# Technology Adoption and Commitments: The Welfare Costs of Industrialization Revisited

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**Abstract**

An individual with time-inconsistent preferences can use technology choice as a commitment device. In order to avoid overwork, he may find it optimal to reject a modern technology. Therefore, having more technologies available never makes an isolated individual worse off.

This is different for groups that interact through markets. Markets destroy the commitment value of non-adoption. The society then adopts the technology even if it lowers the lifetime utility of every member.

This result provides a new interpretation of the health decline that occurred in Britain and the US during the industrialization and is extensively documented in the literature.

*JEL Classification: O14, D91*

## 1 Introduction

An agent whose preferences are time inconsistent may try to constrain his future actions. Since Strotz (1955) had this insight, economists have used it to illuminate the workings of monetary policy (Kydland and Prescott 1977), fiscal policy (Chari and Kehoe 1990), saving (Laibson 1998), growth (Barro 1998), and so on. The point is that decisions which an agent will want to take in the future may reduce his lifetime utility today, and to prevent himself from taking these decisions, he may seek to tie his hands in some way.

One place where the point applies with equal force is in the choice of technology, as adopting a technology usually entails direct costs of training that are largely unrecoverable. In other words, technological decisions are hard to reverse, and it seems

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that their role in providing commitment has not been explored yet. This paper fills this gap, first for one agent, and then, more importantly, for groups of people who can trade with each other.

There is a crucial difference between the case of one agent and the case of a group. When the agent is alone, he may reject a better technology if it involves overwork and reduces his lifetime utility. Therefore, availability of a better technology cannot make the agent worse off. In a group, however, the option to use a better technology may reduce *everyone*'s utility. It turns out that groups will start to adopt the technology even when every member of the group, taken alone, would find it optimal to resist.

To see why isolated individuals behave differently from groups, let's take the case of a 19th century English farmer who considers learning the printing trade and moving to London. Of course, he would earn a higher wage there, but he would also work harder. Perhaps even his children would spend many hours at the factory. Spending too much time at work would make the household members less healthy in the long run, and this may reduce the farmer's lifetime utility. The prospect of all that may well sway the farmer into staying where he is.

Now let us put the farmer in a group. He now can no longer avoid overwork by staying on the farm. If some of his neighbors move to London and become printers, they will need to buy their food from him. As the number of Londoners grows, so does the price of food, and so does the farmer's income per hour spent farming, and he now faces the prospect of overwork *even on the farm*! Too much time is now spent producing food for sale, and this means that the health of household members suffers as well. And this brings us to the main point: If staying on the farm involves overwork too, moving to the city becomes all the more tempting. Groups are therefore quicker to adopt new technology, but they are also prone to overworking and declining health. We demonstrate that incomes do not rise enough to offset the disutility of overwork, and that material progress may bring about a temporary *decline* in welfare.

This has practical consequences for development. We present a large body of evidence that documents the health decline in the US and Britain during the period of industrialization. Is it possible that the living standard was falling? Costa and Steckel (1997, p. 76) find that in the decades before the Civil War in the US the benefits of income growth were not enough to compensate for the higher mortality risk. If they are right about this, one cannot argue that bad health and high mortality were simply a utility-enhancing investment. One could try to argue that Blake's satanic mills were privately utility enhancing and that the rise in mortality was the result of an external effect – northern mill-towns enveloped in poisonous smoke and soot. This may explain why health declined in the cities and near the mills, but it does not explain why the health of farmers deteriorated as well. Economic historians are puzzled by the fact that much of the observed health decline have occurred in the rural population. We offer an explanation for this puzzle. Beyond this, our model predicts that income inequality of the Kuznets type appears between the farmers and the workers; inequality rises at first, but subsequently it falls towards the end of the industrialization.

More broadly, our model adds to "the most famous debate on economic change", as Engerman (1997, p.17) calls it, the debate over whether nineteenth century industrialization reduced welfare for its pioneers. The debate started in 1845 with Engels (1974, p. 10) expressing pessimism about the net benefits of industrialization, and it continues to this day with Williamson (1985, p. 24-28) asserting that welfare rose monotonically and with Costa and Steckel finding that it did not. The matter is far from settled, of course, and the debate will continue. But it is certainly an advance, we think, to have a model in which welfare *can* decline for a while along the equilibrium growth path and this is what our paper delivers.

The plan of the paper is as follows: Section 2 lays out the basic model for the isolated agent; Section 3 characterizes the equilibrium in the multi-agent setting and establishes the main results; Section 4 presents the evidence and provides the interpretation of historical facts on the industrialization in Britain and the US; Section 5 concludes.

# 2 The Model

An agent lives for three periods, labelled 1, 2, and 3. At date 1 the agent chooses an occupation that uses production technology $\alpha$ from an exogenously given menu, the interval $[0, A]$. The choice of $\alpha$ does not involve any cost. The production technology $\alpha$ can be used only next period, perhaps because some training is required. The training delay prevents the agent from revising his choice of technology in period 2 in time to produce with it. This means that the technology choice is irreversible, but $\alpha$ is certainly disposable.

Production happens only in period 2, when the agent has one unit of time that can be used to produce two goods: a consumption good, $y$, consumed at date 2, and an investment good, $x$, consumed at date 3. The investment good $x$ is produced from a single input $z$ and takes one period to mature. This delay seems to make sense when $x$ is the health of oneself and one's children. The state of the investment technology is fixed and not subject to choice.

*Production possibilities*: The agent has a unit of time at his disposal in period 2. Let $t \leq 1$ be the time that he devotes to producing $y$. If the consumption good is produced using technology $\alpha$, consumption output equals

$$y = \alpha t.$$

The agent can either produce the investment input with his own time or, if $z$ is traded, buy it from others. Any agent can make one unit of $z$ with one unit of time. However, the investment input purchased from others is an imperfect substitute for the $z$ that the agent produces on his own. In particular, if $z_{own}$ is produced with the agent's own time and $z_{others}$ is received from other agents, the resulting quantity of

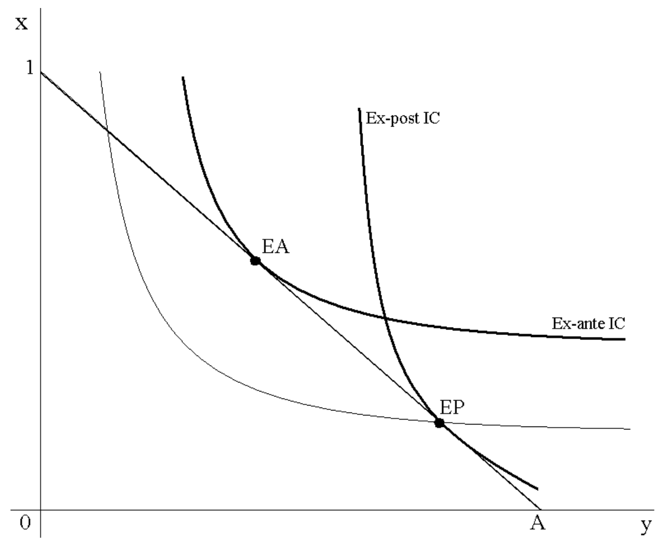


Figure 1: Ex-ante and ex-post utility maximization, fixed $A$.

$x$ equals

$$x = z_{own} + \gamma z_{others},$$

where $0 \leq \gamma \leq 1$.

*Preferences*: We use a special case of preferences that Phelps and Pollak (1968) introduced. Period 1 utility is normalized to zero. Period 2 utility is $u(y)$ and period 3 utility is $v(x)$. Both functions are concave. The agent discounts future utility by the same factor $\beta$, no matter how remote this future is. Therefore, at date 1, the discounted lifetime utility is $\beta\left[u(y) + v(x)\right]$, but at date 2, the utility is $u(y) + \beta v(x)$.

If $\beta < 1$, the mere passage of time changes how the agent ranks various $(y, x)$ bundles, and this induces a conflict between the present (ex-ante) and future (ex-post) self. If $\beta = 1$, no conflict arises because preferences are time-consistent. The marginal rate of substitution between $y$ and $x$ is

$$MRS_{ex-ante} = \frac{u'(y)}{v'(x)} < \frac{u'(y)}{\beta v'(x)} = MRS_{ex-post},$$

which means that the *ex-post* indifference curves in $(y,\, x)$ space are steeper than the *ex-ante* indifference curves. These indifference curves are drawn in Figure 1.

