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Interest rate trends in a global context

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ABSTRACT

Long-term interest rates have been falling globally since the early 1980s and have reached historically low levels. Past forecasts largely missed this secular decline. This paper reviews methodologies for making long-term interest rate projections. We synthesize results from studies that use long historical series and cross-country data to estimate the trend and decompose it into components. We then construct a set of economic indicators that are potentially useful in interest rate forecasting.

We add international forward-looking economic indicators as explanatory variables in a standard macro-finance forecasting model. We find that the model with international variables can outperform the other models by better tracking the falling trajectory of U.S. interest rates in the post-2008 period, a trend that is missed by domestic variables.

1. Introduction

Interest rates have fallen steadily since the early 1980s in advanced economies including the United States. Past forecasts largely missed this global, secular decline in interest rates and tended to predict rate reversals towards the long-run historical average year after year. The goal of this paper is to examine the extent to which global factors improve statistical forecasts of the U.S. long-run interest rate. We find that out-of-sample forecasts of interest rates that make use of information on global growth outperform forecasts based on domestic variables alone. The international forecasts do a better job of tracking the falling trajectory of U.S. interest rates in the post-2008 period, a trend that is missed by domestic variables.

Section 2 begins by summarizing the key macroeconomic determinants of long-run real interest rates. An examination of recent trends suggests that declining productivity, shifts in demographics and an increase in the global demand for safe assets coincide with the long decline in U.S. interest rates. Since U.S .financial markets are globally integrated, one should expect two-way linkages between U.S. interest rates and economic activity abroad (e.g. Obstfeld, 2019).[1] This motivates our inclusion of both domestic and global macroeconomic variables in a forecasting model of the interest rate.

In Section 3, we review the main statistical models that rely on macroeconomic information to forecast interest rates. We focus on two methodologies for constructing long-range interest rate projections: semi-structural methods of interest rate trend decomposition and standard statistical forecasting models with an extended set of explanatory variables, including forward-looking economic indicators. These methodologies use different data and samples, and they provide complementary pieces of information.

In Section 4 we deploy the methodologies discussed above to forecast U.S. interest rates. Our objective is to forecast rates in a framework that is parsimonious, simple to estimate and relies on easily accessible data. We begin by performing a decomposition of the long-run nominal interest rate over the period 1981 to 2019 under the restriction of long-run inflation neutrality. This method helps identify the set of variables that may contain useful information for forecasting interest rates. Three variables, the earnings-price ratio of the stock market, the weighted average of past and forecasted consumption growth and year-on-year productivity growth explain 87% of variation in the 10-year real rate on U.S. Treasuries. The relative importance of the different macroeconomic determinants changes over time, with the earnings-price ratio mattering most in the 1981–1988 period and consumption growth most significant following recessions.

To assess the relative forecasting performance of domestic and international indicators, we estimate a macro-finance affine model of the term structure. An out-of-sample forecasting exercise establishes that a macro-finance model with both domestic and international indicators

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[1] Iacovello and Navarro (2019) document this linkage empirically by measuring the spillover effect from the US interest rate on the economic activity in advanced and developing countries.

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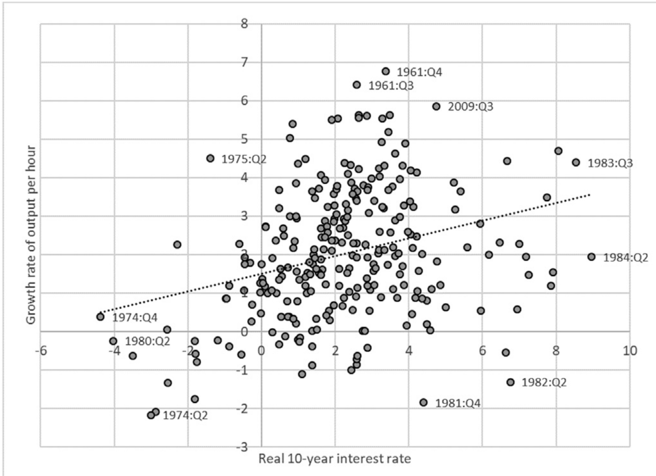

Fig. 1. 10-year U.S. real interest rate and the growth rate of U.S. labor productivity.
Data sources: see the Appendix

outperforms models based on domestic factors alone. In particular, we find that growth indicators for Europe and Asia are strongly significant drivers of the U.S. long-run interest rate.

2. Macroeconomic determinants of long-run interest rates

In equilibrium, the real interest rate is jointly determined by the supply of saving and the demand for investment. All else equal, conditions that induce households to set aside more income today and to postpone consumption for later will increase the supply of saving and shift interest rates down. On the other hand, favorable conditions for investment will put upward pressure on interest rates. Governments can affect both the supply of savings and the demand for investment through spending, taxation and regulatory policies. Finally, as markets become increasingly interconnected, global factors play a role in the determination of U.S. interest rates. In the long-run, monetary policy is to a first-order approximation neutral, so we can abstract from inflation and focus on the long-run "real" determinants of interest rates. Below, we briefly discuss each of the factors that we will include in our analysis in Section 4.

2.1. Labor productivity

A decrease in labor productivity reduces the marginal product of capital, reduces investment demand and lowers the interest rate. Labor productivity has been on a secular decline across the largest advanced economies since the 1980s, and this decline coincides with the general decline in long-run interest rates. Views on future long-run productivity range from pessimistic (Gordon, 2010) to highly optimistic (Mokyr (2014); Bloom et al. (2014)). Our work does not contribute to this debate except to note that historically, the simple link between labor productivity and long-run rates is fairly weak.

Fig. 1 plots the 10-year U.S. real interest rate and the growth rate of U.S. labor productivity (at the quarterly frequency) over the period 1948–2018, with outlier points and recessions labeled. The real interest rate is the nominal rate on a ten-year Treasury note less the year-on-year inflation rate. Labor productivity is the year-on-year growth rate of labor productivity in the U.S. non-farm business sector, again at a quarterly frequency.[2] As the figure indicates, there is a significant positive relationship between the two variables, though the coefficient is fairly small (the unconditional regression coefficient of the real rate on productivity growth is 0.23, with a p-value of $1.2 \cdot 10^{-6}$).[3] Our analysis in Section 4 will confirm that productivity is a significant, though somewhat weak, driver of interest rates.

While textbook macroeconomics predicts a positive correlation between the interest rate and productivity growth, a weak or even a negative correlation can be rationalized in a richer macroeconomic model where households' fertility decisions respond to the level of income (e.g. Barro and Becker, 1989). We turn next to the role of demographic factors in the determination of interest rates in the medium-to long-run.

