ Hipparchus refers and quotes from both of them in his *Exegesis*, but sometimes he simply quotes from Eudoxus without specifying which works he is referring to—which adds a further level of complexity to understanding Eudoxus' contribution to this issue.

In what follows I shall offer a reconstruction of Eudoxus' dealing with simultaneous rising and settings. This reconstruction is based on the analysis of fragments that are highly complex and full of questions—some of which remain unsolved. Yet I think it is worth looking at them and trying to make sense of our evidence, at least raising the question of how Eudoxus treated this topic, and whether his treatment was different from that of Aratus, and how.

Footnote 4 continued

Καρκίνου φησί...[Thus, assuming that the beginning of the Crab is at the point of its rising, Aratus says ...].

⁵ Boll 1903, 59-62 (who suggested that the system of Eudoxus is also adopted by Vettius Valens); Martin 2002-2003, I lxxxix (who concluded that 'Eudoxus' is actually a later prose summary from Aratus); Dekker 2013, 41-43. As Dekker 2013, 41 also explains: "Hipparchus tells us that, in the treatise he attributes to Eudoxus, the author lists those constellations that either rise or set during the time it takes a zodiacal sign to rise. Aratus constructs his lists differently and mentions those constellations that are rising or setting when a zodiacal sign is beginning to rise. When the two versions are compared it appears that the constellations listed by 'Eudoxus' as rising or setting during the rise of a zodiacal sign are actually the same as the ones that are recorded by Aratus as rising or setting when the following zodiacal sign is beginning to rise".

⁶ See Schironi (forthcoming), *passim*.

⁷ See Hipp. 1.2.2 ἀναφέρεται δὲ εἰς τὸν Εὐδόξου δύο βιβλία περὶ τῶν φαινομένων (fr. F 4), σύμφωνα κατὰ πάντα σχεδὸν ἀλλήλοις πλὴν ὀλίγων σφόδρα. τὸ μὲν οὖν ἐν αὐτῶν ἐπιγράφεται Ἐνοπτρον, τὸ δὲ ἕτερον Φαινόμενα. πρὸς τὰ Φαινόμενα δὲ τὴν ποιήσιν συντέταχεν [Two books about the visible phenomena are attributed to Eudoxus (fr. F 4), in agreement with each other in almost everything except for a few details. One of them is entitled *Enoptron* [i.e. *Mirror*] and the other *Phaenomena*. And [Aratus] has put together his poem following the *Phaenomena*].

1 The setting of Boötes

While the scholars mentioned above do not really discuss how they reached the conclusion that the system adopted by Eudoxus was that of Hipparchus, the only clear evidence that they could have used are two verbatim quotations of Eudoxus by Hipparchus (2.2.13), which can be compared with parallel passages in Aratus. The question concerns the setting of the Guardian-of-the-Bear (the Plowman, or Boötes).

Hipparchus starts by saying that (2.2.11), according to him, both Aratus and Eudoxus are completely mistaken (ὄλοσχερῶς δοκοῦσί μοι ἀγνοεῖν) when they say that the Guardian-of-the-Bear sets in opposition to four zodiacal signs: the Ram, the Bull, the Twins, and the Crab. This is of course Hipparchus' system; Aratus describes the setting of the Guardian-of-the-Bear focusing on the zodiacal constellations which are beginning to rise, so that, in his poem, during the time it takes the Guardian-of-the-Bear to set, the Bull, the Twins, the Crab, and the Lion are beginning to rise. Hipparchus then quotes two passages from Aratus:

2.2.12 ὁ μὲν γὰρ Ἄρατος τοῦ Ταύρου πρὸς τῇ ἀνατολῇ ὄντος φησί (*Phaen.* 721–722).

δύνει δ' Ἄρκτοφύλαξ ἤδη πρώτη ἐπὶ μοίρῃ
τάων, αἱ πίσυρές μιν ἄτερ χειρὸς κατάγουσιν·
ὥστε τῷ Κριῶ ἀνατέλλοντι ἄρχεσθαι αὐτὸν ἀντικαταδύνειν. ἀκολούθως
δὲ ταύτῃ τῇ ὑποθέσει καὶ τοῦ Καρκίνου ἀρχομένου ἀνατέλλειν λέγει
ταυτί (*Phaen.* 579–582).

οὐδ' ἂν ἔτ' Ἄρκτοφύλαξ εἶη πολὺς ἀμφοτέρωθεν,
μείων ἡμάτιος, τὸ δ' ἐπὶ πλέον ἐννουχος ἤδη·
τέτρασι γὰρ μοίραις ἄμυδις κατιόντα Βωώτην
ἸΩκεανὸς δέχεται.

For Aratus says that while **the Bull is close to its rising**:

“The Guardian-of-the-Bear is setting already with the first part of those which, in four, bring him down, except for his [left] hand” (*Phaen.* 721–722).

So that [the Guardian-of-the-Bear] starts to set in opposition **to the Ram that is rising**. According to such a hypothesis he also speaks in this way **when the Crab begins to rise**:

“The Guardian-of-the-Bear would not then be large on both sides; [he is] smaller in the day and the greater part of him is already at night. For the Ocean receives the Plowman as he goes down together with four parts” (*Phaen.* 579–582).

Aratus speaks of the zodiacal constellation which is beginning to rise (**bold**), which Hipparchus rephrases focusing on the zodiacal sign which is already visible above the horizon (**bold underlined**). So *Phaen.* 721–722 say that when the Bull is beginning to rise (according to Aratus' system) and the Ram is rising (according to Hipparchus' system), the Guardian-of-the-Bear begins to set. In fact, when ν Boo, which is the first star of Boötes to set (and is placed in the lower leg, according to Ptolemy), sets, the Bull is already rising; however, if we disregard the Pleiades, which indeed for Aratus were an independent constellation (*Phaen.* 254–267), on the horizon there is \omicron Tau,

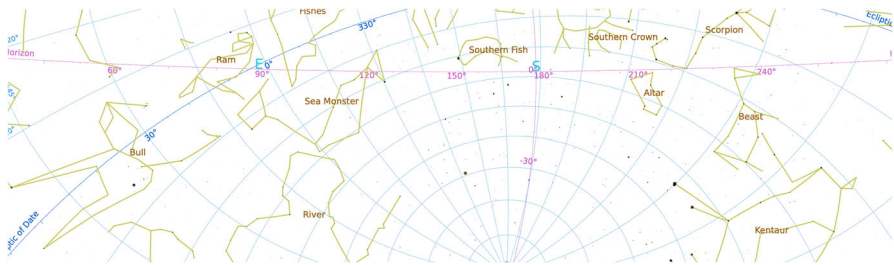


Fig. 1 Setting of the Beast when the Ram is halfway through its rising (Athens, 350 BCE)

which Hipparchus also lists (3.3.11) as the first star of the Bull to rise.⁸ At *Phaen.* 579–582, on the other hand, Aratus says that when the Crab begins to rise most of Boötes has already set.

