American Economic Journal: Macroeconomics 2018, 10(3): 71–117
https://doi.org/10.1257/mac.20160343

The Role of Annuitized Wealth in Post-retirement Behavior†

By John Laitner, Dan Silverman, and Dmitriy Stolyarov*

This paper develops a tractable model of post-retirement behavior with health status uncertainty and state-verification difficulties. The model distinguishes between annuitized and non-annuitized wealth and features means-tested Medicaid assistance with nursing home care. We show how to solve the potentially complex dynamic problem analytically, making it possible to characterize optimal behavior with phase diagrams. The analysis provides an integrated treatment of portfolio composition and consumption/wealth accumulation choices. We show the model can explain both the "retirement-saving puzzle" and the "annuity puzzle." (*JEL* D14, D15, G11, I18, I38, J14, J26)

Interest in the economic behavior of retired households has increased with population aging and the associated strain on public programs for the elderly.[1] Yet post-retirement behavior has proved challenging to understand. Intuition derived from classic theories, which emphasize consumption smoothing and income and longevity risk, does not fit with important features of the data. These features include a lack of wealth depletion after retirement—the "retirement-saving puzzle"—and a low demand for annuities at retirement—the "annuity puzzle." Generalizations of classic theory, partly aimed at addressing these puzzles, face analytic difficulties as they try to accommodate rules governing social insurance aimed at the elderly and interactions of health uncertainty with incomplete financial and insurance markets. This combination of puzzles and analytic difficulties has, so far, restricted research either to two or three period models or to numerical analysis. The purpose of this paper is to develop a new, multi-period workhorse model of post-retirement behavior that captures important uninsured risks and accommodates major puzzles, yet retains

*Laitner: Department of Economics, University of Michigan, 611 Tappan Ave., Ann Arbor, MI 48109 (email: jlaitner@umich.edu); Silverman: Department of Economics, W.P. Carey School of Business, Arizona State University, PO Box 879801, Tempe, AZ 85287 (email: dsilver3@asu.edu); Stolyarov: Department of Economics, University of Michigan, 611 Tappan Ave, Ann Arbor, MI 48109 (email: stolyar@umich.edu). The authors thank Andrew Caplin and Matthew Shapiro, as well as seminar participants at University of California Santa Barbara, MRRC Research Workshop, NBER Summer Institute, BYU Computational Economics Conference, Kansai University Osaka, NETSPAR Conference Amsterdam, "Working Longer and Retirement" Conference at SIEPR, and CIREQ Workshop Montreal. This work was supported by NIH/NIA grant R01-AG030841-01. The opinions and conclusions are solely those of the authors and should not be considered as representing the options or policy of any agency of the federal government.

†Go to https://doi.org/10.1257/mac.20160343 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

[1] E.g., Hubbard, Skinner, and Zeldes (1994, 1995); Palumbo (1999); Sinclair and Smetters (2004); Reichling and Smetters (2015); Dynan, Skinner, and Zeldes (2004); Scholz, Seshadri, and Khitatrakun (2006); Scholz and Seshadri (2009); Ameriks et al. (2011, 2015, 2016); DeNardi, French, and Jones (2010, 2015); Lockwood (2014); Love, Palumbo, and Smith (2009); Laibson (2011); Finkelstein, Luttmer, and Notowidigdo (2013); Poterba, Venti, and Wise (2011a, 2011b); and Pashchenko (2013).

sufficient tractability to be useful for qualitative, as well as quantitative, analysis. The model emphasizes the distinction between annuitized and non-annuitized wealth. With it, we are able to reveal the mechanisms through which portfolio composition interacts with public programs and uninsured risks and affects retiree behavior.

The model captures uncertain health and the correlation of major health changes with changes in mortality risk. Importantly, it assumes that informational asymmetries lead to incomplete private markets for long-term care insurance. It also incorporates a means-tested public alternative, Medicaid nursing home care, which households can use as a fallback during poor health. The model takes into account the inflexible nature of annuities as a form of wealth, as well as their treatment under Medicaid.

Despite its richness, the model is analytically tractable. One key to the tractability is the model's continuous-time formulation, which enables it to sidestep technical challenges related to non-convexities that emerge when accounting for the Medicaid means test. A second key is the simple case-based analytic approach that our formulation allows: although the model's elements and assumptions generate a variety of optimal behavioral patterns, we can partition the domain of observable initial conditions in such a way that outcomes are relatively straightforward on each (partition) element.

We demonstrate the value of the model in two ways. The first way consists of new, qualitative insights revealed by the analytic tractability of the model. Specifically, Propositions 3 and 4 and the associated phase diagrams in Figure 2 show both how portfolio composition plays a critical role in post-retirement behavior and how the level of annuitized wealth is central to the decision (to try) to self-insure or instead to rely on Medicaid for long-term care expenses. We show formally that whether liquid (bequeathable) wealth rises or falls after retirement depends not on total wealth levels but on the ratio of bequeathable to annuitized wealth. We also show how, among those households that might eventually rely on Medicaid, any efforts to self-insure long-term care needs are determined by the level of annuitized wealth.

The second way we demonstrate the formulation's value is to show that, despite its relative simplicity, it is consistent with two well-known puzzles in data. Our framework provides a unified treatment of the two puzzles, and our analytic approach offers detailed, intuitive explanations of each. The "retirement-saving puzzle," to take the first example, consists of evidence that a cohort's average (non-annuitized) wealth often remains roughly constant, or even rises, long into retirement. This contradicts classical life-cycle models, which predict that households save during working years in order to dissave thereafter.[2]

Section IV shows that a reasonable calibration of our model is consistent with rising or flat cohort average wealth profiles. As important, the analytic tractability of the model reveals the mechanisms behind post-retirement saving and the circumstances under which it emerges. Our households begin retirement in good health but subsequently pass into lower health status and then death. On the one hand, if needs for personal services raise the marginal utility of expenditure during poor health, we show that high health-status retirees may husband wealth for the future or even

[2]The present paper considers behavior post-retirement. We do not model changes in consumption just before and just after retirement discussed in other strands of the recent literature, sometimes referred to as the "retirement-consumption puzzle."

continue saving. On the other hand, although a cohort's members all eventually transition to poor health, the outflow of households from poor health to mortality can actually sustain the fraction of survivors in good health at a relatively high level. We show that the combination of the evolution of average health status and incentives to self-insure can dramatically influence cohort trajectories of average wealth.

Section V provides a second example where we generalize our baseline model (which takes annuitization levels as given) to allow endogenous annuities and show that reasonable calibrations are consistent with households' apparent reluctance to annuitize all, or most, of their wealth at retirement—the "annuity puzzle." Households, for instance, often claim Social Security benefits at or below the age for full retirement benefits, thereby forgoing additional actuarially fair annuitization (Brown 2007).

Again, the analytic tractability of the model illuminates the mechanisms behind the puzzle. We find that while households with low lifetime resources find end-of-life Medicaid care acceptable, the middle class is ambivalent. Middle-class households attempt to use their private wealth to delay the standard of living that Medicaid entails—though they reserve, given uncertain longevity, Medicaid as a fall-back option. The generalized analysis in Section V shows that, because asymmetric information precludes health state-contingent annuities, when we allow endogenous levels of annuitization, middle-class households in good health choose portfolios with a mixture of simple (i.e., non-health-contingent) annuities and bonds. (They liquidate the bonds after the arrival of poor health, turning to Medicaid after the bonds are exhausted.) In this way, a substantial demand for liquid wealth can arise among the healthy. Less than complete annuitization at retirement, at least among the middle class, can be fully consistent with the generalized model.

Returning to our baseline specification, Section VI examines two further aspects of optimal life-cycle behavior. As noted, a dichotomy emerges in our analysis: low-resource households tend to accept Medicaid care promptly after their health status declines, whereas middle-class households take steps to delay their reliance upon it. Section VI suggests that this can explain empirical patterns of the timing of Medicaid take-up in different parts of the income distribution (and, in particular, different parts of the distribution of annuity income). Similarly, our analysis shows that accidental bequests arising from self-insurance behavior most frequently occur for households with middle class and above resource levels.

In the end, our model offers new qualitative insights about post-retirement behavior by accommodating important uninsured risks and means-tested social insurance, while maintaining analytic tractability. The model is simple, but offers sufficient flexibility to make quantitative predictions consistent with key empirical puzzles. The model thus provides a potential new workhorse for the analysis of post-retirement behavior.

Relation to the Literature.—This subsection describes a theoretical and empirical backdrop for related literature and compares our approach to leading examples of research in the area. We argue that a recognition of key uninsured risks, complex rules of social insurance, and empirical puzzles inspired researchers to generalize classic life-cycle models. Prior to our paper, however, the generalizations have been restricted either to two to three period models or to numerical analysis.

In the classic life-cycle models (Modigliani 1986, Yaari 1965), households face few uninsured risks and smooth their lifetime consumption by accumulating wealth prior to retirement and decumulating it thereafter. Longevity risk is insured by annuitizing most wealth upon retirement. Subsequent analyses recognized the importance of uninsured income risk and liquidity constraints and generalized the classic models to accommodate different forms of income uncertainty and resulting precautionary savings motives (e.g., Zeldes 1989, Deaton 1991, Carroll 1997, and Gourinchas and Parker 2002).

A variety of empirical regularities have presented puzzles for the classic life-cycle models and their early generalizations. At least since Mirer (1979), for example, evidence has often seemed at variance with simple predictions about post-retirement behavior. Kotlikoff and Summers (1988, 54) noted,

> *Decumulation of wealth after retirement is an essential aspect of the life-cycle theory. Yet simple tabulations of wealth holdings by age … or savings rates by age … do not support the central prediction that the aged dissave.*

More recent work with panel data confirms that mean and median cohort wealth, for either singles or couples, can be stationary or rising for many years after retirement (Poterba, Venti, and Wise 2011a).[3,4]

Similarly, economists long sought to understand with life-cycle models the reasons why the strong Yaari (1965) prediction does not hold and households do not fully annuitize their private wealth at retirement. Benartzi, Previtero, and Thaler (2011, 149) write,

> *The theoretical prediction that many people will want to annuitize a substantial portion of their wealth stands in sharp contrast to what we observe.*

Incomplete markets that leave many forms of uninsured risk, together with these important empirical puzzles, inspired a new generation of life-cycle analysis that emphasizes health risk, the correlation of major health changes with changes in mortality risk, and the influence of means-tested social insurance.

Building on ideas in Hubbard, Skinner, and Zeldes (1995); Kotlikoff (1989); and Palumbo (1999), recent analyses of post-retirement saving such as Ameriks et al. (2011, 2015, 2016) and DeNardi, French, and Jones (2010) include a number of the same elements as our framework, namely, health changes and mortality risk, out-of-pocket expenses in poor health, government guaranteed consumption floors (in our case, Medicaid nursing home care), and fixed annuity income. Since consumption floors can induce non-convexities, the leading multi-period analyses of these

[3] See also, for instance, Ameriks et al. (2015), who observe, "The elementary life-cycle model predicts a strong pattern of dissaving in retirement. Yet this strong dissaving is not observed empirically. Establishing what is wrong with the simple model is vital …." See also DeNardi, French, and Jones (2015, Figure 7) as well as Smith, Soto, and Penner (2009); Love, Palumbo, and Smith (2009); and many others.

[4] Other evidence, however, seems more ambiguous: cohort median wealth is shown to rise with age for 65–79-year-olds and to fall at older ages in Hurd and Rohwedder (2015, Table 14.5a). At the same time, the rate of "active saving," although small, is negative at all ages.

problems, including Ameriks et al. (2011, 2015, 2016) and DeNardi, French, and Jones (2010), rely upon numerical solutions. In explaining household wealth trajectories, both recognize the potential importance of post-retirement precautionary saving.

We are thus not the first to address these important late-life risks or accommodate these important puzzles. Our formulation, however, sidesteps non-convexities and allows us to characterize solutions with first-order conditions that can provide intuitions and comparative-static results. As previously noted, a payoff from being able to avoid numerical analysis is several important refinements for the study of precautionary saving. We show that a (healthy) household's desire to save after retirement depends upon its portfolio composition: given two healthy households with identical total net worth, our model shows that the one with the higher fraction of annuities in its portfolio is the more likely to continue saving. Among those who might eventually turn to Medicaid to pay for long-term care, we show the centrality of annuity-income levels in the decision (to try) to self-insure or instead rely on Medicaid. We thus offer a refined interpretation of the evidence in DeNardi, French, and Jones (2016) showing that wealthier households tend to access Medicaid assistance later in life. Our results are consistent with this finding, and we can characterize Medicaid take-up timing analytically and provide further interpretations of the data.

A recent stream of life-cycle analysis concerned with post-retirement saving emphasizes the role of intentional bequests in sustaining private wealth holdings late in life. See, e.g., Ameriks et al. (2011); DeNardi, French, and Jones (2010); and Lockwood (2014). Our model has no intentional bequests; all bequests here are "accidental." Yet, we find that intentional bequests are not required to fit the evidence on late-in-life saving. Other than for the wealthiest decile of households (see Section V), bequests that emerge in our model are by-products of incomplete annuitization. Survey evidence on intentional bequests is mixed: respondents to direct questions about leaving a bequest split approximately equally between answering that bequests are important and not important (Lockwood 2014, Laitner and Juster 1996). Our analysis allows one to rationalize the post-retirement behavior of the latter group (as well as those for whom an "important" bequest could be a modest family heirloom).

There is also rich literature on the "annuity puzzle" (e.g., Finkelstein and Poterba 2004; Davidoff, Brown, and Diamond 2005; Mitchell et al. 1999; Friedman and Warshawsky 1990; Benartzi, Previtero, and Thaler 2011; and many others). As with the "retirement-savings puzzle," life-cycle multi-period analysis of the "annuity puzzle" has been numerical.

For example, both this paper and Reichling and Smetters (2015) offer new interpretations of the "annuity puzzle." While the studies have a number of assumptions in common, the institutional settings differ, and beyond three periods, the Reichling and Smetters analysis is numerical. Another important distinction is that Reichling and Smetters allow a household whose current health and/or mortality hazards have changed to purchase new annuities reflecting the revised status. Even with these state-contingent annuities, and without liquidity constraints, the annuity puzzle is resolved in Reichling and Smetters. In our model, state-verification problems preclude health-contingent annuities. Nonetheless, a household suffering a decline in health status can access Medicaid nursing home care and that option alone, we show, can substantially reduce the demand for annuities at retirement.

Ameriks et al. (2015) present simulations of a formulation that has health changes and state-dependent utility. Given a 10 percent load factor on annuities and households with \$50–100,000 of existing income and bond wealth up to \$400,000, they find essentially no demand for extra annuities at retirement (Ameriks et al. 2015, Figure 10). We show that this outcome is consistent with the qualitative implications of our model, and we show how and why household initial conditions, health-status realizations, and interest rates affect outcomes.

The organization of this paper is as follows. Section I presents our assumptions and compares our formulation with others in the literature. Sections II–III analyze our model. Section IV considers the retirement-saving puzzle, Section V the annuity puzzle, and Section VI Medicaid take-up and bequests. Section VII concludes.

I. Model

As indicated in the introduction, we follow the recent literature in subdividing a household's post-retirement years into intervals with good and poor health.

We study single-person, retired households. At any age s, a household's health state, h, is either "high," H, or "low," L. The household starts retirement with $h = H$. There is a Poisson process with hazard rate $\lambda > 0$, such that at the first Poisson event the health state drops to low. Once in state $h = L$, a second Poisson process begins, with parameter $\Lambda > 0$. At the Poisson event for the second process, the household's life ends.

We focus on the general "health state" of an individual, rather than his/her medical status. Think of "health state" as referring to chronic conditions. Consider, for example, troubles with activities of daily living (ADLs), such as eating, bathing, dressing, or transferring in and out of bed. Individuals with such difficulties may need to hire assistance or move to a nursing home. The expense can be substantial. It may, in practice, be the largest part of average out-of-pocket (OOP) medical expenses (see, for instance, Marshall, McGarry, and Skinner 2010; and Hurd and Rohwedder 2009).

State-Dependent Utility.—We assume that health state affects behavior through state-dependent utility. In our framework, there are no direct budgetary consequences from changes in h—all retirees have access to Medicare insurance that covers the medical part of long-term care needs. By contrast, we treat all nonmedical long-term care (LTC) expenses (i.e., health-related expenses not covered by Medicare—such as long nursing home stays) as part of consumption. A household with $h = H$ and consumption c has utility flow

$$u(c) = \frac{[c]^{\gamma}}{\gamma}.$$

Following most empirical evidence, let $\gamma < 0$. We assume there is a household production technology for transforming expenditure, x, to a consumption service flow, c:

$$c = \begin{cases} x & \text{if } h = H \\ \omega x & \text{if } h = L \end{cases}. \tag{1}$$

We also assume that the low health state is an impediment to generating consumption services from x; thus,

$$\omega \in (0,1).$$

The loss of consumption services that occurs upon reaching the low health state may be substantial: an agent in need of LTC might lose capacity for home production related to ADLs, and her quality of life may decline precipitously. Utility from consumption expenditure x while in health state $h = L$ is

$$U(x) \equiv u(\omega x) \equiv \omega^{\gamma} u(x). \tag{2}$$

Since $\omega^{\gamma} > 1$, an agent in the low health state has lower utility but higher marginal utility of expenditure. Specifically, marginal utility of consuming X in low health state equals the marginal utility of consuming a smaller amount, X/Ω, in high health state:

$$U'(X) = \frac{\partial u(\omega X)}{\partial X} = \omega u'(\omega X) = u'\Big(\frac{X}{\Omega}\Big), \quad \text{where } \Omega = [\omega]^{\frac{\gamma}{1-\gamma}} > 1. \tag{3}$$

Our specification of household preferences assumes the simplest form of state dependence: utility is $u(x)$ in the high health state and $\omega^{\gamma} u(x)$ in the low health state, where x is a single consumption category that includes the nonmedical part of LTC expenditure.[5] These assumptions are not as restrictive as one might think: for example, state-dependent utility function (2) can be micro-founded with a richer model where nonmedical LTC expenditure is a separate, endogenous variable—see Appendix B.