2.2. Changing demographic factors

The interest rate response to changing demographics is complex because population growth and a changing age structure affect both the demand for investment and the supply of savings and do so at different horizons. Economic theory predicts that the effect of a permanent decrease in the population growth rate on the long-run interest rate depends on the extent of familial altruism in household preferences. In the canonical overlapping generations model, for instance, households save only for their own consumption in retirement and leave no bequests. This model predicts that a decline in population growth and a fall in productivity growth are both associated with a fall in the long-run interest rate. By contrast, in the Ramsey model where households take into account the well-being of their offspring, population growth changes have a small or even no effect on the long-run interest rate, depending on household preferences (e.g. Baker et al., 2005, p. 300).[4]

The term structure of interest rates will reflect both the long-run adjustment to changes in the age composition of the population as well as the transition to the new long-run equilibrium. Life expectancy in advanced economies is projected to rise by about 25 years between 1950 and 2050 while the population growth rate is expected to fall to virtually zero (see Carvalho et al., 2016, Fig. 2). During the transition to an older, longer-lived population, there is downward pressure on interest rates as workers save in anticipation of a longer retirement phase. In the long run, however, the larger share of the elderly in the total population will reduce total private saving and push the interest rate in the opposite direction. Because demographic changes are slow, it is likely that the low rates observed today could persist for some time. However, in the very long run the rising share of the elderly could begin to push interest rates up. How these changes are reflected in the term structure of interest rates depends on the relative strength of these different effects.

The arguments above focus on the impact of demographic changes on the rate of saving. As Geanakoplos et al. (2004) point out, there is a connection between changes in the age composition of the population and the returns to capital as reflected in the earnings-price ratio. This is a relationship that we will explore in section 4.1. Carvalho et al. (2016) argue that the demographic transition can affect the equilibrium real interest rate through three channels. An increase in longevity (or expectations thereof) puts downward pressure on the real interest rate, as agents build up their savings in anticipation of a longer retirement period. A reduction in population growth makes the labor force and output grow more slowly, and thereby reduces investment demand. This

[2] See the Appendix for data sources.

[3] Hansen and Seshadri (2014) consider a longer, 1900–2011 sample and find a *negative* correlation between the real interest rate and productivity growth. This negative correlation appears to be driven by real interest rate volatility early in their sample period that included two wars and the deflation episode associated with the Great Depression. In the 1953–2011 subsample of their data, the correlation between productivity and the interest rate is 0.23, consistent with what we report.

[4] The long-run interest rate depends on the population growth in the variant of the Ramsey model where the dynasty utility does not rise proportionately to its size. In the standard Ramsey model, by contrast, the agent cares about the offspring utility in the same way as their own future utility, and the long-run interest rate is independent of population growth.

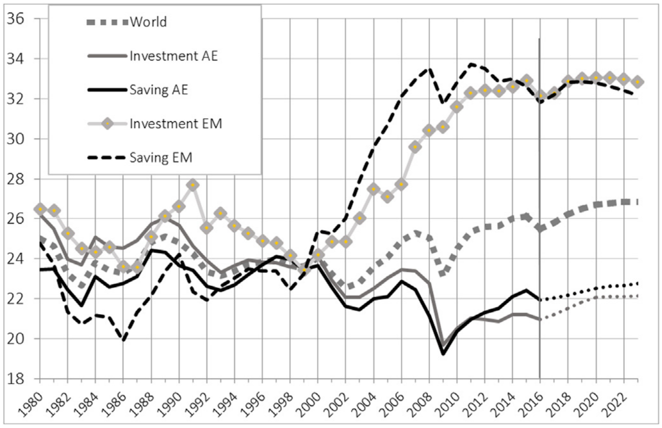

Fig. 2. Saving and investment by country group and for the world economy. Source: IMF World Economic Outlook, various issues.

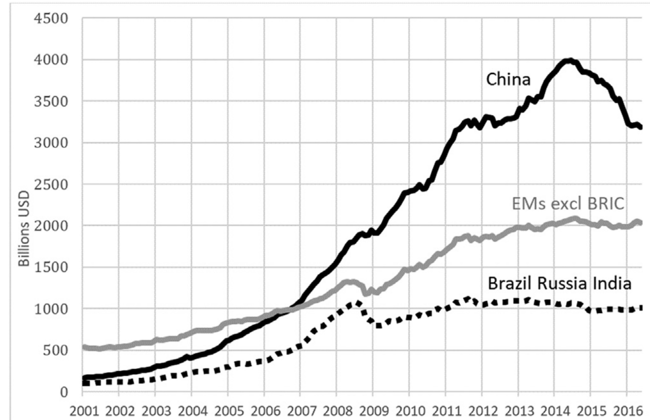

Fig. 3. Stock of foreign exchange reserves in emerging markets. Source: Domanski et al. (2016).

lowers the rate of return on equity in the business sector. When demand growth is slow, both the earnings-price ratio and the return on equity are lower. We will find below that the earnings-price ratio emerges as a significant driver of long-run interest rates, possibly due to the effects of demographic changes.

2.3. Global factors affecting U.S. Interest rates

Global factors are increasingly important for the determination of U.S. interest rates. In a closed economy, the real interest rate is determined by the equality between domestic investment and national saving. In an open economy, a country's savings will seek the highest rate of return in the global financial market, and firms wishing to invest will seek out the lowest cost of capital. Ultimately, if markets are fully integrated, the global interest rate will be determined by saving and investment for the world as a whole, with current account balances reflecting the gaps between saving and investment at the national level.

Fig. 2 illustrates saving and investment rates for advanced economies, for emerging markets and for the world as a whole. The figure reveals the "global imbalances" that emerged during the 2000s – the rise in emerging market saving relative to investment. Many have argued that the expanded pool of excess savings depressed global interest rates. Some have pointed out that the consequent search for yield fueled risk-taking behavior in advanced economies, thereby sowing the seeds of the global financial crisis.

The regulatory response to the financial crisis – chiefly the 2010 Dodd-Frank Act (DFA) – deepened the incentives for holding safe, liquid assets on the part of commercial banks, pensions and insurance companies in the United States. According to Greenwood and Vissing-Jorgensen (2018), the ratio of Treasury holdings to private loans for commercial banks increased five-fold after the DFA and bond prices *rose* even as the supply of debt expanded. They also document that pension and insurance companies increased their holdings, further driving down yields.

Demand for safe assets is not restricted to U.S. financial institutions. Indeed, by 2017 foreign investors held $6 trillion of long-maturity bonds, compared to the $500 billion held by U.S. commercial banks. Fig. 3 below shows the rise in foreign exchange reserves held by the central banks in emerging markets that account for a substantial fraction of foreign holdings of U.S. Treasuries. Domanski et al. (2016) observe that since the global financial crisis, two developments in particular may have increased financial stability concerns in in emerging markets and therefore a greater need for large, liquid reserves. One is the rapid growth of emerging market foreign-currency denominated debt. A second, related trend is growth in emerging market securities held by foreign institutional investors. Both factors increase the exposure of emerging markets to swings in capital flows and large changes in exchange rates.