Next, Hipparchus quotes Eudoxus verbatim:

2.2.13 ὁ δὲ Εὐδόξος γράφει οὕτως (fr. F 113): “ὅταν ὁ Κριὸς δύνῃ [sic, leg. ἀνατέλλῃ], τῶν μὲν πρὸς ἄρκτους τοῦ Ἀρκτοφύλακος οἱ πόδες δύνουσι, τῶν δὲ πρὸς νότον τοῦ Θηρίου τὸ λοιπόν.”

And Eudoxus writes as follows (fr. F 113): “**when the Ram sets [sic., leg. rises],** [the following constellations] set: of those toward the north, **the feet of the Guardian-of-the-Bear;** of those toward the south, the rest of the Beast”.

As Eudoxus speaks of the feet of Boötes (i.e. ν Boo) setting, and these are the first parts of this constellation to set, this passage refers to *Phaen.* 721–722. However, Eudoxus speaks of the Ram rising, so he is following the system of Hipparchus. He also adds that “the rest of the Beast” also sets, even if the Beast has set much earlier than ν Boo. Indeed, Aratus does not mention the Beast in the section that discusses the Bull ‘close to its rising’ (*Phaen.* 712–723)—which makes sense, because the Beast does not set when the Bull is at the point of its rising but much earlier, when the Ram is halfway through its rising (Fig. 1).

This suggests that in this passage Eudoxus is listing all the constellations that rise and set during the entire time it takes the Ram to rise, from beginning to end. This seems to be confirmed by the other direct quotation of Eudoxus, which follows right away:

2.2.13 ...ἐν δὲ τῇ ἀρχῇ τοῦ περὶ τῶν ἀνατολῶν λόγου διελθὼν, ὅσα ἐπὶ <τῆς> δύσεως καὶ ἀνατολῆς ἐστὶν ἀρχομένου τοῦ Καρκίνου ἀνατέλλειν, ἐξῆς φησὶν (fr. F 82): “ὅταν δ’ ὁ Καρκίνος ἀνατέλλῃ, τῶν μὲν πρὸς ἄρκτους οὐθὲν ἀνίσχει, τῶν δὲ πρὸς νότον ὁ Λαγῶς, καὶ τοῦ Κυνοῦ τὰ ἐμπρόσθια, καὶ ὁ Προκύων, καὶ τοῦ Ὑδροῦ ἡ κεφαλὴ· δύνει δὲ τῶν μὲν πρὸς ἄρκτους ἢ τοῦ Ἀρκτοφύλακος κεφαλῇ.”

In the beginning of his discussion of the risings, reviewing those [constellations] that are at <the> point of their setting and rising when the Crab starts to rise, he next says (fr. F 82): “**when the Crab rises,** [the following constellations] rise up:

⁸ In fact, even not counting the Pleiades, σ Tau is not the first star of the Bull to rise. See Schironi (forthcoming), at Hipp. 3.3.11.

of those toward the north, none; of those toward the south, the Hare; of the Dog, the front parts; the Forerunning-Dog; and, of the Water-Snake, the head. And of the [constellations] toward the north, the head of the Guardian-of-the-Bear sets".

One might think that the passage just read corresponds to *Phaen.* 579–582, where Aratus describes the constellations rising and setting when the Crab is beginning to rise since Hipparchus quotes these passages in the same section (2.2.12–13). If so, Eudoxus would follow here Aratus' system and focus on the Crab when it is 'beginning to rise'. However, Eudoxus' list of constellations rising and setting 'with the Crab' is the following:

- Rising: the Hare; of the Dog, the front parts; the Forerunning-Dog; and, of the Water-Snake, the head.
- Setting: the head of the Guardian-of-the-Bear".

The head of Boötes (β Boo), which is among the last stars of Boötes to set, sets when the Crab is up in the sky and the Lion is halfway through its rising (Fig. 2).

In Aratus' system, this precise moment would not be described since it does not coincide with any zodiacal constellation beginning to rise. However, the head of Boötes would still be close to its setting a bit earlier, when the Lion is beginning to rise, which is discussed at *Phaen.* 590–595:

Ἐρχομένῳ δὲ Λέοντι τὰ μὲν κατὰ πάντα φέρονται
Καρκίνῳ ὅσσ' ἐδύοντο, καὶ Αἰετός. Αὐτὰρ ὄγε γνῶξ
ἦμενος ἄλλα μὲν ἤδη, ἀτὰρ γόνυ καὶ πόδα λαιδὸν
οὐπω κυμαίνοντος ὑποστρέφει ὠκεανοῖο.
Ἄντέλλει δ' Ὕδρης κεφαλὴ χαροπὸς τε Λαγωγὸς
καὶ Προκύων πρότεροί τε πόδες Κυνὸς αἰθομένοιο.