Available Insurance Instruments.—Households in our model face correlated longevity and health-status risks. If asset markets were complete, agents would optimally rely on state-contingent annuities and insurance contracts as follows. (i) At retirement, a household would buy an annuity paying a fixed benefit stream for the duration of the high health state. (ii) The household would also buy an insurance policy paying a lump-sum benefit when the high health state ends. (This is referred to as "long-term care insurance.") (iii) The household would use the insurance payout to purchase a low health-state annuity (the return on which would reflect the low health-state mortality rate Λ). A household could complete financial steps (i)–(iii) at the moment of retirement, and it would have no demand for liquid wealth.

Crucially, however, our analysis assumes that state-verification problems for h are much greater than for medical status (which we assume is insured through Medicare). An agent knows when he/she enters state $h = L$, but the transition from

[5] Hubbard, Skinner, and Zeldes (1995) and DeNardi, French, and Jones (2010) use a similar specification of preferences but assume that nonmedical LTC expenditure is an exogenously fixed parameter not subject to choice and not directly affecting utility.

$h = H$ is not legally verifiable. This prevents agents from obtaining health-state insurance.[6] Marshall, McGarry, and Skinner (2010, 26) write,

> *Indeed, the ultimate luxury good appears to be the ability to retain independence and remain in one's home … through the use of (paid) helpers …. These types of expenses are generally not amenable to insurance coverage ….*

Put differently, in our model, state-verification difficulties preclude private long-term care insurance and health state-contingent annuities, thus transactions (i)–(iii) are infeasible. With incomplete markets, households, we show, use simple (not state-contingent) annuities to insure longevity in the high health state, and they carry liquid wealth to (partially) self-insure higher expenditure needs associated with the low health state. In addition, households in the low health state are assumed to have access to social insurance, as described later.

Means-Tested Public Assistance.—In our framework, a household with health status $h = L$ can qualify for Medicaid-provided nursing home care. State-verification difficulties affecting private LTC insurance markets may be less relevant for the Medicaid program because it provides only a basic level of in-kind benefits and access is rigorously means tested. The means test for this program requires the household to forfeit all of its bequeathable wealth and annuities to qualify for assistance.[7] Let Medicaid nursing home care correspond to expenditure flow $X_M > 0$. In practice, elderly households often view Medicaid nursing home care as a relatively unattractive option.[8] Accordingly, our model assumes that the utility flow from Medicaid nursing home care is $U(\bar{X})$, where $\bar{X} \leq X_M$ is the expenditure flow adjusted for disamenities.

Household Financial Assets.—Households retire with endowments of two assets, annuities, with income a, and bequeathable net worth b (i.e., liquid wealth).[9] Major components of annuitized wealth include Social Security, defined benefit pensions, and Medicare benefits. Bequeathable wealth b pays real interest rate $r > 0$. Let $\beta \geq 0$ be the subjective discount rate. We assume $r \geq \beta$. If we think of the analysis as beginning at age 65, the average interval of $h = H$ might be about 12 years,

[6] On the use of long-term care insurance, which is analogous to health-state insurance in our model, see Miller, Mor, and Clark (2010); Brown and Finkelstein (2007, 2008); Brown, Goda, and McGarry (2012); Congressional Budget Office (2004); and Pauly (1990). Private insurance covers less than 5 percent of long-term care expenditures in the United States (Brown and Finkelstein 2007). For a discussion of information problems and the long-term care insurance market, see, for example, Norton (2000).

[7] In practice, a household may be able to maintain limited private assets after accepting Medicaid—for example, under some circumstances a recipient can transfer her residence to a sibling or child (see Budish 1995, 43). This paper disregards these program details.

[8] Ameriks et al. (2011) refer to disamenities of Medicaid-provided nursing home care as *public care aversion*. Indeed, the level of service is very basic, access is rigorously means tested, and many households strongly prefer to live in familiar surroundings and to maintain a degree of control over their lives (Schafer 1999).

[9] In the model, liquid wealth includes home equity. This interpretation assumes that a household can borrow up to 100 percent of the value of its house at rate r and that the real estate market is frictionless. Both of the previously mentioned assumptions are standard in optimal consumption models with one good, including those in related literature on post-retirement behavior.

and the average duration of $h = L$ about 3 years.[10] With a Poisson process, average duration is the reciprocal of the hazard. We assume $\Lambda > \lambda > r - \beta$.

Summary.—Recapping our baseline assumptions:

ASSUMPTION 1: "Health state" is not verifiable; hence, there is no health-state insurance. Annuities are exogenously set at retirement.

ASSUMPTION 2: If b_s is bequeathable net worth when $h = H$ and B_s is the same for $h = L$, we have $b_s \geq 0$ and $B_s \geq 0$ all $s \geq 0$.

ASSUMPTION 3: $\gamma < 0$, and $\omega \in (0, 1)$.

ASSUMPTION 4: A household transitions from $h = H$ to $h = L$ with Poisson hazard λ and from health state $h = L$ to death with Poisson hazard Λ. We assume $\Lambda > \lambda$.

ASSUMPTION 5: The real interest rate is r, with $0 \leq \beta \leq r < \lambda + \beta$.

ASSUMPTION 6: A household in the low health state can turn to Medicaid nursing home care. The consumption value of the latter is a flow $\bar{X}$.

II. Low Health Phase

We solve our model backward, beginning with the last phase of life. In that period, the household is in the low health state $h = L$ and faces mortality hazard Λ. The corresponding optimal consumption problem has two state variables: B, the liquid wealth at the onset of poor health, and a, the exogenous annuity-income flow. Solving the problem yields the value function $V(B, a)$, which we then use as a continuation value describing behavior during the initial healthy phase. Importantly, we show that the value function $V(B, a)$ is strictly concave despite the presence of the Medicaid-provided consumption floor $\bar{X}$. Concavity makes it possible to derive analytical results based on phase diagram characterizations.

Without loss of generality, set the age at which the $h = L$ state begins to $t = 0$. At $t = 0$, let bequeathable net worth be $B \geq 0$. Annuity income is $a > 0$, X_t is consumption expenditure at age t, and $U(X_t)$ is the corresponding utility flow. The expected utility of the household is

$$\int_0^{\infty} \Lambda e^{-\Lambda \cdot S} \int_0^{S} e^{-\beta t} U(X_t)\, dt dS = \int_0^{\infty} e^{-(\Lambda+\beta)t} U(X_t)\, dt.$$

Later, we show that the household will optimally plan to exhaust its liquid wealth within a finite time, which we denote by T. If the household dies before reaching age T, it leaves an accidental bequest. If the household is alive at age T, it becomes

[10] E.g., Sinclair and Smetters (2004).

liquidity constrained and chooses one of two courses of action: it either relinquishes its annuity income a and accepts Medicaid-provided consumption flow $\bar{X}$, or it sets its consumption equal to its annuity income for the remainder of its life. Households with $a \geq \bar{X}$ will prefer to live on their annuity income (case (i)), while households with $a < \bar{X}$ will accept Medicaid assistance (case (ii)). To simplify the exposition, it is convenient to analyze the two cases separately.

Case (i): $a \geq \bar{X}$. Starting from an initial wealth level B, the household chooses a consumption expenditure path X_t all $t \geq 0$ to solve

$$V(B, a) \equiv \max_{X_t} \int_0^{\infty} e^{-(\Lambda+\beta)t} U(X_t)\, dt \tag{4}$$

subject to

$$\dot{B}_t = r \cdot B_t + a - X_t,$$

$$B_t \geq 0 \quad \text{all } t \geq 0,$$

$$B_0 = B, \quad \text{and } a \text{ given.}$$

Case (i) is thus described by a standard, infinite horizon optimal control problem with exponentially discounted utility and a state-variable constraint $B_t \geq 0$. The strict concavity of problem (4) ensures that if a solution exists, it is unique.

We start by separately characterizing the solution to (4) in the liquidity constrained and unconstrained regimes. In the constrained regime, the optimal consumption trajectory is flat, as the household consumes its annuity-income flow in every period. In the unconstrained regime, the optimal consumption falls at a constant rate. We further show—in Proposition 1—that the household spends the first T periods unconstrained, and subsequently, it enters the liquidity constrained regime for the rest of its life.

LEMMA 1: *Suppose that the liquidity constraint binds at date T; that is, $B_T = 0$. Then $(B_t^*, X_t^*) = (0, a)$ solves (4) for all $t \geq T$. Moreover, for any t with $B_t > 0$, the optimal consumption trajectory obeys*

$$\frac{\dot{X}_t}{X_t} = \sigma, \quad \textit{where } \sigma \equiv \frac{r - (\Lambda + \beta)}{1 - \gamma} < 0. \tag{5}$$

PROOF:

See Appendix B.

The idea of the proof is as follows. Households in (4) behave as if their subjective discount rate is $\Lambda + \beta > r$; so, a household without a binding liquidity constraint desires a falling time path of consumption expenditure. Standard arguments in this case lead to the Euler equation (5).

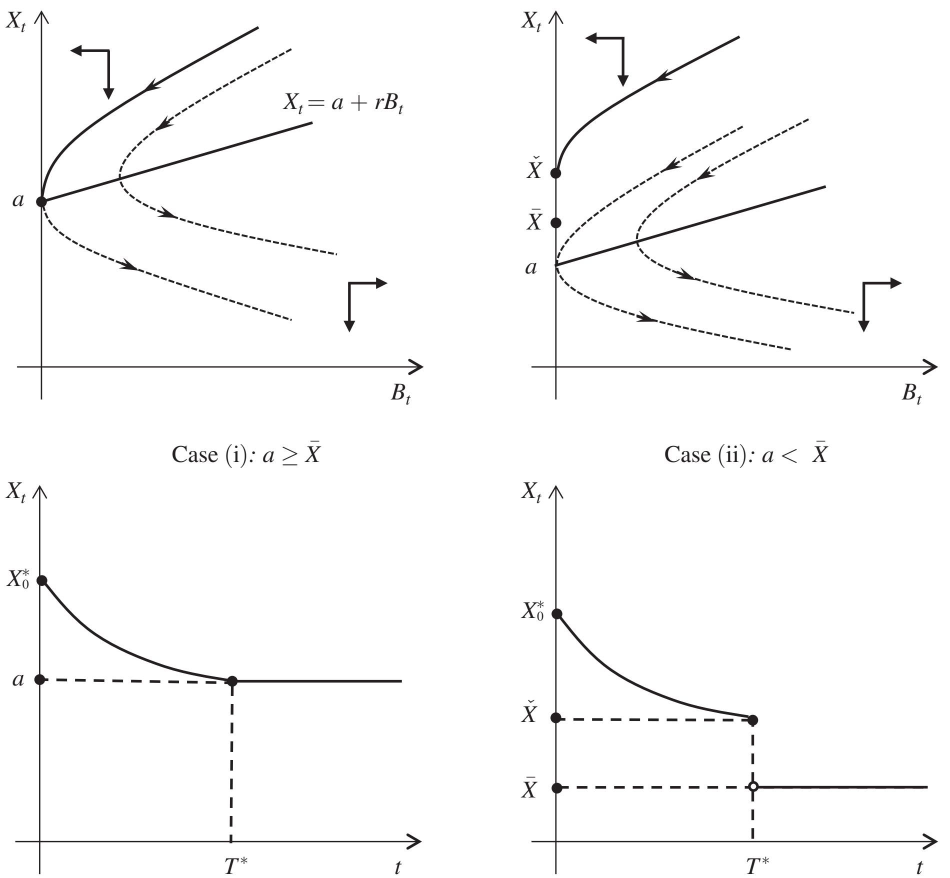

Figure 1. Phase Diagrams and Consumption Trajectories in Low Health State

When $B_t = 0$, however, only $X_t \leq a$ is feasible. Choosing $X_t < a$ makes (5) a necessary condition. But, a permanently falling consumption path cannot be optimal because the household's liquid wealth would then expand until its death, with the final balance left unused. The solution is instead to consume the annuity income and maintain the constrained regime.

The phase diagram of Figure 1, case (i) depicts candidate solutions in the unconstrained regime. Each dotted curve in Figure 1, case (i) is a trajectory satisfying the budget constraint, the liquidity constraint $B_t \geq 0$, and the Euler equation (5). However, we can rule out the optimality of most of the trajectories a priori. A given trajectory intersects the vertical line $B_0 = B > 0$ at two points. Starting at the point with higher consumption should clearly be preferred. By the same reasoning, following the higher trajectory indefinitely is inferior to stopping at its intersection with the line $X_t = rB_t + a$. Yet, the latter cannot be optimal since bequeathable wealth is never exhausted. The exception is the trajectory that stops at point $(0, a)$ and stays there indefinitely. Lemma 1 shows that the transversality condition is then satisfied.

PROPOSITION 1: *In case* (*i*), *the trajectory in Figure* 1, case (*i*) *that reaches* $(B_t, X_t) = (0, a)$ *from above and then remains at* $(0, a)$ *forever solves problem* (4). *The solution,* (B_t^*, X_t^*), *is continuous in t. There exists* $T^* = T^*(B, a) \in [0, \infty)$, *such that both* B_t^* *and* X_t^* *are strictly decreasing in t for* $t \leq T^*$, *but* $(B_t^*, X_t^*) = (0, a)$ *for* $t > T^*$. *The value function* $V(B, a)$ *is strictly increasing, strictly concave, and continuously differentiable in B.*

PROOF:

See Appendix B.

The concavity of the value function in Proposition 1 is a straightforward consequence of the concavity of the maximization problem (4). It plays an important role in our analysis, enabling Section III to rely upon first-order conditions, for example.

The case (i) solution does not depend on $\bar{X}$ since the household never turns to Medicaid. Accordingly, optimal behavior in the absence of public assistance would also be as in Proposition 1.

Case (ii): $a < \bar{X}$. Case (ii) obtains when the value of Medicaid nursing home care exceeds a household's annuity income. If such a household fully depletes its bequeathable wealth, accepting Medicaid nursing home assistance is attractive.

To be more precise, $(B_t^*, X_t^*) = (0, \bar{X})$ is the optimal trajectory in the constrained regime. If $B_t = 0$, the household must either accept Medicaid or choose $X_t \leq a$. The logic of Lemma 1 shows that in the latter case, setting $X_t = a$ for all subsequent ages is optimal. But in case (ii), permanently accepting Medicaid nursing home care is better. Once Medicaid is accepted, there is no advantage to ever leaving it.

Let T denote the age when the household exhausts its liquid wealth and turns to Medicaid (with $T = \infty$ corresponding to the option of never using Medicaid). Then case (ii) behavior can be described with a standard free-endpoint salvage value problem (Kamien and Schwartz 1981, sect. 7):

$$V(B, a) = \max_{X_t, T} \left(\int_0^T e^{-(\Lambda+\beta)t} U(X_t)\, dt + e^{-(\Lambda+\beta)T} \frac{U(\bar{X})}{\Lambda + \beta} \right) \tag{6}$$

subject to

$$\dot{B}_t = r \cdot B_t + a - X_t,$$

$$B_t \geq 0 \quad \text{all } t \geq 0,$$

$$B_0 = B, \quad \text{and} \quad a \text{ given.}$$

The main difference from case (i) is that the optimal consumption trajectory experiences a discontinuous drop at date T when the household becomes liquidity constrained. The value function $V(B, a)$ in (6) is nevertheless concave and continuously differentiable, just as in case (i). The following proposition characterizes the solution.

PROPOSITION 2: *In case* (*ii*), *there is a unique* $\check{X} = \check{X}(a) \in (\bar{X}, \infty)$, *independent of* B, *such that the trajectory in Figure* 1, case (*ii*) *that reaches* $(B_t, X_t) = (0, \check{X})$ *from above*, *then jumps to the point* $(0, \bar{X})$ *and remains there forever solves problem* (6).

There exists $T^* = T^*(B, a) \in [0, \infty)$, *such that both* B_t^* *and* X_t^* *are strictly decreasing in t for* $t \leq T^*$, *but* $(B_t^*, X_t^*) = (0, \bar{X})$ *for* $t > T^*$. (B_t^*, X_t^*) *is continuous in t except at* $t = T^*$, *when* X_t^* *drops abruptly. Specifically*,

$$X_t^* = \begin{cases} \check{X} \cdot e^{\sigma(t - T^*)} & \text{for } t \in [0, T^*] \\ \bar{X} & \text{for } t > T^* \end{cases},$$

with σ *defined in* (5).