We will now describe the utility maximization problem and the optimal technology choice for an isolated individual.

## 2.1 A one-agent economy

The stand-alone agent cannot purchase $z$ from others, so $z_{others} = 0$ for now. Since he has to produce his own investment input, the agent will allocate the time between producing $y$ and $z$. Spending time $t$ producing $y$ leaves $1-t$ units of time for producing $z$. Therefore, the lifetime utility of someone who chooses technology $\alpha$ in period 1 is

$$U^1(\alpha, t) \equiv \beta \left[ u(\alpha t) + v(1-t) \right]$$

For the level of $\alpha$ fixed at the technological frontier $\alpha = A$, Figure 1 shows the production possibility set and two types of optima. If the agent could commit to a certain value of $t$, he would choose to be at the point $EA$ which involves the use of the frontier technology $A$. The point $EA$ is optimal from the standpoint of *ex-ante* preferences. In contrast, the point $EP$ is optimal with respect to the agent's preferences *at the time* he chooses $t$. This solution is obtained if the agent cannot commit to a particular value of $t$ at date 1. Since the point $EP$ lies on a lower *ex-ante* indifference curve than the point $EA$, the difference in welfare between these two curves is the value of being able to commit to a decision on $t$ holding the technology fixed at $A$.

*Choice of $t$*: The technology, however, is not fixed, but is chosen at date 1. We shall analyze the decision problem "backwards." First, we shall describe how, at date 2, the agent chooses $t$, taking as given the technology level $\alpha$ that was already picked at date 1. After that, we shall determine $\alpha$. The agent chooses $t$ at date 2, when his lifetime utility is

$$U^2(\alpha, t) = u(y) + \beta v(x).$$

he therefore chooses $t$ to equal

$$t(\alpha) = \arg\max_t \left\{ u(\alpha t) + \beta v(1-t) \right\}.$$

*Choice of $\alpha$*: Let $U(\alpha) = U^1(\alpha, t(\alpha))$ be the ex-ante utility of adopting technology $\alpha$ and choosing $t(\alpha)$ optimally at date 2. The agent chooses $\alpha$ to maximize $U(\alpha) = U^1(\alpha, t(\alpha))$.

Figure 2 shows how the use of an inferior technology $A' < A$, may raise *ex-ante* utility. *Ex-post* utility is now maximized at the point $EP'$, and *ex-ante* utility is higher at that point than at the point $EP$. When, precisely, will such a reduction in technology raise *ex-ante* utility? The first derivative of $U(\alpha)$ is

$$\frac{dU}{d\alpha} = \beta \left[ tu' + (\alpha u' - v') \frac{dt}{d\alpha} \right].$$

The necessary condition that $t(\alpha)$ satisfies is

$$\alpha u' - \beta v' = 0. \tag{1}$$

The second order condition $D \equiv \alpha^2 u'' + \beta v'' < 0$ is met since $u$ and $v$ are concave. Taking total derivatives in (1),

$$\frac{dt}{d\alpha} = -\frac{u' + t\alpha u''}{D}.$$

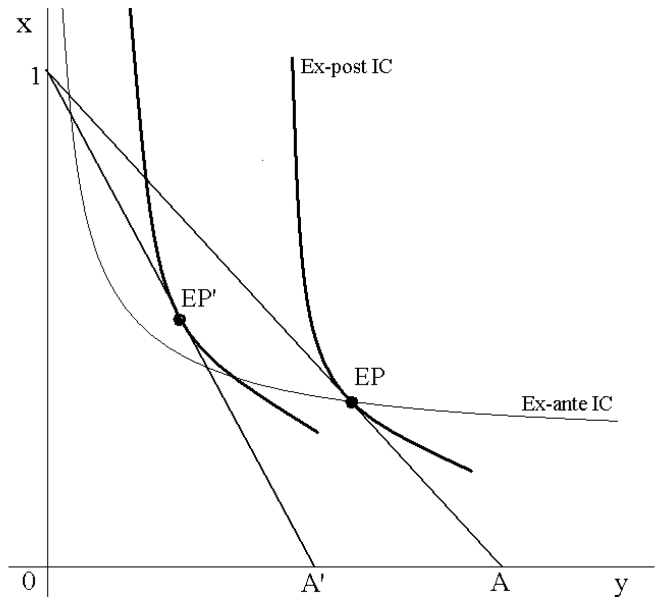


Figure 2: Ex-ante utility is higher under inferior technology $A' < A$.

From (1), $u' = \beta v'/\alpha$, so that

$$\frac{dU}{d\alpha} = \beta v' \left( \beta \frac{t}{\alpha} - (1-\beta) \frac{dt}{d\alpha} \right)$$

Since $\alpha$ is the return per unit of time devoted to producing $y$, it is the analog of the wage. Therefore, we have proved the following result involving $\beta$ and the wage-elasticity of labor supply, $\frac{\alpha}{t}\frac{dt}{d\alpha}$, which we denote by $\varepsilon(t(\alpha), \alpha)$:

**Proposition 1** *If $\alpha$ is such that*

$$\varepsilon(t(\alpha), \alpha) > \frac{\beta}{1-\beta}, \tag{2}$$

*then*

$$\frac{dU}{d\alpha} < 0.$$

In order for him to prefer a lower $\alpha$, the agent's preferences must be time-inconsistent with $\beta < 1$. But this is not enough, because his labor supply must also be relatively elastic. The parameter $\varepsilon$ is the *uncompensated* wage-elasticity of labor supply which depends inversely on the curvature of the indifference curves in $(y, x)$ space. To see this more clearly, consider an example.

*Example*: Let $u(y) = y^{1-\theta}$, and $v(x) = x^{1-\theta}$, where

$$0 < \theta < 1. \tag{3}$$

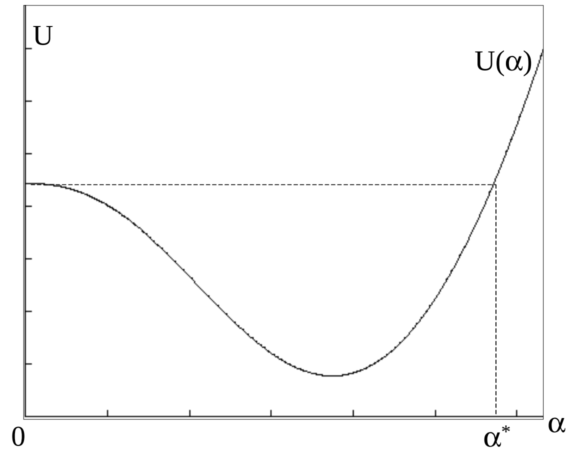


Figure 3: Value of technology: reduced form ex-ante utility

From (1), labor supply to the goods market is

$$t(\alpha) = \frac{1}{1 + \alpha^{1-\frac{1}{\theta}} \beta^{\frac{1}{\theta}}},$$

which rises monotonically from zero to one as $\alpha$ increases. Differentiating, we find that the labor supply elasticity is

$$\varepsilon = \frac{1-\theta}{\theta} \left(1 - t(\alpha)\right),$$

and it is decreasing in $\alpha$. When $\alpha = 0$, $t = 0$, and $\varepsilon = \frac{1-\theta}{\theta}$, and, so, by Proposition 1, we find that $U(\alpha)$ *has a decreasing portion in the neighborhood of* $\alpha = 0$ if and only if

$$\theta + \beta < 1. \tag{4}$$

We can make stronger statements about the shape of $U(\alpha)$. It is not much harder to also show that if $v(x) = x^{1-\sigma}$ and

$$0 < \sigma < 1 \tag{5}$$

$U(\alpha)$ has a unique minimum at a strictly positive value of $\alpha$ if and only if (4) holds. In this case, too, $U(\alpha)$ is U-shaped as in Figure 3, which shows $U(\alpha)$ for $\theta = \sigma = 0.3$ and $\beta = 0.6$.

Suppose that feasible technologies lie in the interval $[0, A]$. Then, the optimal technology choice will be either at $\alpha = 0$, or at $\alpha = A$. Which choice is better? Let $\alpha^* > 0$ be the smallest positive number such that $U(\alpha^*) \geq U(0)$, as shown in Figure 3. As long as $A < \alpha^*$, the agent would reject the frontier technology, and opt for $\alpha = 0$. But if $A > \alpha^*$, the agent would choose the frontier technology $A$. The equilibrium utility that the agent attains equals $\min\{U(0), U(A)\}$. That is, although some technology choices reduce his utility, the technological progress *does not harm* the agent. The next section will demonstrate that this result does not survive in a group setting.