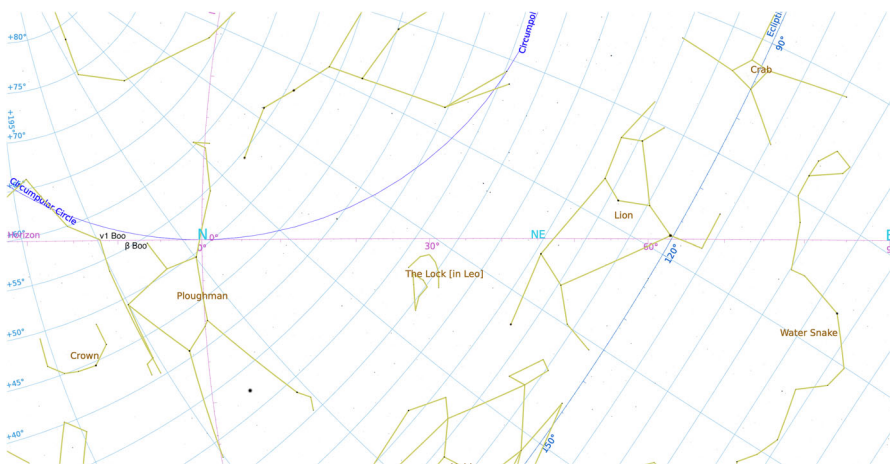


Fig. 2 β Boo setting when the Crab has already risen and the Lion is halfway through its rising (Athens, 350 BCE)

With the Lion coming, these [constellations], which were setting with the Crab, are all borne down, and the Eagle too. And the one who is on his knees is already [set] for the rest but does not yet withdraw his knee and left foot under the swelling Ocean. The head of Water-Snake rises, and the gray-blue Hare, and the Forerunning-Dog, and the front feet of the blazing Dog.

In Aratus' description, when the Lion is beginning to rise, the Water-Snake's head, the Hare, the Forerunning-Dog, and the forefeet of the Dog are rising—which corresponds exactly to Eudoxus' list above. All these constellations are above the horizon, or well visible on it, when the Crab is rising (in Hipparchus' system) and when the Lion is beginning to rise (in Aratus' system). Aratus does not mention Boötes here, but this is not necessary. When talking of the beginning of the rising of the Bull (*Phaen.* 721–723; see above) he says that this is the first of the four 'parts' which accompany the setting of Boötes (these parts are the Bull, the Twins, the Crab, and the Lion); so one can assume that Boötes keeps setting while all these other constellations (Twins, Crab, and Lion) are beginning to rise. While Aratus does not mention the setting of parts of Boötes in the section on the rising of the Twins (*Phaen.* 724–732), nor in the one on the rising of the Lion (*Phaen.* 590–595), he does mention the setting of Boötes when discussing the beginning of the rising of the Crab (*Phaen.* 569–589). This is necessary because the Crab is the first zodiacal constellation Aratus treats in the *Simultaneous Risings and Settings*. There he specifies (*Phaen.* 581–582) that “the Ocean receives the Plowman when he is setting [in the time taken] by four parts [to rise]” (τέτρασι γὰρ μοίραις ἄμυδις κατιόντα Βοώτην / Ὠκεανὸς δέχεται).

Thus, Hipparchus seems to have quoted two passages of Eudoxus that refer, one, to the beginning of the setting of Boötes (with the Ram rising and the Bull beginning to rise), the other, to the end of its setting (with the Crab rising and the Lion beginning to rise). Let us collect the evidence discussed so far in the following synoptic table:

When the Ram is rising (and the Bull is beginning to rise)

Eudoxus, fr. F 113 = Hipp. 2.2.13

Aratus, *Phaen.* 712–723

- the feet of the Guardian-of-the-Bear set (while the Ram is rising)
 - the Guardian-of-the-Bear begins his setting (721)
-

When the Bull is rising (and the Twins are beginning to rise)

Eudoxus

Aratus, *Phaen.* 724–732

- [the Guardian-of-the Bear continues to set]
-

When the Twins are rising (and the Crab is beginning to rise)

Eudoxus

Aratus, *Phaen.* 569–589

- Most of the Guardian-of-the Bear has set (579–580)
-

When the Crab is rising (and the Lion is beginning to rise)

Eudoxus, fr. F 82 = Hipp. 2.2.13

Aratus, *Phaen.* 590–595

- the head of the Guardian-of-the-Bear sets (while the Crab is rising)
 - [the Guardian-of-the Bear has set completely]^a
-

^aThis can be assumed from what Aratus says at *Phaen.* 590–591: Ἐρχομένων δὲ Λέοντι τὰ μὲν κατὰ πάντα φέρονται / Καρκίνῳ ὅσσ' ἐδύοντο [With the Lion coming, these [constellations], which were setting with the Crab, are all borne down].

Hence, Hipparchus can conclude:

2.2.13 ... δῆλον οὖν ὅτι καὶ κατὰ τὸν Εὐδόξου τὰ μὲν πρῶτα μέρη τοῦ Ἄρκτοφύλακος ἀντικαταδύνει τῷ Κριῶ, τὰ δὲ ἔσχατα τῷ Καρκίνῳ.

Thus, it is clear that according to Eudoxus as well the first parts of the Guardian-of-the-Bear set in opposition to the Ram and the last parts in opposition to the Crab.

Hipparchus summarizes here the two quotations of Eudoxus. In this case, then, Eudoxus seems to have used the same system as Hipparchus, namely, he focused on the zodiacal constellation that was rising, not on the one which was beginning to rise—which is what Boll, Martin and Dekker stated.

Another fragment of Eudoxus, still connected with the rising of the Crab, however, tells a different story. Let's start with Hipparchus' discussion of Aratus first:

2.2.2 Ὑποθέμενος τοίνυν ὁ Ἄρατος ἐπὶ τῆς ἀνατολῆς τὴν ἀρχὴν τοῦ Καρκίνου, φησὶ τοῦ μὲν Στεφάνου δεδυκέσθαι τὸ ἥμισυ, τοῦ δὲ Νοτίου

Ἰχθύος τὸ ἕως τῆς ῥάχεως, τοῦ δὲ Ἐνγόνασι τὰ ὑπὲρ τὴν γαστέρα, τὸν δὲ Ὀφιοῦχον ἕως τῶν ὤμων, τὸν δὲ Ὀφιν, ὃν ἔχει ὁ Ὀφιοῦχος, ἕως τοῦ αὐχένος, τοῦ δὲ Ἀρκτοφύλακος πλέον μέρος ἢ τὸ ἥμισυ. **2.2.3** ἀνατέλλειν δέ φησι τοῦ Ὠρίωνος τὴν ζώνην καὶ τοὺς ὤμους. τὰ εἰρημένα τοίνυν ἄστρα **τοῖς** Διδύμοις κατ' αὐτὸν ἃ μὲν συνανατέλλει, ἃ δὲ ἀντικαταδύνει.