The value function $V(B, a)$ *is strictly increasing in B; strictly concave; and, except at* $B = 0$, *continuously differentiable*.

PROOF:

See Appendix B.

Discussion.—Once we fix the optimal T^*, problem (6) has, for $t < T^*$, the same first-order conditions and budget constraint as (4). Hence, for $t \in [0, T^*]$, the same trajectories in Figure 1 apply as before. As in case (i), only the upper part of a trajectory ending at a point with $B = 0$ is of potential interest as a candidate solution.

Proposition 2 shows that optimal behavior (B_t^*, X_t^*) in case (ii) leads to convergence to $(0, \check{X})$ followed by a discontinuous drop to $(0, \bar{X})$ and subsequent stationarity—see Figure 1, case (ii). The intuition for the discontinuous drop in expenditure at time T^* when the household becomes liquidity constrained is as follows.

If $B = 0$, we have argued that the household can do no better than immediately accepting Medicaid nursing home care and never leaving it. Thus, $T^* = 0$. Without loss of generality, we can think of $X_0^* = \check{X}$ and $X_t^* = \bar{X}$ all $t > 0$.[11]

The consumption discontinuity arises in case (ii) because at time T^* the household exchanges its annuity-income flow a for a Medicaid-provided consumption flow $\bar{X} > a$. Consider the household's trade-offs just prior to Medicaid acceptance, in the interval $[T^* - dt, T^*]$. Over this interval, the optimal consumption trajectory provides utility $U(\check{X})\,dt$. Suppose instead that the household accepts Medicaid an instant earlier, at time $T^* - dt$. Its utility then drops to $U(\bar{X})dt$, but its liquid wealth—available for consumption at prior times—rises by $[\check{X} - a]\,dt$. The value of this wealth in units of utility is $U'(\check{X}) \cdot [\check{X} - a]\,dt$. Optimality requires that accepting Medicaid at time T^* or an instant prior yields equal net benefit

$$U(\check{X}) - U(\bar{X}) = U'(\check{X}) \cdot [\check{X} - a]. \tag{7}$$

Since the optimal consumption expenditure never drops below the floor $\bar{X} > a$, it must be $\check{X}$ exceeds a so that the RHS of (7) is positive. Then, for the LHS of (7) to be positive, we must have $\check{X} > \bar{X}$.

[11] Note that changing consumption expenditure at a single point does not affect any of this paper's criterion integrals.

In other words, the previous analysis shows that age T^* will, in practice, be particularly unhappy: at $t = T^*$, a household's bequeathable wealth runs out, and as the household transits from privately funded LTC to Medicaid, its utility flow takes a permanent, discrete step downward. Section I notes the public's seeming aversion to Medicaid nursing home care, and the decline in utility predicted by the model at age T^* might rationalize this aversion.

Our analytical results and phase diagram characterization depend on the value function $V(B,a)$ being concave and smooth. These properties obtain despite the presence of a consumption floor in case (ii) because of our continuous-time formulation. To see the role of continuous time, compare our framework to one where time is discrete. Suppose that the last period of life lasts for one discrete unit of time and that the household carries liquid wealth B to its last period. The value function is then

$$V(B,a) = \max\{U(\bar{X}), U(B+a)\}. \tag{8}$$

Medicaid creates a welfare floor $U(\bar{X})$, which renders $V(B,a)$ non-concave, with a kink at $B = \bar{X} - a$.

In contrast, with our continuous-time framework, any wealth amount $B > 0$ can temporarily generate a consumption flow greater than $\bar{X}$. Optimal Medicaid take-up then never occurs until $B_t = 0$. Roughly speaking, the flat segment $B \leq \bar{X} - a$ in (8) collapses to a single point $B = 0$, and concavity of the value function is thus preserved. Nor does the discontinuity of the consumption decision rule $X_t^*(B,a)$ at $B = 0$ interfere with the value function's concavity—as the discontinuity occurs on the boundary and takes the form of a decline. Given the maximization with respect to T in (6), the familiar envelope theorem holds

$$\frac{\partial V(B,a)}{\partial B} = U'\big(X_0^*(B,a)\big),$$

and Figure 1, case (ii) shows that $X_0^*(\cdot)$ increases in B.

Summary.—Our low health-state analysis yields four results. We show that a household optimally decumulates its liquid wealth and depletes it in finite time; that it subsequently sets its consumption expenditure equal to its annuity income or accepts Medicaid nursing home assistance; and that at the moment a household accepts means-tested nursing home care, its consumption expenditure drops discontinuously. Furthermore, the value function is smooth and concave in liquid wealth, with or without the Medicaid consumption floor.

III. High Health-State Phase

Turn next to households in the healthy phase of their retirement, where $h = H$. Without loss of generality, rescale household ages to $s = 0$ at the start of this phase. The household problem has two state variables: initial bequeathable net worth (i.e., liquid wealth), $b \geq 0$, and annuity income, $a > 0$. With Poisson rate λ, the household's health state changes to $h = L$, and it receives (recall Section II) the continuation value $V(b_s, a)$, where b_s is its liquid wealth at the time of the transition. Accordingly, a household in state $h = H$ solves[12]

$$v(b,a) = \max_{x_s}\left(\int_0^{\infty} e^{-(\lambda+\beta)s}\left[u(x_s)\,ds + \lambda V(b_s,a)\right] ds\right) \tag{9}$$

subject to

$$\dot{b}_s = r \cdot b_s + a - x_s,$$

$$b_s \geq 0 \quad \text{all } s \geq 0,$$

$$a > 0, \quad \text{and} \quad b_0 = b \text{ given.}$$

Concavity of $V(\cdot)$, shown in the previous section, assures that the integrand in (9) is strictly concave in (x_s, b_s). First-order conditions yield a consumption Euler equation

$$\frac{u''(x_s)}{u'(x_s)}\dot{x}_s + \lambda\left(\frac{U'(X_0^*) - u'(x_s)}{u'(x_s)}\right) - \beta = -r \tag{10}$$

that can be interpreted as follows. Along the optimal consumption trajectory, the expected growth rate of discounted marginal utility is set equal to the growth rate of the relative price of future consumption $(-r)$. Euler equation (10) has a non-standard second term in the left-hand side. This extra term accounts for the jump in marginal utility upon the transition to the low health state. The marginal-utility growth rate in (10) depends on the value function $V(\cdot)$ through the expression $U'(X_0^*(b_s,a)) = \frac{\partial V}{\partial B}(b_s,a)$—a familiar envelope condition discussed in Section II.

The Euler equation (10) and the law of motion for liquid wealth

$$\dot{b}_s = r \cdot b_s + a - x_s \tag{11}$$

determine the phase diagram. The isocline $\dot{b} = 0$ is a straight line with slope r and intercept a:

$$x = \Gamma_b(b) \equiv r \cdot b + a. \tag{12}$$

[12] Our baseline model assumes that λ does not vary with household age s. This assumption can be relaxed by letting $\tau_s ds$ denote transition probability to the low health state at age s and $p_s = 1 - \int_0^s \tau_z\,dz$ be the probability to remain in healthy state at age s. The expected utility is then

$$v(b,a) = \max_{x_s}\left(\int_0^{\infty} e^{-\beta s}\left[p_s U(x_s) + \tau_s V(b_s,a)\right] ds\right),$$

and the corresponding consumption Euler equation—the analog of (10)—is

$$\frac{u''}{u'}\dot{x} + \frac{\tau_s}{p_s}\left(\frac{U' - u'}{u'}\right) - \beta = -r.$$

The baseline case (9) assumes $p_s = e^{-\lambda s}$ and $\tau_s = \lambda e^{-\lambda s}$.

To derive the $\dot{x} = 0$ isocline, we set $\dot{x}_s = 0$ in (10) and use expression (3) relating marginal utilities u' and U'. The isocline shape follows the consumption decision rule in the low health state—recall Figure 1—compressed by a factor $\theta \in (0, 1)$:

$$\dot{x} = 0: \; x = \Gamma_x(b) \equiv \theta \cdot X_0^*(b, a), \tag{13}$$

where

$$\theta \equiv \frac{1}{\Omega}\left[1 - \frac{r - \beta}{\lambda}\right]^{\frac{1}{1-\gamma}} \in (0, 1). \tag{14}$$

To interpret (13)–(14), consider a special case $r = \beta$.[13] Setting $r = \beta$ and $\dot{x}_s = 0$ in (10) shows that marginal utilities in high and low health states are equal along the $\Gamma_x(b)$ isocline:

$$u'(x) = U'(X_0^*) \Leftrightarrow \Gamma_x(b) = \frac{1}{\Omega} X_0^*(b, a).$$

Put differently, when $r = \beta$, the steady-state expenditure level corresponds to the household fully self-insuring its health status.

Several distinct phase portraits can arise depending on the shape of $\Gamma_x(b)$ and the values of exogenous parameters. We begin our analysis of phase diagrams with a lemma that allows us to limit the eventual number of cases.

LEMMA 2: *$\Gamma_x(b)$ and $\Gamma_b(b)$ cross at most once.*

PROOF:

See Appendix B.

Given Lemma 2, the phase portrait of the high health-state period depends on the relative magnitudes of $\Gamma_b(0)$ and $\Gamma_x(0)$ and on their asymptotic slopes $\Gamma_b'(\infty)$ and $\Gamma_x'(\infty)$. Recall that Propositions 1 and 2 imply

$$\Gamma_b(0) = a, \quad \Gamma_x(0) = \begin{cases} \theta a & a \geq \bar{X} \\ \theta \check{X}(a) & a < \bar{X} \end{cases}.$$

Later, we show there exists $\bar{a} \in (0, \bar{X})$, such that

$$\Gamma_b(0) < \Gamma_x(0) \Leftrightarrow a < \bar{a}. \tag{15}$$

Turning to the asymptotic slopes of the isoclines, Proposition 3 shows that there exists $\bar{r} \in (0, \beta + \lambda)$, such that

$$\Gamma_b'(\infty) < \Gamma_x'(\infty) \Leftrightarrow r < \bar{r}. \tag{16}$$

[13] The special case also informs on the generality of the model. When $r = \beta$, our phase diagram analysis can incorporate age-dependent transition probability of the low health state, $\lambda = \lambda_s$. Indeed, imposing $r = \beta$ makes θ in (14) independent of λ; hence, the isoclines are independent of λ as well. The optimal consumption trajectory in (10) would still depend on λ_s. Nevertheless, our results in Propositions 3 and 4 will not be affected.

Accordingly, four phase portraits are possible depending on the signs of inequalities (15) and (16). We distinguish between the high annuity case $a > \bar{a}$ (labelled **A**) and low annuity case $a < \bar{a}$ (labelled **a**) based on the sign of (15). Similarly, the standard interest rate case (labelled **r**) will obtain when $r < \bar{r}$, and the high interest rate case (labelled **R**) will obtain when $r > \bar{r}$. Summarizing, we have the following.

PROPOSITION 3: *The solution* (x_s^*, b_s^*) *to* (9) *is a dotted trajectory on one of the four phase diagrams on Figure* 2. *The phase portrait depends on the parameter values as follows:*

		High annuity $a > \bar{a}$	Low annuity $a < \bar{a}$
Standard interest rate	$r < \bar{r}$	(**Ar**)	(**ar**)
High interest rate	$r > \bar{r}$	(**AR**)	(**aR**)

where $\bar{r}$ *is the unique root in* $(0, \beta + \lambda)$ *of*

$$\frac{r}{r-\sigma} = \frac{1}{\Omega}\left(1 - \frac{r-\beta}{\lambda}\right)^{\frac{1}{1-\gamma}}$$

and

$$\bar{a} = \bar{X} \cdot \theta\big(1 - \gamma(1-\theta)\big)^{-\frac{1}{\gamma}}.$$

PROOF:

See Appendix B.

Proposition 3 and Figure 2 characterize consumption and wealth trajectories for all initial conditions (b, a) and partition the state space into regions with distinct wealth accumulation patterns. The new insight emerging from our analysis is that a household's annuity-income level and its initial portfolio composition matter greatly for subsequent wealth holdings. Post-retirement wealth trajectories that vary with b and a reflect, in part, different strategies that households use for insuring late-life risks. For instance, in the standard interest rate case, high annuity households $(a \geq \bar{X})$ rely on self-insurance while low annuity households $(a < \bar{a})$ rely on Medicaid. The middle group with $a \in (\bar{a}, \bar{X})$ self-insures at first and uses Medicaid as a fall-back option if it lives long enough. As a consequence, saving behavior in the middle and low groups is shaped in important ways by responses to the Medicaid means test. The strength of the self-insurance motive is shown to vary significantly with a (see Proposition 4). We can build further intuitions for Figure 2 by examining behavior for households in different circumstances.

Self-Insurance.—Consider behavior of households with $a \geq \bar{X}$ who never find it optimal to resort to Medicaid (equivalently, one could set $\bar{X} = 0$, $\bar{a} = 0$, and analyze behavior without Medicaid). With $a \geq \bar{X} > \bar{a}$, we are left with just two phase diagrams—(**Ar**) for the standard interest rate case and (**AR**) for the high interest rate case.

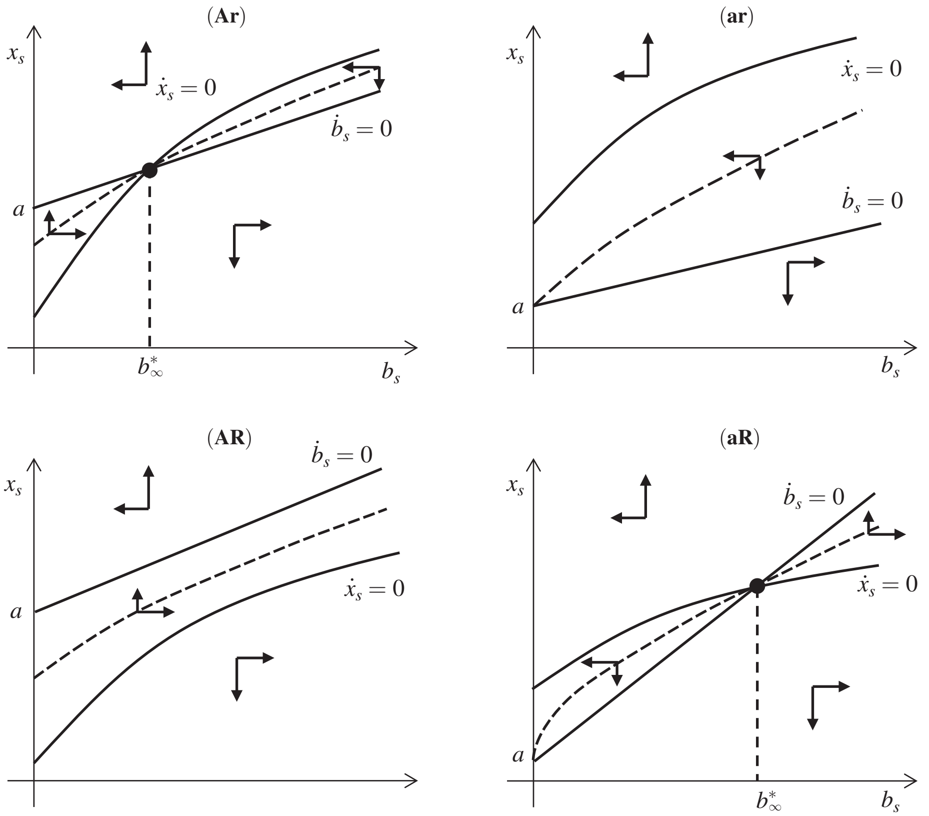

Figure 2. Possible Phase Diagrams for Optimal Behavior in High Health State

Note: See Proposition 3.

During poor health, a household's subjective discount rate, $\Lambda + \beta$, exceeds the rate of return on wealth. This makes the household prefer a falling consumption profile until age T when liquid wealth is exhausted. After age T, the household becomes liquidity constrained and consumes just its annuity income a (recall Proposition 1). Liquid wealth and annuities thus play complementary roles in the low health state: liquid wealth offers the flexibility to adust expenditure timing while annuity income provides longevity insurance.

With b and a playing complementary roles, households seek a balance of liquid wealth and annuities. In the standard interest rate case (i.e., phase diagram (**Ar**)), all households relying on self-insurance target the same long-run ratio of liquid wealth to annuities $b^*_\infty / a = \bar{\rho}$—see Proposition 4.

If r is high (i.e., phase diagram (**AR**)), liquid wealth is an attractive investment. At first, households may desire more liquid wealth in preparation for poor health. As liquid wealth grows, interest income can be used to save for the future and as well as to increase current expenditure. In fact, on phase diagram (**AR**) saving continues as long as high health status lasts.

Effects of Social Insurance.—The decision whether to accept Medicaid public assistance becomes relevant if the household outlives its liquid wealth. At that point, self-insurance would provide a standard of living a, and the Medicaid program would confiscate annuity income and provide a consumption floor $\bar{X}$. Accordingly, the gain from Medicaid is $\bar{X} - a$. When annuity income is below a threshold, $a < \bar{a}$, the gain from Medicaid is great enough to induce some households to systematically dissave. On phase diagram (**aR**), for instance, saving behavior is dichotomous. Low-resource households (i.e., those with $b < b^*_\infty$) decumulate wealth and anticipate accepting Medicaid quickly upon reaching the low health state. High-resource households ($b > b^*_\infty$), however, retain the self-insurance motive, but they count on using Medicaid as a fallback in the event they outlive their liquid wealth.