# 3 Equilibrium with many agents

We have shown that an isolated agent who cannot commit on the level of $t$ can still raise his lifetime utility by exercising his option of "free disposal" of technology. A single agent may always reject the option of high earnings and thereby ensure that he ends up with more of the investment good. But what if $z$, the input for producing the investment good, can be purchased on the market? This makes the agents' market earnings harder to control. When $z$ is traded, the commitment not to produce $y$ may become ineffective, since the agent can always produce $z$ and exchange it for $y$. Then, in a group setting, the effectiveness of commitment will be determined endogenously, and will depend on technology choices made by the rest of the society. In general, inability to commit effectively should raise the appeal of earning a large income producing $y$ and then spending a part of it on $z$. In other words, the opening of a market for the investment input should induce technology adoption in the $y$ sector.

Let us now develop this point formally. Let there be many agents in a three-period economy. Suppose that all agents choose $\alpha = 0$ in period one. Each of them then sets ex-post labor supply to $t = 0$, produces 1 unit of the investment good and zero units of the consumption good, and thereby enjoys a lifetime utility of $U(0) = \beta[u(0) + v(1)]$. In this situation, the last unit of time spend producing the investment good yields a utility level $\beta v'(1)$. If ex-post an agent were offered a price $p$ to give up some of his $z$, he would do so if

$$p \geq \frac{\beta v'(1)}{u'(0)}.$$

The right-hand side of this inequality is the supply price of investment input in an economy where no one is able to produce the consumption good, and this supply price tends to zero as $u'(0)$ becomes large relative to $v'(1)$. Then any agent who at date 1 deviates to adopting the frontier technology $A$ will have a chance to purchase the investment input from everybody else in the economy for close to a zero price. Clearly, such an agent can attain a higher utility than $U(0)$, which makes this deviation profitable. Thus we have proved the following result:

**Proposition 2** *If $u'(0) = +\infty$ and $A > 0$, then in a group setting, all agents choosing $\alpha = 0$ cannot be an equilibrium.*

Proposition 2 says that if $u'(0)$ is large enough, there is no equilibrium where everyone rejects technology. The mechanism that drives these results is the same as before - the inability to commit to a limited level of market participation. Choosing $\alpha = 0$ no longer removes the temptation to spend resources on the consumption good, as the market for $z$ forms in the second period. We have shown that when $A \in [0, \alpha^*]$, the isolated agent invariably chooses $\alpha = 0$. But, by Proposition 2, a group of agents will always contain some members who choose $\alpha > 0$. In this sense, groups of agents should be technologically more progressive than isolated individuals.

The question remains whether the introduction of a market for the investment input will entirely eliminate the incentive to reject technology. To answer this question we need to specify how an economy with the investment good market would work.

## 3.1 The market for $z$

Demand and supply for $z$ are determined in period 2 and they, therefore, must maximize *ex-post* utility taking market price and the period 1 technological choices as given. Suppose that ex-ante an agent has adopted technology $\alpha$ and ex-post is faced with the time allocation problem. As before, let $t$ be the time that the household uses to operate the technology $\alpha$, and let $z_s$ be the amount of $z$ that it supplies to the market. The household can also buy $z_d$ units of investment input at a price of $p$ per unit. We continue to assume that $z_d$ is an inferior, though not entirely useless substitute for one's own time in producing the investment good. In particular, each unit of $z_d$ yields only $\gamma < 1$ units of $x$.[1] Substituting the effective input into the production function for $x$ yields the following output of the investment good

$$x = 1 - t - z_s + \gamma z_d.$$

The case $\gamma = 0$ corresponds to autarchy, where only the agent himself can produce the investment input, and in this case there can be no trade among agents. In the case $\gamma = 1$, the model cannot determine how much $z$ changes hands on the market and how much is produced "in house".[2]

In period 2, the agent takes the market price $p$ as given, and the technology, $\alpha$, is predetermined. The *ex-post* decision rules, $t = T(\alpha, p)$, $z_s = Z_s(\alpha, p)$, and $z_d = Z_d(\alpha, p)$, solve the problem

$$\max_{t, z_s, z_d} \{u[\alpha t + p(z_s - z_d)] + \beta v(1 - t - z_s + \gamma z_d)\}. \tag{6}$$

Substituting these decision rules into the ex-ante utility function, we obtain the reduced form ex-ante utility:

$$\begin{aligned} V(\alpha, p) = {} & \beta u[\alpha T(\alpha, p) + p(Z_s(\alpha, p) - Z_d(\alpha, p))] + \\ & + \beta v[1 - T(\alpha, p) - Z_s(\alpha, p) + \gamma Z_d(\alpha, p)]. \end{aligned} \tag{7}$$

[1] We will interpret this assumption in Section 4.

[2] When $\gamma = 1$ everyone who produces $y$ does so using the frontier technology $A$, and the market price of $z$ equals $A$ as well.

### 3.2 Definition of equilibrium

Equilibrium is a distribution of technology choices which is some measure $\mu$ over $[0, A]$, and a price $p$ that clears the market in the second period. Since all agents are ex-ante the same, they must be indifferent between any of the technologies that are chosen in equilibrium:

$$\text{supp}(\mu) = \arg\max_{\alpha} V(\alpha, p) \tag{8}$$

For now we shall assume – and later on we shall prove – that any equilibrium distribution of technology choices belongs to a class of two-point distributions. It has a mass $\pi > 0$ of agents choosing the frontier technology $\alpha = A$. The rest of the agents (fraction $1-\pi$) dispose of their technology (i.e. choose $\alpha = 0$[3]) and produce $z$. Condition (8) implies that in any equilibrium with an open market for $z$ both choices must yield the same utility:

$$V(0, p) = V(A, p) \tag{9}$$

As shown in the Lemma 1 of the Appendix, agents will be either suppliers or demanders of $z$, not both. The high-$\alpha$ households will buy $z$ from the low-$\alpha$ households, unless we are in an equilibrium in which the market is inactive. The first order conditions for the two types of households then simplify.

Anyone who chooses $\alpha = 0$ supplies $z$ and does not produce $y$. Such an agent has $T(0, p) = 0$, $Z_s(0, p) = z_s > 0$ and $Z_d(0, p) = 0$. His consumption is $pz_s$, and his supply at price $p$ is $Z_s(0, p)$:

$$Z_s(0, p) = \arg\max_{z} \left[u(pz) + \beta v(1 - z)\right]. \tag{10}$$

A "high-tech" type has $\alpha = A$, $T(A, p) = 1$, $Z_s(A, p) = 0$ and $Z_d(A, p) = z_d > 0$.[4] his consumption is $A - pz_d$. His demand for investment input, $Z_d(A, p)$, maximizes the ex-post utility. Decision variable, $z_d$, will be in the interior when $p \leq \gamma A$. The first-order condition of the high-tech type reads

$$Z_d(A, p) = \arg\max_{z} \left[u(A - pz) + \beta v(\gamma z)\right]. \tag{11}$$

Let $\pi$ denote the fraction of people who choose $\alpha = A$, in which case the fraction $1 - \pi$ choose $\alpha = 0$. In this case, the supply equals demand condition for $z$ reads

$$\pi Z_d(A, p) = (1 - \pi) Z_s(0, p). \tag{12}$$

[3]Strictly speaking, the agent need not choose $\alpha = 0$ ex-ante in order to decide to dispose of his technology ex-post. The agents who ex-post become the producers of $z$ are indifferent between choosing any $\alpha \in [0, p)$, since $\alpha$ is irrelevant for their second period payoff. Because of this, it is possible that the same ex-post equilibrium is supported by multiple ex-ante distributions over $\alpha$. However, introducing an arbitrarily small adoption cost $\delta$ rules out this multiplicity and leaves the only equilibrium which involves the distribution with two mass points: $\alpha = 0$ (disposal) and $\alpha = A$ (frontier).

[4]See Lemma 1 of the Appendix for the detailed proof.

We can now define the equilibrium.

