Turnover of Used Durables in a Stationary Equilibrium: Are Older Goods Traded More?

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This paper develops a dynamic model with transaction costs to determine the equilibrium resale pattern in a market for a durable good. The key result is that the probability of resale is nonmonotonic in the age of the good. Trade volume is relatively low in the very beginning and in the middle of a good's life. This result helps explain observed variations of resale rates across vintages for the U.S. market of used cars.

I. Introduction

The distinguishing feature of a durable good is its potential for resale. Resale is very active in many markets. For example, nearly 66 percent of all cars bought in the United States in 1995 were used (U.S. Department of Transportion 1997), more than half of all Boeing 707 aircraft changed owners during their lifetime (Goolsbee 1998), and 68 percent of all machine tools sold in the United States in 1960 were used (Waterston 1964).

When goods are long-lived and are actively traded, the demand for new goods is shaped by consumers' decisions when to trade. Consequently, patterns of resale are very important in understanding how the market operates. This paper presents a model that determines the equilibrium trade pattern in a market for a durable good. This pattern can

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be summarized by the relationship between the good's age and its probability of resale.

The evidence on probability of resale for autos by age shows an interesting regularity. It turns out that the resale pattern has a "double-hump" shape. Young vehicles have low resale rates. Probability of resale subsequently rises, peaks at around three to five years, falls, and then rises again for vehicles about 10 years old.

Previous literature did not focus on explaining resale patterns for durables. Most existing models of durable goods belong to three main categories. It the first category, there are equilibrium models without frictions, where 100 percent of all goods are traded at every moment (Rust 1985; Kursten 1991; Jovanovic 1998). In the second category, there are models with infrequent replacement but no trade (Eberly 1994; Caplin and Leahy 1999; Adda and Cooper 2000). The third category includes the models in which goods live for just two periods, so that used goods of all ages are lumped together (Bond 1983; Hendel and Lizzeri 1999; Porter and Sattler 1999).[1] Clearly, one needs a different model to account for the observed resale pattern.

This paper develops a dynamic model with transaction costs in which individual optimal replacement cycles are embedded into an equilibrium framework. Transaction costs are central to the model since they make replacement infrequent. Trade occurs because goods deteriorate with age and because consumer types differ in their marginal utility of service provided by the good. Trade allows both buyer and seller to adjust to their optimal vintage. Prices and quantities are determined simultaneously in a stationary equilibrium.

The model captures the observed resale pattern for autos. The key result behind this is that the optimal holding time is hump-shaped in consumer type: in equilibrium, high and low types hold the good for a short time, whereas middle types hold it for the longest time. Goods are not resold right away because of transaction costs, so young cars have low resale rates. When cars have depreciated sufficiently, high types sell them, generating the first peak of resale. Sellers of intermediate-age used cars are middle types who have the longest holding times. Hence they supply fewer goods per period, generating a trough in the resale rate at intermediate ages. Finally, old used cars are just a few

[1] Beyond these three categories, related work includes the paper by Holmes and Schmitz (1990), who consider a stationary equilibrium in the market for durable capital goods (businesses) with transfer costs. In their model the size of the secondary market and the range and distribution of qualities traded are determined endogenously. Konishi and Sandfort (2002) prove the existence of a stationary equilibrium in a generalized version of Rust's (1985) model with transaction costs. House and Leahy (2000) consider a model with three-period-lived goods to show how adjustment costs arise endogenously because of adverse selection.

years from being scrapped, so the low types who buy them again turn over quickly. Therefore, the resale rates for old used cars are high.

Beyond this, the model produces comparative statics that are consistent with stylized facts about the auto market documented in the literature (Hendel and Lizzeri 1999; Porter and Sattler 1999). In particular, cars that deteriorate faster have more convex used price profiles and are resold more frequently and earlier in life.

II. The Model

Goods.—Durable goods provide a useful service for the first T periods of their life. The good that has age $t = 0, \ldots, T-1$ at the beginning of the period provides the flow of service equal to x_t during this period. Older goods provide less service:

$$0 < x_{t+1} < x_t, \quad t = 0, \ldots, T-2.$$

Every good becomes useless at age T: $x_t = 0$, $t \geq T$.

Consumers are infinitely lived and can use only one durable good at a time. They differ in their marginal utility of x, denoted $h \in [0, h_{\max}]$, distributed according to an atomless density $n(h)$. The current-period utility to a consumer of type h from having a good x and c units of the numeraire is

$$u_h(x, c) = xh + c. \tag{1}$$

Consumers have the same rational expectations about future prices of all vintages and maximize their lifetime utility of owning an infinite sequence of durable goods. For each good in this sequence, they choose what vintage to buy and how long to hold it.

New goods are homogeneous, and their price, p_0, is exogenous. Any quantity can be supplied at this price.

Trade happens at the end of every period. The buyer pays the price p_t for the good of age $t \geq 0$, and the seller receives $p_t - \tilde{\Lambda}_t$, where $\tilde{\Lambda}_t$ is the transaction cost. Transaction cost is a random variable that assumes either a zero value with probability $\alpha > 0$ or a nonzero value

$$\Lambda(t, p_t) = \lambda_t p_t, \quad \lambda_t < 1, \tag{2}$$

with probability $1 - \alpha$. Since many infinitesimal sellers independently draw $\tilde{\Lambda}_t$ from the same distribution, there is no aggregate uncertainty in the model. Any good can be scrapped at zero value. Since useless goods are free and yield zero utility, consumers whose present value of participating in the market is negative do not participate.

III. Steady-State Equilibrium

For the model economy to be in the *steady state,* all markets must clear every period and all prices and quantities traded must stay constant over time. Prices and quantities that give rise to the steady-state equilibrium are to be determined simultaneously. The analysis proceeds as follows. First, the optimization problem of any consumer h is solved given the prices of all vintages. Then the optimal decision rules are aggregated to determine the steady-state distribution of durable goods holdings, that is, the state of the economy that replicates itself indefinitely when all consumers follow their decision rules. The steady-state holdings distribution generates constant resales and constant purchases for every vintage. These resales and purchases make up steady-state supply and demand, conditional on vintage prices. Finally, supply is equated to demand for each vintage in order to determine the equilibrium price vector.

A. *Optimal Replacement Problem*

In a stationary environment, prices of used goods are constant over time and depend only on the age of the good. Consumer h arrives at the beginning of the current period with a good of age $t \in \{0, \ldots, T-1\}$. The state variable for the consumer is the age of her current good. In the current period, this good yields utility flow $x_t h$. At the end of the period, the consumer sees the realization of the transaction cost and then decides whether to sell the good and purchase a replacement or keep the good for another period. Since $\lambda_t < 1$, resale is always preferred to scrappage.