It is unclear whether global demand for safe assets will be sustained going forward. As Fig. 3 shows, since 2013, major EMEs have on balance sold FX reserves – note especially the selloff by China starting in 2015. If this trend continues, it would exert an upward pressure on U.S. interest rates going forward.

Investors generally perceive government debt issued by the largest advanced economies as a relatively low risk despite the recent dramatic rises in debt-to-GDP ratios in the United States and much of Europe. Economic theory suggests that, unless Ricardian equivalence holds (i.e. conditions such that households anticipate the higher taxes needed to service the debt and respond by offsetting government dissaving with private saving), increases in government borrowing should result in a downward shift in total savings and an increase in the interest rate. The rise in the interest rate will crowd out private investment and reduce economic activity.

Despite this theoretical prediction, there is little empirical evidence of a secular trend in long-run rates due to the rise in public sector borrowing or of a crowding out effect on investment.[5] Indeed, in his 2019 Presidential Address to the American Economic Association (Blanchard, 2019), Olivier Blanchard argued that in this low interest rate environment that seems set to last, increased public debt may come at no fiscal cost and at only a limited cost to overall economic welfare. This is not to say that there is *no* connection between interest rates and government debt. Concerns about solvency did produce spikes in risk premia for some countries in Europe and there is ample evidence from emerging markets that low economic growth coupled with high public debt can trigger sudden capital flow reversals, high interest rates and deep economic recessions. Nevertheless, many have argued that financial sector demand for low-risk, highly liquid assets has played an important role in suppressing interest rates despite high levels of public debt.

An overview of recent trends suggests that changes in demographics, productivity growth, and demand for safe assets by private and institutional investors are important determinants of long-run interest rates. It is also evident that global factors have become more important over time. This suggests a possibility that global macroeconomic variables may contain useful information for long-run interest rate forecasts.

3. Interest rate forecasting methodologies

Forecasting interest rates has proven to be a difficult challenge. For

[5] Ehrlich and Thapar (2020) find a large effect of debt on the interest rate – a 5.6 basis points increase for every percentage point rise in the debt-to-GDP ratio – and they argue that previous estimates may have been biased downwards due to simultaneity issues in the determination of debt and interest rate.

decades, forecasters missed the decline in long-run rates. While forecasted rates shifted downward somewhat over time, there remained a strong tendency to predict that the nominal long-run rate would revert back to a range of four to six percent, reflecting an underlying real rate of 2–4 percent and a target inflation rate of around 2 percent (e.g. Obstfeld and Tesar (2015, Fig. 5)). This is despite the fact that the 10-year Treasury rate had not consistently remained in that range since the early 2000s.

3.1. Econometric models used in yield forecasting

Forecasting the interest rate is a question of long-standing interest among academics and practitioners alike. In this section, we provide a brief overview of the literature on interest rate forecasting. For the purposes of this discussion, it is useful to differentiate statistical forecasting models along two dimensions: (i) the set of explanatory variables for bond prices or yields (also referred to as state variables, risk factors or pricing factors) and (ii) the features of the term structure model, particularly, the inclusion of no-arbitrage conditions.

3.1.1. State variables

Researchers use several methods in both the selection and construction of state variables. Most commonly, information contained in the yield curve is summarized by a small set of linear combinations of yields, typically the first three principal components of the covariance matrix of yields at different maturities (see Section 4.2 below for details). This reduces the dimensionality of the state vector, an essential step in controlling the number of parameters to estimate.

While the principal component approach is the most common, the dynamic Nelson-Siegel model uses a somewhat different method for summarizing the yield curve. The model fits a functional form for the yield curve to the cross-sections of yields to estimate the latent state variables. One advantage of this method is that latent variables thus constructed have a clearer interpretation as level, slope and curvature factors, whereas the same interpretation for principal components is less precise.

Statistical models of the term structure fall into two categories with respect to the set of the state variables they use. Yields-only models use only the information contained in yield curves themselves. Macro-finance models add other observables, such as measures of real activity, inflation, and information from macroeconomic forecasts. We discuss the macro-finance models in more detail in Section 3.1.3.

Table 1 presents a taxonomy of statistical models commonly used in interest rate forecasting. Besides using different sets of state variables,

Table 1
Categories of statistical models for interest rate forecasting.

Model type	State variables		
	Yields or latent variables deriving from yields	Latent and macro-finance variables	Latent, demographic and macroeconomic variables
No-arbitrage affine model	Adrian et al. (2013)	Ang and Piazzesi (2003) Wright (2011)	
Reduced form affine model	Abbritti et al. (2018)	Ludvigson and Ng (2009) Joslin et al. (2014) This paper	Favero et al. (2016)
Reduced form dynamic Nelson-Siegel model	Diebold and Li (2006) Diebold et al. (2008)	Coroneo et al. (2014)	
Regime-switching VAR model	Ang dnd Bekaert (2002)	Guidolin and Timmerman (2006), Nyberg (2018)	

common models in the literature differ with respect to imposing no-arbitrage conditions on the time series of bond prices. Reduced form models require that bonds be consistently priced in a cross-section. No-arbitrage models require, in addition, that bonds be consistently priced over time. In the next subsection, we discuss no-arbitrage conditions in more detail.

Some VAR-based forecasting models feature state-dependent coefficients that switch depending on the phase of the business cycle (e.g. Ang and Bekaert, 2002) or the state of the stock market (e.g. Guidolin and Timmerman, 2006). Nyberg (2018) additionally incorporates a time-varying regime switching probability to capture the predictability of business cycle regime.

3.1.2. No-arbitrage conditions

Forecasting methods grounded in finance theory start with the premise that asset prices incorporate all information available to investors and that arbitrage opportunities are either absent or transitory. The information available to investors at date t is contained in the state vector X_t. The state vector follows a Markov process that captures the evolving set of information relevant for computing conditional expectations of future interest rates and bond prices. The model specifies an intertemporal no-arbitrage condition of the form

$$P_{n,t} = \mathbb{E}_t\left(M_{t+1}P_{n-1,t+1}\middle|X_t\right). \tag{1}$$

where $P_{n,t}$ is the price of a bond with maturity n at date t and M is the stochastic discount factor, also referred to as the pricing kernel. Estimating the model involves estimating the stochastic process for the state vector as well as the functional form for the stochastic discount factor, $M(X_t)$. A widely used class of empirical no-arbitrage models used to obtain joint forecasts of future yields, future returns, and risk premia derive from the class of affine yield-factor term structure models introduced in Duffie and Kan (1996) and categorized in Dai and Singleton (2000).