**Definition:** *Equilibrium is a pair of scalars* $(\pi, p)$ *and a pair of functions* $Z_s(0,p)$, $Z_d(A,p)$ *such that:*

1. Agents have an incentive to make their respective technology choices:

$$\arg\max_{\alpha} V(\alpha, p) = \begin{cases} \{0, A\}, & \text{if } \pi < 1 \\ A, & \text{if } \pi = 1 \end{cases};$$

2. Given the fraction of agents $\pi$ who have chosen $\alpha = A$ ex-ante, price $p$ clears the investment good market ex-post:

$$\pi Z_d(A,p) = (1-\pi) Z_s(0,p);$$

3. Supply and demand functions $Z_s(0,p)$ and $Z_d(A,p)$ are given by (10) and (11), respectively.

## 3.3 Properties of the equilibrium

Let the technological frontier $A$ increase exogenously with the passage of time. Depending on the value of $A$, different types of equilibria will arise in the model. The purpose of this section is to characterize them. It turns out that when $A$ is sufficiently low, the equilibrium is an asymmetric one, with some agents adopting the frontier technology and other agents disposing of their technology and producing $z$. Investment input $z$ is traded in equilibrium for sufficiently low $A$. If $A$ is large enough, *all* agents adopt the frontier technology and there is no trade in equilibrium.

The set of $A$ which supports equilibria with trade is determined by the shape of $U(\alpha)$, the reduced form ex-ante utility of choosing the technology $\alpha$ in autarchy. We will continue to use the assumptions regarding $U(\cdot)$ made in Proposition 1 guaranteeing that it is initially decreasing and will have a unique minimum. The following proposition, proved in the Appendix, says that when frontier technology $A$ is below a certain cutoff point $\hat{\alpha}$, then investment input is traded in equilibrium, and a positive fraction of agents choose $\alpha = 0$. When the frontier technology is above the cutoff, the only equilibrium is for *all* agents to adopt the frontier technology $A$.

**Proposition 3** *Let* $\beta$, $u(y)$ *and* $v(x)$ *satisfy (4), (3), (5) and let* $\gamma \in (0,1)$. *Then there is a unique number* $\hat{\alpha} \in (0, \alpha_*)$ *such that*

*(i) For any* $A \leq \hat{\alpha}$, *any equilibrium has a fraction* $\pi(A) < 1$ *of agents choose* $\alpha = A$ *and a fraction* $1 - \pi(A)$ *choose* $\alpha = 0$. *Investment input* $z$ *is traded in equilibrium, and the equilibrium price is* $0 < p < \gamma A$.

*(ii) For any $A > \hat{\alpha}$, the only equilibrium is for all agents to choose the frontier technology $\alpha = A$, i.e. $\pi(A) = 1$. Investment input $z$ is not traded in equilibrium, and the price that supports this outcome is $p = \gamma A$.*

*(iii) A distribution of technology choices where some agents choose $\alpha \in (0, A)$ cannot be part of an equilibrium.*

**Proof:** See Appendix.

The equilibrium distribution of technology choices has two mass points, because the ex-ante payoff $V(\alpha, p)$ evaluated at the equilibrium price is non-monotonic in $\alpha$. It is initially falling in $\alpha$, and then rising for the agent who demands $z$. Then, $V(\alpha, p)$ reaches its maximum either at $\alpha \in \{0, A\}$ or at $\alpha = A$, depending on the value of $A$.

The investment good market eventually breaks down because the opportunity cost of supplying $z$ grows with $A$. The more attractive it is to produce $y$, the higher is the reservation price for someone to supply $z$. Eventually, any candidate supplier of $z$ prices himself out of the market next period. Then, no one supplies $z$, and no one expects to purchase $z$ from the outsiders.

Note that the cutoff $\hat{\alpha} < \alpha^*$. The proposition implies that for any $A \in [\hat{\alpha}, \alpha^*]$, as well as for any $A > \alpha^*$, the equilibrium is for all agents to choose the frontier technology $\alpha = A$. By contrast, in autarchy, when $A \in [\hat{\alpha}, \alpha^*]$, all agents choose $\alpha = 0$. In other words, when the investment input is tradeable, the mass adoption of frontier technology happens *earlier* (i.e. for a smaller value of $A$) that in autarchy. This result may seem surprising , because when $A \in [\hat{\alpha}, \alpha^*]$, $U(0) > U(A)$[5]. Why can't the agent collect a higher ex-ante utility if he chooses not to participate in the market? Our point is that the presence of the ex-post market deprives him of such a commitment. Even having chosen $\alpha = 0$ ex-ante, the agent knows that ex-post he will find it optimal to produce $z$ and offer it for sale. This decision will yield the ex-ante utility level that is even lower than $U(A)$. Knowing that he cannot commit to non-participation, the agent chooses $\alpha = A$ as the next best alternative.

For the asymmetric equilibrium to arise, it is necessary that the preferences are time-inconsistent. What if condition (4) is violated? Then $U(A)$ is everywhere increasing in $A$, and the isolated agent will never reject the frontier technology. This still holds in a group setting for any $A$ because everyone must have the same lifetime utility. In equilibrium, the incentive compatibility condition (8) can only hold when $\pi = 1$.

Figure 4 summarizes the types of equilibria for different values of $A$ and $\gamma$. The curve on the plot shows the locus of the cutoff points $\alpha = \hat{\alpha}(\gamma)$ for $\gamma \in (0, 1)$. It turns out that $\lim_{\gamma \to 0} \hat{\alpha}(\gamma) = \alpha^*$ and $\lim_{\gamma \to 1} \hat{\alpha}(\gamma) = \alpha_{\min} = \arg\min_\alpha U(\alpha)$. For any $\gamma \in (0, 1)$ there is an asymmetric equilibrium for $A \leq \hat{\alpha}(\gamma)$, and there is a symmetric equilibrium for $A > \hat{\alpha}(\gamma)$.

[5] See Figure 3.

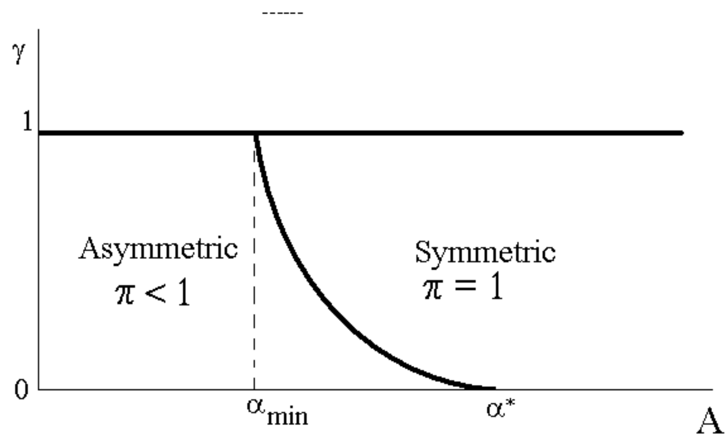


Figure 4: Type of equilibria depending on $A$ and $\gamma$.

An open market for $z$ clearly makes people better off *ex-post*, but worse off *ex-ante* because they cannot restrain themselves from working and are thus unable to collect the autarchy *ex-ante* utility $\max\{U(0), U(A)\}$. The bold line on Figure 5 shows the equilibrium utility: when $A \leq \alpha^*$, the agents collect *lower* utility than in autarchy.

The figure also shows that when equilibrium is asymmetric, the agents' lifetime utility *decreases* with $A$. Because the relative price of $z$ grows with $A$, the agents spend even more time producing $z$ for sale. This reduces ex-ante utility of the low-tech types. The equilibrium utility of a high-tech type is, of course, identically equal to that of the low-tech type. The symmetric equilibrium utility is still below the autarchy utility when $A \in [\hat{\alpha}, \alpha^*]$, because no agent can commit to non-participation in the market when he knows he can produce and sell $z$ ex-post. Finally, when $A > \alpha^*$, the symmetric equilibrium utility is at its autarchy level $U(A)$.

Since consumption output $y = \pi(A)A$ rises with $A$, the figure suggests that a society can get a lower lifetime utility at the same time that it experiences economic growth. It is true that the present utility of the agents who are in period 2 of their lives is increasing in $A$. However, a policymaker who maximizes the *lifetime* utility of households will have $u(y) + v(x)$ as a social welfare function. A policymaker who maximizes the sum of "present perspective" utilities of all his constituents will use a criterion that puts an even bigger weight on $x$:

$$W = \frac{1}{3}\left[\beta u(y) + \beta v(x)\right] + \frac{1}{3}\left[u(y) + \beta v(x)\right] + \frac{1}{3}v(x) =$$
$$= \frac{\beta+1}{3}\left[u(y) + \left(\frac{2\beta+1}{\beta+1}\right)v(x)\right].$$

By any of these two criteria, the welfare falls when $A < \hat{\alpha}$. The next section presents the evidence that this phenomenon may have occurred during the early years of industrialization and provides the interpretation of the model's equilibrium.