In equilibrium, all consumers will follow a stationary decision rule: they will sell their goods either when they draw a zero transaction cost or when the good reaches a threshold age, whichever happens first. After resale, each consumer type will update to her optimal vintage, and the holding cycle will start again. Let $V(h;\ t)$ be the discounted present value of consumer h with the good of age t at the beginning of the current period[2] and let $\beta < 1$ be the discount factor. The Bellman

[2] Technically speaking, the value function V and the optimal decision rules will depend on the price vector as well, but this dependence is suppressed in the notation for convenience.

equation describing this optimal replacement problem reads as

$$V(h;\ t) = x_t h + \beta\alpha \max\left\{\max_S\,[V(h;\ S) - p_S] + p_{t+1},\ V(h;\ t+1)\right\}$$
$$+\ \beta(1-\alpha)\max\left\{\max_S\,[V(h;\ S) - p_S] + p_{t+1}(1-\lambda_{t+1}),\right.$$
$$\left. V(h;\ t+1)\right\}. \qquad (3)$$

The first term is the value of service provided by the good during the current period, the second term is the expected value of the resale opportunity with a zero transaction cost, and the third term is the expected value of the resale opportunity with a positive transaction cost. Let

$$S_h = \arg\max_S\,[V(h;\ S) - p_S] \qquad (4)$$

be the optimal vintage for consumer h. Since the problem is stationary, consumer h will purchase the good of age S_h every time she replaces her durable. I shall call S_h the *buying point* for consumer h. Since for every t

$$\max_S\,[V(h;\ S) - p_S] \geq V(h;\ t+1) - p_{t+1},$$

any consumer who draws a zero transaction cost resells her good and returns to her buying point. A consumer who draws a positive transaction cost replaces her good when the gain from resale exceeds the transaction cost:

$$\max_S\,[V(h;\ S) - p_S] - [V(h;\ t+1) - p_{t+1}] > \lambda_{t+1} p_{t+1}.$$

Let

$$\tau_h = \min\{t : V(h;\ S_h) - p_{S_h} - [V(h;\ S_h + t) - p_{S_h+t}] > \lambda_{S_h+t} p_{S_h+t}\}. \qquad (5)$$

Scrapping is free, so any good will be replaced before age T. Expression (5) says that consumer h will sell her good at age $S_h + \tau_h$ even if she draws a positive transaction cost. I shall therefore use the term *selling point* for $S_h + \tau_h$ and the term *holding time* for τ_h. The interval $[S_h,\ S_h + \tau_h - 1]$ will be called the *holding interval*.

If consumer h follows a decision rule with the buying point S and the holding time τ, we can compute her lifetime utility by substituting the decision rule into the Bellman equation (3). This yields the following expressions for the value function on the holding interval:

$$\begin{aligned} V(h;\ S+i-1) &= x_{S+i-1}h + \alpha\beta[V(h;\ S) - p_S + p_{S+i}] \\ &\quad + (1-\alpha)\beta V(h;\ S+i), \quad i = 1, \dots, \tau-1; \\ V(h;\ S+\tau-1) &= x_{S+\tau-1}h + \alpha\beta[V(h;\ S) - p_S + p_{S+\tau}] \\ &\quad + (1-\alpha)\beta[V(h;\ S) - p_S + p_{S+\tau} - \lambda_{S+\tau}p_{S+\tau}]. \end{aligned}$$

Using the notation

$$\gamma = (1-\alpha)\beta, \quad \alpha\beta = \beta - \gamma$$

and making recursive substitution, we can find the expression for the optimal value function at the buying point:

$$V(h;\ S) - p_S = \max_{\tau} U(h,\ S,\ \tau), \tag{6}$$

where

$$U(h,\ S,\ \tau) = \frac{h\sum_{i=1}^{\tau} x_{S+i-1}\gamma^{i-1} - p_S + (\beta-\gamma)\sum_{i=1}^{\tau} p_{S+i}\gamma^{i-1} + \gamma^{\tau}p_{S+\tau}(1-\lambda_{S+\tau})}{(1-\beta)[(1-\gamma^{\tau})/(1-\gamma)]}. \tag{7}$$

In this expression, $U(h,\ S,\ \tau)$ equals the consumer's expected lifetime utility measured at the buying point. The optimal decision rule must yield the maximum expected utility:

$$(S_h,\ \tau_h) = \arg\max_{S,\tau} U(h,\ S,\ \tau) \tag{8}$$

subject to

$$0 \le S \le T-1,$$

$$1 \le \tau \le T-S.$$

If initially the consumer has a good of age $t_0 \in [S_h,\ S_h + \tau_h - 1]$, then by construction she will find it optimal to follow the decision rule (S_h, τ_h). However, since we have not determined the optimal value function outside the holding interval, we do not know how the consumer will behave if her initial state is not in $[S_h,\ S_h + \tau_h - 1]$.[3] Nevertheless, (8) will be sufficient for all consumers to abide by the equilibrium behavior,

[3] The optimal policy may not be an $(S,\ s)$ rule because, depending on the values of x_t and λ_t for different t, the value function may not be concave. If, e.g., a consumer is given a good whose age is greater than her selling point, she may not want to return to the buying point immediately, but may instead choose to keep holding the good.

since in equilibrium the ages of goods owned by consumers will *always* belong to their respective holding intervals.

I conclude the discussion of decision rules by proving that they are monotonic in h. It turns out that no matter what the prices are, higher types prefer to buy younger goods and resell them earlier. Formally, the buying point S_h and the selling point $S_h + \tau_h$ are step functions that map $[0, h_{\max}]$ into $\{0, \ldots, T\}$. The following proposition says that these step functions are nonincreasing functions of h.

PROPOSITION 1. *Monotonicity of decision rules.*—Let (S_h, τ_h) be the solution to the optimal replacement problem. Then S_h and $S_h + \tau_h$ are nonincreasing functions of h for almost all prices.[4]

Proof. See the Appendix.

The essential assumption on $u_h(x, c)$ that is required for proposition 1 is that it is linear in c.[5] The result in the proposition will be important in characterizing the equilibrium.

I shall now turn to describing the distribution of durable goods across consumer types that gives rise to the steady-state equilibrium.

B. *Steady-State Holdings Distribution*

In the steady state, the distribution of durable goods across consumer types must stay the same every period and replicate itself indefinitely. This holdings distribution will generate constant purchases and constant resales for every vintage, which will make up steady-state supply and demand.

Let $f(h, t)$ be the number of consumers of type $h \in [0, h_{\max}]$ who hold the goods of age $t = 0, \ldots, T-1$ at the beginning of the current period. Since trade happens at the *end* of the period, no one holds goods of age T at the beginning of the period. In the steady state, $f(h, t)$ must be the same every period. The optimal decision rules (S_h, τ_h) impose a certain law of motion on the holdings distribution $f(h, t)$. In particular, consumers do not own goods whose ages are outside of their holding interval:

$$f(h, t) = 0, \quad t < S_h \text{ or } t > S_h + \tau_h - 1. \tag{9}$$

[4] Except for price vectors for which all three functions S_h, $S_h + \tau_h$, and τ_h have a discontinuity at the same point $\hat{h} \in [h_{\min}, h_{\max}]$. However, the subset of such price vectors has measure zero.