One advantage of no-arbitrage models is that they can interpret information contained in a panel of bond prices rather than working off repeated cross-sections. Another advantage is that a no-arbitrage model can separately quantify the effects of individual risk factors on bond prices by estimating a functional form for the stochastic discount factor. This is useful, in particular, for pricing derivative securities. The main disadvantage of no-arbitrage models is that their estimation is usually computationally intensive, and computational constraints may limit the size of the state vector for which estimation is practical.[6]

Reduced-form affine models posit a linear relationship between yields and pricing factors, but they do not impose intertemporal no-arbitrage conditions requiring that bonds be priced consistently at different dates. No-arbitrage conditions do not affect the dynamics of the state variables but they do affect the mapping from state variables to yields.

In general, it is not clear whether reduced-form and no-arbitrage models produce significantly different interest rate forecasts despite the different specification of the mappings from states to yields. Pericoli and Taboga (2012), for example, show that the fitted yields almost coincide between a no-arbitrage affine term structure model and its reduced-form counterpart. In contrast, Ang and Piazzesi (2003) show that the model with no-arbitrage conditions forecasts better than one without.

In some cases, it is possible to show theoretically that omitting no-arbitrage conditions involves no loss of information. For instance, Joslin et al. (2011) provide conditions when the no-arbitrage restrictions

[6] Adrian et al. (2013) estimate the affine term structure model without imposing cross-parameter bond pricing restrictions derived from no-arbitrage conditions. They instead incorporate a return pricing error into equation (3). The resulting system of equations can be estimated with a multi-step linear procedure. Their procedure does not rely on a constructed yield curve, as it can use coupon bond prices directly as a data input.

have no effect on the maximum likelihood parameter estimates within a class of yields-only affine models.

The approach we take in this paper is to use a reduced-form affine model. This is partly because our focus is interest rate forecasting rather than asset pricing. The advantage is that the model can be consistently estimated with simple ordinary least squares (OLS), and we thus avoid computational complexities stemming from a non-linear estimation procedure.

3.1.3. Macro-finance models

The role of macroeconomic factors in interest rate forecasting in addition to yield-only factors is a subject of ongoing investigation and debate. To frame our discussion, it is convenient to use a decomposition of bond yields into a sequence of expected short rates and expected excess returns. One can show that (see e.g. Duffee, 2013) the current yield on an n-period bond, $y_{n,t}$, equals the sum of expected future short rates and the risk (or term) premium that depends on expected future excess returns:

$$y_{n,t} = \frac{1}{n}\sum_{\tau=1}^{n-1}\left[\mathbb{E}_t\left(y_{1,t+\tau}\middle|X_t\right) + \mathbb{E}_t\left(rx^{n-\tau}_{t+\tau-1,t+\tau}\middle|X_t\right)\right], \tag{2}$$

where the excess return is defined as

$$rx^n_{t,t+1} = ln\left(\frac{P_{n-1,t+1}}{P_{n,t}}\right) - ln\left(\frac{1}{P_{1,t}}\right) \tag{3}$$

The yield decomposition in equation (2) is helpful for understanding the role of information contained in the state variables. In particular, equation (2) illustrates an interesting possibility that there can be so-called “hidden factors” – state variables that have opposite and offsetting effects on expected short rates and expected excess returns (see e.g. Duffee (2011) and Joslin et al. (2014)). Such factors can have a small effect on the cross section of yields but potentially large effects on the dynamics of yields themselves. For this reason, if hidden factors are present, the information contained in the current cross-section of yields is not sufficient for forecasting future yields. As it turns out, macro-finance models can account for some hidden factors. According to Ludvigson and Ng (2009) and Joslin et al. (2014), macroeconomic variables are not “spanned” by the cross-section of yields, perhaps because they influence investor's expectations of future yields and risk premia and yet are unaffected by current yields.

A large body of research focuses on incorporating macroeconomic variables in econometric frameworks for yield forecasting. It is not clear at the outset which variables are good candidates as forecasting indicators. Accordingly, one approach in the recent literature has been to start with a large number of macroeconomic time series and employ “shrinkage” methods to summarize them with a few linear combinations that deliver the smallest forecasting error within a particular sample (e.g. Li and Chen, 2014). The advantage of this approach is that it strikes a balance between extracting information from a large number of predictors and keeping in check the number of free parameters in the econometric model.

Our analysis in Section 4.3 takes a different approach. We use a standard, parsimonious econometric model and add a small set of macroeconomic indicators informed by economic theory. The main contribution to the macro-finance body of research is to expand the set of macroeconomic variables to include not just domestic economic indicators and forecasts, as would be appropriate in a closed economy, but also measures of global real activity.

In addition to potentially improving forecasting, macro-finance forecasting models can be used to quantify the relationship between yields and macroeconomic fundamentals. Economic theory predicts that macroeconomic variables may help forecast the level component of yields in particular. Prior research on yields-only models has cast doubt on whether the current term structure contains information about changes in the future level of yields (e.g. Duffee, 2011, Table 1). It remains an open question whether macro-finance models improve forecasts of the level component of the yield curve. Ang and Piazzessi (2003), for instance, obtain virtually the same dynamics of the level factor whether or not macroeconomic variables are included. This suggests that macroeconomic variables add little to the model's ability to predict the changes in the level of yields. Our results in Section 4.4 are more nuanced – we show that forecasting performance with respect to the level of yields depends on the set of the state variables, and that global forward-looking indicators do better in this respect than domestic state variables.

Overall, the literature on macro-finance forecasting models seems to suggest that domestic real activity indicators are useful in understanding risk premia on bonds, but there is less evidence on whether forecasting models can predict changes in interest rate levels. Below, we draw insights from analyses that specifically focus on interest rate trends to help identify other potential forecasting indicators.

3.1.4. Interest rate trend decompositions

A semi-structural method proposed in Del Negro et al. (2018) jointly estimates trends in real rates for seven advanced economies using long historical time series on short- and long-term interest rates, inflation, and consumption starting in 1870. The method decomposes real rates and term premia into a common component, a country-specific component and a convenience yield which is a rate cut that investors are willing to take in exchange for holding a safe and liquid asset. Convenience yields are identified with the assumption that all assets are priced with the same stochastic discount factor that is tied to data on consumption growth.

The analysis points to three major drivers of falling interest rates since the 1980s: increasing convenience yield, a slowdown in global growth and an increase in desired saving. The rising convenience yield accounts for over half of the world real interest rate decline (over 90 basis over the past 25 years), and it makes a larger contribution to declining rate since 1997. The slowdown in global growth accounts for about one-third of the decline, or 60 basis points. The rest of the decline, about 40 basis points, is attributed to a rising desire to save.

The rise in convenience yield along with the increased desire to save as drivers of the lower world interest rate appears to be consistent with “safe asset shortage” as an explanation for low rates (e.g. Caballero et al., 2017). According to this view, the rise in the emerging economies' wealth may have changed the composition of international investors in terms of their risk attitudes and the overall desire to save, and this change brought about rising convenience yields and falling rates.[7] If changes in risk attitudes are an important driver of interest rates, global forward-looking indicators – such as measures of consumer and business confidence – may contain valuable information for statistical forecasting.