Thus, assuming that **the beginning of the Crab is at the point of its rising**, Aratus says that [at this point] [the following constellations] have set: of the Crown, half; of the Southern Fish, the part up to the spine; of the Kneeler, the parts above the belly; the Serpent-Bearer up to the shoulders; the Serpent that the Serpent-Bearer bears up to the neck; **of the Guardian-of-the-Bear, more than half** (cf. *Phaen.* 569–580). And he says that, of Orion, the belt and the shoulders rise (cf. *Phaen.* 586–589). Thus, as far as concerns the constellations in question, according to him, **some rise together with the Twins, some set in opposition to them.**

Hipparchus paraphrases *Phaen.* 569–589, in which Aratus first lists a number of constellations that are setting or that have already set when the Crab is beginning to rise (**bold**), then one constellation that rises, namely, Orion.⁹ Hipparchus converts this description into his system, which refers to the Twins rising (**bold underlined**). He then quotes Eudoxus:

2.2.4 Ὁ δὲ Εὐδόξος (fr. F 81) ἐξαριθμούμενος, ὅσα ἐστὶν ὑπὲρ γῆς τοῦ Καρκίνου ἀρχομένου ἀνατέλλειν, φησὶ τὸν Ἐνγόνασιν ὅλον εἶναι φανερόν, τοῦ δὲ Στεφάνου τὸ ἥμισυ, καὶ τὴν τοῦ Ἀρκτοφύλακος κεφαλὴν, καὶ τὴν τοῦ Ὀφιοῦχου κεφαλὴν, καὶ τὴν οὐρὰν τοῦ ἐχομένου Ὀφειως, τῶν δὲ πρὸς νότον <τὸν> Ὠρίωνα ὅλον, καὶ τοῦ Νοτίου Ἰχθύος τὸ πρὸς τὴν κεφαλὴν.

Eudoxus (fr. F 81), listing all the [constellations] which are above the earth **when the Crab begins to rise**, says that the entire Kneeler is visible; [also visible are]: of the Crown, half; **the head of the Guardian-of-the-Bear**; the head of the Serpent-Bearer; the tail of the Serpent borne [by the Serpent-Bearer]; and, of the [constellations] toward the south, <the> entire Orion and, of the Southern Fish, the part toward the head.

⁹ *Phaen.* 569–589: When the Crab rises, not the faintest stars lie on both sides [of the Ocean], turning around, some setting and some coming up on the other [side]. The Crown sets and the [Southern] Fish sets along its spine; you would see half of the descending Crown up in the air but the farthest edges [of the horizon] already throw down half of it. But the one who is turned backward [i.e. the Kneeler] is not yet [set] in the other parts up to the lower belly, while the upper parts are borne in the night. The Crab also brings down the wretched Serpent-Bearer from the knees to the shoulders and brings down the Serpent near the neck. The Guardian-of-the-Bear would not then be large on both sides [i.e. above and below the horizon]; [he is] smaller in the day [i.e. above the horizon] and the greater part of him is already at night [i.e. below the horizon]. For the Ocean receives the Plowman as he goes down together with four parts [i.e. four zodiac constellations]; and after he is satiated with light, he occupies more than half of the passing night in the unyoking of his oxen when he sets with the sun going down. Those nights are also named after his late setting. So these [stars] set, and on the opposite side, not at all worthless, but very bright in his belt and both his shoulders, Orion, trusting in the strength of his sword, stretches along the other horn [i.e. limit] [of the Ocean], carrying the entire River.

Eudoxus speaks of Crab *beginning to rise*, just like Aratus. His list is slightly different from Aratus' because Eudoxus focuses on the constellations which are visible, while Aratus is mostly concerned with those that are setting or have set. Yet they have almost the same list with little differences, as the following table shows (I have converted Eudoxus' description so that it focuses on what is setting and rising, not on what is still visible):

When the Twins are rising (and the Crab is beginning to rise)

Eudoxus (Hipp. 2.2.4 = fr. F 81)	Aratus, <i>Phaen.</i> 569–589 (cf. Hipp. 2.2.2–3)
-------------------------------------	--

Rising

- | | |
|--|--|
| <ul style="list-style-type: none"> • the entire Orion | <ul style="list-style-type: none"> • Orion [all of it?] • the entire River |
|--|--|

Setting

- | | |
|--|--|
| <ul style="list-style-type: none"> • of the Crown, half • of the Southern Fish, all set except the part toward the head • [[the entire Kneeler is still visible]] • the Serpent-Bearer all set except the head • the Serpent all set except the tail • the Guardian-of-the-Bear all set except the head | <ul style="list-style-type: none"> • of the Crown, half • of the Southern Fish, along the spine • of the Kneeler, the parts above the belly • the Serpent-Bearer up to the shoulders • the Serpent up to the neck^a • of the Guardian-of-the-Bear, more than half |
|--|--|
-

^aThe tail and the neck of the Serpent are indeed the most northern parts of this constellation, which is almost a semi-circle in the hands of the Serpent-Bearer. Yet the tail sets later than the head.

As seen above, at **2.2.13** Hipparchus has also Eudoxus speak of the *Crab rising*, but the list of rising and setting constellations is different. Most importantly, in **2.2.4** Eudoxus says that 'when the Crab begins to rise' the head of the Guardian-of-the-Bear is visible, while in **2.2.13** he says that 'when the Crab is rising' the head of the Guardian-of-the-Bear sets. The comparison of **2.2.4** and **2.2.13** thus suggests that Eudoxus gave two different set of data:

- (1) The constellations rising and setting when a zodiacal constellation is beginning to rise (Aratus' system)
- (2) The constellations rising and setting during the time it takes a zodiacal constellation to rise from beginning to end (Hipparchus' system)

Before we proceed to draw conclusions, another set of examples should be examined.