[5] Linearity in h and x is not essential because the utility function can be transformed by an appropriate choice of units of h and x.

Inside the holding interval, the transitions of consumer h across states can be summarized by the following $\tau_h \times \tau_h$ matrix:

$$\mathbf{\Pi}_h = \begin{pmatrix} \alpha & 1-\alpha & 0 & \dots & 0 \\ \alpha & 0 & 1-\alpha & \dots & 0 \\ \dots & \dots & 0 & \dots & \dots \\ \alpha & 0 & \dots & \dots & 1-\alpha \\ 1 & 0 & \dots & 0 & 0 \end{pmatrix}.$$

The first row of $\mathbf{\Pi}_h$ corresponds to state S_h and the last row corresponds to state $S_h + \tau_h - 1$. In every state, the consumer draws a zero transaction cost with probability α and returns to her buying point S_h next period. With probability $1 - \alpha$ the consumer moves to the state in which her good is one period older. When the consumer is one period away from her selling point, she will return to the buying point for sure. For each consumer type, the steady-state holdings distribution is the stationary distribution of her transition matrix $\mathbf{\Pi}_h$:

$$f(h, t) = \begin{cases} 0 & t < S_h \text{ or } t > S_h + \tau_h - 1 \\ \dfrac{\alpha(1-\alpha)^{t-S_h}}{1-(1-\alpha)^{\tau_h}} n(h) & S_h \leq t \leq S_h + \tau_h - 1. \end{cases} \tag{10}$$

We can now compute steady-state flows of goods as functions of consumer type[6] and age of the good.

Type h consumers demand goods only at the buying point S_h. Therefore, their contribution to demand for goods is simply

$$q_d(h, t) = \begin{cases} 0 & t \neq S_h \\ f(h, S_h) & t = S_h. \end{cases} \tag{11}$$

Similarly, type h consumers supply used goods at every point of their holding interval, because whenever the transaction cost is zero, they sell their current good. Accordingly, the supply function for type h consumers whose decision rule is (S_h, τ_h) can be written as

$$q_s(h, t) = \begin{cases} 0 & t < S_h + 1 \text{ or } t > S_h + \tau_h \\ \alpha f(h, t-1) & S_h + 1 \leq t \leq S_h + \tau_h - 1 \\ f(h, t-1) & t = S_h + \tau_h. \end{cases} \tag{12}$$

Using the expressions (9)–(12) and integrating over consumer types with the same decision rule, we can determine the steady-state supply and demand for every vintage. The steady-state supply and demand depend on the price vector $\mathbf{p} = (p_t)_{t=0}^{T}$ through the decision rules $(S_h,$

[6] More precisely, consumer type and (S_h, τ_h), which are themselves functions of h.

τ_h). All the elements are now in place to define the steady-state equilibrium.

C. *Equilibrium*

Definition. Steady-state equilibrium consists of the price vector $\mathbf{p} = (p_0, p_1, \ldots, p_{T-1}, 0)$, the steady-state holdings distribution $f(h, t)$, the marginal consumer $h_{\min}$, and the optimal decision rule (S_h, τ_h), $h \in [h_{\min}, h_{\max}]$ such that the following conditions hold:

1. The steady-state holdings distribution $f(h, t)$ is given by (10) for every participating consumer $h \in [h_{\min}, h_{\max}]$ and $f(h, t) = 0$ for nonparticipating consumers $h \in [0, h_{\min})$.
2. Prices for all useful goods are positive:

$$p_t > 0, \quad t = 0, \ldots, T-1,$$

$$p_T = 0,$$

and supply equals demand for any used good that is not scrapped:

$$Q_s(t) = \int_{h_{\min}}^{h_{\max}} q_s(h, t)dh = \int_{h_{\min}}^{h_{\max}} q_d(h, t)dh = Q_d(t),$$

$$t = 1, \ldots, T-1. \tag{13}$$

3. Consumers choose the decision rule that maximizes their lifetime utility:

$$(S_h, \tau_h) = \arg\max_{(S,\tau)} U(h, S, \tau).$$

4. The marginal consumer[7] is indifferent between buying a used good and taking a useless good for free:

$$h_{\min} = \frac{p_{T-1}}{x_{T-1}}.$$

For the applications, it makes sense to restrict attention to steady-state equilibria with positive prices. Positive equilibrium prices of all useful vintages can be guaranteed if new goods price p_0 is large enough relative to the values of T and x_t.

Although the equilibrium can be computed only numerically, it is possible to characterize it in an important way.

Proposition 2. In any steady-state equilibrium with positive prices,

[7] Nonnegativity of utility $U(h, S_h, \tau_h) \geq 0$ is equivalent to $h \geq p_{T-1}/x_{T-1}$. The proof is in the Appendix.

the holding time τ_h is (weakly) increasing in h around $h_{\min}$ and (weakly) decreasing in h around $h_{\max}$.

Proof. The proof will use two properties of the steady-state equilibrium: that equilibrium decision rules are monotonic functions of consumer type (proposition 1) and that goods are not scrapped before age T. The latter statement follows from the fact that the net proceeds from every resale are positive,

$$p_t - \Lambda(t, p_t) = (1 - \lambda_t)p_t > 0 \quad \text{for every } t,$$

so that every consumer prefers resale to scrappage. Therefore, scrapping the good before age T cannot be anyone's optimal decision rule.

In the steady state, the total number of goods held in the economy at a moment in time equals the number of participating consumers, $1 - N(h_{\min})$. Because all goods are retired at age T, $[1 - N(h_{\min})]/T$ consumers must buy new goods each period, and an equal number of consumers must retire their goods. Proposition 1 then implies that there must be an interval of types (including $h_{\max}$) whose buying point is zero (new goods) and an interval of types (including $h_{\min}$) whose selling point is T. Take two consumers, h_1 and $h_2 < h_1$, whose buying point is zero. The higher type h_1 must hold the good for a shorter time: $\tau(h_1) \leq \tau(h_2)$. Since the types whose buying point is zero are at the top of the type distribution, holding time $\tau(h)$ will be *decreasing* in h for some interval including $h_{\max}$. Similarly, take two consumers, h'_1 and $h'_2 < h'_1$, whose selling point is T. They must be at the bottom of the type distribution, and the higher type h'_1 must buy a younger good: $S(h'_1) \leq S(h'_2)$. As a result, holding time $\tau(h)$ will be *increasing* in h near the bottom of the type distribution. Q.E.D.