In the next section, we experiment with both domestic and global forward-looking indicators and compare, in particular, the model's ability to forecast the level component of yields across different sets of state variables. We show that a model with global forward-looking economic indicators does better in forecasting the interest rate level out-of-sample than either a yields-only model or a model with just the U.S. indicators.

4. Economic indicators potentially useful in interest rate forecasting

Our approach is to use a model that is standard, minimal and simple to estimate and we focus on the core set of macroeconomic data that is useful in forecasting. We start with an analysis of the 10-year U.S. Treasury rate to understand interest rate dynamics since 1981, the point at which nominal rates in the United States and in foreign markets began their decline.

[7] Caballero et al. (2008) and Hall (2016) propose stylized theoretical frameworks tailored to illustrating the impact of risks faced by emerging economies on the interest rate trends.

4.1. 10-year Treasury rate decomposition

We rely on economic theory to guide our choice of variables to include in the analysis, although we do not take a firm stand on the precise specification of a model. We use quarterly data from 1981:Q3 to 2019:Q1. The dependent variable to be explained is the 10-year nominal Treasury rate $y_{10,t}$.

The discussion in Section 2 largely focused on the determinants of the long-run real interest rate. The variable we observe, however, is the long-run *nominal* interest rate. While most economists would agree that the role of inflation is likely to be small if not zero in the very long run, inflation will affect yields at shorter maturities as we will show below. We therefore include inflation as a control. Because expectations about future inflation are important for the current interest rate, we include a time-smoothed measure of inflation that includes both expected inflation as well as past inflation. Our measure is:

$$\pi_t = \omega\pi_{t-4} + (1-\omega)\pi^e_{t+4}$$

where π_{t-4} is CPI inflation rate over the past four quarters and π^e_{t+4} is expected inflation over the next 4 quarters.[8]

The second explanatory variable included in our analysis is the growth rate of consumption. Macroeconomic theory interprets the "stochastic discount factor" M in section 3.1.3 as the expected intertemporal marginal rate of substitution in consumption. We therefore include the growth rate of real private consumption expenditure (PCE), which is again smoothed over time to include both past and future expected consumption growth:

$$g_{C,t} = \frac{1}{2}g_{C,t-4} + \frac{1}{2}g^e_{C,t+4}$$

where $g_{C,t-4}$ is the growth rate of the real private consumption expenditure over the past four quarters and, and $g^e_{C,t+4}$ is the forecast of the same, four quarters forward.[9] Note that because we are using total real PCE and not per capita PCE, changes in population growth will also be picked up by this variable.

We also include a measure of labor productivity growth, $g_{y,t-4}$, measured as the growth rate of business sector real output per hour over the previous four quarters. Although we saw in Fig. 1 that there is only a weak unconditional relationship between productivity growth and the long-run rate, we include it in the analysis because it is possible the productivity growth will play a stronger role after conditioning on other variables.

Finally, we include in the regression the inverse of Robert Shiller's CAPE ratio – that is, the ratio of average real earnings 10 years back to the current inflation-adjusted S&P index – as our financial market variable, denoted EP_t. In the textbook macroeconomic model, the long-run value of this ratio is equal to the real rate of return of equity in the business sector. If equity returns are high, this could be a signal that the returns to investment are high and interest rates will also be high.

Another reason for inclusion of the CAPE ratio is that, as we noted above, cyclical variations in EP_t seem to capture the effects of population dynamics on asset returns (Geanakoplos et al. (2004)). In particular, Geanakoplos et al. (2004) suggest that the relevant demographic variable is the ratio of young workers (ages 20–29) to workers in the middle of the age distribution (ages 40–49), denoted the YM ratio.

Fig. 4 depicts the relation between Shiller's EP, the real 10-year rate and the YM ratio since 1953. The YM ratio, EP and real rate all track one another well since the 1960s. However, the YM ratio bottomed out in 2002 and has been rising since. If anything, the demographic argument suggests that the changing age composition of the population should be putting an upward pressure on the real rate and on EP. Del Negro et al. (2018) point out the divergence between YM and interest rates for other countries as well.

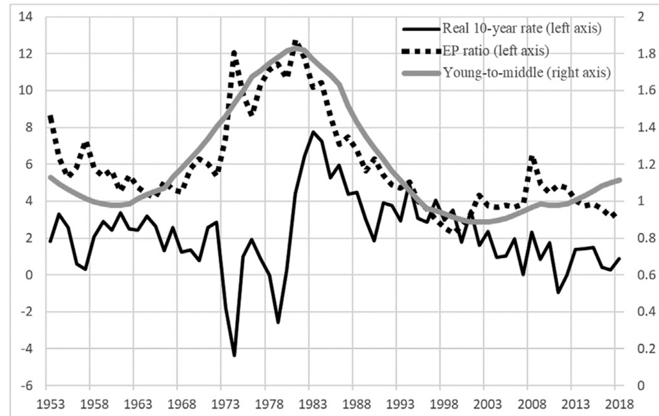

Fig. 4. Earnings-to-price ratio, 10-year real rate and young-to-middle (20–29 y.o. over 40–49 y.o.) ratio.
Data sources: see the Appendix.

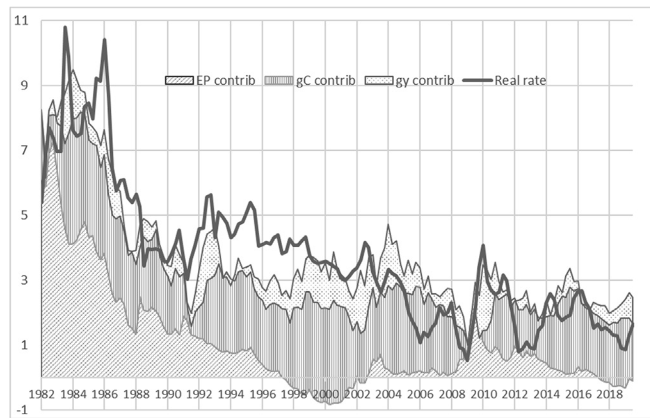

Fig. 5. Real interest rate decomposition. Predicted values based on regression coefficients in Table 2. Real rate is $y_{10,t} - \beta_\pi\pi_t$, EP contribution is $\beta_{EP}EP_t + \beta_0$, gC contribution is $\beta_C g_{C,t}$, gy contribution is $\beta_y g_{y,t-4}$.

We perform the yield decomposition with an estimating equation

$$y_{10,t} = \beta_0 + \beta_\pi\pi_t + \beta_C g_{C,t} + \beta_y g_{y,t-4} + \beta_{EP}EP_t.$$

We take the stand that in the long run inflation is neutral, so that a one percentage point increase in inflation that is expected to be permanent should lead to a one percentage point rise in the long-run nominal rate. Accordingly, we choose the weight ω so that the coefficient $\beta_\pi = 1$. Under this restriction, Table 2 reports the estimates for the coefficients on consumption growth, productivity growth, and the EP ratio (the units of interest rates and growth rates are annual percent).