Whenever a household's annuity income is below $\bar{X}$, there is a state of the world when accepting Medicaid is attractive. Accordingly, saving disincentives associated with the Medicaid means test—commonly thought to affect just the poor—may extend to high-resource households with high liquid wealth but low annuity income. Phase diagram (**ar**) provides a stark illustration. In it, all low annuity households start decumulating wealth after retirement regardless of their initial wealth level.

Saving Motives of the Middle Class.—As previously noted, saving incentives of the middle group with annuity income in the range $a \in (\bar{a}, \bar{X})$ are the most complex. In the high interest rate case (phase diagram (**AR**)), the middle group chooses self-insurance at first and relies on Medicaid as a backstop. In the standard interest rate case (phase diagram (**Ar**)), saving behavior depends on both the annuity level and the initial composition of wealth.

In Figure 2, phase diagram (**Ar**) has a stationary point at $b = b^*_\infty = b^*_\infty(a)$. We can view $b^*_\infty(a)$ as a healthy household's "target level" of liquid wealth: if the household begins retirement with $b <(>) \; b^*_\infty(a)$, it will save (dissave) until reaching the target—or falling to health status $h = L$. The following proposition characterizes $b^*_\infty(a)$ in the standard interest rate case ((**Ar**) and (**ar**)).

PROPOSITION 4: *Assume $r < \bar{r}$ and let*

$$\rho(a) = \frac{b^*_\infty(a)}{a} = \frac{1}{a} \lim_{t \to \infty} b^*_t(b, a)$$

be the long-run optimal ratio of liquid wealth to annuities. Then

$$\rho(a) = \begin{cases} \bar{\rho} & a \geq \bar{X} \\ \varsigma(a) & a \in (\bar{a}, \bar{X}), \\ 0 & a \leq \bar{a} \end{cases}$$

where $\varsigma'(a) > 0, \varsigma(\bar{a}) = 0, \varsigma(\bar{X}) = \bar{\rho}$, and

$$\dot{b}^*_t > 0 \Leftrightarrow \frac{b}{a} < \rho(a).$$

PROOF:

See Appendix B.

Proposition 4 summarizes behavior in the standard interest rate case and shows how the long-run target wealth level $b_\infty^*(a)$ depends on the the household's annuity endowment. The contrast between the high annuity (top) group $(a \geq \bar{X})$ and the middle group $(a \in (\bar{a}, \bar{X}))$ reveals new insights about the incentive effects of public assistance.

The top and middle groups both possess self-insurance motives, and thus, they seek a balance of liquid wealth and annuities. The top group targets a long-run wealth level proportionate to the annuity endowment, $b_\infty^*(a) = \bar{\rho} a$. The middle group, in addition, responds to anticipated public benefit, and it accumulates less wealth than the top group (i.e., $b_\infty^*(a) < \bar{\rho} a$). At the same time, the self-insurance motive for the middle group is more sensitive to the annuity-income level: $b_\infty^*(a) = \rho(a)\, a$ rises more than proportionately with a. The steep rise of $b_\infty^*(a)$ results from the interaction of the means test with the self-insurance motive: if a is higher, the gain from Medicaid, $\bar{X} - a$, is less, and this, in turn, strengthens the incentive to self-insure.

Our analysis thus explains why incentive effects of the Medicaid means test may extend beyond the poorest households and why behavior of the middle class may be especially responsive to these incentives. Proposition 4 provides an intuitive explanation for numerical results in the recent literature (e.g., Amerkis et al. 2011, Figure 1; and DeNardi, French, and Jones 2010) that shows the sensitivity of saving behavior to the consumption floor across broad ranges of the wealth distribution.

Summary.—With our analytically tractable model, we are able to characterize wealth trajectories for all initial conditions (b, a) and to partition the state space into regions with distinct post-retirement wealth accumulation patterns. The patterns correspond to different strategies that households choose to insure late-life risks. A novelty of our results is that a household's annuity-income level and its wealth composition matter greatly for precautionary saving. Annuity income matters, in part, because of the incentive effects of the Medicaid means test. The analysis explains why these incentives may be particularly strong for the middle class.

We turn now to several important puzzles that challenged classical life-cycle analyses.

IV. Saving after Retirement

Although the standard life-cycle model implies that households will systematically dissave late in life, survey data often seem to show cohort post-retirement average liquid wealth declining only slowly with age or, perhaps, even increasing. The Introduction refers to this inconsistency as the "retirement-saving puzzle." The present section suggests that as we enhance our modeling framework with Medicaid, multiple health states, and asymmetries of health information, the discrepancy between the theory's predictions and evidence diminishes.

Section III shows that healthy households may continue to save after retirement, or at least, may want to husband their existing liquid wealth. Here, we demonstrate that healthy households can remain a significant fraction of cohort survivors long after retirement. Combining the two results, we then show that a cohort's average liquid wealth need not decline with age.

Post-retirement Saving.—Section III finds that some households may, while their health status remains favorable, want to continue accumulating wealth after retirement due to concerns about future consumption needs in the low health state. Initial conditions, in particular, a household's annuity income, are an important factor.

Proposition 3 partitions households into three groups. We have a low-resource group, $a \leq \bar{a}$; a middle-class group, $a \in (\bar{a}, \bar{X})$; and a top group, $a \geq \bar{X}$. Households in the low-resource group tend to spend their liquid wealth promptly, beginning during good health. They then subsist on their annuity income until poor health makes them eligible for Medicaid, which they find relatively attractive.[14] The middle-class group, in contrast, builds a nest egg of liquid wealth $b_{\infty}^{*}(a) > 0$. The target nest egg is increasing in a. If a household in this category begins retirement with liquid wealth $b < b_{\infty}^{*}(a)$, it saves until $b = b_{\infty}^{*}(a)$ or $h = L$. After the onset of poor health, it spends the liquid wealth and, after the latter is gone, accepts Medicaid. The $a \geq \bar{X}$ group also has a liquid-wealth target during good health. In poor health, after spending down the liquid wealth, these households live on their annuity income.

The richness of the set of possible behaviors hints that the model may be able to rationalize otherwise paradoxical post-retirement outcomes. We now examine that possibility further.

Cohort Composition.—The evidence on post-retirement wealth that has attracted the most attention measures average (liquid) wealth, at different ages, for an individual birth cohort's survivors. Fortunately, our model allows a detailed description of cohort wealth trajectories. We begin with an examination of the evolution of a cohort's mixture of health states.

Consider a cohort of retired, single-person households. In the model, all begin retirement with health status $h = H$. Each subsequently transitions to $h = L$, then to death. As the households age, the cohort size steadily diminishes. Somewhat paradoxically, however, the ratio of survivors in high versus low health converges to a positive constant. We have the following result.

LEMMA 3: *The fraction of cohort's survivors having high health-status t periods after retirement is*

$$f_t \equiv \frac{1}{1 + \frac{\lambda}{\Lambda - \lambda} \cdot \left(1 - e^{-(\Lambda-\lambda)\cdot t}\right)}. \tag{17}$$

[14] This description is somewhat oversimplified if $r > \bar{r}$—see case (**aR**) in Figure 2.

PROOF:
See Appendix B.

Provided $\Lambda > \lambda, f_t$ falls monotonically from $f_0 = 1$ to $f_\infty = (\Lambda - \lambda)/\Lambda > 0$. With $\lambda = 1/12$ and $\Lambda = 1/3$ (recall the illustration in Section I), for instance, $f_\infty = 3/4$.

Although our Poisson processes may only be approximations, they illustrate that healthy households can comprise a substantial fraction of cohort survivors long into retirement. This is important because, as previously noted, retirees in good health can behave quite differently from those whose health is poor.

Cohort Average Wealth.—We now characterize a cohort's long-run average liquid wealth for each of the four phase portraits in Proposition 3. For the short run, simulations illustrate that many outcomes are possible— including, as we shall see, outcomes resembling those in the data.

Long-Run Outcomes: Begin with a cohort of single-person, healthy households each with the same endowment (b, a). Normalize the cohort size to one. Let $\bar{b}(t; b, a)$ denote the cohort average liquid-wealth t periods after retirement, that is, the total liquid wealth of survivors divided by the total number of age-t survivors.[15] An analytic characterization for $\bar{b}^*(a) \equiv \lim_{t\to\infty} \bar{b}(t; b, a)$ is possible.

COROLLARY TO PROPOSITION 3: *The long-run cohort average wealth*, $\bar{b}^*(a)$, *depends on exogenous parameters as follows.*

		High annuity $a > \bar{a}$	Low annuity $a < \bar{a}$
Standard interest rate	$r < \bar{r}$	$\bar{b}^*(a) \in b^*_\infty(a) \cdot [f_\infty, 1]$	$\bar{b}^*(a) = 0$
High interest rate	$r > \bar{r}$	$\bar{b}^*(a) \to \infty$	$\bar{b}^*(a) = \begin{cases} 0 & b < b^*_\infty(a) \\ \infty & b > b^*_\infty(a) \end{cases}$

The proof is straightforward. The cases in which $\bar{b}^*(a)$ is zero or infinity follow directly from Proposition 3 and Figure 2. The case in which $\bar{b}^*(a)$ is positive and finite corresponds to phase diagram (**Ar**). The bounds are intuitive. In the long run, new entrants to the low health group have liquid wealth no greater than $b^*_\infty(a)$; consequently, members of the $h = L$ group have wealth that is nonnegative but bounded above by $b^*_\infty(a)$. The long-run contribution of the $h = L$ group to cohort average liquid wealth is between 0 and $(1 - f_\infty)\, b^*_\infty(a)$. The wealth of the high health-status group converges to $f_\infty \cdot b^*_\infty(a)$. The sum of the two contributions, $\bar{b}^*(a)$, therefore must lie in the interval $b^*_\infty(a) \cdot [f_\infty, 1]$. This establishes the Corollary.

[15] See Appendix A for analytic expressions that relate households' optimal wealth trajectories and $\bar{b}(t; b, a)$.

If $r < \bar{r}$, we can see that in the long run, a cohort with some high annuity households should have positive stationary average liquid wealth. For $r > \bar{r}$, long-run average liquid wealth should approach ∞. The possibility of a level, or rising, cohort wealth trajectory depends on the asymptotic stationarity of (17) and on Section III's finding that healthy retirees may husband their wealth or continue to accumulate more.

Simulated Wealth Trajectories: narrow wealth ranges. We utilize numerical simulations in illustrating our model's ability to match empirical outcomes in the short run. We consider two comparisons.

Section I suggests parameter values $\lambda = 1/12$, $\Lambda = 1/3$, and $\bar{X} = \xi \cdot X^M$ for $\xi \in (0,1]$. Appendix A calibrates Ω. Appendix A also suggests cross-sectional quantiles for a—see Table A1—and notes values for γ, r, and β familiar from the literature. Table A2 determines corresponding phase diagrams for the model.

We first compare post-retirement cohort trajectories of average (liquid) wealth for the model with empirical profiles from DeNardi, French, and Jones (2015, Figure 4). We simulate age-wealth profiles for the model for selected parameters within the Appendix A domain. DeNardi, French, and Jones derive graphs of cohort wealth from HRS/AHEAD panel data on single-person households aged 74 or older in 1996.[16] Convenient features of the empirical graphs are that they segregate the underlying sample into narrow annuity-income bands (i.e., into quintiles of the cross-sectional distribution of a) and that, because the median age of retirement in the United States is about 62, even the youngest households in the graphs have often been retired for over a decade. The latter implies that the ratio of health types may well have virtually completed its convergence to $f(\infty)$ in (17).[17] A complication, however, is that the number of data points is fairly small, especially at higher ages (c.f., DeNardi, French, and Jones 2015, fn. 4). The asymptotic stationarity of our ratio $f(t)$ depends on large samples. Accordingly, we ignore the jagged regions at the right-hand ends of the empirical graphs.

Figure 3, left panel, presents illustrative simulations from the model. The simulations assume $f(t) = f(\infty)$, $r = \beta = 0.02$, and $\bar{X} = 52.5$, and they consider $\gamma = -0.75$, -1.0, and -1.25. In all cases, $r < \bar{r}$. For a below the median of Table A1, Table A2 then implies phase diagram (**ar**)—i.e., $a < \bar{a}$—with prompt spend-down of liquid wealth regardless of health. That behavior is consistent with the low and declining wealth balances evident in the two bottom-quintile empirical graphs. The model provides an intuitive explanation, namely, that low annuity households do not perceive that they can do better, once stricken with poor health status, than to depend upon Medicaid. For a near the (Table A1) median, similar parameter values yield $b \approx b^*_\infty(a) > 0$ in Table A2. Hence, by age 74, the corresponding simulated wealth trajectory is nearly horizontal. Again, that seems broadly consistent with the empirical graphs. Finally, for simulations of the top 30 and top

[16] DeNardi, French, and Jones' panel data can avoid birth-cohort fixed effects and complications from correlations of survival probability with portfolio size.

[17] With $\lambda = 1/12$ and $\Lambda = 1/3$, for example, convergence to $f(\infty)$ is over 98 percent complete after 12 years.

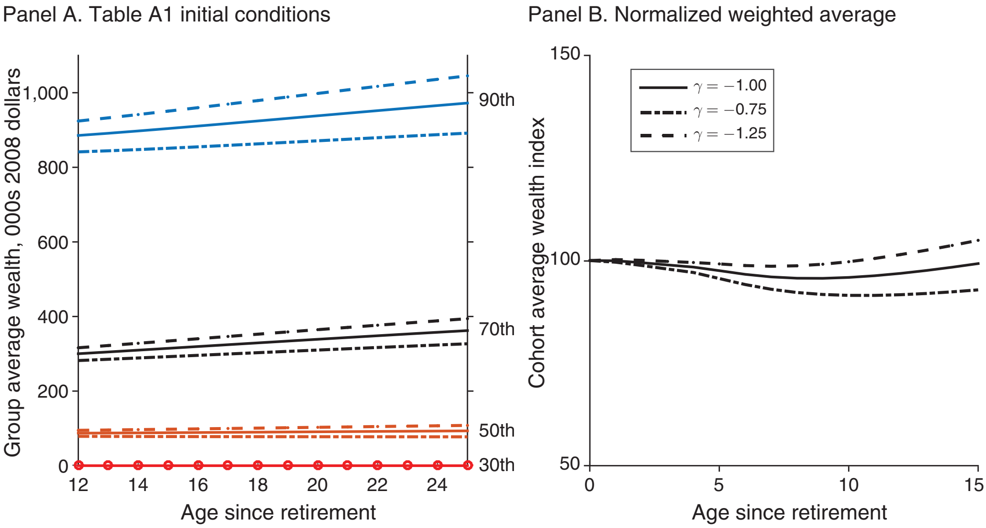

FIGURE 3. COHORT AVERAGE WEALTH TRAJECTORIES BY INITIAL WEALTH LEVEL

Note: $\lambda = 1/12, \Lambda = 1/3, r = \beta = 0.02, \Omega = 5.25, \bar{X} = 52.5$

10 percent annuity groups, Table A2 implies much higher values of $b^*_\infty(a)$—as Proposition 4 would predict. In particular, $b^*_\infty(a)$ tends to be large relative to b, leading to simulated age wealth trajectories that rise for a number of years.[18] Intuitively, high annuity households demand high liquid-wealth balances to reduce their future reliance upon Medicaid.

Figure 3 suggests that, for plausible parameter values, simulations from the model can match empirical trajectory shapes and that, through Propositions 3–4, our theoretical analysis can provide explanations for the behavior arising in practice.

Simulated Wealth Trajectories: broad population averages. Second, we compare the model with empirical figures from Poterba, Venti, and Wise (2011a). Single graphs from the latter summarize a full cross section of annuity incomes. And the data tend to begin at the empirical retirement age so that the convergence of (17) runs its course as we move along a graph. Nonetheless, this has been an important form for evidence in the literature, and we can again use our model to interpret the data's patterns.

As discussed earlier, Poterba, Venti, and Wise use panel data. They link average liquid-wealth holdings in adjoining survey waves, including only households with data in both waves. They process the data extensively, using trimmed means and medians. We focus on the graphs of Poterba, Venti, and Wise (2011a, Figures 1.10

[18] The top-quintile graph of the data rises from age 74 to 84–86 at a rate of 0.5–1.5 percent/year. For the simulations, the average wealth of the top 30 and 10 percent of households rises 0.5 to 1.7 percent/year (cf. DeNardi, French, and Jones 2015, Figure 4 and the left panel of our Figure 3).

and 1.11), which combine five age groups. These graphs include only single households—though, as noted later, they are not limited to retirees.