Remark.—Proportional transaction costs assumed in (2) guarantee that resale is always preferred to scrappage. Proposition 2 will still hold if we instead assume that transaction cost $\Lambda(t, p_t)$ is bounded above by a function $\bar{\Lambda}(t)$. Although there is no closed-form expression for this bound, $\bar{\Lambda}(t)$ can be computed from the primitives of the model in a straightforward manner. For details, see the discussion in Section IV*B*3.

IV. Application: Used Automobiles

Proposition 2 says that consumers at the extremes of the type distribution turn over their goods faster than the ones in the middle. This property will play an important role in interpreting the observed holding patterns for cars. Before turning to the numerical results, I shall discuss the evidence from the U.S. market for used automobiles.

A. *Evidence on Resale of Automobiles*

The evidence on resale of automobiles in the United States comes from the 1995 Nationwide Personal Transportation Survey (NPTS), a data set with more than 75,000 observations. The data presented pertain to six major automobile manufacturers in the model year range 1982–96, which covers more than 75 percent of all cars in the sample. Figure 1 shows resale rates as a function of model year, which is taken as a proxy for the vehicle's age. The vertical axis of each plot shows the observed fraction of vehicles of a particular age purchased *in used condition* in 1995. The horizontal axis shows the vehicle's age, with model year 1996 at age 0 through model year 1982 at age 14.

There are several factors that can make the resale rate vary with age. Since the resale rate is a *fraction,* the variations in resale rates across vintages are not due to high or low sales of new vehicles in the past. Resale rates may also vary because some vintages of cars are very popular. The "good vintage" effects are controlled for by pooling together all car models by the same manufacturer. Because the introduction of new models is usually staggered, only a fraction of observations for a particular model year can conceivably come from the good vintage. Besides, the effect on quantity traded is ambiguous: a good vintage is not only something that consumers want to *buy* (increased demand) but also something that other consumers want to *keep* (reduced supply).

All makes exhibit similar regularities in resale patterns: the resale rate is very low for young cars and it peaks when vehicles are three to four years old; then the resale rate stays relatively low for several years, and then goes up again when the vehicle is about 10 years old.

The patterns in figure 1 are confirmed by tests. The tests are based on the difference in resale rates for two consecutive years. The shaded bars in figure 1 show the first year, when the resale rate drops significantly, and the first subsequent year, when the resale rate rises significantly. The hypothesis that the resale rate for cars that are more than two years old is monotonic in the vehicle's age can be rejected at 10 percent significance or better for all makes but Nissan. The p-values for each test are reported on top of the shaded bars in figure 1.

The observations record only purchases, but not the vehicle ownership histories. Ideally, one would like to exclude fleet sales by rental car companies because they come from agents who hold multiple vehicles at a time and thus face a different decision problem. However, since fleet sales usually involve one-year-old cars, excluding them from the sample would likely lower the resale rate at one year and make the nonmonotonic pattern in figure 1 even more pronounced.

The evidence also shows how frequency of trade varies by vehicle make. On average, Hondas and Toyotas are traded significantly less than other makes considered.[8] Subsection *B* explains why resale rates are relatively low in the middle of a car's life and why some automobiles are traded more frequently than others.

B. Results

The computation proceeds along the steps described in Section III. Each iteration starts with computing the optimal decision rules using some fixed price vector as an input. Next, using the decision rules just obtained, one can solve the supply equals demand equations (13) to find the market-clearing price. This market-clearing price is used as an input for the next iteration, and so forth. Essentially, each iteration computes the value of an operator that maps price vectors into price vectors. By construction, the fixed point of this operator is the equilibrium price.

1. Choice of Parameter Values

The physical lifetime of a car is taken to be $T = 15$ years. Deterioration is assumed to be exponential, with a constant rate δ and $x_0 = 1$ as a normalization:

$$x_t = x_0(1-\delta)^t, \quad t = 1, \dots, T-1.$$

The transaction cost of selling a used vehicle is measured with the difference between its market price and the trade-in value. In the model, the ratio of this difference to market price is equal to λ. It turns out that the ratio of transaction cost to price rises with the vehicle's age and roughly doubles over a vehicle's lifetime.[9] Accordingly, I set

$$\lambda_t = \lambda_1 2^{(t-1)/(T-2)}, \quad t = 2, \dots, T-1. \tag{14}$$

In the model, goods of all ages are traded with a probability of at least α. Typically, resale rates are minimal (i.e., equal to α) for young vintages. We can therefore choose the value of α to match the resale rate for one- to two-year-old vehicles in the data.[10] This implies $\alpha = 0.1$.

The values of p_0, $h_{\max}$, λ_1, and δ are chosen by fitting the equilibrium price predicted by the model[11] to the normalized price series for used automobiles reported in Porter and Sattler (1999, table 4). The resulting

[8] Hendel and Lizzeri (1999) find a similar result: they use the data from the 1991 Consumer Expenditure Survey to conclude that Fords are traded more often than Hondas.

[9] This calculation was performed using trade-in and market values reported by Edmunds.com (http://www.edmunds.com/used/).

[10] The average resale rate for used one-year-old vehicles is 0.058 and for used two-year-old vehicles it is 0.144. Source: 1995 NPTS.

[11] If one assumes that $n(h)$ is uniform.

a

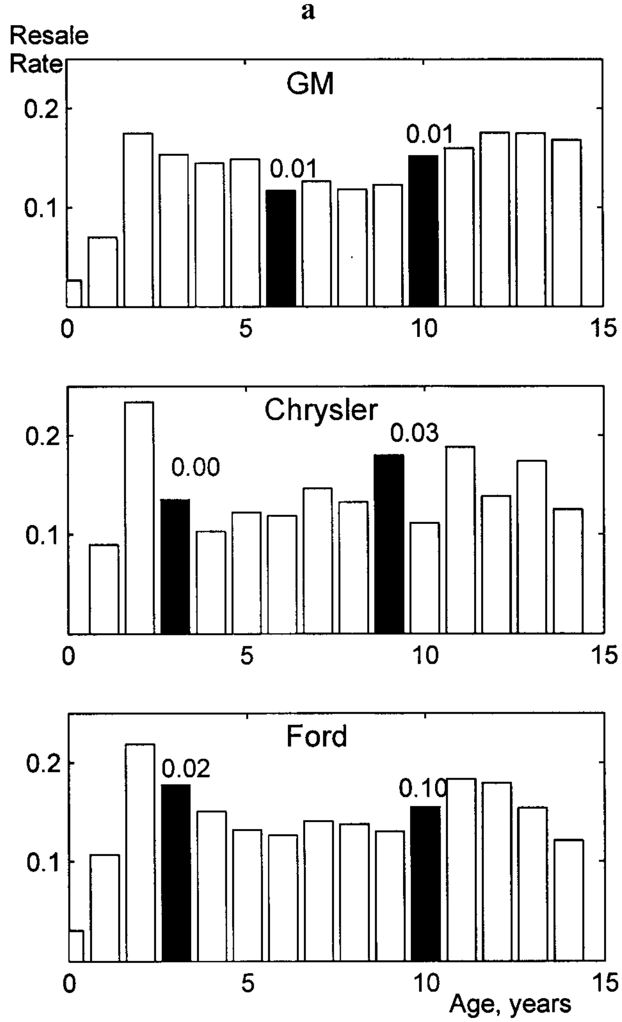

FIG. 1.—Resale rates as a function of age for major car makes. Shaded bars show ages when resale rate drops or rises significantly; numbers on top of the shaded bars are p-values.