2 Different systems? *Phaenomena* and *Enoptron* on the rising of Perseus

Other data offered by Eudoxus exhibit a similar discrepancy. The case concerns the rising of Perseus when the Fishes are beginning to rise (and the Water-Pourer is rising). Aratus does not mention Perseus rising in the section dedicated to the Fishes beginning to rise (*Phaen.* 699–708).¹⁰ However, commenting on this section of the *Phaenomena* (Hipp. 2.3.11–17) Hipparchus quotes Eudoxus:

2.3.12 ὁ δὲ Εὐδοξὸς (fr. F 108) ἐν μὲν τῷ ἐνὶ συντάγματι, ᾧ δὴ καὶ ὁ Ἄρατος κατακολουθεῖ, καὶ τοῦ Περσέως τὴν δεξιὰν χεῖρά φησιν ἀνατέλλειν, ἐν δὲ τῷ Ἐνόπτρῳ (fr. F 112a), ὅλον αὐτὸν ἀνατεταλκέναι πλὴν μικροῦ τινος· τοῦ δὲ Ὑδροῦ τὸ μέγρι πρὸς τὸν Κόρακα δύνειν.

Eudoxus (fr. F 108) in the one treatise that is also the one that Aratus follows, says that the right hand of Perseus also rises; but in the *Enoptron* [he says] (fr. F 112a) that the entirety of [Perseus] has risen except some small part; [and that], of the Water-Snake, the part up to the Raven sets.

Thus, in the *Phaenomena*, Eudoxus said that only the right hand of Perseus rises when the Fishes begin to rise, while in the later treatise, the *Enoptron*,¹¹ he claimed that the entire Perseus rises; he also said that the Water-Snake sets in the part towards the Raven. In fact, when the Fishes are close to their rising, Perseus' right hand and arm are above the horizon (Fig. 3), as Eudoxus said in the *Phaenomena*.

Perseus has almost fully risen only when the Fishes have almost all risen, and the Ram is halfway through its rising (Fig. 4), which is at variance with what Eudoxus said in the *Enoptron*.

The other fact mentioned by Eudoxus in the *Enoptron*, namely, that at the moment when most of Perseus has risen the Water-Snake has set up to the Raven, is also incorrect, for when Perseus (and the Fishes) are almost fully risen, the Water-Snake has been under the horizon for a while. As the question of the rising of Perseus is quite easy to settle, one wonders whether what Eudoxus said in the *Phaenomena* and in the *Enoptron* might be made to match by understanding these data in a different way.

In 2.3.12 there are no temporal determinations attached to the state of affairs Eudoxus is describing. Since Hipparchus mentions Eudoxus' views in the section about the Fishes beginning to rise (and the Water-Pourer rising), it is natural to assume

¹⁰ *Phaen.* 699–708: Yet much of it [i.e., the Water-Snake] still remains behind, but when the Fishes rise, [night] carries it away entirely together with the Centaur itself. With the Fishes comes the [Southern] Fish, which lies below dark Capricorn itself—not all of it, but a bit of it waits for the twelfth other part. So also the unhappy hands and the knees and the shoulders of Andromeda are all stretched in two directions, some in front and others behind, when both Fishes first show themselves from the Ocean.

¹¹ For the *Enoptron* being later than the *Phaenomena*, see Hipp. 1.3.10 and Schironi (forthcoming), at Hipp. 1.3.5–12.

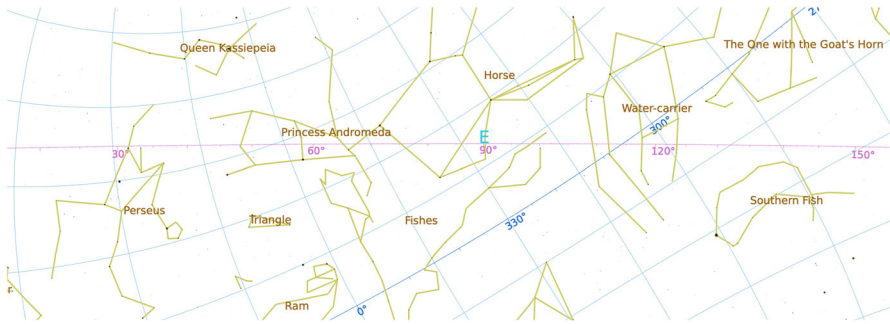


Fig. 3 Perseus' right hand and arm rising when the Fishes are beginning to rise (Athens, 350 BCE)

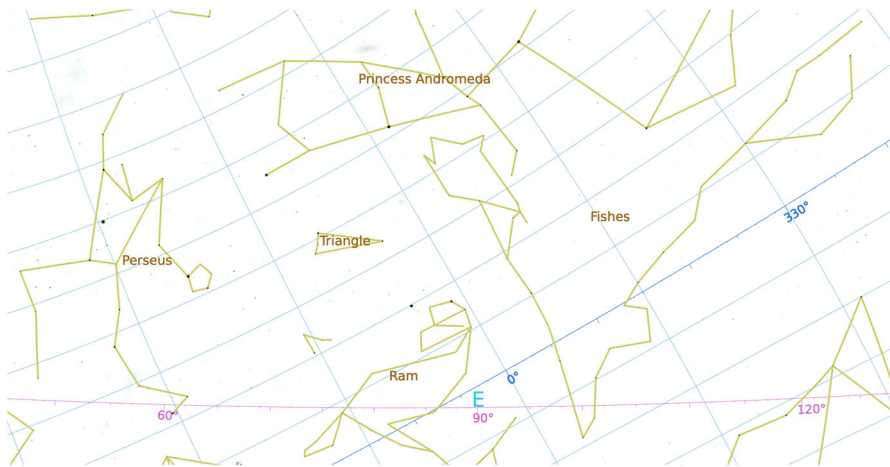


Fig. 4 Perseus almost fully risen when the Fishes have almost fully risen, and the Ram is halfway through its rising (Athens, 350 BCE)

that this applies to Eudoxus' description in both works. This assumption I would like to challenge.