We can compare simulated median liquid wealth with Poterba, Venti, and Wise (2011a, Figure 1.11). Roughly speaking, the empirical graph is horizontal for households in their late 60s, and falls -0.6 percent/year for households in their 70s. Medians may be less sensitive to non-retirees than means. Our comparison group from the model is healthy households with median initial conditions (i.e., $(b, a) = (21, 100)$). We use single age groups. We set the same parameters as those previously mentioned. Thus, we have $r < \bar{r}$. And, for a median household, $a > \bar{a}$. The phase diagram is (**Ar**). Outcomes are straightforward as follows: for $b \;<(>)\; b^*_\infty(a)$, the liquid wealth of healthy households monotonically rises (falls) until becoming stationary at the target level, $b^*_\infty(a)$. For $\gamma = -1.25$, -1.0, and -0.75, the 15-year growth rates of simulated liquid wealth are, respectively, 1.4 percent/year, 0.7 percent/year, and -0.04 percent/year. The last is the best match.[19]

Figure 3, right panel, simulates cohort mean wealth trajectories from the model. The simulations use Table A1 endowments $(b, a) = (14, 15)$, $(100, 21)$, $(272, 34)$, and $(57, 892)$ with weights 1/3, 1/3, 2/9, and 1/9, respectively. Parameter values continue to be as in the preceding subsection. For conformity with Poterba, Venti, and Wise (2011a, Figure 1.10), our simulations present 5-year moving averages. The empirical graphs reveal a growth rate of about 1.3 percent/year for 5 years and 1.4 percent/year for the next 10. The simulated curves show a brief dip from the large initial (percentage) increases in low health-status households.[20] Thereafter, they manifest growth at rates 1.1 percent/year, 0.9 percent/year, and 0.6 percent/year for $\gamma = -1.25$, -1.0, and -0.75, respectively. The presence of non-retired households in the data may, in part, explain remaining discrepancies.

Thus, even in the most challenging case, Poterba, Venti, and Wise (2011a, Figure 1.10), the illustrative simulations can match the data quite well. Our qualitative analysis shows why. If $r < \bar{r}$, poor health or very low annuity income leads to declining liquid wealth (see Proposition 3). Households with moderate annuity income accumulate wealth more slowly than high annuity, healthy households (see Figure 3 and Proposition 4). We show the rising and falling segments can counterbalance one another in the weighted average, and the time-varying cohort composition can flatten the initial portion of the average trajectory.

Discussion.—For decades, evidence of the "retirement-saving puzzle" has raised questions about the validity of the life-cycle model. We, however, argue that several elaborations of the standard framework, which are interesting and realistic in their own right, can greatly improve the model's performance. The enhanced model's ability to match the evidence includes both aggregative data and data on separate income groups.

[19] For $\gamma = -0.50$, the simulated (15-year) growth rate would drop to -1.1 percent/year.

[20] The total peak-to-trough is 0.7 percent, 2.3 percent, and 5.0 percent for $\gamma = -1.25$, -1.0, and -0.75, respectively.

Our analysis uses both qualitative results and straightforward numerical simulations. The latter utilizes plausible parameters' values. The former enables us to shed light on the possible causes of otherwise surprising cohort wealth trajectory patterns evident in practice.

V. Demand for Annuities

Standard life-cycle theories addressing self-insurance of longevity risk—starting with the well-known work of Yaari (1965)—have been hard to reconcile with households' apparent lack of demand for annuities at retirement. Using our model, we now reconsider this "annuity puzzle."

To study demand for annuities, this section deviates from our baseline specification to allow households to reallocate their portfolios at retirement. Let

$$r_A = \frac{(\lambda + r)(\Lambda + r)}{\lambda + \Lambda + r} \tag{18}$$

denote the actuarially fair rate of return used to capitalize annuity income (see Appendix B for the derivation of (18)). Then household total initial wealth, w_0, can be expressed through its endowment of liquid wealth and annuities (b_0, a_0) prior to portfolio choice as follows:

$$w_0 = b_0 + \frac{a_0}{r_A}.$$

At retirement, a household reallocates its endowed total wealth between bonds and annuities to maximize its post-retirement value function. The resulting optimal allocation (b, a) becomes the initial state for the household optimization problem (9). To formulate the household portfolio choice problem, it is convenient to define

$$\alpha_0 = \frac{a_0/r_A}{w_0} = \frac{a_0}{a_0 + r_A b_0},$$

the initial share of annuitized wealth at retirement. Then the household problem is

$$\hat{\alpha}(w_0) = \operatorname*{arg\,max}_{\alpha \in [0,1]} v\big((1-\alpha)\, w_0, r_A\, \alpha\, w_0\big). \tag{19}$$

If the desired annuity share, $\hat{\alpha}$, exceeds the endowed share α_0, a household will exhibit demand for annuities at retirement.

To develop intuitions, first consider the role of annuities for low-resource households, that is, the group following phase diagram (**ar**) or phase diagram (**aR**) with $b < b_\infty^*$. Medicaid provides better support in the low health state than these households could otherwise afford; therefore, they are content to accept Medicaid promptly after poor health begins. Annuities provide insurance against outliving one's resources during good health; Medicaid provides longevity protection once $h = L$. But, Medicaid usurps a household's annuity income, causing annuities to lose part of their appeal. Using (18), the capitalized value of annuity flow a is $A = a/r_A$. The expected present value of an annuity stream useful only during good health is

$$a \cdot \int_0^{\infty} e^{-(\lambda+r)\cdot s}\, ds = \frac{A \cdot r_A}{\lambda + r} = A \frac{\Lambda + r}{\lambda + \Lambda + r} < A.$$

With $\lambda = 1/12$ and $\Lambda = 1/3$ and $r = 0.02\ (0.03)$, for example, we have

$$\frac{\Lambda + r}{\lambda + \Lambda + r} = 0.81\,(0.81) < 1.$$

In other words, an actuarially fair annuity carries, roughly speaking, an inherent user cost (or "load"), which is likely to be nontrivial.

The inherent cost can be even larger for middle-class households with $a \in (\bar{a}, \bar{X})$. The one-size-fits-all Medicaid benefit $\bar{X}$ leaves them dissatisfied. Hence, they carry resources to the low health state to postpone reliance upon Medicaid. To do so, a middle-class household augments its annuities with bonds. Upon Medicaid take-up, a household must relinquish its annuities and remaining bonds to the public authority. As in Section II, a household can consume both the income and principal of its bonds prior to accepting Medicaid. Unlike bonds, however, annuities are illiquid. Recall that utility in state $h = L$ is

$$\int_0^{\infty} e^{-(\beta+\Lambda)\cdot t} \cdot U(X_t)\, dt.$$

Since Λ tends to be large, even if bond wealth is used up rather quickly after the onset of poor health, total utility can significantly benefit. Relying exclusively on accumulating bonds during good health is risky as the good health phase may turn out to be brief. Starting with a mixture of annuities—to protect against a long span of $h = H$— and bonds—to delay the need to accept Medicaid if the span of $h = H$ turns out to be short—becomes attractive.

Put another way, purchasing an annuity income a at retirement costs $A = a/r_A$. When the low health state arrives, the actuarially fair capitalized value of the annuity-income flow drops to $a/(\Lambda + r)$. The capital loss can be substantial: the value of a after $h = L$ as a fraction of its initial cost is

$$\frac{a/(\Lambda + r)}{a/r_A} = \frac{r_A}{\Lambda + r} = \frac{\lambda + r}{\lambda + \Lambda + r}. \tag{20}$$

Letting $\Lambda = 1/3$ and $\lambda = 1/12$, for instance, the relative value in (20) is 0.24 (0.25) when $r = 0.02\ (0.03)$—a roughly 75 percent capital loss. If the household subsequently turns to Medicaid, it must relinquish a to the Medicaid program. At that moment, the value to the household of the annuity income declines further, to zero. These are steep drops. What is more, their timing is extremely inopportune: at the onset of $h = L$, a household's marginal-utility-of-consumption function rises abruptly. And Proposition 3 shows that as a household accepts Medicaid, its consumption (discontinuously) drops. Evidently, annuities subject a household to severe capital losses exactly at times when the household values consumption highly. Bond values, in contrast, are unrelated to health. At Medicaid take-up, a household essentially must hand over its remaining bonds. But, as Section II shows,

TABLE 1—PORTFOLIOS AT RETIREMENT:
ACTUAL, OPTIMAL (*no Medicaid*), OPTIMAL (*with Medicaid*)

a_0 (1)	b_0 (2)	w_0 (3)	$\alpha_0 = \frac{a_0}{a_0 + r_A b_0}$ (4)	$\hat{\alpha}\|_{\bar{X}=0}$ (5)	$\hat{\alpha}$ (6)
15	14	177	0.92	0.93	0.92
21	100	328	0.70	0.93	0.67
34	272	641	0.58	0.93	0.48
57	892	1,571	0.43	0.93	0.93

households can spend their bond wealth completely prior to that moment. Roughly speaking, in the last stage of life, annuities and Medicaid are substitutes, whereas bonds and Medicaid are complements.

The previous arguments do not apply to very high annuity households, that is to say, those with $a \geq \bar{X}$. The latter households never use Medicaid. Their total wealth must exceed $\bar{w}$ with

$$\bar{w} \geq \frac{a}{r_A} \geq \frac{\bar{X}}{r_A}.$$

With λ and Λ as previously stated and $r = 0.02$ (0.03),

$$\frac{\bar{X}}{r_A} = 11.96 \cdot \bar{X} \; (10.85 \cdot \bar{X}).$$

In Table A1, only top-decile households have $w \geq \bar{w}$.

Illustrative Examples.—We present solutions to the household portfolio choice problem for different initial wealth levels to illustrate the impact of Medicaid availability on the demand for annuities at retirement. Table 1 shows the initial wealth components and initial share of annuitized wealth, as well as the solutions to (19), for the thirtieth, fiftieth, seventieth, and ninetieth percentiles of the empirical wealth distribution of Table A1. The exogenous parameters are set to $r = \beta = 0.02, \gamma = -1$, $\lambda = 1/12$, $\Lambda = 1/3$, $\bar{X} = 52.5$, and $\Omega = 5.25$, consistent with Figure 3.

For comparison, column 5 illustrates the case without Medicaid, with $\bar{X} = 0$. Proposition 3 shows that $\bar{X} = 0$ implies $a > \bar{a} = 0$; hence, phase diagram (**Ar**) applies. Without Medicaid long-term care, the model is homothetic in (b, a), and the optimal share of annuitized wealth at retirement, $\hat{\alpha}\,|_{\bar{X}=0}$, is independent of household total wealth. Evidently, absent Medicaid, the model exhibits the annuity puzzle in rows 2–4: the desired share of annuitized wealth, $\hat{\alpha}\,|_{\bar{X}=0}$, exceeds the initial share, α_0 in all rows except the first.[21]

The last column reports the optimal share of annuitized wealth, $\hat{\alpha}$, with Medicaid. For rows 1–3, the annuity puzzle has disappeared: in rows 1–2, actual annuitization

[21] Our framework differs from Yaari (1965) in that the mortality hazard is correlated with state-dependent marginal utility—and this explains why households desire less than 100 percent annuitization. However, the deviation from full annuitization is slight—an outcome that is reminiscent of other recent analyses, e.g., Davidoff, Brown, and Diamond (2005).

is equal or slightly larger than desired; and in row 3, the actual is 20 percent above desired. Our model provides an interpretation. As in Section IV, we have $r < \bar{r}$. According to the model, row 1 households, with $a < \bar{a}$, will find standard annuities attractive. And Table A1 shows they are heavily annuitized in practice as well. In the middle class, with $a \in (\bar{a}, \bar{X})$, the model implies households will desire mixed portfolios, using liquid assets to postpone reliance on Medicaid.

Annuity-puzzle behavior does emerge in row 4 of Table 1. One possibility is that a somewhat higher choice of ξ would make $a < \bar{X} = \xi X_M$ for the top group (recall that $X_M = 70$ and $a = 57$). Another is that the millionaires in the top decile want to leave intentional bequests—behavior which is outside the scope of our modeling.

The analysis suggests a possible resolution of the annuity puzzle, at least for households with middle-class annuity incomes: the limited annuitization that households have in practice may accurately reflect their preferences, given the availability of Medicaid long-term care and the treatment of annuity income in the Medicaid means test.

VI. Medicaid Take-up and Accidental Bequests

The basic assumptions of our model enable it to offer interpretations of interesting phenomena in addition to the retirement-saving and annuity puzzles. This section briefly describes two examples.

The Timing of Medicaid Take-up.—DeNardi, French, and Jones (2016) present evidence that even households with relatively high annuity income sometimes use Medicaid nursing home assistance very late in life, though households with lower a tend to access Medicaid more frequently and at younger ages. Our model offers an intuitive explanation for these outcomes.

Proposition 3 shows that any household with $a < \bar{X}$ will access Medicaid if it survives long enough. The model determines Medicaid take-up time as a function of a retiree's initial condition (b, a) and age at the onset of poor health. If S is the time spent in good health, then the optimal age of Medicaid take-up is $S + T^*(b_S^*(b,a), a)$, where the function $T^*(\cdot)$ is as in Section II. The model thus provides a mapping between portfolio composition at retirement, household health history, and Medicaid take-up age—making a comprehensive treatment possible.

Consider the standard interest rate case. Households with $a < \bar{a}$ want to accept Medicaid promptly once $h = L$. Households with $a > \bar{a}$, however, hold liquid wealth to postpone their resort to Medicaid. These households are more likely to die before Medicaid take-up and to take up Medicaid only at advanced ages.

Bequest Behavior.—Households in the model leave accidental bequests if they die before spending down their liquid wealth. Survey questions suggest that such bequests may be important in practice, while evidence on intentional bequests has been mixed.[22]

[22] E.g., Altonji, Hayashi, and Kotlikoff (1992, 1997); Laitner and Ohlsson (2001); and others.

In our model, a household begins health state $h = L$ with liquid wealth $B \geq 0$, and it spends the latter at a rapid rate (with its consumption flow exceeding $\bar{X}$). If it dies before exhausting B, the residual constitutes a bequest. If it lives longer, it has no bequest and finishes life relying upon Medicaid or its annuity income, whichever is larger.

Proposition 3 determines interest rate and annuity thresholds, $\bar{r}$ and $\bar{a}$. Consider first a low-resource household, i.e., one following phase diagram (**ar**) or phase diagram (**aR**) with $b < b_{\infty}^{*}$. Section III shows the household will dissave in the good health state. If $h = H$ lasts long enough, it will begin poor health with liquid wealth $B = 0$. Since all households dissave in poor health, the household would then die with no estate. If $B > 0$, it would subsequently decumulate its liquid wealth rapidly, leaving a bequest only if it died before B_t^* reached 0. In general, households in the low-resource group will tend to leave estates only if their life span in both segments of retirement is short.

Alternatively, suppose a household has (i) $r > \bar{r}$, $b > b_{\infty}^{*}$, and $a < \bar{a}$; (ii) $r > \bar{r}$ and $a > \bar{a}$; (iii) $r < \bar{r}$, $a > \bar{a}$, and $b < b_{\infty}^{*}$; or (iv) $r < \bar{r}$, $a > \bar{a}$, and $b > b_{\infty}^{*}$. In case (iv), the household dissaves during good health with a lower limit b_{∞}^{*}. In the remaining three cases, it saves while $h = H$. It begins $h = L$ with liquid wealth $B > 0$. It fully dissaves its liquid wealth after (finite) time span $T^*(B, a)$ (recall Section II). The function $T^*(\cdot)$ is increasing in B and decreasing in a. The household leaves an estate if it dies within $T < T^*(B, a)$ years. In cases (i)–(iii), a longer time spent in good health leads to a higher probability of leaving an estate. In cases (i)–(iv), a shorter life span after $h = L$ makes a positive estate more likely. For the same B, a higher annuity income a makes a bequest less likely. Our propositions offer a full characterization of the timing and magnitude of such transfers.

VII. Conclusion

This paper presents a life-cycle model of post-retirement household behavior emphasizing the roles of changing health status (correlated with changes in mortality), annuitized wealth, and Medicaid assistance with long-term care. Despite the presence of health-status uncertainty and the non-convexities introduced by the Medicaid means test, our analysis yields a deterministic optimal control problem where the solution can be characterized with phase diagrams.

Qualitatively (and quantitatively in calibrated examples), we show the model is consistent with the gently rising cohort post-retirement wealth trajectories that tend to appear in data. Similarly, we show that a sizeable fraction of households may not wish to buy additional annuities at retirement—with both Medicaid LTC and existing primary annuitization from Social Security and DB pensions playing important roles in the outcome. The model can, in other words, offer a unified explanation for two long-standing empirical puzzles, the "retirement-saving puzzle" and the "annuity puzzle."

The model shows that after retirement but while in good health, middle-class households may want to maintain, or continue to build, their non-annuitized net worth. Households value primary annuities for the income that they provide, bonds for flexibility of access to funds, and Medicaid LTC for backstop protection against extreme longevity. Primary annuities and bonds can assume complementary roles: middle-class households may, during good health, save part of their annuity income to (temporarily) support a higher living standard later, after poor health strikes, than Medicaid nursing home care provides. In the model, this behavior can be understood to be a consequence of state-dependent utility and incomplete financial and insurance markets.

Appendix A. Calibration and Numerical Results

Calibration.—Our model has a limited number of parameters. We set $\lambda = 0.0833$ and $\Lambda = 0.3333$, corresponding to time intervals of 12 and 3 years, respectively, as in Sinclair and Smetters (2004). The literature has a variety of estimates of $\gamma \leq 0$ (see, for example, Laitner and Silverman 2012) and generally uses $\beta \in [0, 0.04]$. We consider values $\gamma \in [-0.5, -3.0]$, corresponding to a coefficient of relative risk aversion $1 - \gamma \in [1.5, 4]$, and values r, $\beta \in [0.02, 0.03]$.