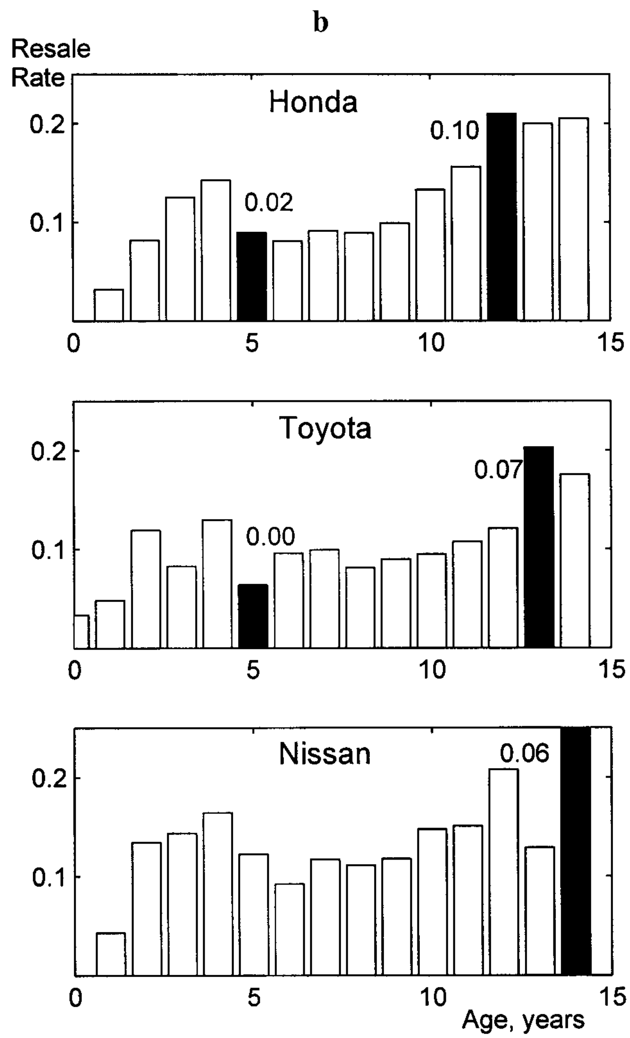

FIG. 1.—*Continued*

benchmark parameter values are $p_0 = 3.95$, $h_{\max} = 1.9$, $\lambda_1 = 0.075$, and $\delta = 0.085$. The unit of measurement is the stream of service from the brand new good, $x_0 = 1$. To be more specific, the price of a new car approximately equals the present value of five years of its service to the median consumer. The real interest rate is set to 0.04,[12] which implies $\beta = 0.96$.

[12] Since utility is linear in the numeraire commodity, utility and wealth are equivalent,

2. Numerical Results

The simulations for prices and quantities traded are made for different plausible assumptions about the distribution of types $n(h)$. When $n(h)$ is uniform, variations in resale rates cannot be due to different densities of consumers who prefer particular vintages. This case clearly illustrates how the variation in holding times across consumer types translates into the variations in resale rates. Take, for example, consumers at the top of the type distribution who buy new cars. New-good buyers sell their cars at age 4, 5, 6 or 7, with lower types holding cars for a longer time (lower left panel of fig. 2*a*). Accordingly, the resale rate falls for five- to seven-year-old cars (upper left panel of fig. 2*a*). Next, take the consumers at the bottom of the type distribution who hold the good until it falls apart. This group buys goods in the 11–14-year age interval. Accordingly, the resale rate rises with age for these vintages. Roughly speaking, the number of consumers who buy a particular vintage is in the numerator of the resale rate, and their holding time is in the denominator.[13] The number of consumers with the same buying point differs from vintage to vintage, which also affects the resale rate. For example, there are very few consumers who buy 14-year-old cars, which pushes their resale rate slightly down.

3. Robustness

Distribution of consumer types.—Consumer type in the model most likely corresponds to some increasing function of income. Because type is determined outside the model, we must explore how the resale pattern is affected by alternative assumptions about the type distribution. Let us consider two cases: when the distribution of types is extremely skewed to the left and when it is extremely concentrated in the middle. Even these extreme[14] changes in the distribution of types preserve the basic two-hump resale pattern, as shown in figure 2.

At first glance, it may seem that if there are many low-type consumers, the resale rates for old used goods should also be high. In fact, this is not the case. The intuition for this result becomes transparent when

so $\beta = 1/(1+r)$.

[13] Observe that the number of consumers at the buying point $f(h, S_h)$ is approximately equal to

$$\lim_{\alpha \to 0} f(h, S_h) = \frac{n(h)}{\tau_h}.$$

[14] The distribution that is skewed to the left is a truncated normal with mean $h = 0$ (the bottom of the market) and standard deviation $h_{\max}/4$. This implies that $n(h_{\max})$ is almost $e^8 \approx 3{,}000$ times less than $n(0)$. The distribution that is concentrated in the middle is a truncated normal with mean $h_{\max}/2$ (symmetric) and standard deviation $h_{\max}/8$, so that $h_{\max}$ is again four standard deviations from the mean.

one takes into account how the change in the distribution of types affects equilibrium prices (middle right panel of fig. 2*a*). Because the demand for old used goods is now relatively high, holding a used good (that will sell for a high price when old) becomes more attractive. As a result, the prices for *all* used goods go up. Since used goods now depreciate slowly, first and second owners hold them for a longer time (see the corresponding decision rule plot), driving down the supply of old used goods. Higher prices for old used goods decrease market participation and sustain this low supply in equilibrium.

Next, take the case in which there are a lot of consumers in the middle of the type distribution. This drives up demand for midlife used goods and their prices. Prices for younger used goods become higher as well. Because of lower depreciation, first owners now hold their goods for a longer time. By contrast, second owners who buy midlife used goods face a thinning resale market. Therefore, prices for old used goods are low. Second owners now face *faster* depreciation and resell their goods faster, driving up the supply of old used goods. Finally, low prices for old used goods increase market participation and sustain the high demand for these goods.

Transaction costs.—A different transaction cost level will affect the equilibrium trade pattern mainly through changes in optimal holding times. When transaction costs are extremely high, no one resells young or midlife used goods unless the transaction cost is zero, so the resale rate for all vintages but the last few is constant at α. The trough of resale at middle ages starts to disappear when transaction costs reach about 80 percent of the sale price. It is doubtful that the auto market has this level of transaction costs, because the implied share of used goods in total sales is unrealistically low.