The regression coefficients in Table 2 can be interpreted as the elasticities of the nominal 10-year rate with respect to the rate of inflation,

Table 2
10-year rate decomposition.

10-year rate	$\beta_\pi(\omega = 0.6)$	β_C	β_y	β_{EP}	β_0
Coeff.	1.00	0.74	0.34	0.64	−2.28
Std. error	0.12	0.09	0.07	0.06	0.35
$R^2 = 0.87$	$N = 151$				

[8] See the Appendix for data sources.

[9] Casey (2020) finds that professional forecasts are consistent with standard relationships from economic theory. This suggests that professional forecast data may be a good proxy for expectations of agents in standard macroeconomic models.

growth rate of private consumption expenditure, productivity growth and the earnings-to-price ratio. Each of the variables enters with the expected sign, with increases in productivity growth, consumption growth and the EP ratio all contributing to an increase in the interest rate. The largest elasticities as estimated over the full sample are with respect to private consumption growth and the EP ratio.

Fig. 5 illustrates the changing contribution of each factor over time. The solid line is the predicted real rate, $y_{10,t} - \beta_\pi \pi_t$. We net out the impact of inflation, so that we can focus on the drivers of the long-run real interest rate. In the 1980s, the real interest rate and the contribution of the EP ratio ($\beta_{EP} EP_t + \beta_0$) were both high. Recall from Fig. 4 that the young-to-middle ratio was high in that period, consistent with a high ratio of dis-savers in the economy and a therefore a high interest rate. This demographic factor falls off by 1988, and coincides with the drop in the real rate over the 1980s. The contribution of productivity growth to the real interest rate, $\beta_y g_{y,t-4}$, is small throughout the sample. The figure illustrates the collapse and then recovery of consumption, $\beta_C g_{C,t}$, in each of the recessions, accompanied by a fall and then an increase in the interest rate. By the end of the sample, the real interest is lower than what is predicted given the rate of consumption and productivity growth. To summarize, we find that the EP ratio (that captures demographic change), productivity growth and consumption growth emerge as significant determinants of the long-run real interest rate over the 1981:Q3 to 2019:Q1 period. The relative importance of the macroeconomic determinants changes over time, with the demographic factor mattering most in the 1981–1988 period and consumption growth most significant following recessions.

4.2. Principal components and long-run interest rates

The analysis above focused on the long-run rate and its determinants. We now turn to an analysis of the yield curve, which conveys information both about the interest rate on long-term bonds as well as the compensation investors demand for holding a ten-year bond relative to holding bonds with shorter maturities (i.e. the term structure). By making use of the full yield curve it is possible to extract a set of factors that capture interest rate dynamics in both the short and the long run.

Litterman and Scheinkman (1991) proposed a technique for summarizing the yield curve with common factors that account for co-movement of yields at different maturities. A common approach is to construct these factors using principal component analysis. The principal components are linear combinations of yields at different maturities that account for the maximum portion of the variance-covariance matrix of yields. Constructing the principal components amounts to finding a rotation that diagonalizes the variance-covariance matrix of a panel of yield curves.

The principal components are a statistical method of describing patterns in yield curves. The components have no economic content, in and of themselves. An interesting question, and one frequently asked in analyses of this type, is whether the principal components reflect a relationship between changes in macroeconomic variables that could be used to better understand the dynamics of yields in sample, as well as for forecasting out of sample. To this end, we examine whether the macroeconomic variables we found to be statistically significant for explaining the long-run interest rate in the previous section are associated with the principal components.

As a first step in this analysis, we extract principal components on U.S. yield curves from quarterly data over the 1981:Q3-2019:Q1 sample period. Our results are comparable to what is typically found in the literature. The first principal component accounts for almost all of the variation in yields (98.1 percent), while the second and third principal components account for 1.5 percent and 0.4 percent of the variation, respectively. The variation accounted for by additional components are at least an order of magnitude smaller so we drop them from our discussion.

Fig. 6 depicts the time series for the three principal components. The first component has a clear downward time trend. Over this period nominal interest rates were falling as was the rate of inflation. The fact that so much of the co-movement in yield curves is captured by a secular downward trend is an indication that the low frequency drivers of the interest rate discussed above have a reasonable chance of explaining yield curves since the early 1980s.

The second component is harder to interpret, with sharp swings and an irregular cyclical pattern between the 1980s and later in the sample. It has been suggested that the second component is related to the business cycle (e.g. Abbritti et al., 2018). The troughs in the second component occur in 1983:Q1, 1985:Q3, 1993:Q2, 2003:Q3 and 2010:Q4, roughly two years following a recession. It appears that there may be a connection to business cycle downturns, though the connection is tenuous.

The third component exhibits fluctuations at a higher frequency. Note that although the second and third components explain less of the variation in yield curves in the estimation sample, this does not preclude the possibility that these factors are important for forecasting yields out of sample. For example, information that the economy may be shifting out of a boom into a recession may have only a slight impact on the trend, but could well be picked up by the second or third components and could therefore help forecast short-term yields.

We next estimate the “factor loadings” of each component by running a regression of yields on the three principal components:

$$y_{n,t} = a_n + b_{1,n} X_{1,t} + b_{2,n} X_{2,t} + b_{3,n} X_{3,t} + \varepsilon_{n,t} \tag{4}$$

This is done for yields of different maturities.

The loadings are plotted in Fig. 7 for maturities ranging from 1 month to 10 years. The first loading, $b_{1,n}$, (for each maturity n) is shown by the solid grey line. It is quite flat, indicating that most of the variation in yields can be explained by a factor that shifts the entire yield curve and is therefore often referred to as a “level factor.” The second factor (the dashed line) has a loading that is high at short maturities and low at long maturities. This factor is referred to as a “slope factor,” and a rise in this factor will make the yield curve flatter as it will raise short-term yields more than long-term yields. The third factor (in solid black), the “curvature factor,” is higher at short- and long-maturities and lowest at middle maturities of 3–6 years.

We examine the correlations between principal components and the set of macroeconomic variables with simple regressions. The results are shown in Table 3. The regression coefficients have the same units as the standard deviation of the first principal component. For example, according to the estimates in the table, a 1 percent permanent rise in private consumption expenditure will shift the first principal component of yields up by about one-fifth of its standard deviation. Note that the smoothing weight for inflation is kept the same as in the previous specification. This means that we are not imposing that inflation be neutral and we allow the principal components to reveal the impact of inflation on the nominal interest rate at different maturities.