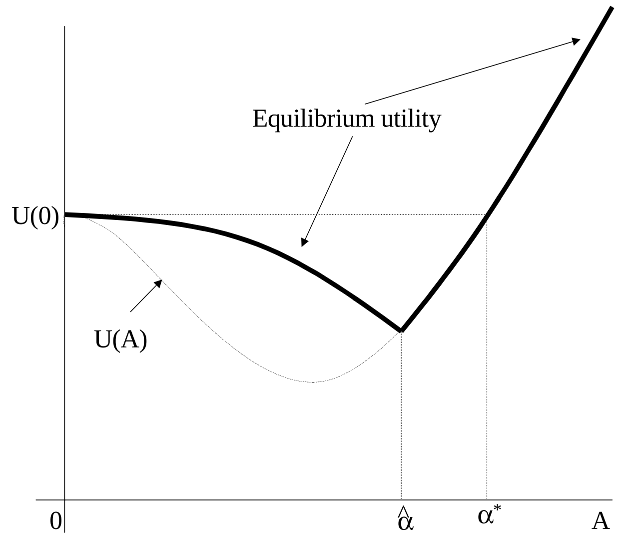


Figure 5: Equilibrium utility

# 4 Applications

In this section we use the model to interpret some aspects of industrialization in Britain and the US. Several features of the model – a falling price of industrial output, urbanization, increased work effort, deterioration of health and rising and then falling income inequality – are consistent with historical evidence that we discuss below.

## 4.1 Industrial revolution and welfare

Most economists now agree that the early industrialization of Britain and the US brought a rise in real wages. But it is still unclear whether living standards also rose, because during that period health and life expectancy declined in both countries. For example, Costa and Steckel (1997) and Fogel (1986) find that the average height of the native-born white males in the United States declined steadily from the 1830 cohort through the 1880 birth cohorts, and was shorter by an inch in 1860. Similarly, in 1790, life expectancy in the US started a decline that continued for half a century (Fogel, 1986). British males experienced a similar decline in height. Nicholas and Steckel (1991) report that the height downturn in Britain started with the 1780 birth cohort, and the terminal heights of urban workers born in 1802 were a full 1.25 inches less that of the 1780 cohort.

In an attempt to calculate whether wage growth more than offset the health decline, Williamson (1981) calculated the wage premium that factory workers commanded over farmers, and he took it to be the differential that compensated them for a poorer quality of life in the city. Since this differential was a lot smaller than the wage growth over the period in question, Williamson (1981) argues that the net effect on welfare in Britain was positive. But this approach ignores the evidence (which our model is consistent with) that industrialization made the farmers worse off. In fact, the health of farmers declined as well – all studies on heights find that the decline occurred in *both* urban and rural populations. Indeed, Fogel (1986, p. 500) notes that "about 4/5 of the [height] decline was due to deterioration of conditions affecting [human] growth in the rural areas". The rise in mortality risk associated with the lower stature may have been sufficient to reduce the lifetime utility even in the face of the robust income growth. Recently, Costa and Steckel (1995, p. 76) estimated that this was in fact the case for the US prior to the Civil War.

If a utility decline indeed occurred, and if moving to the city involved a net loss, why did workers do it? And if agricultural commerce also brought increased work effort and higher exposure to disease, why didn't the farmers stay isolated? Our model provides the explanations. Let us interpret $y$ as industrial output and $z$ as agricultural output. Throughout the nineteenth century, nutrition was a key factor affecting fertility and mortality, and so it seems appropriate to think of $z$ as a health-producing good.[6] The link between nutrition and health is even stronger in children, whose well-being we also count as $x$. Let us think of households who choose $\alpha = A$ as the industrial workers living in the city. In equilibrium, they do not grow any food, and they buy their food from the "low-tech" farmers at relative price $p$. Without doubt, urban dwellers are more exposed to health-threatening conditions, such as diseases, poor sanitation and pollution. Hence their investments in health are less productive ($\gamma < 1$).

Now, according to our model, it is the opening of a market for food that caused both workers *and* farmers to under-invest in health. The city dweller who earns a high wage will substitute away from health investment – this is true even for an isolated agent. But the emergence of markets erodes the lifetime utility of a farmer because, as factory wages grow and more workers move to the city, demand for food and its price rise. In other words, the high earnings that modern skills afford in the city are passed on to the farmers in the form of higher equilibrium food prices. The farmers, in turn, respond by selling too much food and leaving too little for their own households' nutritional needs, which explains why their heights declined. This resolves the height puzzle roughly along the lines argued by Stephen B. Webb and cited in Fogel (1986, p. 500): "Giving farmers the opportunity to trade corn for calico may raise their utility, or at least the parents' utility, but may also reduce caloric intake".

Our model takes nutrition to be the channel through which health declined, and direct evidence supporting this assumption is in Komlos (1987) who finds that Americans' diet deteriorated during the years of industrialization, and that the caloric

[6] McKeown (1978, p. 535) argues that "...medical intervention was relatively unimportant in determining the level of mortality, and ... nutritional influences were predominant".

intake per adult male equivalent fell by as much as 10% during the late antebellum period. The reason, according to Komlos (1987, p. 915), is rising relative prices of food: "between the 1820s and 1850s, the price of grain rose 30 percent and the price of meat 22 percent relative to industrial products". In our equilibrium, the relative price $p$ rises with $A$ as more people move to the city.

In equilibrium, workers and farmers must have the same utility. Farmers are paid less than workers and consume less $y$, so they must end up with more $x$ instead. Farmers will therefore be healthier than city-dwellers. Costa and Steckel (1997, fig. 2.5) find that farmers indeed were taller than workers in other occupations; farmers born in 1820-40 were taller than laborers, artisans or professionals. Nicholas and Steckel (1991, Table 4) report, similarly, that around 1790, the English farmer was taller than his urban countryman.

## 4.2 Industrialization and the ability to commit

Our results require that preferences be time-inconsistent, and a fair amount of experimental evidence supports this assumption (Laibson, 1998). We do not offer any new evidence on this score and turn, instead, to evidence on the proposition that time-inconsistent preferences lead people to make commitments. In particular, policies and regulations that serve as commitment devices are signs of *revealed* time-inconsistency.

If people cannot resolve the overwork problem on their own, we may expect that their government will enact policies and laws that will force the ex-post the behavior that is desirable ex-ante. Out model suggests that a government may enact laws that will put an upper bound on $t$, the time spent in market work. Chosen properly, an upper bound on $t$ would lead people to adopt advanced technology without fear of it pushing them into overwork. A well known enactment of an effective upper bound on $t$ is the British Factory Act of 1833 which banned the employment of children under nine, and restricted hours worked for a person under eighteen[7]. And, as early as the fourteenth century in England, rural children under twelve and *all* children of parents that did not own enough land were forbidden to enter urban industries (Derry, 1965, p. 79). Note how the latter policy specifically targets the households who cannot employ all their children on their own farm and thus have to supply labor at the outside market. In the United States the legal limit on weekly hours and the ban on child labor came with the Fair Labor Standards Act of 1938. Since the British policy precedes the US policy at least by a century, it may have helped Britain develop earlier.

If it is not feasible to limit $t$, a second-best policy is to force everyone to stay on the farm for as long as $A < \alpha_*$. This follows from Figure 5. Governments may achieve this outcome by banning occupational mobility (and thereby forcing people to stay on the farm), restricting agricultural commerce (and thereby closing the market

[7] The Factory Act was drafted years before, but all earlier attempts to pass it had been unsuccessful (Marvel, 1977).

for $z$) and restricting the import of technology (and thereby setting the effective $A$ close to zero during the period that the frontier $A$ is below $\alpha_*$). Interestingly, *all three* of these policies were enacted in pre-industrial Japan from the 1640 to the 1860s. The sale of land was prohibited from 1643 on, in order to perpetuate the system of independent peasant proprietors. The peasants owned the land and paid rice-denominated taxes on it to the feudal lords (Yamamura, 1973, p. 511). The decree effectively prevented the landholding peasants from leaving his farm. During the early pre-industrial period in Japan, the commerce was local, tightly controlled by feudal lords and limited to castle towns within each domain (Yamamura, 1973, p. 512). Only in the middle of the 18th century did merchants come to villages to trade rice for other goods. At the same time, Japan closed its ports to foreign trade for more than 250 years. When this self-imposed isolation from the outside world ended in 1858, Japan rapidly industrialized. All three policies would reduce welfare in a society where people have time-consistent preferences. Our model suggests, by contrast, that these policies may have been necessary in order to avoid the drop in welfare like one on Figure 5. Consistent with this view, Honda (1997, p. 259) concludes: "The health and welfare of the Japanese people during the 250 years prior to industrialization were good relative to that of the population of contemporary Europe and early industrial Japan".