The question of Perseus' rising comes back when, at **2.3.18–32**, Hipparchus discusses the section of the *Phaenomena* which deals with the Fishes rising and the Ram beginning to rise (*Phaen.* 708–714):

τὰ δ' ἀριστερὰ νειόθεν ἔλκει
 Κριὸς ἀνερχόμενος τοῦ καὶ περιτελλομένοιο
 ἔσπερόθεν κεν ἴδοιο Θουτήριον, αὐτὰρ ἐν ἄλλῃ
 Περσεὸς ἀντέλλοντος ὅσον κεφαλὴν τε καὶ ὤμους·
 αὐτὴ δὲ ζώνη καὶ κ' ἀμφήριστα πέλοιτο
 ἢ Κριῶ λήγοντι φαίνεται ἢ ἐπὶ Ταύρω.
 σὺν τῷ πανσυδίῃ ἀνελίσσεται.

...while the Ram, coming up, pulls her [i.e. Andromeda's] left parts from below. **When [the Ram] turns around [above the horizon],** you would see the Incense-Burner in the west, and in the other direction [you would see], **of the**

rising Perseus, only the head and the shoulders. As for the belt itself, it may be disputed whether it is visible with the Ram ending [its rising], or with the Bull, with which he [i.e. Perseus] rolls up [in the sky] completely.

According to Aratus, when the Ram is beginning to rise, Perseus' head and shoulders are above the horizon. He also adds that it is not clear whether the belt has already risen or will do so when the Bull is beginning to rise. According to Hipparchus (2.3.29–31), Aratus had found conflicting evidence in Eudoxus:

2.3.29 Δοκεῖ δέ μοι διηπορηῆσθαι ὁ Ἄρατος παρὰ τὴν αὐτὴν αἰτίαν, παρ' ἣν καὶ ὁ Εὐδόξος (fr. F 111), ᾧ καταηκολούθηκεν ὁ Ἄρατος. ἐν μὲν γὰρ τῷ συντάγματι τῷ περὶ τῶν φαινομένων γράφει, ὅτι τοῖς Ἰχθύσι συνανατέλλει τὰ δεξιὰ τοῦ Περσέως, ὥστε τῆς ἀρχῆς τοῦ Κριοῦ πρὸς τῆ ἀνατολῆς οὐσῆς τὰ δεξιὰ μόνον τοῦ Περσέως μέρη κατ' αὐτὸν ὑπὲρ γῆν φαίνεσθαι. ἐν δὲ τῷ ἑτέρῳ συντάγματι, ὃ ἐπιγράφεται Ἐνοπτρον, τοῖς Ἰχθύσι φησὶν (fr. F 112b) αὐτὸν ὅλον πλὴν ὀλίγου συνανατέλλειν.

Aratus seems to me to have been at a loss for the same reason as Eudoxus (fr. F 111), whom Aratus has followed, was. For in his treatise on the visible phenomena, he writes that the right [parts] of Perseus rise together with the Fishes, so that when the beginning of the Ram is close to its rising, according to him only the right parts of Perseus are visible above the earth. But in the other treatise, which is entitled *Enoptron*, he (fr. F 112b) says that **the entirety of [Perseus] except for a small part rises together with the Fishes**.

According to Hipparchus, Eudoxus did not offer the same data in his two works. In the *Phaenomena*, he stated that Perseus' right parts rise together with the Fishes—which means that these are the only parts of Perseus above the horizon when the Ram is beginning to rise. In the *Enoptron*, he said that the entire Perseus rises almost entirely together with the Fishes. This is surprising, Hipparchus goes on (2.3.30), because these two treatises generally agree.

If we compare Eudoxus' quotes in 2.3.29 and in 2.3.12 and we equate Perseus' 'right hand' (which can include the right arm) with the 'right parts' of Perseus, in the *Phaenomena* Eudoxus said that, when the Fishes are rising (and the Ram is beginning to rise), Perseus rises in the right side/arm; in the *Enoptron*, he said that most of Perseus is above the horizon when the Fishes are rising (and the Ram is beginning to rise). Hence the doubts of Aratus about whether the belt of Perseus was rising with the Ram beginning to rise, as in the *Enoptron*, or with the Bull—as in the *Phaenomena*.

However, if this is correct and Eudoxus was in both cases talking about the Fishes rising and the Ram beginning to rise, then why did Hipparchus quote the same evidence when, at 2.3.11–17, he discusses the Fishes beginning to rise (and the Archer rising)? As a matter of fact, the data associated with the *Phaenomena* in 2.3.29 are wrong: when the Fishes are rising (and the Ram is close to its rising), not only the right parts of Perseus are above the horizon but also half of his body up to the legs (Fig. 5).

In this case, thus, what Eudoxus says in the *Enoptron* is more correct. Because of this and of the incorrect statement about the Water-Snake in 2.3.12, we might wonder whether Hipparchus' quotations are misleading.

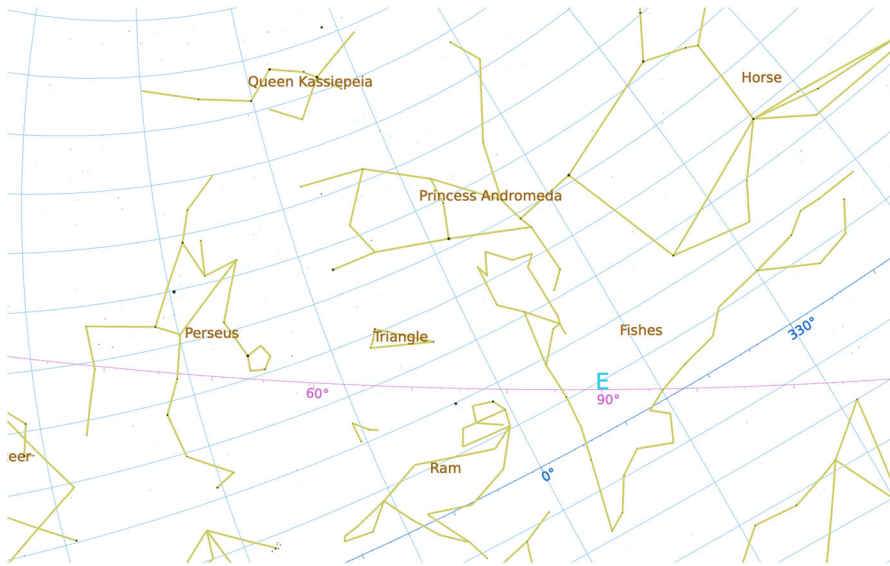


Fig. 5 Perseus with half of his body up to the legs above the horizon when the Fishes are rising (and the Ram is close to its rising) (Athens, 350 BCE)