The model includes two parameters that are less familiar: Ω—defined in (3)—which captures the rise in marginal utility associated with the low health state, and $\bar{X}$, which measures the value to a recipient household of Medicaid nursing home care.

The proposed calibration exploits the fact that Medicaid is a social-insurance program. Theoretically, $\bar{X}$ might be thought of as a choice variable for a social planner who seeks to insure the target recipient of public long-term care. Accordingly, a comparison of $\bar{X}$ with the normal expenditure of a healthy target recipient identifies the difference in marginal utility across states that would rationalize $\bar{X}$.

Think of the target recipient as a household that would quickly turn to Medicaid upon reaching the low health state, and let $\bar{x}$ denote the recipient's expenditure level while still healthy. Efficiency requires equalizing marginal utilities of expenditure across health states:

$$U'(\bar{X}) = u'(\bar{x}). \tag{A1}$$

In the model, households that are quick to accept Medicaid enter the low health state, say, at age s, with nearly zero liquid wealth, $b_s = B \simeq 0$ (see phase diagram (**ar**)). Since $b_s \simeq 0$, the typical recipient's consumption just prior to s must be $\bar{x} \simeq a$ so that $U'(\bar{X}) = u'(a)$ in (A1). Optimality condition (A1) then relates $\bar{X}$ and Ω as follows (recall (3)):

$$\Omega = \frac{\bar{X}}{a}. \tag{A2}$$

Condition (A2) enables us to use data on Medicaid nursing home reimbursement amounts and target-recipient annuity incomes to evaluate Ω. To calibrate a, we assume, as shown earlier, that a target Medicaid recipient has low initial liquid wealth and an annuity income substantially below the population median. We set $a = \underline{a} = 10{,}000$, which is about one-half of population median and about 2/3 of the annuity income

Table A1—Primary Annuities and Bequeathable Wealth (*000s of 2008 dollars*) for Single-Person Households Aged 65–69

	Annuitized wealth		Annuity	Bequeathable
	Social Security, b_{ss}	DB pension, b_{DB}	income, a_0	wealth, b_0
Net worth	(1)	(2)	(3)	(4)
30th percentile	166	0	15	14
Median	230	0	21	100
70th percentile	299	73	34	272
90th percentile	388	292	57	892

Source: Poterba, Venti, and Wise (2011b), Table 2 and p. 99

of the thirtieth percentile among single-person retired households (see Table A1, column 4).[23]

To estimate the effective long-term care consumption flow $\bar{X}$, we start with a direct measure of nursing home care cost, X_M. In MetLife Mature Market Institute (2009), annual average expenditures for nursing home care in 2008 are \$69,715 for a semi-private room and \$77,380 for a private room. Accordingly, we set $X_M = 70{,}000$. Prior studies (e.g., Ameriks et al. 2011, and Schafer 1999) suggest that $\bar{X}$ might be much lower than X_M. Reasons might include the disutility of living in an institution and/or accepting government welfare. Accordingly, for a fixed $X_M = 70{,}000$, let

$$\bar{X} = \xi \cdot X_M, \quad \xi \leq 1.$$

We report results for $\xi \in \{0.5, 0.75, 1\}$, which imply $\bar{X} \in \{35{,}000, 52{,}500, 70{,}000\}$ and $\Omega = \{3.5, 5.25, 7.0\}$. The resulting middle estimate, $\bar{X} = 52{,}500$, is close to the calibrated consumption floor in the nursing home eligible state in Ameriks et al. (2011)—their estimate of $\bar{X}$ is 56,300 (2008 dollars).

Numerical Results.—Table A2 provides calculations that illustrate Proposition 3 and the qualitative results of Section IV. Each panel of the table corresponds to a distinct vector of exogenous parameters $(r, \beta, \Omega, \bar{X})$ consistent with (A2) and reports the values of $\bar{a}$, $\bar{r}$, and b_∞^* for a set $\gamma \in \{-0.5, -1, -2, -3\}$.

We can see that all four phase diagrams of Figure 2 obtain for empirically relevant parameter values. To illustrate the model's predictions, we take several initial conditions (b, a) from the balance sheets of single-person households aged 65–69 reported in Poterba, Venti, and Wise (2011b, Table 2). Table A1 shows the corresponding components of annuitized and non-annuitized wealth at selected points of the wealth distribution.[24]

[23] By way of comparison, the chosen value of $\underline{a} = 10{,}000$ is somewhat higher than the annual SSI amount (7,644 2008 dollars) that acts as a lower bound on household annuity income in practice. All else equal, calibrating from a lower $\underline{a}$ would produce a higher Ω and supply a stronger self-insurance motive. We prefer calibrations of Ω on the low side to stack the cards against the post-retirement-saving behavior that our model is trying to explain.

[24] Poterba, Venti, and Wise (2011b) use the actuarially fair rate of return on annuities to capitalize annuity flows. Consistent with this, we use the actuarially fair rate of return r_A from (18) to convert between annuity wealth and income flow.

Table A2—Phase Diagram Types for Various Parameter Combinations

			$a = 15$		$a = 21$		$a = 34$		$a = 57$	
γ	$\bar{a}$	$\bar{r}$	PD type	b^*_∞	PD type	b^*_∞	PD type	b^*_∞	PD type	b^*_∞
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Panel A			$r = 0.02, \beta = 0.02, \Omega = 7.0, \bar{X} = 70$							
−0.5	20.4	0.032	ar	0	Ar	38.4	Ar	809.6	Ar	1,986.7
−1	18.6	0.026	ar	0	Ar	326.5	Ar	1,861.3	Ar	4,183.9
−2	16.5	0.019	aR	980.2	AR	∞	AR	∞	AR	∞
−3	15.3	0.014	aR	48.5	AR	∞	AR	∞	AR	∞
Panel B			$r = 0.02, \beta = 0.02, \Omega = 5.25, \bar{X} = 52.5$							
−0.5	19.7	0.040	ar	0	Ar	44.8	Ar	456.5	Ar	1,017.7
−1	18.1	0.034	ar	0	Ar	162.8	Ar	795.0	Ar	1,661.2
−2	16.2	0.025	ar	0	Ar	757.0	Ar	2,411.1	Ar	5,038.3
−3	15.1	0.020	aR	206.7	AR	∞	AR	∞	AR	∞
Panel C			$r = 0.02, \beta = 0.02, \Omega = 3.5, \bar{X} = 35$							
−0.5	18.4	0.052	ar	0	Ar	45.7	Ar	232.6	Ar	397
−1	17.1	0.047	ar	0	Ar	93.1	Ar	339.4	Ar	576.6
−2	15.6	0.038	ar	0	Ar	224.3	Ar	625.2	Ar	1,488.4
−3	14.6	0.031	Ar	28.7	Ar	442.1	Ar	1,081.4	Ar	2,585.5
Panel D			$r = 0.03, \beta = 0.03, \Omega = 7.0, \bar{X} = 70$							
−0.5	20.4	0.035	ar	0	Ar	92.2	Ar	1,830.0	Ar	4,387.0
−1	18.6	0.028	aR	1,545	AR	∞	AR	∞	AR	∞
−2	16.5	0.020	aR	134	AR	∞	AR	∞	AR	∞
−3	15.3	0.015	aR	18.4	AR	∞	AR	∞	AR	∞
Panel E			$r = 0.03, \beta = 0.03, \Omega = 5.25, \bar{X} = 52.5$							
−0.5	19.7	0.043	ar	0	Ar	64.5	Ar	639.1	Ar	1,406.6
−1	18.1	0.037	ar	0	Ar	328.5	Ar	1,526.6	Ar	3,133.8
−2	16.2	0.027	aR	314.2	AR	∞	AR	∞	AR	∞
−3	15.1	0.021	aR	8.0	AR	∞	AR	∞	AR	∞
Panel F			$r = 0.03, \beta = 0.03, \Omega = 3.5, \bar{X} = 35$							
−0.5	18.4	0.057	ar	0	Ar	53.2	Ar	265.5	Ar	452.8
−1	17.1	0.051	ar	0	Ar	116.6	Ar	415.2	Ar	704.9
−2	15.6	0.040	ar	0	Ar	358.8	Ar	963.1	Ar	3,084.9
−3	14.6	0.032	Ar	110.9	Ar	1,421.7	Ar	3,123.6	Ar	4,989.4

Notes: See Figure 2 and Proposition 3. Fixed parameters: $\Lambda = 1/3$, $\lambda = 1/12$. Wealth units: 000s 2008 dollars.

Consider a household at the thirtieth percentile of the annuitized wealth distribution in Table A1 with $a = 15$ and $b = 14$. Table A2, columns 4 and 5, show the phase diagram types and the values of b^*_∞ corresponding to $a = 15$. For all $\gamma > -3$ (CRRA less than 4), the model predicts that households with $a = 15$ and $b = 14$ should dissave, either because they follow phase diagram (**ar**) or because they follow phase diagram (**aR**) and have a low initial wealth $b = 14 < b^*_\infty$.

Next, take a household with a median annuity income $a = 21$ and the corresponding liquid wealth $b = 100$. Table 2, columns 6 and 7 show that the model's predictions with respect to wealth accumulation depend on the risk aversion parameter. When risk aversion is low (e.g., $\gamma = -0.5$), the phase diagram type is (**Ar**) with $b > b^*_\infty$, where Proposition 4 would imply wealth decumulation. As risk aversion rises, so does b^*_∞. Accordingly, for higher levels of risk aversion ($\gamma \leq -1$), we have $b < b^*_\infty$, and the model predicts post-retirement saving.

The previously discussed logic extends to the behavior of households all the way to the top of the wealth distribution. For instance, take a household with $a = 34$ and $b = 272$ corresponding to the seventieth percentile. Table A2, column 9 shows that b_∞^* is large, as $b_\infty^*(a)$ in Proposition 4 rises rapidly with a. The model then predicts $b < b_\infty^*$, at least for $\gamma < -0.5$. At higher levels of risk aversion (CRRA 3 or 4), column 8 shows that the phase diagram switches to (**AR**), where the model predicts wealth accumulation starting from arbitrarily large b.

Broadly, the patterns of Table A2 seem consistent with observations on wealth accumulation behavior of single-person households showing post-retirement saving at higher wealth levels and flat or falling wealth at lower wealth levels (e.g., Poterba, Venti, and Wise 2011a; and De Nardi, French, and Jones 2015).

Our analysis stresses portfolio composition at retirement as an important determinant of post-retirement saving. It is therefore worth explaining why Table A1 data might show households at retirement with annuity-heavy portfolios. If agents anticipate a need to save after retirement, then why did they not save more before retirement? We think that one answer has to do with the composition of single-person households by marital status. According to the US Census (2012), 42 percent of single-person households aged 65–74 are widowed, and an additional 40 percent are divorced. Thus, the Table A1 wealth distribution used as the initial condition for the model describes mostly single households who experienced a past shock to family status. Both divorce and death of a spouse deplete wealth: Poterba, Venti, and Wise (2011b, Figures 2, 4) show a sharp drop in non-annuity financial assets following a transition from a two- to one-person household. By contrast, married couples and continuing singles show a rising wealth-age profile. In line with this, the data show that single-person households are more heavily annuitized than couples—70 percent annuitization for a median single household versus 57 percent for a median married couple (Poterba, Venti, and Wise 2011b, Table 2).

Simulation of Cohort Average Wealth.—Here, we provide analytic expressions that relate households' optimal liquid wealth trajectories and the cohort average wealth $\bar{b}(t; b, a)$. Using the notation of Sections II–III, a household remaining in high health-status t periods after retirement has liquid wealth $b_t^* = b_t^*(b, a)$. The total wealth of a cohort of agents, of measure one, who remain healthy is

$$b_H(t; b, a) = e^{-\lambda t} \cdot b_t^*(b, a).$$

The wealth of cohort survivors in the low health state depends on the age at which their health status changed. If a household enters low health-status $s \leq t$ periods after retirement, its initial wealth upon entering that state is $B = b_s^*(b, a)$. The household subsequently follows the low health-status optimal wealth trajectory (recall Section II). At time t, it has passed $t - s$ years in low health status, and its wealth is $B_{t-s}^*(B, a)$. The fraction of a cohort entering the low health state at age s and surviving until age t is $\lambda \cdot e^{-\lambda \cdot s} \cdot e^{-\Lambda \cdot (t-s)}$. Accordingly, the total wealth of agents who are in low health t periods into retirement is

$$b_L(t; b, a) = \int_0^t \lambda e^{-\lambda s} e^{-\Lambda(t-s)} B_{t-s}^*\big(b_s^*(b, a), a\big)\, ds.$$

Table A3—List of Variables

State variables	
a	Annuity endowment
b	Liquid wealth in healthy state
B	Liquid wealth in low health state
s	Household current age in healthy state
S	Time spent in healthy state, Section VI
t	Household current age in low health state
w_0	Household initial total wealth, Section V
α_0	Household initial portfolio share of annuitized wealth, Section V
Control variables	
x	Consumption in healthy state
X	Consumption in low health state
$\check{X}$	Consumption just prior to accepting Medicaid
T	Age when liquid wealth is exhausted
$\hat{\alpha}$	Optimal portfolio share of annuitized wealth at retirement
Exogenous parameters	
γ	Curvature of the utility function
ω	Rate of transformation between expenditure and consumption in low health state
β	Rate of time preference
λ	Transition hazard from healthy state to low health
Λ	Transition hazard from low health to death
r	Rate of return on liquid wealth
$\bar{X}$	Medicaid consumption floor
X_M	Medicaid program expenditure per recipient
Functions of endogenous variables	
$u(x)$	Utility function in healthy state
$U(X)$	Utility function in low health state
$v(b, a)$	Household's value function in healthy state
$V(B, a)$	Household's value function in low health state
$\Gamma_x(b)$	Graph of $\dot{x} = 0$ isocline
$\Gamma_b(b)$	Graph of $\dot{b} = 0$ isocline
f_t	Fraction of survivors in healthy-state t periods after retirement, Section IV
$\bar{b}(t, b, a)$	Cohort average liquid-wealth t periods after retirement, Section IV
Functions of exogenous parameters	
$\xi = \bar{X}/X_M$	Rate of transformation between expenditure and consumption floor for the Medicaid program
$\Omega = \omega^{\frac{\gamma}{1-\gamma}}$	Marginal-utility conversion factor between low and high health states
$\sigma = \frac{r - (\Lambda + \beta)}{1 - \gamma}$	Growth rate of expenditure in low health state
$\theta = \frac{1}{\Omega}\left(1 - \frac{r - \beta}{\lambda}\right)^{\frac{1}{1-\gamma}}$	Conversion factor between consumption decision rule and $\dot{x} = 0$ isocline
$\bar{a}$	Threshold annuity income separating high-annuity and low-annuity phase diagrams
$\bar{r}$	Threshold interest rate separating standard and high interest rate phase diagrams
$\bar{\rho}$	Long-run optimal ratio of liquid wealth to annuities for households relying on self-insurance
$\rho(a)$	Long-run optimal ratio of liquid wealth to annuities, standard interest rate case, Proposition 4
r_A	Actuarially fair rate of return on annuities
$\bar{w}$	Minimum initial total wealth of households who never use Medicaid, Section V

Cohort average wealth is total wealth divided by the number of survivors:

$$\bar{b}(t; b, a) = \frac{b_H(t; b, a) + b_L(t; b, a)}{f_H(t) + f_L(t)}.$$

Appendix B. Proofs

PROOF OF LEMMA 1:

The present-value Hamiltonian for (4) is

$$\mathcal{H} \equiv e^{-(\Lambda+\beta)t} U(X_t) + M_t(rB_t + a - X_t) + N_t B_t, \tag{B1}$$

with costate M_t and Lagrange multiplier N_t for the state-variable constraint $B_t \geq 0$.

The first-order condition for optimal expenditure is

$$\frac{\partial \mathcal{H}}{\partial X_t} = 0 \Leftrightarrow e^{-(\Lambda+\beta)t} U'(X_t) = M_t, \tag{B2}$$

and the costate equation is

$$\dot{M}_t = -\frac{\partial \mathcal{H}}{\partial B_t} \Leftrightarrow \dot{M}_t = -rM_t - N_t. \tag{B3}$$

The transversality condition is

$$\lim_{t \to \infty} M_t \cdot B_t = 0. \tag{B4}$$

Provided $M_t \geq 0$, first-order conditions and (B4) will be sufficient for optimality. The strict concavity of problem (4) ensures that if an interior solution exists, it is unique.

We start by checking that $(B_t, X_t) = (0, a)$ satisfies the first-order conditions and (B4) for all t. Substituting $X_t = a$ into (B2) and eliminating M_t from (B2)–(B3) gives the expression for N_t:

$$N_t = [(\Lambda + \beta) - r] e^{-(\Lambda+\beta)t} U'(a).$$

By assumption, $\Lambda + \beta > r$. So, $N_t \geq 0$, and the time path of the Lagrange multiplier is continuous. In this lemma, $B_t = 0$ all t. Hence, $N_t \cdot B_t = 0$. Similarly, we can see that transversality condition (B4) also holds.