The first panel of Table 3 shows the coefficients of a regression of the first principal component on macroeconomic variables. The R^2 on this regression is high at 0.87, an indication that the macroeconomic variables are successful in capturing the in-sample variation in yields. An increase in each of these variables results in a significant shift in the yield curve, with inflation having the largest elasticity, followed by consumption and the *EP* ratio.

Macroeconomic variables explain much less of the variance in the second and third principal components of yields (the second and third panels of the table). Inflation enters with a positive coefficient, indicating that a rise in inflation will have a stronger, positive impact on the yield curve at shorter maturities. The *EP* ratio has the opposite sign, perhaps an indication that the *EP* ratio makes the yield curve steeper. The third principal component is explained most strongly by consumption growth.

To summarize, principal components analysis of U.S. yield curves over the 1981:Q3-2019:Q1 period generates the standard result that the

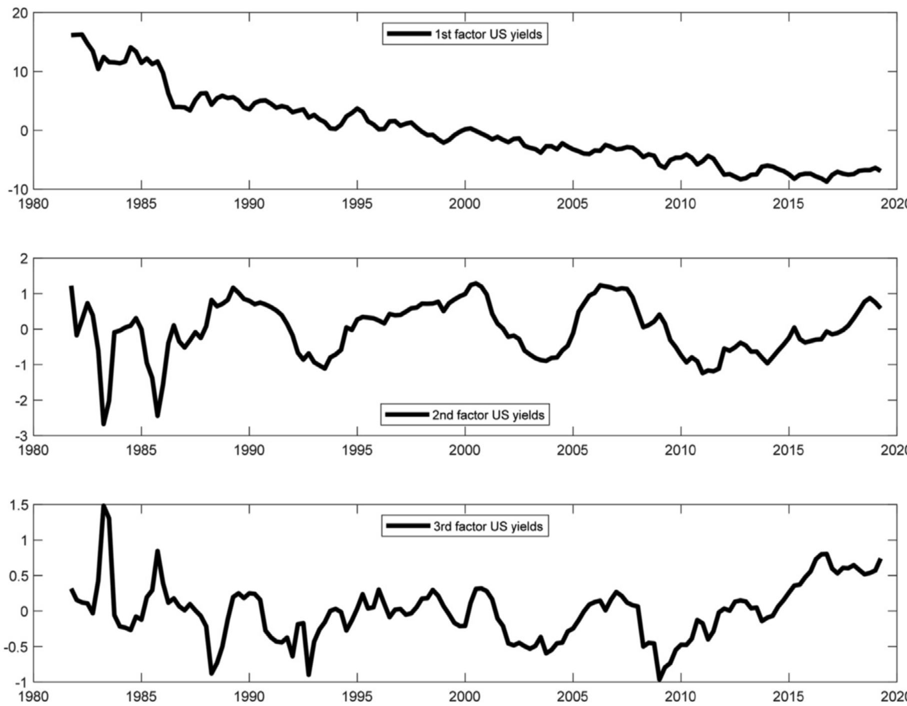

Fig. 6. Three principal components of US yields, 1981:Q3-2019:Q1.
Data source: see the Appendix.

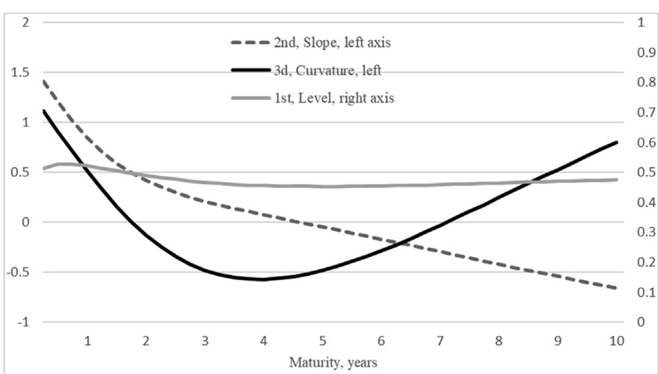

Fig. 7. Loadings b_n on the first (level), second (slope) and third (curvature) principal components as functions of maturity, n. See equation (4).

Table 3
Decomposition of the first three principal components of yields. The table reports the coefficients from the regressions $X_{i,t} = \beta_{i0} + \beta_{i\pi}\pi_t + \beta_{iC}g_{C,t} + \beta_{iy}g_{y,t-4} + \beta_{EP}EP_t + \varepsilon_{it}$, $i = \{1, 2, 3\}$.

1st principal component of yields	β_π ($\omega =$ 0.6)	β_C	β_y	β_{EP}	β_0
Coeff.	0.45	0.20	0.13	0.16	−3.02
Std. error	0.04	0.03	0.02	0.02	0.11
$R^2 = 0.87$	$N = 151$				
2nd principal component of yields	β_π ($\omega =$ 0.6)	β_C	β_y	β_{EP}	β_0
Coeff.	0.59	−0.06	−0.01	−0.30	0.18
Std. error	0.09	0.07	0.05	0.04	0.27
$R^2 = 0.26$	$N = 151$				
3^d principal component of yields	β_π ($\omega =$ 0.6)	β_C	β_y	β_{EP}	β_0
Coeff.	−0.05	0.39	−0.20	0.01	−0.60
Std. error	0.10	0.08	0.05	0.05	0.28
$R^2 = 0.19$	$N = 151$				

first component accounts for almost all of the variation in yields. The first component has a clear downward time trend, consistent with falling nominal rates and a declining rate of inflation over this period. We find strong evidence of a relationship between the first principal component and the macroeconomic drivers discussed in Section 2. That is, level shifts in the yield curve are largely driven by changes in inflation, consumption growth and the EP ratio.

Table 3 suggests that the yields-only model may capture some of the co-movement between the interest rates and the U.S. macroeconomic variables.

4.3. Results from the dynamic model

An advantage of using the full yield curve is that it has the potential to reveal information not only about the level of interest rates but also about the dynamic adjustment of interest rates to shocks. We implement a dynamic reduced form affine term structure model with a set of three latent variables stacked into a (3×1) vector X_t and three economic indicators in a separate (3×1) vector F_t. The model is similar to that in Abbritti et al. (2018) but the state vector is different.[10] The latent variables are the three principal components of the yield curves from 3 months to 10 years maturity. The model's equations are

$$y_{n,t} = a_n + b_n X_t \tag{5}$$

$$X_t = \Phi X_{t-1} + \Lambda F_t + v_t \tag{6}$$

$$F_t = \Gamma F_{t-1} + \eta_t$$

We assume that the shocks to economic indicators, η_t, and the shocks to the principal components, v_t are uncorrelated. We estimate $(\Lambda, \Phi, \Gamma, \Sigma_v, \Sigma_\eta)$ with OLS and inverse bootstrap bias correction. The term structure

[10] We thank Mirko Abbritti for sharing the MATLAB code for estimating the model. We report results with inverse bootstrap bias correction (Bauer et al., 2012).