In fact, the astronomical configuration is described with precision by Eudoxus if we assume that in the *Phaenomena* he was focusing on the Fishes beginning to rise, while in the *Enoptron* he was describing the Fishes rising, from beginning to end. If we accept this hypothesis, we could also accommodate the fact that, according to **2.3.12**, in the *Enoptron* he also stated that, when the Fishes were rising, the Water-Snake set up to the Raven. This entails that, in the *Enoptron*, Eudoxus looked at the entire time it takes each zodiacal constellation to rise. If this is the case, the astronomical configurations described in these two fragments are as follows:

Phaenomena: when the Fishes are beginning to rise (and the Water-Pourer is rising):

- the right hand of Perseus rises (fr. F 108: **2.3.12**) = the right parts of Perseus rise (fr. F 111: **2.3.29**)

Enoptron: when the Fishes are rising (from beginning to end):

- **Beginning of the Fishes' rising:** of the Water-Snake, the part up to the Raven sets (fr. F 112a: **2.3.12**)
- **End of the Fishes' rising:** the entirety of [Perseus] has risen except some small part (fr. F 112a: **2.3.12**) = the entirety of [Perseus] except for a small part (fr. F 112b: **2.3.29**).

As is clear (Fig. 6), the point in the *Phaenomena* corresponds to the first point of the *Enoptron* in time—yet the *Enoptron* gives a more complete description covering the entire rising of the Fishes.

If this is correct, at **2.3.29** Hipparchus might have confused the data offered by Eudoxus. For the passage in the *Phaenomena* Hipparchus says “the right [parts] of

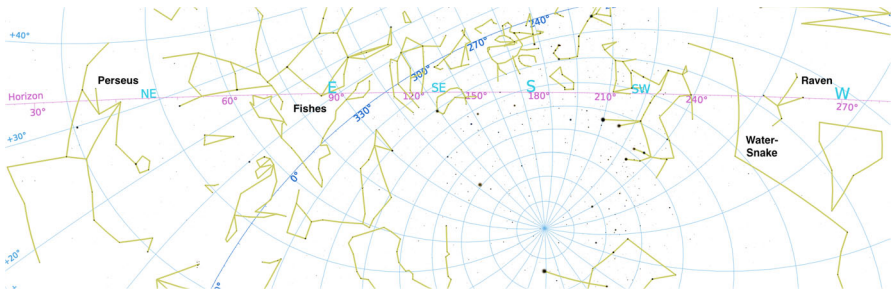


Fig. 6 The right hand (= arm) of Perseus rising and the Water-Snake up to the Raven setting when the Fishes are beginning to rise (Athens, 350 BCE)

Perseus rise together with the Fishes, so that when the beginning of the Ram is close to its rising, according to him only the right parts of Perseus are visible above the earth”, while for the passage of the *Enoptron* he speaks of almost all of Perseus rising ‘together with the Fishes’. Hipparchus does not seem to have realized that Eudoxus refers to two different temporal determinations, for ‘together with the Fishes’ in the *Phaenomena* means ‘when the Fishes are beginning to rise’. If this is correct, the further specification about the beginning of the Ram being close to its rising, must be Hipparchus’ own addition—which indeed might be the case given the phrasing (it is in a consecutive clause).

Thus, Eudoxus used two different temporal determinations in the two treatises Hipparchus cites. Hipparchus never says so explicitly, and he might not have realized this. Maybe Aratus also had a look at both texts, but he got confused about the discrepancy of the data, not realizing that different conventions were adopted. So perhaps here we could trust Hipparchus’ reconstruction of the problem concerning the belt of Perseus.

We must now go back to the question of Boötes and see whether our working hypothesis can be brought to bear on it too. The two passages are **2.2.4**, which focuses on the Crab beginning to rise, and **2.2.13**, which focuses on the zodiacal constellations of the Crab and of the Ram rising, from start to end. The two texts have already been quoted above; let us read them again in a synoptic table:

2.2.4: when the Crab is beginning to rise

Eudoxus (fr. F 81), listing all the [constellations] which are above the earth **when the Crab begins to rise**, says that the entire Kneeler is visible; [also visible are]: of the Crown, half; the head of the Guardian-of-the-Bear; the head of the Serpent-Bearer; the tail of the Serpent borne [by the Serpent-Bearer]; and, of the [constellations] toward the south, <the> entire Orion and, of the Southern Fish, the part toward the head

2.2.13: when the Ram is rising and when Crab is rising

And Eudoxus writes as follows (fr. F 113): “**when the Ram sets [sic., leg. rises]**, [the following constellations] set: of those toward the north, the feet of the Guardian-of-the-Bear; of those toward the south, the rest of the Beast”

In the beginning of his discussion of the risings, reviewing those [constellations] that are at <the> point of their setting and rising when the Crab starts to rise, he next says (fr. F 82): “**when the Crab rises**, [the following constellations] rise up: of those toward the north, none; of those toward the south, the Hare; of the Dog, the front parts; the Forerunning-Dog; and, of the Water-Snake, the head. And of the [constellations] toward the north, the head of the Guardian-of-the-Bear sets”. Thus, it is clear that according to Eudoxus as well the first parts of the Guardian-of-the-Bear set in opposition to the Ram and the last parts in opposition to the Crab

Hipparchus does not say from which work these two quotations were taken, but the example of Perseus suggests they were taken from different works, **2.2.4** being

a quotation from the *Phaenomena* and **2.2.13** a quotation from the *Enoptron*. If we focus on the time it takes for Boötes to set, the sequence is:

Enoptron (quoted in **2.2.13**).

when the Ram is rising [the following constellations] **set**:

- o of those toward the north, **the feet of the Guardian-of-the-Bear** [beginning of Boötes' setting];
- o of those toward the south, the rest of the Beast.