Now suppose that the state-variable constraint does not bind so that $N_t = 0$. Taking the logarithm of (B2) and differentiating with respect to t gives

$$\frac{\dot{M}_t}{M_t} = \frac{U''(X_t)}{U'(X_t)} \dot{X}_t - (\Lambda + \beta) = (\gamma - 1) \frac{\dot{X}_t}{X_t} - (\Lambda + \beta).$$

Substituting the previous expression together with $N_t = 0$ into (B3) gives

$$\frac{\dot{X}_s}{X_s} = \sigma, \quad \text{where } \sigma \equiv \frac{r - (\Lambda + \beta)}{1 - \gamma} < 0. \blacksquare$$

PROOF OF PROPOSITION 1:

Refer to Hamiltonian (B1). Let (B_t^*, X_t^*) be the trajectory that converges to the aforementioned $(0, a)$. Equation (5) shows the vertical motion in Figure 1, case (i) is strictly negative. Let $T^* < \infty$ be the time (B_t^*, X_t^*) reaches $(0, a)$. For $t \leq T^*$, the budget constraint of (4) together with (5) determine the shape of (B_t^*, X_t^*); (B2) determines M_t. Set $N_t = 0$.

For $t > T^*$, set N_t, M_t, X_t^*, and B_t^* as in the proof of Lemma 1. Then the first-order condition for X_t, the costate equation, the budget equation, and the state-variable constraint all hold for $t \geq 0$; we have $N_t \geq 0$ all t; the path of N_t is piecewise continuous; $N_t \cdot B_t = 0$, all t, by construction; the costate variable is nonnegative, all t, and continuous by construction; and transversality condition (B4) holds. Hence, (B_t^*, X_t^*) is optimal. Continuity of (B_t^*, X_t^*) in t follows by construction.

To show that the value function $V(B, a)$ is continuously differentiable and concave, we first establish the following Lemma.

LEMMA A1: *Let T^*, B_t^*, and X_t^* be as in Proposition* 1. *Then $T^*(B, a)$ is strictly increasing and continuous in B*:

$$T^*(0, a) = 0, \quad \textit{and} \quad \lim_{B \to \infty} T^*(B, a) = \infty.$$

As a function of B, $X_0^* = X_0^*(B, a)$ is continuous, strictly increasing, and strictly concave:

$$X_0^*(0, a) = a, \quad \textit{and} \quad \lim_{B \to \infty} \frac{\partial X_0^*(B, a)}{\partial B} = r - \sigma > 0.$$

PROOF OF LEMMA A1:

Expression (5) shows

$$X_{T^*}^* = X_0^* \cdot e^{\sigma T^*}.$$

By construction, $X_{T^*}^* = a$. So,

$$X_0^* = a \cdot e^{-\sigma \cdot T^*}. \tag{B5}$$

Budget accounting then implies

$$B = \int_0^{T^*} e^{-rt}\left(a \cdot e^{-\sigma(T^* - t)} - a\right) dt, \tag{B6}$$

which determines $T^* = T^*(B, a)$. From (B6), we can see that $T^*(B, a)$ is a strictly increasing and continuous function of B, with

$$\lim_{B \to \infty} T^*(B, a) = \infty, \tag{B7}$$

and

$$T^*(0, a) = 0. \tag{B8}$$

Turning to the properties of $X_0^*(B, a)$, we can then see from (B5) that $X_0^*(B, a)$ is continuous and strictly increasing in B; (B8) implies $X_0^*(0, a) = a$.

Differentiating (B6) with respect to B gives $\frac{\partial T^*}{\partial B}$:

$$1 \;=\; \int_0^{T^*} a \cdot e^{-rt} \cdot (-\sigma) \cdot e^{-\sigma(T^*-t)} \cdot \frac{\partial T^*}{\partial B} \cdot dt \;\Leftrightarrow$$

$$\frac{\partial T^*}{\partial B} \;=\; \frac{1}{-a \cdot \sigma \cdot e^{-\sigma T^*} \cdot \int_0^{T^*} e^{(\sigma-r)t}\,dt}.$$

Differentiating (B5),

$$\frac{\partial X_0^*}{\partial B} \;=\; -\sigma \cdot a \cdot e^{-\sigma \cdot T^*} \cdot \frac{\partial T^*}{\partial B}.$$

Combining the last two expressions,

$$\text{(B9)} \qquad \frac{\partial X_0^*}{\partial B} \;=\; \frac{1}{\int_0^{T^*} e^{(\sigma-r)\cdot t}\,dt}.$$

Since $T^*(B,a)$ is increasing in B, (B9) implies X_0^* is concave in B. Given (B7), (B9) also establishes

$$\lim_{B\to\infty} \frac{\partial X_0^*(B,a)}{\partial B} \;=\; r - \sigma. \;\blacksquare$$

Turning now to the value function $V(B,a)$, the envelope theorem shows

$$\frac{\partial V}{\partial B}(B,a) \;=\; U'\big(X_0^*(B,a)\big).$$

From Lemma A1, $X_0^*(B,a)$ is continuously differentiable and strictly increasing in B. Hence, $V(B,a)$ is continuously differentiable and strictly concave in B. ∎

PROOF OF PROPOSITION 2:

Step 1: Fix a and $\bar{X}$. In case (ii), we have $a < \bar{X}$. Define a function

$$\text{(B10)} \qquad \pi(X) \;\equiv\; U(X) - U(\bar{X}) + U'(X) \cdot (a - X), \quad \text{all } X > a.$$

This function is continuous and strictly increasing in X, and it has opposite signs at the ends of the interval $[\bar{X}, \infty)$:

$$\pi'(X) \;=\; U''(X) \cdot (a - X) \;>\; 0,$$

$$\pi(\bar{X}) \;=\; U'(\bar{X}) \cdot (a - \bar{X}) \;<\; 0,$$

$$\lim_{X\to\infty} \pi(X) \;=\; -U(\bar{X}) + \lim_{X\to\infty}\Big(U(X) + \gamma \cdot U(X) \cdot \frac{a-X}{X}\Big) \;=\; -U(\bar{X}) \;>\; 0.$$

It follows that on $(\bar{X}, \infty)$, $\pi(X)$ has a unique root. Denote this root $\check{X}$.

Step 2: For any initial $B > 0$, the Hamiltonian is (B1) with $N_t = 0$. The first-order condition for X_t, costate equation, and budget equation are as in case (i). Hence, the phase diagram is as in Figure 1, case (ii). Choose the trajectory at the top of the diagram that converges to $(0, \check{X})$. As in case (i), convergence takes a finite time (which we denote T^*). Assume Medicaid take-up for $t > T^*$, with $X_t^* = \bar{X}$.

Optimality requires that once $B_t = 0$, the household permanently accepts Medicaid. If the household enters the liquidity constrained regime at age T, its continuation value is

$$W(T) = e^{-(\Lambda+\beta)\cdot T} \frac{U(\bar{X})}{\Lambda+\beta}.$$

Kamien and Schwartz (1981, 143) show that the first-order conditions for the optimal acceptance date T^* are

$$\text{(B11)} \quad B_{T^*} \geq 0, \quad M_{T^*} \geq \frac{\partial W(T^*)}{\partial B_{T^*}} \geq 0, \quad B_{T^*} \cdot \left[M_{T^*} - \frac{\partial W(T^*)}{\partial B_{T^*}}\right] = 0,$$

$$\text{(B12)} \qquad \mathcal{H}_{t=T^*} + \frac{\partial W(T^*)}{\partial T} = 0,$$

where we use the Hamiltonian from (B1) without the state-variable constraint. Our proposed solution has

$$\text{(B13)} \qquad B_{T^*} = 0.$$

From (B2), $M_{T^*} > 0$. Notice, $W(T)$ is not a function of B_T, making its partial derivative zero. Hence, our proposed solution is consistent with (B11). Evaluating the left-hand side of (B12) at $T = T^*$ yields

$$\pi(\check{X}) \cdot e^{-(\Lambda+\beta)T^*} = 0,$$

so that step 1 establishes (B12).

By construction, we have $\check{X} = \lim_{t \to T^* - 0} X_t^*$, and

$$X_t^* = \begin{cases} \check{X} \cdot e^{\sigma(t-T^*)} & \text{for } t \in [0, T^*] \\ \bar{X} & \text{for } t > T^* \end{cases}.$$

It remains to show that the first-order condition for T^* is sufficient. We have argued that the root of $\pi(\cdot)$ is unique. Suppose we choose a larger (smaller) T^*. The trajectories of Figure 1, case (ii) remain as before. Thus, budgetary accounting implies we must lower (raise) $\check{X}$ for our stationary point accordingly, leading to $\pi(\check{X}) < (>) \ 0$. Hence, the right-hand side of first-order condition (B12) yields a maximum at our original T^*.

To show that the value function $V(B, a)$ is continuously differentiable and concave, we first establish the following lemma.

LEMMA A2: *Let T^*, B_t^*, and X_t^* be as in Proposition 2. Then $T^*(B,a)$ is strictly increasing and continuous in B:*

$$T^*(0,a) \;=\; 0, \quad \textit{and} \quad \lim_{B\to\infty} T^*(B,a) \;=\; \infty.$$

As a function of B, $X_0^* = X_0^*(B,a)$ is continuous (except at $B = 0$) and strictly increasing; we have

$$X_0^*(B,a) \;=\; \begin{cases} \textit{convex in } B & \left(1-\frac{a}{\check{X}}\right)\left(1-\frac{r}{\sigma}\right) \;>\; 1 \\ \textit{concave in } B & \left(1-\frac{a}{\check{X}}\right)\left(1-\frac{r}{\sigma}\right) \;<\; (0,1) \end{cases}, \quad \textit{all } B \;>\; 0,$$

and

$$\lim_{B\to\infty} \frac{\partial X_0^*(B,a)}{\partial B} \;=\; r - \sigma \;>\; 0.$$

PROOF OF LEMMA A2:

The proof parallels that of Lemma A. The analog of (B5) for case (ii) is

$$X_0^* \;=\; \check{X} \cdot e^{-\sigma T^*}, \tag{B14}$$

and

$$\frac{\partial X_0^*}{\partial B} \;=\; -\sigma \cdot \check{X} \cdot e^{-\sigma T^*} \cdot \frac{\partial T^*}{\partial B}. \tag{B15}$$

Budgetary accounting implies

$$B \;=\; \int_0^{T^*(B,a)} e^{-rt} \cdot \left(X_0^*(B,a)\, e^{\sigma t} - a\right) dt.$$

Differentiating the previous equation with respect to B yields

$$1 = e^{-rT^*}\left(X_0^* \cdot e^{\sigma T^*} - a\right)\frac{\partial T^*}{\partial B} + \frac{\partial X_0^*}{\partial B} J(T^*), \quad \text{where } J(T) \;\equiv\; \int_0^T e^{-(r-\sigma)t}\, dt. \tag{B16}$$

Substituting from (B14)–(B15) into (B16), we have

$$\frac{\partial X_0^*}{\partial B} \;=\; \frac{1}{D(T^*)}, \quad \text{where} \quad D(T^*) \;\equiv\; -\frac{1}{\sigma} \cdot \frac{\check{X} - a}{\check{X}} \cdot e^{-(r-\sigma)T^*} + J(T^*).$$

The asymptotic behavior of $\partial X_0^*/\partial B$ follows:

$$\lim_{B\to\infty} D\left(T^*(B,a)\right) \;=\; \lim_{T^*\to\infty} D(T^*) \;=\; \lim_{T^*\to\infty} J(T^*) \;=\; \frac{1}{r-\sigma}.$$

The convexity or concavity of $X_0^*(B,a)$ follows as well:

$$\begin{aligned} D'(T^*) \;&=\; \frac{r-\sigma}{\sigma} \cdot \frac{\check{X} - a}{\check{X}} \cdot e^{-(r-\sigma)T^*} + e^{-(r-\sigma)T^*} \\ &=\; \left[1 - \left(1 - \frac{a}{\check{X}}\right)\left(1 - \frac{r}{\sigma}\right)\right] \cdot e^{-(r-\sigma)T^*}. \;\blacksquare \end{aligned}$$

From Lemma A2, $X_0^*(B,a)$ is continuously differentiable and strictly increasing in B, except at $B = 0$. Hence, $V(B,a)$ is continuously differentiable and strictly concave in B, except at $B = 0$. ∎

PROOF OF LEMMA 2:

Suppose (b^*, x^*) is a solution to (12)–(13) for a fixed a and $\bar{X}$.

Step 1: Start with case (ii), $a < \bar{X}$.

Proposition 2 and (13) imply

$$x^* = \theta \cdot X_0^*(b,a) = \theta \cdot \check{X} \cdot e^{-\sigma \cdot T},$$

where $T = T^*(b^*, a)$. Let $Z = \check{X}/a$. Then the equation for b^* reads

$$\theta a Z e^{-\sigma T} = r b^* + a \Leftrightarrow \frac{b^*}{a} = \frac{1}{r}[\theta Z e^{-\sigma T} - 1]. \tag{B17}$$

As in the proof of Lemma A2, budgetary accounting yields

$$b^* = \int_0^T e^{-rt}\left(aZe^{\sigma t}e^{-\sigma T} - a\right) dt \Leftrightarrow$$

$$\frac{b^*}{a} = Z\frac{e^{-\sigma T} - e^{-rT}}{r - \sigma} - \frac{1 - e^{-rT}}{r}. \tag{B18}$$

Equating b^*/a in (B17)–(B18), we have

$$\frac{1}{r}\left(\theta Z e^{-\sigma T} - 1\right) = Z\frac{e^{-\sigma T} - e^{-rT}}{r - \sigma} - \frac{1 - e^{-rT}}{r} \Leftrightarrow$$

$$e^{-rT}\left(\frac{Z}{r - \sigma} - \frac{1}{r}\right) = Ze^{-\sigma T}\left(\frac{1}{r - \sigma} - \frac{\theta}{r}\right) \Leftrightarrow$$

$$e^{(r-\sigma)T}Z\left(\frac{1}{r - \sigma} - \frac{\theta}{r}\right) = \left(\frac{Z}{r - \sigma} - \frac{1}{r}\right). \tag{B19}$$

The last expression depends on b^* only through T. Equation (B19) either has a unique solution $T > 0$ or no solution. If $T > 0$ exists, Lemma A2 shows that $T = T^*(b,a)$ is strictly increasing in b; hence, b^* must be unique if T is unique.

Step 2: If $a \geq \bar{X}$, repeat the step 1 argument setting $Z = 1$—recall Proposition 1. ∎

PROOF OF PROPOSITION 3:

Step 1: Define a function:

$$\xi(r) = \frac{r}{r - \sigma} - \frac{1}{\Omega}\left(1 - \frac{r - \beta}{\lambda}\right)^{\frac{1}{1-\gamma}}.$$

Assumption 5 has $r < \lambda + \beta$. On the interval $[0, \beta + \lambda]$, $\xi(r)$ is continuous (recall that $\sigma < 0$) and strictly increasing, with $\xi(0) < 0$ and $\xi(\beta + \lambda) > 0$. Hence, it has a unique root $\bar{r} \in (0, \beta + \lambda)$. Then, using monotonicity of $\xi(r)$ and the definition of θ in (14), we have

$$r < \bar{r} \Leftrightarrow r < \theta(r - \sigma) \Leftrightarrow \Gamma_b'(\infty) < \Gamma_x'(\infty),$$

where the last inequality follows from Lemmas A1–A2 and (12).

Step 2: Suppose $a \geq \bar{X}$. Then

$$a \geq \bar{X} > \theta \cdot \bar{X} \cdot \left(1 - \gamma(1 - \theta)\right)^{-\frac{1}{\gamma}} = \bar{a},$$

and

$$\Gamma_b(0) = a > \theta X^*(0, a) = \theta a = \Gamma_x(0),$$

so we have left-hand side diagrams on Figure 2. The asymptotic slope from Lemma A1 establishes the cases that obtain for $r < (>) \ \theta \cdot (r - \sigma)$.