## 4.3 Development, growth and income inequality

In our model, a gradual rise in $A$ induces industrialization. While $A$ is small, most people are in the agricultural sector. Next, as $A$ rises, people start to move to the cities, and income inequality appears between those who work on the factory and those who remain on the farm. During this phase, output grows, but lifetime utility falls. As $A$ rises in this phase, more and more people adopt the frontier technology. Finally, when $A$ reaches the point $\hat{\alpha}$, everyone enters the industrial sector "en masse". The economy then suddenly catches up with the industrial leaders and thereafter keeps pace with them.[8] From that point on, welfare rises.

We now plot the equilibrium fraction of industrial labor $\pi$ as a function of the position of the frontier technology $A$. This is the "industrialization path" of an economy that faces an exogenously growing technological frontier. Figure 6 shows two $\pi(A)$ plots for different values of $\gamma$. A country where the city environment is healthier should have a higher value of $\gamma$. This country will have a higher GDP per capita. Since $\frac{\partial \hat{\alpha}}{\partial \gamma} < 0$, the countries with higher $\gamma$ will also industrialize earlier. The

[8] One should not take too literally our "prediction" that the commercial agriculture shuts down after industrialization. This implication is only as good as the assumption that there is no technological progress in agriculture. While this assumption seems to be justified for the 19th century US (see Komlos, 1987, p. 910), the productivity gap across sectors cannot widen indefinitely. If we relax this assumption, we can keep all of our other implications, as well as have an agricultural sector in equilibrium.

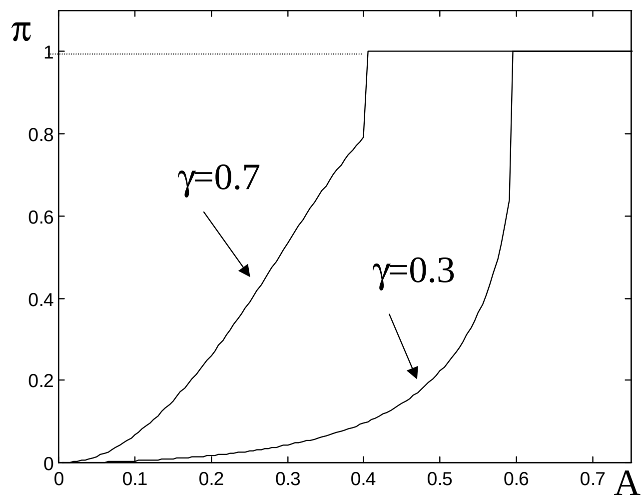


Figure 6: Development paths $\pi(A)$ for diffeent values of $\gamma$.

case in point is Britain versus the US. In 1800-30, the crude death rates in London and New York City were roughly the same at 25-27 persons per thousand (Deane and Cole, 1967, Table 28 and Haines, 1998, Figure 2). While urban death rates in England and Wales declined throughout the nineteenth century (Floud and Harris (1997, p. 97), the opposite occurred in the New York City. According to Haines (1998, p. 3), "The pattern is one of rising mortality from about 1820, sharply rising for the 1840s and 1850s, and a gradual subsidence after 1865". The rapid growth of population density[9] and poor sanitation in New York City are the likely reasons for this. We can interpret the US as the country with lower $\gamma$. Consistent with this, the United States *did* industrialize later, and did not reach the British level of GDP per capita until around 1900.

While only a fraction of the population is in the industrial sector, workers are earning more than farmers. The farming wage equals to the relative price of food $p$, and it is less than the industrial wage $A$. The wage income disparity follows a Kuznets-type relationship as $A$ grows. The income inequality first increases, because there is entry in high-wage industrial sector. While $\pi(A)$ is sufficiently small, the wage in agriculture grows slower than $A$. However, as $A$ increases, demand for food grows, which commands a higher price. At some point $p$ starts growing faster than $A$, which makes income distribution increasingly more equal. Inequality disappears permanently at the point where $A = \hat{\alpha}$.

Figure 7 provides a numerical illustration of the model's implications for income

[9] The population of New York City grew from 79,000 in 1800 to 1,441,000 (just Manhattan) in 1900 (Haines, 1998).

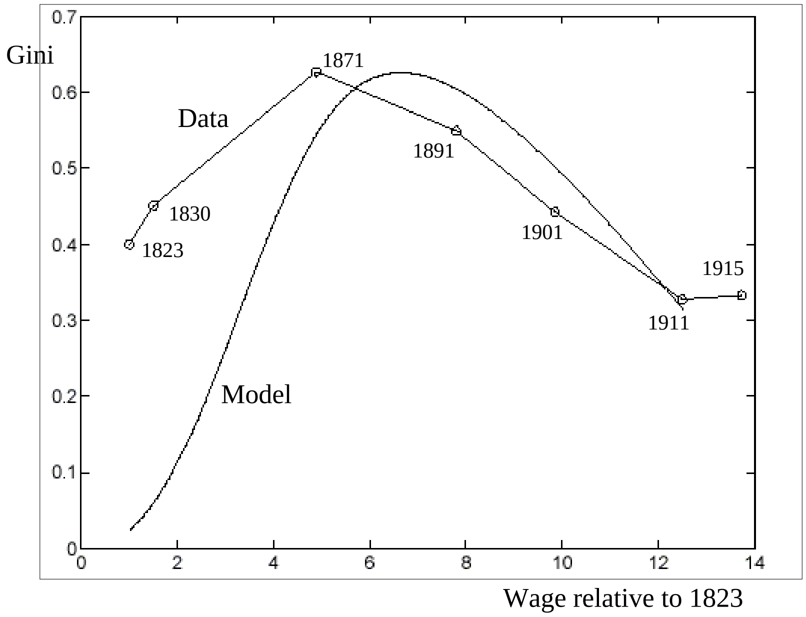


Figure 7: Simulated and actual Kuznets curves for Britain, 1823-1915

inequality. The figure depicts the time path of the Gini coefficient that the model implies and the actual historical series of the Gini in Britain reported in Williamson (1985)[10]. The vertical axis measures the values of the Gini coefficient at various dates. On the horizontal axis, however, instead of calendar time, we plot the average real wage[11] for 1823-1915. The plot is scaled so that the British real wage in 1823 is 1. For our model, the implied average wage is

$$w_t = \pi(A_t) A_t + [1 - \pi(A_t)] p_t.$$

Because we do not know the mapping from calendar time to $A_t$, we scale our Kuznets curve so that it flattens out (becomes zero) at the same wage as the historical series do, and also starts at the wage equal to 1. This determines the initial and the final values of $A$ for the simulation. We set $\theta = 0.3$ and $\beta = 0.6$ at the empirically plausible values for purposes of numerical illustration, and choose the value of $\gamma = 0.39$ to match the peak Gini coefficient of 0.62.

[10]The Gini coefficient for 1823-1915 is estimated from tax data and reported in Table 4.2, p. 61.

[11]ibid, Appendix Table C.1

# 5 Conclusion

We have shown that it is sometimes optimal for an agent to reject technology in order to commit to a certain behavior in the future. If that were all, it would be an example of results that Carrilo and Mariotti (2000) have proved in a more general setting. What we have added is the analysis of groups and the finding that groups behave very differently from isolated agents. This difference arises because a commitment value of non-adoption depends on the actions of other agents. Open markets destroy commitments and raise the temptation to adopt, which makes groups more technologically progressive. However, the early adoption makes everyone worse off, and the lifetime utility declines for a while as technology advances.

In interpreting the results, we focused on historical evidence. But our results on self-control and technology adoption in groups apply today as well. Technologies like the Internet and the cell-phone appear to be addictive.[12] Moreover, these technologies are more valuable if adopted by large groups. Our framework can be applied to analyze the adoption of addictive network technologies and to evaluate the public policy towards the next generation of communication devices, such as, for example, Web-enabled wireless phones.