Phaenomena (quoted in **2.2.4**)

when the Crab begins to rise, [the following constellations] **are visible**:

- o the entire Kneeler; of the Crown, half of it; **the head of the Guardian-of-the-Bear**; the head of the Serpent-Bearer; the tail of the Serpent held [by the Serpent-Bearer];
- o and, of the [constellations] toward the south, the entire Orion and, of the Southern Fish, the part toward the head.

Enoptron (quoted in **2.2.13**).

when the Crab is rising [the following constellations] rise:

- o of those toward the north, none;
 - o of those toward the south, the Hare; of the Dog, the front parts; the Forerunning-Dog; and, of the Water-Snake, the head.
- [the following constellations] **set**:
- o And of the [constellations] toward the north, **the head of the Guardian-of-the-Bear** [end of Boötes' setting].

The only problem with this reconstruction is the way Hipparchus introduces the quotation of Eudoxus at **2.2.13**:

2.2.13 ... ἐν δὲ τῇ ἀρχῇ τοῦ περὶ τῶν ἀνατολῶν λόγου διελθὼν, ὅσα ἐπὶ <τῆς> δύσεως καὶ ἀνατολῆς ἐστὶν ἀρχομένου τοῦ Καρκίνου ἀνατέλλειν, **ἐξῆς** φησιν (fr. F 82). “ὅταν δ’ ὁ Καρκίνος ἀνατέλλῃ, ...

... In the beginning of his discussion of the risings, reviewing those [constellations] that are at <the> point of their setting and rising when the Crab starts to rise, he **next** says (fr. F 82): “when the Crab rises, ...

Hipparchus seems to suggest that *within one and the same work* Eudoxus listed first the constellations that are rising and setting when a zodiacal constellation begins to rise, and then listed the constellations that are rising and setting during the time it takes the same zodiacal constellation to rise. One could wonder whether this is simply another of Hipparchus' misunderstandings of Eudoxus. Alternatively, this could be just an imprecise way of referring to the two different works of Eudoxus, with ἐξῆς ‘next’ meaning ‘in the second treatise of Eudoxus’, that is, in the *Enoptron*. However since in this paragraph Hipparchus is allegedly only quoting from the *Enoptron*, the former hypothesis (Hipparchus' misunderstanding of Eudoxus' two different systems) is probably correct.

Another element that emerges from the above-listed quotations is that, in his list of simultaneous risings and settings, Eudoxus appears to distinguish the northern and the southern constellations in a more systematic way than what Aratus did. This is confirmed by other (non-verbatim) quotations from Eudoxus, in which he always seems to list first the northern constellations and then the southern ones.

3 Conclusions

Reconstructing how Eudoxus dealt with the simultaneous risings and settings is difficult because of the scanty documentary record and because Eudoxus sometimes appears to follow Hipparchus' system, focused on the time it takes a zodiacal constellation/sign to rise from beginning to end, and sometimes Aratus' system, focused on the zodiacal constellation which is beginning to rise only. The above analysis suggests a possible origin of this discrepancy. In his analysis of simultaneous risings and settings, Eudoxus, just like Aratus, spoke of zodiacal constellations (not zodiacal signs) rising. The analysis of two cases (the only ones which allow such an analysis) suggests that Eudoxus provided two sets of data for the simultaneous risings and settings:

- a. The constellations which either rose or set when a zodiacal constellation was *beginning to rise*.
- b. The constellations which either rose or set during the time it takes a zodiacal constellation to rise (*from beginning to end*).

Aratus selected the first set of data because they were suitable for roughly figuring out when sunrise would occur, which is the goal of *Phaen.* 559–732. Thus, Aratus did not change a dataset keyed on the time it takes a zodiacal constellation to rise from beginning to end (as listed by Eudoxus, just like Hipparchus) to a dataset keyed on a zodiacal constellation beginning to rise—which is what the reconstruction of Boll and Dekker implies.¹² The present reconstruction suggests that Aratus only selected a set of data from Eudoxus without adding anything of his own—as we should expect from a poet—and explains why Hipparchus can so often insist that Aratus, in his *Simultaneous Risings and Settings*, has copied from Eudoxus. Assuming that Eudoxus gave both sets of data also explains why Eudoxus intermittently seems to use the same system as Aratus, something Boll and Dekker left unexplained.

This discrepancy in Eudoxus' data might originate from different conventions adopted in his two different treatises, namely, the *Phaenomena* and the *Enoptron*. This might explain why these two works had these titles. The *Phaenomena* dealt with the 'visible phenomena', which any stargazer could observe when the sun was still below the horizon—this is why their focus is on zodiacal constellations beginning to rise. The *Enoptron* was a more advanced text, intended for professional astronomers, who could work with a globe: its focus was on the constellations which *were rising and setting simultaneously* during the time it takes a zodiacal constellation to rise. This might be indeed the meaning for the title *Enoptron*, 'mirror': the treatise worked

¹² On the other hand, Martin thinks that 'Eudoxus' is later than Aratus, so 'Eudoxus' must have changed the system.

on a system in which the rising (or setting) of a specific constellation was 'mirrored' by a zodiacal constellation which was also rising.

As for Hipparchus, it is not clear whether he did not realize that Eudoxus adopted different conventions. This could explain why he made mistakes in quoting from Eudoxus. On the other hand, given that oftentimes Hipparchus is less than honest when criticizing Aratus and Eudoxus,¹³ we could suppose that he did realize the difference between Eudoxus' *Phaenomena* and *Enoptron*, but did not explicitly recognize it when their (inconsistent) evidence was functional to his polemics against Aratus (and Eudoxus). If this is correct, we could also explain why Hipparchus often says that Aratus copied from the *Phaenomena* and not from the *Enoptron*: this is so because Aratus, just like Eudoxus in his *Phaenomena*, focused on a zodiacal constellation beginning to rise.

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Declarations

Conflict of interest The author states that there is no conflict of interest.

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¹³ See Schironi (forthcoming).