Step 3: Suppose $a < \bar{X}$. We show that there exists a unique $\bar{a} \in (0, \bar{X})$, such that

$$\Gamma_b(0) = a < \theta \check{X}(a) = \Gamma_x(0) \Leftrightarrow a < \bar{a}.$$

Consider $\pi(\cdot)$ from (B10), and make a change of variables

$$\check{X}(a) = aZ(a). \tag{B20}$$

Since $\pi(\check{X}(a)) = 0$ implies that $\check{X}(a) > a$, we have $Z(a) > 1$. Using (B20), equation $\pi = 0$ can be written as

$$(1 - \gamma) Z^{\gamma} + \gamma Z^{\gamma - 1} = \left(\frac{a}{\bar{X}}\right)^{-\gamma}. \tag{B21}$$

The left-hand side of (B21) is strictly decreasing in Z for all $Z \geq 1$, with

$$Z(\bar{X}) = 1, \quad \lim_{a \to 0} Z(a) = \infty, \quad \text{and } Z'(a) < 0.$$

Hence, there is a unique $\bar{a} \in (0, \bar{X})$, with

$$\bar{a} = \theta \check{X}(\bar{a}) \Leftrightarrow Z(\bar{a}) = \frac{1}{\theta}. \tag{B22}$$

Evaluating (B21) at $Z = 1/\theta$ and $a = \bar{a}$ gives

$$\bar{a} = \bar{X} \cdot \theta \left(1 - \gamma(1 - \theta)\right)^{-\frac{1}{\gamma}}.$$

Since $Z(a)$ is strictly decreasing,

$$a < \theta \check{X}(a) \Leftrightarrow \frac{1}{\theta} < Z(a) \Leftrightarrow a < \bar{a}.$$

Step 4: Step 3 shows that $a > (<)\ \bar{a}$ separates the left- and right-hand side diagrams in Figure 2. The asymptotic slopes in Lemma A2 complete the proof. ∎

PROOF OF PROPOSITION 4:

Suppose $\theta \cdot (r - \sigma) > r$ (i.e., standard interest rate case). Let $\check{X}(a) = aZ(a)$ as in (B20). Define $Z(a)$ for all $a \geq \bar{a}$ as follows:

$$Z(a) = \begin{cases} \frac{1}{a}0\check{X}(a) & \text{if } a \in [\bar{a}, \bar{X}) \\ 1 & \text{if } a \geq \bar{X} \end{cases}.$$

Then Proposition 3 shows $Z(a)$ is continuous for all $a \geq \bar{a}$ and strictly decreasing for $a \in [\bar{a}, \bar{X})$. From (B22), we have

$$\text{(B23)} \qquad 1 \leq Z(a) \leq \frac{1}{\theta}.$$

In the proof of Lemma 2, (B17) shows

$$\text{(B24)} \qquad \frac{b^*}{a} = \frac{\theta Z(a)\, e^{-\sigma T^*} - 1}{r}.$$

And (B19) relates T^* and Z:

$$e^{(r-\sigma)T^*} = \frac{\frac{1}{r} - \frac{Z}{r-\sigma}}{\frac{\theta Z}{r} - \frac{Z}{r-\sigma}}.$$

In the standard interest rate case, $\theta \cdot (r - \sigma) > r$ and (B23) imply that both the numerator and the denominator of the aforementioned expression are positive. Define

$$\psi(Z) \equiv Ze^{-\sigma T^*} = Z\left[\frac{\frac{1}{Zr} - \frac{1}{r-\sigma}}{\frac{\theta}{r} - \frac{1}{r-\sigma}}\right]^{-\frac{\sigma}{r-\sigma}}.$$

Then, from (B24),

$$\frac{d}{da}\left(\frac{b^*}{a}\right) = \frac{\theta}{r}\psi'(Z) \cdot Z'(a).$$

Showing that $\psi'(Z) < 0$ for all $Z \in (1, 1/\theta]$ and $\psi'(1) = 0$ would complete the proof. Indeed,

$$\frac{d}{dZ}\ln\psi(Z) = \frac{1}{Z} + \frac{\sigma}{r-\sigma}\frac{\frac{1}{rZ^2}}{\frac{1}{rZ} - \frac{1}{r-\sigma}} = \frac{1}{Z}\frac{\frac{1}{Z} - 1}{\frac{r-\sigma}{rZ} - 1} < 0.$$

The numerator of the previous expression is negative for all $Z > 1$ and 0 for $Z = 0$. The denominator is positive when $r < \bar{r}$ and $Z < 1/\theta$. ∎

PROOF OF LEMMA 3:

Normalize the initial cohort size (at $t = 0$) to 1. Then the number of households remaining alive and in good health t years after retirement is $f_{H,t} = e^{-\lambda t}$. Similarly, let the fraction alive at t but in low health status be

$$f_{L,t} \equiv \int_0^t \lambda \cdot e^{-\lambda \cdot s} \cdot e^{-\Lambda(t-s)} ds.$$

Combining expressions, the fraction of survivors in high health status is

$$f_t = \frac{f_{H,t}}{f_{H,t} + f_{L,t}} = \frac{1}{1 + \frac{\lambda}{\Lambda - \lambda} \cdot (1 - e^{-(\Lambda-\lambda)t})}. \blacksquare$$

Micro-foundation for State-Dependent Utility.—A richer model where nonmedical LTC expenditure is a separate, endogenous variable would produce an indirect utility function of form (2). To see this, assume that a household has two remaining periods of life and that $h = H$ in the first period and $h = L$ in the last period.[25] Set $r = 0$ and $\beta = 1$; disregard annuities, Medicaid, and uncertain mortality. Then a newly retired household solves

$$\max_x \{u(x) + U(b - x)\}. \tag{B25}$$

To endogenize the choice of nonmedical LTC expenditure, l, replace $U(b - x)$ in (B25) with

$$U(b - x) \equiv \kappa \cdot \max_l \{\varphi \cdot u(b - x - l) + (1 - \varphi) \cdot u(l)\}, \tag{B26}$$

where $\kappa > 0$ and $\varphi \in (0, 1)$ are preference parameters. Maximization with respect to l in (B26) yields exactly the reduced form utility function (2):

$$U(b - x) = \omega^\gamma \cdot u(b - x),$$
$$\omega^\gamma \equiv \kappa \cdot \left([\varphi]^{\frac{1}{1-\gamma}} + [1 - \varphi]^{\frac{1}{1-\gamma}}\right).$$

Derivation of the Actuarially Fair Rate of Return on Annuities.—Let A be the market value of an annuity with income a. Then

$$a = A r_A. \tag{B27}$$

If $E_T[\cdot]$ is the expectation over the stochastic life-span $\tilde{T}$, we have

$$A = E_T\left[\int_0^{\tilde{T}} e^{-rt} dt\right] = a \int_0^\infty \lambda e^{-\lambda T} \int_0^T e^{-rt} dt dT + a \int_0^\infty \lambda e^{-\lambda T} \int_T^\infty \Lambda e^{-\Lambda(S-T)} \int_T^S e^{-rs} ds dS dT. \tag{B28}$$

[25] The two-period example is also convenient for direct comparisons with other two-period models, such as Finkelstein, Luttmer, and Notowidigdo (2013) and Hubbard, Skinner, and Zeldes (1995).

The first right-hand side term registers annuity income during the healthy phase of retirement; the second term gives income during the last phase of life. Performing the integration and combining (B27)–(B28), we have

$$r_A = \frac{(\lambda + r)(\Lambda + r)}{\lambda + \Lambda + r}.$$

REFERENCES

Altonji, Joseph G., Fumio Hayashi, and Laurence J. Kotlikoff. 1992. "Is the Extended Family Altruistically Linked? Direct Tests Using Micro Data." *American Economic Review* 82 (5): 1177–98.

Altonji, Joseph G., Fumio Hayashi, and Laurence J. Kotlikoff. 1997. "Parental Altruism and Inter Vivos Transfers: Theory and Evidence." *Journal of Political Economy* 105 (6): 1121–66.

Ameriks, John, Joseph S. Briggs, Andrew Caplin, Matthew D. Shapiro, and Christopher Tonetti. 2015. "Long-Term-Care Utility and Late-in-Life Saving." National Bureau of Economic Research (NBER) Working Paper 20973.

Ameriks, John, Joseph Briggs, Andrew Caplin, Matthew D. Shapiro, and Christopher Tonetti. 2016. "Late-in-Life Risks and the Under-Insurance Puzzle." National Bureau of Economic Research (NBER) Working Paper 22726.

Ameriks, John, Andrew Caplin, Steven Laufer, and Stijn Van Nieuwerburgh. 2011. "The Joy of Giving or Assisted Living? Using Strategic Surveys to Separate Public Care Aversion from Bequest Motives." *Journal of Finance* 66 (2): 519–61.

Benartzi, Shlomo, Alessandro Previtero, and Richard H. Thaler. 2011. "Annuitization Puzzles." *Journal of Economic Perspectives* 25 (4): 143–64.

Brown, Jeffrey R. 2007. "Rational and Behavioral Perspectives on the Role of Annuities in Retirement Planning." National Bureau of Economic Research (NBER) Working Paper 13537.

Brown, Jeffrey R., and Amy Finkelstein. 2007. "Why is the market for long-term care insurance so small?" *Journal of Public Economics* 91 (2007): 1967–91.

Brown, Jeffrey R., and Amy Finkelstein. 2008. "The Interaction of Public and Private Insurance: Medicaid and the Long-Term Care Insurance Market." *American Economic Review* 98 (3): 1083–1102.

Brown, Jeffrey R., Gopi Shah Goda, and Kathleen McGarry. 2012. "Long-Term Care Insurance Demand Limited By Beliefs About Needs, Concerns About Insurers, And Care Available From Family." *Health Affairs* 31 (6): 1294–1302.

Budish, Armond D. 1995. *Avoiding the Medicaid Trap: How Every American Can Beat Catastrophic Costs of Nursing Home Care*. New York: Avon Books.

Carroll, Christopher D. 1997. "Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis." *Quarterly Journal of Economics* 112 (1): 1–55.

Congressional Budget Office (CBO). 2004. *Financing Long-Term Care for the Elderly*. US Congress. Washington, DC, April.

Davidoff, Thomas, Jeffrey R. Brown, and Peter A. Diamond. 2005. "Annuities and Individual Welfare." *American Economic Review* 95 (5): 1573–90.

Deaton, Angus. 1991. "Saving and Liquidity Constraints." *Econometrica* 59 (5): 1221–48.

De Nardi, Mariacristina, Eric French, and John B. Jones. 2010. "Why Do the Elderly Save? The Role of Medical Expenses." *Journal of Political Economy* 118 (1): 39–75.

De Nardi, Mariacristina, Eric French, and John B. Jones. 2015. "Savings after Retirement: A Survey." National Bureau of Economic Research (NBER) Working Paper 21268.

De Nardi, Mariacristina, Eric French, and John Bailey Jones. 2016. "Medicaid Insurance in Old Age." *American Economic Review* 106 (11): 3480–3520.

Dynan, Karen E., Jonathan Skinner, and Stephen P. Zeldes. 2004. "Do the Rich Save More?" *Journal of Political Economy* 112 (2): 397–444.

Finkelstein, Amy, Erzo F. P. Luttmer, and Matthew J. Notowidigdo. 2013. "What Good Is Wealth without Health? The Effect of Health on the Marginal Utility of Consumption." *Journal of the European Economic Association* 11 (S1): 221–58.

Finkelstein, Amy, and James Poterba. 2004. "Adverse Selection in Insurance Markets: Policyholder Evidence from the U.K. Annuity Market." *Journal of Political Economy* 112 (1): 183–208.

Friedman, Benjamin M., and Mark J. Warshawsky. 1990. "The Cost of Annuities: Implications for Saving Behavior and Bequests." *Quarterly Journal of Economics* 105 (1): 135–54.

Gourinchas, Pierre-Olivier, and Jonathan A. Parker. 2002. "Consumption over the Life Cycle." *Econometrica* 70 (1): 47–89.

Hubbard, R. Glenn, Jonathan Skinner, and Stephen P. Zeldes. 1994. "The importance of precautionary motives in explaining individual and aggregate saving." *Carnegie-Rochester Conference Series on Public Policy* 40 (Supplement C): 59–125.

Hubbard, R. Glenn, Jonathan Skinner, and Stephen P. Zeldes. 1995. "Precautionary Saving and Social Insurance." *Journal of Political Economy* 103 (2): 360–99.

Hurd, Michael D., and Susann Rohwedder. 2009. "The Level and Risk of Out-of-Pocket Health Care Spending." Michigan Retirement Research Center Working Paper 2009–218.

Hurd, Michael D., and Susann Rohwedder. 2015. "Wealth Dynamics and Active Saving at Older Ages." In *Improving the Measurement of Consumer Expenditures*, edited by Christopher D. Carroll, Thomas F. Crossley, and John Sabelhaus, 388–413. Chicago: University of Chicago Press.

Kamien, Morton I., and Nancy L. Schwartz. 1981. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*. Amsterdam: North-Holland.

Kopecky, Karen A., and Tatyana Koreshkova. 2014. "The Impact of Medical and Nursing Home Expenses on Savings." *American Economic Journal: Macroeconomics* 6 (3): 29–72.

Kotlikoff, Laurence J. 1989. *What Determines Savings?* Cambridge, MA: MIT Press.

Kotlikoff, Laurence J., and Lawrence H. Summers. 1988. "The Contribution of Intergenerational Transfers to Total Wealth: A Reply." In *Modelling the Accumulation and the Distribution of Wealth*, edited by Denis Kessler and Andre Masson, 53–67. Oxford, UK: Oxford University Press.

Laibson, David. 2011. "Family Status Transitions, Latent Health, and the Post-Retirement Evolution of Assets: Comment." In *Explorations in the Economics of Aging*, edited by David A. Wise, 69–73. Chicago: University of Chicago Press.

Laitner, John. 1988. "Bequests, Gifts, and Social Security." *Review of Economic Studies* 55 (2): 275–99.

Laitner, John, and F. Thomas Juster. 1996. "New Evidence on Altruism: A Study of TIAA-CREF Retirees." *American Economic Review* 86 (4): 893–908.

Laitner, John, and Henry Ohlsson. 2001. "Bequest motives: A comparison of Sweden and the United States." *Journal of Public Economics* 79 (1): 205–36.

Laitner, John, and Dan Silverman. 2012. "Consumption, retirement and social security: Evaluating the efficiency of reform that encourages longer careers. *Journal of Public Economics* 96 (7–8): 615–34.

Laitner, John, and Amanda Sonnega. 2010. "Intergenerational Transfers in the Health and Retirement Study Data." Michigan Retirement Research Center Working Paper 2010–238.

Lockwood, Lee M. 2014. "Incidental Bequests and the Choice to Self-Insure Late-Life Risks." National Bureau of Economic Research (NBER) Working Paper 20745.

Love, David A., Michael G. Palumbo, and Paul A. Smith. 2009. "The trajectory of wealth in retirement." *Journal of Public Economics* 93 (1–2): 191–208.

Marshall, Samuel, Kathleen M. McGarry, and Jonathan S. Skinner. 2010. "The Risk of Out-of-Pocket Health Care Expenditure at End of Life." National Bureau of Economic Research (NBER) Working Paper 16170.

MetLife Mature Market Institute. 2009. *The 2009 MetLife Survey of Nursing Home, Assisted Living, Adult Day Services, and Home Care Costs.* Westport, October.

Miller, Edward Alan, Vincent Mor, and Melissa Clark. 2010. "Reforming Long-Term Care in the United States: Findings From a National Survey of Specialists." *Gerontologist* 50 (2): 238–52.

Mirer, Thad W. 1979. "The Wealth-Age Relation among the Aged." *American Economic Review* 69 (3): 435–43.

Mitchell, Olivia S., James M. Poterba, Mark J. Warshawsky, and Jeffrey R. Brown. 1999. "New Evidence on the Money's Worth of Individual Annuities." *American Economic Review* 89 (5): 1299–1318.

Modigliani, Franco. 1986. "Life Cycle, Individual Thrift, and the Wealth of Nations." *American Economic Review* 76 (3): 297–313.

Norton, Edward C. 2000. "Long-Term Care." In *Handbook of Health Economics*, Volume 1B, edited by Anthony J. Culyer and Joseph P. Newhouse, 955–94. Amsterdam: North-Holland.

Palumbo, Michael G. 1999. "Uncertain Medical Expenses and Precautionary Saving Near the End of the Life Cycle." *Review of Economic Studies* 66 (2): 395–421.

Pashchenko, Svetlana. 2013. "Accounting for non-annuitization." *Journal of Public Economics* 98 (Supplement C): 53–67.

Pauly, Mark V. 1990. "The Rational Nonpurchase of Long-Term-Care Insurance." *Journal of Political Economy* 98 (1): 153–68.

Poterba, James M., Steven F. Venti, and David A. Wise. 2011a. "Family Status Transitions, Latent Health, and the Post-Retirement Evolution of Assets." In *Explorations in the Economics of Aging*, edited by David A. Wise, 23–69. Chicago: University of Chicago Press.

Poterba, James, Steven Venti, and David Wise. 2011b. "The Composition and Drawdown of Wealth in Retirement." *Journal of Economic Perspectives* 25 (4): 95–118.

Reichling, Felix, and Kent Smetters. 2015. "Optimal Annuitization with Stochastic Mortality and Correlated Medical Costs." *American Economic Review* 105 (11): 3273–3320.

Schafer, Robert. 1999. "Determinants of the Living Arrangements of the Elderly." Joint Center for Housing Studies Harvard University Working Paper W99–6.

Scholz, John Karl, and Ananth Seshadri. 2009. "What Replacement Rates Should Households Use?" Michigan Retirement Research Center Working Paper 2009–214.

Scholz, John Karl, Ananth Seshadri, and Surachai Khitatrakun. 2006. "Are Americans Saving 'Optimally' for Retirement?" *Journal of Political Economy* 114 (4): 607–43.

Sinclair, Sven H., and Kent A. Smetters. 2004. "Health Shocks and the Demand for Annuities." Congressional Budget Office Technical Paper 2004–9.

Smith, Karen E., Mauricio Soto, and Rudolph G. Penner. 2009. "How Seniors Change Their Asset Holdings During Retirement." Center for Retirement Research at Boston College Working Paper 2009–31.

US Census Bureau. 2012. *Current Population Survey, 2012 Annual Social and Economic Supplement*. Suitland, MD: US Census Bureau.

Yaari, Menahem E. 1965. "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer." *Review of Economic Studies* 32 (2): 137–50.

Zeldes, Stephen P. 1989. "Consumption and Liquidity Constraints: An Empirical Investigation." *Journal of Political Economy* 97 (2): 305–46.

Zhao, Kai. 2015. "The impact of the correlation between health expenditure and survival probability on the demand for insurance." *European Economic Review* 75 (Supplement C): 98–111.