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[12] The self-control problem that some Internet users experience is so widespread that it has even merited an official name: the Internet Addiction Disorder (IAD). Among the primary symptoms of the IAD is accessing the Internet for a longer time than intended, and neglecting marital and occupational duties. See http://www.iucf.indiana.edu/~brown/hyplan/addict.html for details.

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# 6 Appendix

**Lemma 1** Let $\gamma \in (0,1)$. The optimization problem (6) yields the following optimal decision rules $t = T(\alpha, p)$, $z_s = Z_s(\alpha, p)$, and $z_d = Z_d(\alpha, p)$ that are in the interior or in the corner depending on which of three regions $\alpha$ falls in:

| Values of $(\alpha, p)$ | Decision | Reduced form ex-ante utility |
|---|---|---|
| $\alpha \leq p$ | Supply $z$ to the market and produce $z$ for own household $t = 0$, $z_s > 0$, $z_d = 0$ | $U(p)$ |
| $\alpha \in \left(p, \frac{1}{\gamma}p\right)$ | Produce both $y$ and $z$, not participate in the market $t \in (0,1)$, $z_s = 0$, $z_d = 0$ | $U(\alpha)$ |
| $\alpha \geq \frac{1}{\gamma}p$ | Produce only $y$ and buy $z$ on the market $t = 1$, $z_s = 0$, $z_d > 0$ | $V(\alpha, p)$ |

**Proof:** The proof follows from inspection of the first order conditions for the ex-post optimization problem (6):

$$t: \qquad \alpha u'(y) - \beta v'(x) \lesseqgtr 0 \tag{13}$$

$$z_s: \qquad pu'(y) - \beta v'(x) \lesseqgtr 0 \tag{14}$$

$$z_d: \qquad -\frac{p}{\gamma}u'(y) + \beta v'(x) \lesseqgtr 0 \tag{15}$$

When $\alpha \leq p$ and (14) holds as equality, (13) is negative, and so is (15). This implies that $t = 0$, $z_s > 0$, $z_d = 0$. Therefore, when $\alpha \leq p$ the household does not operate technology $\alpha$ ($t = 0$) and does not buy $z$ from the market ($z_d = 0$). When $p < \alpha \leq \frac{p}{\gamma}$ and (13) holds as equality, (14) is negative, and (15) is still negative. Therefore, $t \in (0,1)$, $z_s = 0$, $z_d = 0$. The household spends part of the time operating technology $\alpha$ and spends the rest of the time producing its own $z$. Finally, when $\frac{p}{\gamma} < \alpha$ and (15) holds as equality (14) is negative and (13) is positive, which means that $t = 1$, $z_s = 0$, $z_d > 0$. The household spends all its time operating technology $\alpha$, does not supply $z$, but buys it from the market.

Substituting the decision rules back into expression (6), and comparing it with the definition of $U(\cdot)$ yields the following identity:

$$V(\alpha, p) \equiv \begin{cases} U(p), & \alpha \leq p \\ U(\alpha), & p < \alpha < \frac{1}{\gamma}p \end{cases} . \blacksquare$$

Before we prove Proposition 3, let us prove the following lemma:

**Lemma 2** Suppose that the function $U(\cdot)$ satisfies (4), (3), (5). Let $\gamma \in (0,1)$ and $\hat{\alpha} > 0$ be the (unique) positive root of the equation

$$U(\gamma\hat{\alpha}) = U(\hat{\alpha}).$$

Then, for any $A \leq \hat{\alpha}$ the equation

$$V(A,p) = U(p) \tag{16}$$

has a solution for $p \in [0, \gamma A]$, and for any $A > \hat{\alpha}$ this equation has no solution for $p \in [0, \gamma A]$.

**Proof:** Let us first establish that $U(\gamma\hat{\alpha}) = U(\hat{\alpha})$ has a unique positive root. This follows from the fact that $U(\cdot)$ has a unique minimum. Suppose that $\alpha_{\min} = \arg\min_\alpha U(\alpha)$. Then

$$U(\gamma\alpha) > U(\alpha) \text{ for any } \alpha < \alpha_{\min}$$

and

$$U(\gamma\alpha) < U(\alpha) \text{ for any } \alpha > \frac{\alpha_{\min}}{\gamma}.$$

Then the root $\hat{\alpha}$ can only be on the interval $\left[\alpha_{\min}, \frac{\alpha_{\min}}{\gamma}\right]$. On this interval, $U(\alpha)$ is strictly increasing, but $U(\gamma\alpha)$ is strictly decreasing. Besides,

$$U(\gamma\alpha_{\min}) > U(\alpha_{\min})$$

and

$$U(\gamma\frac{\alpha_{\min}}{\gamma}) = U(\alpha_{\min}) < U(\frac{\alpha_{\min}}{\gamma}).$$

This implies that there is a unique positive root $\hat{\alpha} \in \left[\alpha_{\min}, \frac{\alpha_{\min}}{\gamma}\right]$ and that

$$\begin{aligned} U(\gamma\alpha) &> U(\alpha) \text{ for any } \alpha < \hat{\alpha}, \\ U(\gamma\alpha) &< U(\alpha) \text{ for any } \alpha > \hat{\alpha}. \end{aligned} \tag{17}$$

Next, we need to show that for any $A \leq \hat{\alpha}$ the equation (16) has a solution on $p \in [0, \gamma A]$. Observe that $V(A,p)$ is strictly decreasing in $p$ (ex-ante utility of a high-tech type falls as price of food rises):

$$\frac{dV(A,p)}{dp} = \frac{d}{dp}\left(u(A - z_d(A,p)p) + v(\gamma z_d(A,p))\right) = u'(c)\left(-p\frac{\partial z_d}{\partial p} - z_d\right) + \gamma v'(x)\frac{\partial z_d}{\partial p} =$$

$$= -u'(c)z_d + pu'(c)\frac{\partial z_d}{\partial p}\left(\frac{1}{\beta} - 1\right) < 0.$$

Also, when $p \to 0$, $V(A,p)$ tends to infinity. Clearly,

$$\lim_{p \to 0} V(A,p) > U(0) = v(1). \tag{18}$$

At price $p = \gamma A$, the high-tech type is indifferent between buying $z$ on the market and producing $z$ with his own time. Therefore,

$$V(A, \gamma A) \equiv U(A), \text{ for any } A$$

This implies that for any $A < \hat{\alpha}$

$$V(A, \gamma A) \equiv U(A) < U(\gamma A). \tag{19}$$

It follows immediately from (18) and (19) that for any $A < \hat{\alpha}$ there is a $p \in [0, \gamma A]$ that solves (16). Figure 8 depicts the solution to (16) for some $A_1 < \hat{\alpha}$. On the figure, the curve $V(A_1, p)$ crosses $U(p)$ at the point where $p < \gamma A$. Of course, if $U(\cdot)$ is not convex, it is conceivable that there are three roots, not one. However, if either $U(\cdot)$ is convex or if $\frac{dV}{dp} < \frac{dU}{dp}$ for every $p$, there is just one root. For simplicity, let us assume that this is the case[13]. When $A = \hat{\alpha}$, equation (16) has a unique root $p = \gamma\hat{\alpha}$ (see Figure 8), because

$$V(\hat{\alpha}, p) > U(p) \text{ for } p < \gamma\hat{\alpha}.$$

Suppose now that $A > \hat{\alpha}$. Take any $A_2 > \hat{\alpha}$. Since $\frac{\partial V}{\partial A} > 0$,

$$V(A_2, p) > V(\hat{\alpha}, p) \geq U(p) \text{ for any } p \leq \gamma\hat{\alpha}.$$

Therefore, (16) has no solutions on $[0, \gamma\hat{\alpha}]$. It is left to check whether there can be any solutions on $[\gamma\hat{\alpha}, \gamma A_2]$. According to (17) any $p > \gamma\hat{\alpha}$,

$$U(p) < U(\hat{\alpha})$$

On the other hand, $\frac{dV}{dp} < 0$ implies that

$$V(A_2, p) > V(A_2, \gamma A_2) \equiv U(A_2) > U(\hat{\alpha}).$$

Hence there cannot be a solution on $[\gamma\hat{\alpha}, \gamma A_2]$ either. This situation is depicted on Figure 8 which shows that $V(A_2, p)$ never crosses $U(p)$ on $[0, \gamma A_2]$. ■

**Proof of Proposition 3:** Let $A > 0$. First, we must find the equilibrium on the ex-post market for $z$ for a given $\pi$. For any $\pi \in (0, 1)$ there exists a unique equilibrium where aggregate supply for $z$ equals aggregate demand. To see this, note that the left

[13] We have never encountered the three roots during numerical simulations, but we cannot rule this case out without further restricting the class of utility functions. For the three-root case, the results are essentially the same, except $\hat{\alpha}$ will have to be defined as a point where (16) has exactly one root. It will still be true that $\hat{\alpha} < \alpha_*$, so the statement of Proposition 3 and all the implications will remain unchanged.