The transaction cost function $\Lambda(t, p_t)$ need not be proportional to price. The equilibrium can be computed in exactly the same manner whenever transaction costs are bounded:

$$\Lambda(t, p_t) \leq \bar{\Lambda}(t) \quad \text{for all } t.$$

For a given set of primitives, $\bar{\Lambda}(t)$ can be found using the property that equilibrium market prices of all vintages increase in Λ and are the lowest when transaction costs are zero.[15] Therefore, if we set $\bar{\Lambda}(t)$ equal to the equilibrium price at zero transaction cost,

$$\bar{\Lambda}(t) = p_t|_{\Lambda=0} \quad \text{for all } t,$$

we can guarantee that resale is preferred to scrappage in *any* equilibrium

[15] This is intuitive: as transaction costs increase, the supply of used goods of all ages is reduced because sellers hold their goods for a longer time. This implies that prices must rise to bring the market in equilibrium.

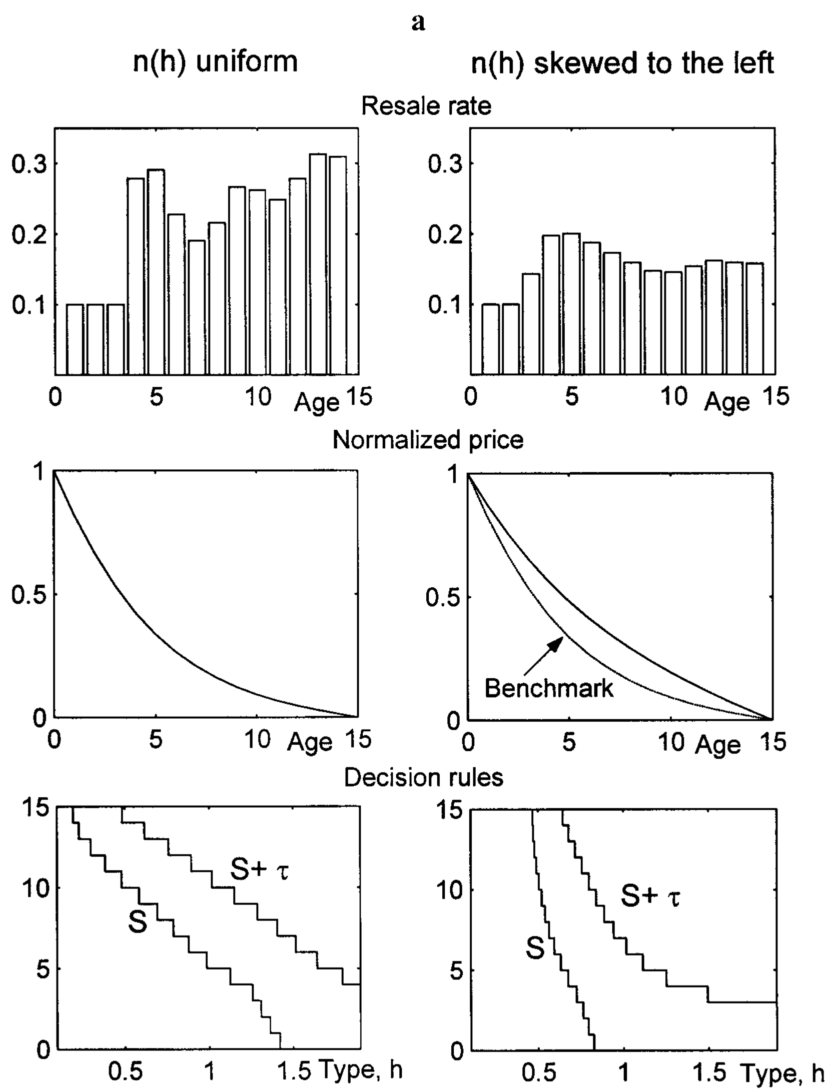

FIG. 2.—Simulations of resale patterns and prices for different assumptions about $n(h)$.

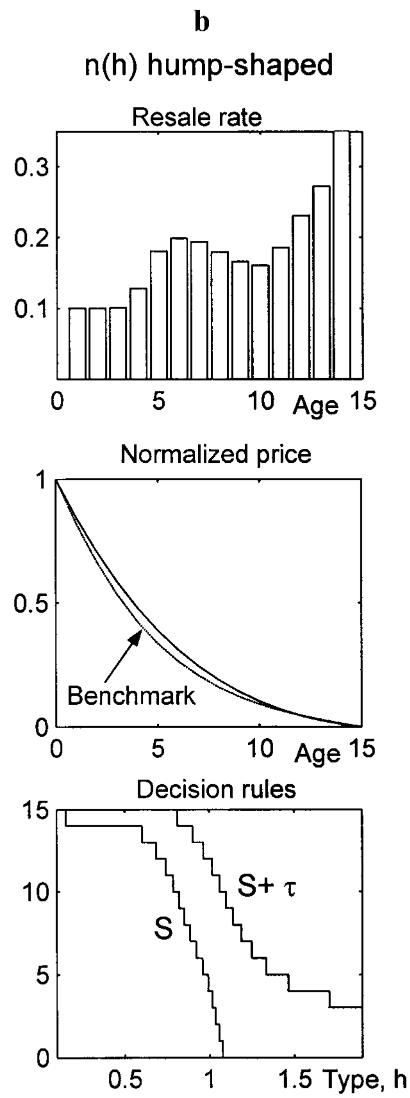

FIG. 2.—*Continued*

with bounded $\Lambda(t, p_t)$:

$$p_t - \Lambda(t, p_t) \geq p_t - \bar{\Lambda}(t) = p_t - p_t|_{\Lambda=0} > 0.$$

4. Comparative Statics

Different vintages of the durable are essentially different goods. Faster deterioration (higher δ) makes vintages more different and increases the gains from trade. Resale rates go up as a result. Now the goods are passed down to the lower types faster, which implies that prices should fall faster as well. With all other parameters held constant, cars that deteriorate faster have more convex prices and shorter holding intervals.

If we assume that a fixed fraction of each consumer type is loyal to a certain brand of car, the model would predict that less reliable brands should be traded more frequently and earlier in life. The evidence supports these predictions. Porter and Sattler (1999, tables 6–8) report that unreliable vehicles are traded more frequently. They also find that "the rate of decline of a used car model's prices is negatively and significantly correlated with the length of ownership tenure" (p. 3). In the model, more convex prices imply shorter holding intervals.

Hendel and Lizzeri (1999) have a related finding. They consider a simple model with two brands of two-period-lived cars and show that the brand that deteriorates faster has a larger volume of trade and a steeper price decline.

There is also evidence that reliable vehicles are traded later in life. According to Porter and Sattler (1999, table 3), two makes with the highest reliability are Honda and Toyota. The median[16] selling age for a used Honda or Toyota is 7.1 years. In contrast, the median selling age for a Pontiac or General Motors car, two of the less reliable makes, is 6.1 years.

V. Conclusion

This paper developed a dynamic full equilibrium model of durable goods markets. The unique feature of the model is that it allows multiple holding periods for durables and at the same time considers equilibrium interactions in the aftermarkets. Prices and quantities traded for every vintage are determined endogenously in a stationary equilibrium. The model can account for the variations in resale rate for used automobiles in the United States.

This work contributes to the durable goods literature by showing how

[16] The median is computed with respect to the distribution of resale rates by age. Source: 1995 NPTS.

to incorporate the used market into a model with periodic replacement and how to compute the equilibrium in such a model. Another important contribution is that the model provides an explicit aggregation of microeconomic consumption patterns for durable goods.

Possible extensions may have applications beyond explaining the trade volume for used autos. For example, one can use a related model in an environment in which workers are heterogeneous and firms face hiring costs. Human capital is a durable asset whose services can be rented to firms. Firms use different technologies, and workers possess different skills. Human capital may depreciate as a result of technological progress or it may appreciate as a result of learning. Either way, this will induce labor market transactions. Slow learners will eventually fall behind their firm's technology and will be rented by technological laggards. Fast learners will "outlearn" the firms they are currently with and leave to work for technological leaders. Such a model can generate variations of labor turnover by skill level as well as predict how skill premium depends on the rate of technological progress and the learning response of workers.

Appendix

Proof of Proposition 1

The proof is based on the following lemma.

LEMMA. Suppose that $h > h'$. Then

$$U(h,\ S,\ \tau) - U(h,\ S+1,\ \tau-1) > U(h',\ S,\ \tau) - U(h',\ S+1,\ \tau-1) \quad \text{for any } S \geq 0,\ \tau > 1,$$

$$U(h,\ S,\ \tau) - U(h,\ S,\ \tau+1) > U(h',\ S,\ \tau) - U(h',\ S,\ \tau+1) \quad \text{for any } S \geq 0,\ \tau \geq 1,$$

and

$$U(h,\ S,\ \tau) - U(h,\ S+1,\ \tau) > U(h',\ S,\ \tau) - U(h',\ S+1,\ \tau) \quad \text{for any } S \geq 0,\ \tau \geq 1.$$

Proof. When the terms in (7) are rearranged, a consumer's lifetime utility can be expressed as

$$U(h,\ S,\ \tau) = h\frac{X(S,\ \tau)}{R(\tau)} + \sum_{t=0}^{T} b_t(S,\ \tau)p_t - \lambda_{S+\tau}p_{S+\tau}b_{S+\tau}(S,\ \tau),$$

where

$$X(S,\ \tau) = \sum_{i=1}^{\tau} x_{S+i-1}\gamma^{i-1}$$

is the discounted stream of service from the good,

$$R(\tau) = 1 - (\beta - \gamma)\sum_{i=1}^{\tau}\gamma^{i-1} - \gamma^{\tau} = (1-\gamma^{\tau})\frac{1-\beta}{1-\gamma}$$

is the discount factor for a τ-period replacement cycle, and

$$b_t(S, \tau) = \begin{cases} 0 & t < S \text{ or } t > S+\tau \\ \dfrac{-1}{R(\tau)} & t = S \\ \dfrac{(\beta-\gamma)\gamma^{t-1-S}}{R(\tau)} & S+1 \le t \le S+\tau-1 \\ \dfrac{\beta\gamma^{\tau-1}}{R(\tau)} & t = S+\tau \end{cases}$$

is the discount factor for prices. With this notation,

$$U(h, S, \tau) - U(h', S, \tau) = \frac{X(S, \tau)}{R(\tau)}(h-h'),$$

$$U(h, S+1, \tau-1) - U(h', S+1, \tau-1) = \frac{X(S+1, \tau-1)}{R(\tau-1)}(h-h').$$

To prove the first inequality of the lemma, we must establish that

$$\frac{X(S, \tau)}{X(S+1, \tau-1)} > \frac{R(\tau)}{R(\tau-1)} = \frac{1-\gamma^{\tau}}{1-\gamma^{\tau-1}}.$$

This follows directly from the fact that $x_S > x_{S+i}$:

$$\frac{X(S, \tau)}{X(S+1, \tau-1)} = \frac{x_S + \gamma X(S+1, \tau-1)}{X(S+1, \tau-1)} = \frac{x_S}{\sum_{i=1}^{\tau-1} x_{S+i}\gamma^{i-1}} + \gamma$$

$$> \frac{1-\gamma}{1-\gamma^{\tau-1}} + \gamma = \frac{1-\gamma^{\tau}}{1-\gamma^{\tau-1}}.$$

Similarly, to prove the second inequality of the lemma, we must establish that

$$\frac{R(\tau+1)}{R(\tau)} = \frac{1-\gamma^{\tau+1}}{1-\gamma^{\tau}} > \frac{X(S, \tau+1)}{X(S, \tau)}.$$

Rewriting the right-hand side and using the monotonicity of x_t, we obtain

$$\frac{X(S, \tau+1)}{X(S, \tau)} = \frac{X(S, \tau) + \gamma^{\tau}x_{S+\tau}}{X(S, \tau)} < 1 + \frac{\gamma^{\tau}(1-\gamma)}{1-\gamma^{\tau}} = \frac{1-\gamma^{\tau+1}}{1-\gamma^{\tau}}.$$

To prove the third inequality, simply observe that because x_t is decreasing,

$$X(S, \tau) > X(S+1, \tau),$$

which implies that

$$U(h, S, \tau) - U(h', S, \tau) = \frac{X(S, \tau)}{R(\tau)}(h - h') > \frac{X(S+1, \tau)}{R(\tau)}(h - h')$$
$$= U(h, S+1, \tau) - U(h', S+1, \tau).$$

Q.E.D.

COROLLARY. For any $t > 0$ and $h > h'$,

$$U(h, S, \tau) - U(h, S, \tau + t) > U(h', S, \tau) - U(h', S, \tau + t),$$
$$U(h, S, \tau) - U(h, S+t, \tau - t) > U(h', S, \tau) - U(h', S+t, \tau - t),$$
$$U(h, S, \tau) - U(h, S+t, \tau) > U(h', S, \tau) - U(h', S+t, \tau).$$

Proof. Observe that

$$U(h, S, \tau) - U(h, S, \tau + t) = [U(h, S, \tau) - U(h, S, \tau + 1)]$$
$$+ [U(h, S, \tau + 1) - U(h, S, \tau + 2)] + \cdots$$
$$+ [U(h, S, \tau + t - 1) - U(h, S, \tau + t)],$$

and according to the lemma, the desired inequality holds for each of the terms. Q.E.D.