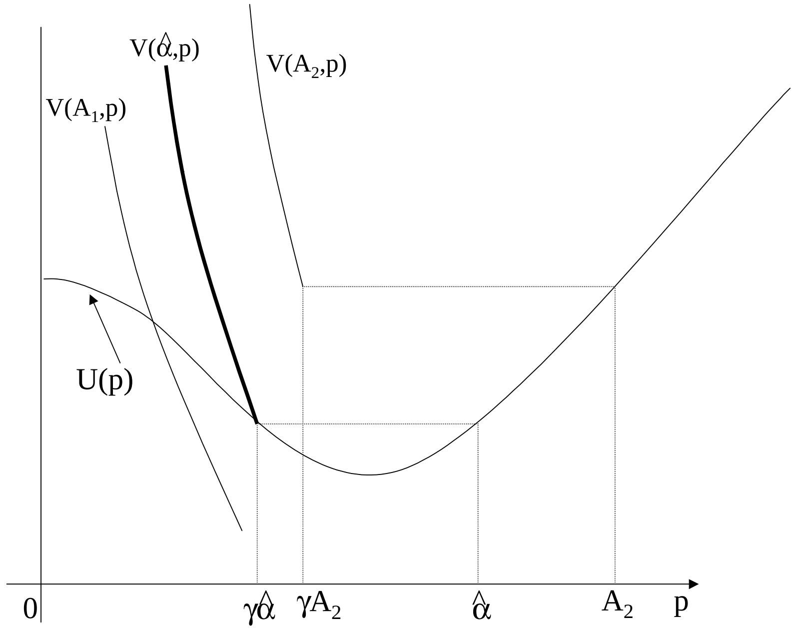


Figure 8: Roots of the equation $V(A,p) = U(p)$ for different values of $A$.

hand side of (12) is continuous and strictly decreasing in $p$ for $p \in (0, \gamma A]$, and the right-hand side of (12) is continuous and strictly increasing in $p$ on the same interval Moreover, since

$$\pi Z_d(A, 0) = +\infty > 0 = (1-\pi) Z_s(0, 0)$$

and

$$\pi Z_d(A, \gamma A) = 0 < (1-\pi) Z_s(0, \gamma A),$$

for every $\pi \in (0,1)$ there exists a unique price

$$P_e(\pi) : [0,1] \to [0, \gamma A]$$

implicitly defined by the supply equals demand condition.

$$\pi Z_d(A, P_e(\pi)) = (1-\pi) Z_s(0, P_e(\pi)). \tag{20}$$

The function $P_e(\cdot)$ is continuous, differentiable and strictly increasing. For $\pi = 0$ and $\pi = 1$ the quantity traded equals zero, and the supporting equilibrium price can be defined as the limit of $P_e(\cdot)$:

$$\begin{aligned} \pi &= 0: \; z = 0, \, p = \lim_{\pi \to 0} P_e(\pi) = 0 \\ \pi &= 1: \; z = 0, \, p = \lim_{\pi \to 1} P_e(\pi) = \gamma A. \end{aligned}$$

We have characterized the second period equilibrium given any $\pi$.

We will now turn to the proof of part (i). Suppose that $0 < A \leq \hat{\alpha}$. For the incentive compatibility condition (8) to hold, it is necessary that the high-tech type and the low-tech type have the same ex-ante utilities in equilibrium. That is,

$$V(A, p) = V(0, p) \equiv U(p).$$

According to Lemma 2, this equation has a solution on $[0, \gamma A]$. We can therefore define

$$\pi = P_e^{-1}(p)$$

as the distribution of agents across technology choices that supports the second period price $p$. It is left to show that

$$\{0, A\} = \arg\max_{\alpha} V(\alpha, p).$$

Let $p$ be the equilibrium price. Then, according to Lemma 1,

$$V(\alpha, p) \equiv \begin{cases} U(p), & \alpha \leq p \\ U(\alpha), & p < \alpha < \frac{1}{\gamma}p \\ V(\alpha, p) & \frac{1}{\gamma}p \leq \alpha \leq A \end{cases} .$$

Since $p < \gamma\hat{\alpha}$, (17) implies that

$$V(0, p) \equiv U(p) > U(\alpha) \equiv V(\alpha, p) \text{ for any } \alpha \in \left[p, \frac{1}{\gamma}p\right] \tag{21}$$

Because $V(\alpha, p)$ is continuous everywhere and is increasing for $\alpha \in \left[\frac{1}{\gamma}p, A\right]$, it follows that

$$V(\alpha, p) < V(A, p) \text{ for any } \alpha \in \left[\frac{1}{\gamma}p, A\right]$$

Therefore,

$$\arg\max_{\alpha} V(\alpha, p) = [0, p] \cup \{A\}.$$

Assuming an arbitrarily small adoption cost $\delta$, we can rule out $(0, p]$ from the arg max. This leaves us with

$$\arg\max_{\alpha} V(\alpha, p) = \{0, A\}.$$

We have just showed that for any $A \leq \hat{\alpha}$ there is an equilibrium where $\pi < 1$. Can there be other equilibria where $\pi \in \{0, 1\}$? By Proposition 2, $\pi = 0$ is never an equilibrium. But $\pi = 1$ is not an equilibrium either, because disposal of technology is a profitable deviation. If $\pi = 1$, then every agent earns $U(A)$. However, the first agent who deviates to producing $z$ will secure the ex-ante utility $U(\gamma A) > U(A)$.

Proving part (ii) is now easy. Because (16) has no solutions when $A > \hat{\alpha}$, an equilibrium where $\pi < 1$ cannot arise. By proposition 2, $\pi = 0$ cannot arise either. However, $\pi = 1$ is now incentive-compatible, since for $A > \hat{\alpha}$

$$U(\gamma A) < U(A).$$

In other words, the deviation to producing $z$ is no longer profitable when $A > \hat{\alpha}$.

It is left to prove part (iii). Suppose that $\mu$ is a measure that specifies the equilibrium allocation of agents over technology choices $[0, A]$. Assume that $\mu$ has no atoms on $(0, A)$ (it is still allowed to have atoms at $\alpha = 0$ and $\alpha = A$). Lemma 1 still applies, so we can define the second-period supply of $z$ as

$$\zeta_s(p) = \int_{\alpha \leq p} Z_s(0, p) d\mu$$

and second-period demand for $z$ as

$$\zeta_d(p) = \int_{\alpha \geq \frac{p}{\gamma}} Z_d(\alpha, p) d\mu$$

Because $\mu$ has no atoms on $(0, A)$, $\zeta_s(p)$ and $\zeta_d(p)$ are continuous. Also observe that $\zeta_s(p)$ is increasing and $\zeta_d(p)$ is decreasing, with $\zeta_d(p) = 0$ for any $p \geq \gamma A$. This implies that for every $\mu$, $\zeta_s(p)$ and $\zeta_d(p)$ cross at some second-period equilibrium price

$$p(\mu) \in (0, \gamma A].$$

If $z$ is traded in equilibrium, demand must be positive, so there are at least some agents whose $\alpha \geq \frac{p}{\gamma}$. All of them are strictly better off choosing $\alpha = A$, so any incentive-compatible equilibrium with traded $z$ must have all agents who demand $z$ choose $A$. Also, (if there is a trivial adoption cost), all agents who supply $z$ must choose $\alpha = 0$. Finally, for any $p(\mu) \in (0, \gamma A]$, condition (21) still applies. Therefore, all agents who participate in the market are strictly better off than those who do not participate. Hence when $z$ is traded in the second period, any choice $\alpha \in (0, A)$ is not incentive compatible.

To finish the proof, observe that either $z$ is traded in the second period (in which case $\alpha \in (0, A)$ is not incentive compatible), or the only incentive-compatible equilibrium is for everybody to choose $A$. This implies that the support of $\mu$ can have at most two points: $\alpha = 0$ and $\alpha = A$. ■