To prove proposition 1, we shall prove the following statements: (i) Over any interval in h in which S_h is constant, $S_h + \tau_h$ is nonincreasing in h. (ii) Over any interval in which $S_h + \tau_h$ is constant, S_h is nonincreasing in h. (iii) Over any interval in which τ_h is constant, S_h is nonincreasing in h.

i) Take two consumers with $h > h'$ who have the same buying point, $S_h = S_{h'}$. The lemma shows that a higher type $h > h'$ must be strictly better off holding her good for $\tau_{h'}$ periods than for $\tau_{h'} + t$ periods:

$$U(h, S_{h'}, \tau_{h'}) - U(h, S_{h'}, \tau_{h'} + t) > U(h', S_{h'}, \tau_{h'})$$
$$- U(h', S_{h'}, \tau_{h'} + t) \geq 0 \quad \forall t > 0.$$

Then it is optimal for h to sell her good earlier: $\tau_h \leq \tau_{h'}$.

ii) Take two consumers with $h > h'$ who have the same selling points, $S_h + \tau_h = S_{h'} + \tau_{h'}$. The lemma shows that a higher type $h > h'$ will be strictly better off buying the good of age $S_{h'}$ than any older good of age $S_{h'} + t$:

$$U(h, S_{h'}, \tau_{h'}) - U(h, S_{h'} + t, \tau_{h'} - t) > U(h', S_{h'}, \tau_{h'})$$
$$- U(h', S_{h'} + t, \tau_{h'} - t) \geq 0 \quad \forall t > 0.$$

Then it is optimal for the higher type to buy a younger good: $S_h \leq S_{h'}$.

iii) Similarly, if holding times for two consumers h' and $h > h'$ are the same, the higher type is better off with $S_{h'}$ than any buying point $S_{h'} + t$:

$$U(h, S_{h'}, \tau_{h'}) - U(h, S_{h'} + t, \tau_{h'}) > U(h', S_{h'}, \tau_{h'})$$
$$- U(h', S_{h'} + t, \tau_{h'}) > 0 \quad \forall t > 0.$$

The three statements just proved would imply monotonicity if not for one nongeneric case. Suppose that for some price vector there exists a point $\hat{h}$ such that S_h, $S_h + \tau_h$, and τ_h are all discontinuous at the same point $\hat{h}$. Then the proposition does not tell anything about the relationship between $S_{\hat{h}+0}$ and $S_{\hat{h}-0}$. Although it is possible to choose such a special price vector, the set of these vectors will be a measure zero subset of the price space.

Claim. In any steady-state equilibrium, the market participation constraint reads $h_{\min} = p_{T-1}/x_{T-1}$.

Proof. The marginal consumer who participates in the market must get zero utility, because she can otherwise take a useless ($x_T = 0$) good for free every period.

Consumers whose utility is positive belong to an interval $[h_{\min}, h_{\max}]$ since, for any $h > h_{\min}$,

$$U(h, S_h, \tau_h) > U(h, S_{h_{\min}}, \tau_{h_{\min}}) > U(h_{\min}, S_{h_{\min}}, \tau_{h_{\min}}) = 0.$$

Next, we must show that either $S_{h_{\min}} = T - 1$ and $\tau_{h_{\min}} = 1$ or $p_{T-1} = 0$ and $h_{\min} = 0$.

Since $\alpha > 0$, goods of every age are offered for sale, so that $Q_s(t) > 0$ for all $t < T$. If $p_{T-1} > 0$, market clearing requires

$$Q_s(T-1) = Q_d(T-1) > 0.$$

Since $h_{\min}$ is the lowest type that participates in the market, proposition 1 implies that she will demand the oldest vintage

$$S_{h_{\min}} = \max_h (S_h).$$

If $S_{h_{\min}} < T - 1$, markets cannot clear because then $Q_d(T-1) = 0$. Therefore, it must be the case that $S_{h_{\min}} = T - 1$, and it is straightforward to check that $\tau_{h_{\min}} = 1$. Observe that

$$U(h_{\min}, T-1, 1) = \frac{x_{T-1}h_{\min} - p_{T-1}}{1 - \beta},$$

which proves the claim for $p_{T-1} > 0$.

Finally, if $p_{T-1} = 0$, all consumers participate in the market, that is, $h_{\min} = 0$. Q.E.D.

References

Adda, Jérôme, and Cooper, Russell. “Balladurette and Juppette: A Discrete Analysis of Scrapping Subsidies.” *J.P.E.* 108 (August 2000): 778–806.

Bond, Eric W. “Trade in Used Equipment with Heterogeneous Firms.” *J.P.E.* 91 (August 1983): 688–705.

Caplin, Andrew, and Leahy, John V. “Durable Goods Cycles.” Working Paper no 6987. Cambridge, Mass.: NBER, February 1999.

Eberly, Janice C. “Adjustment of Consumers’ Durables Stocks: Evidence from Automobile Purchases.” *J.P.E.* 102 (June 1994): 403–36.

Goolsbee, Austan. “The Business Cycle, Financial Performance, and the Retirement of Capital Goods.” *Rev. Econ. Dynamics* 1 (April 1998): 474–96.

Hendel, Igal, and Lizzeri, Alessandro. “Adverse Selection in Durable Goods Markets.” *A.E.R.* 89 (December 1999): 1097–1115.

Holmes, Thomas J., and Schmitz, James A., Jr. “A Theory of Entrepreneurship and Its Application to the Study of Business Transfers.” *J.P.E.* 98 (April 1990): 265–94.

House, Christopher L., and Leahy, John V. “An sS Model with Adverse Selection.” Working Paper no. 8030. Cambridge, Mass.: NBER, December 2000.

Jovanovic, Boyan. “Vintage Capital and Inequality.” *Rev. Econ. Dynamics* 1 (April 1998): 497–530.

Konishi, Hideo, and Sandfort, Michael T. “Existence of Stationary Equilibrium in the Markets for New and Used Durable Goods.” *J. Econ. Dynamics and Control* 26 (June 2002): 1029–52.

Kursten, Wolfgang. “A Theory of Second-Hand Markets: The Rapid Depreciation of Consumer Durables and Product Differentiation Effects.” *J. Inst. and Theoretical Econ.* 147 (September 1991): 459–76.

Porter, Robert H., and Sattler, Peter. “Patterns of Trade in the Market for Used Durables: Theory and Evidence.” Working Paper no. 7149. Cambridge, Mass.: NBER, May 1999.

Rust, John. “Stationary Equilibrium in a Market for Durable Assets.” *Econometrica* 53 (July 1985): 783–805.

U.S. Department of Transportation. Federal Highways Administration. *1995 Nationwide Personal Transportation Survey*. Publication no. FHWA-PL-98-002. Research Triangle Park, N.C.: Research Triangle Inst., October 1997. http://www-cta.ornl.gov/npts/1995/Doc/index.shtml.

Waterston, Albert. “Good Enough for Developing Countries?” *Finance and Development* 1 (September 1964): 89–96.