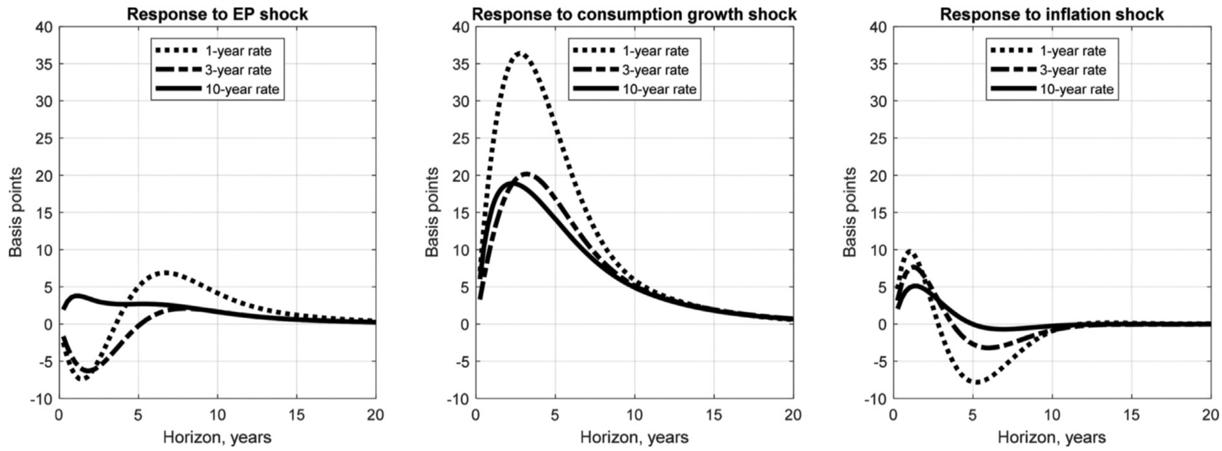

Fig. 8. Impulse responses of yields to F_t^1 indicators.

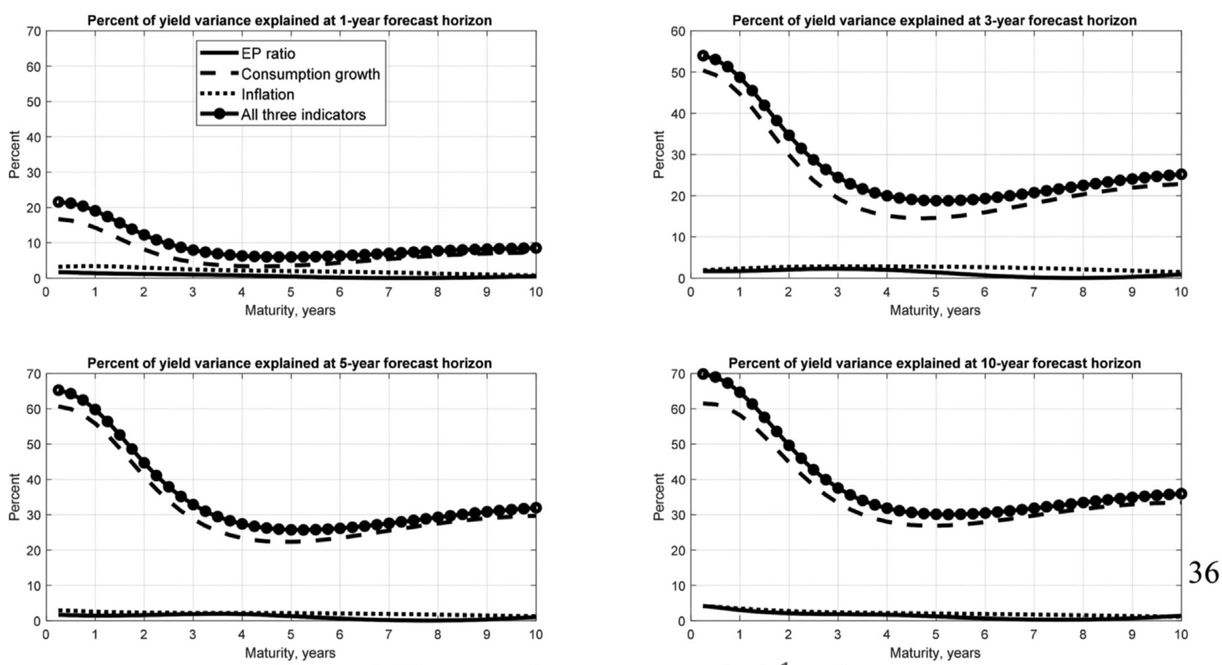

Fig. 9. Variance decomposition for F_t^1 indicators.

model allows us to ask, what is the role of various macroeconomic indicators in explaining the dynamics of yield curves, after accounting for the role of the principal components? In particular, the VAR shows the change in yields at different maturities and the rate at which that impact dies out over time.

4.3.1. U.S. economic indicators

We estimate the dynamic model with U.S. economic indicators using the 1981:Q3-2019:Q1 sample, the longest time period for which both inflation and consumption growth forecasts are available. The addition of shocks raises the dimensionality at an exponential rate, so we limit the set of economic indicators to three. In the first set, we include the *EP* ratio, consumption growth and inflation where consumption growth and inflation are again smoothed. In the second set of indicators, we replace the *EP* ratio with productivity growth, that is

$$F_t^1 = \begin{bmatrix} EP_t \\ g_{C,t} \\ \pi_t \end{bmatrix}, \; and \; F_t^2 = \begin{bmatrix} g_{y,t-4} \\ g_{C,t} \\ \pi_t \end{bmatrix}.$$

Fig. 8 depicts the impulse response of yields at different maturities to a one standard deviation shock to each of the three indicators. The panels show the estimated response of yields to innovations in the particular indicator at different maturities (1-year, 3-year and 10-year) and at different horizons (0–20 years). For example, the solid line in the left-most panel of Fig. 8 shows that a rise in the EP ratio has little effect on the 10-year rate on impact. In addition, the VAR does not pick up dynamic effects of the EP ratio on the 10-year rate over time, as the rate is not affected at out-horizons of five to 20 years. There does appear to be a weak negative relationship between the EP ratio and short-term yields of one to five years, which turn positive at a five-year horizon.

Consumption growth, the middle panel of Fig. 8, has a strong positive effect on yields, with the largest impact on the very short end of the yield curve. The impact on all yields is strongest at the three-year horizon and then quickly dies out. The dynamic response of yields to innovations in inflation is weak with most of the action in the 1-year rate.

Fig. 9 plots the variance decomposition that shows the percent of variance in yields explained by each indicator in F_t^1 at different forecasting horizons. The line with circle markers shows the percent of variance explained by all three indicators together. Looking across the figures we see that consumption growth (the dashed line) explains up to

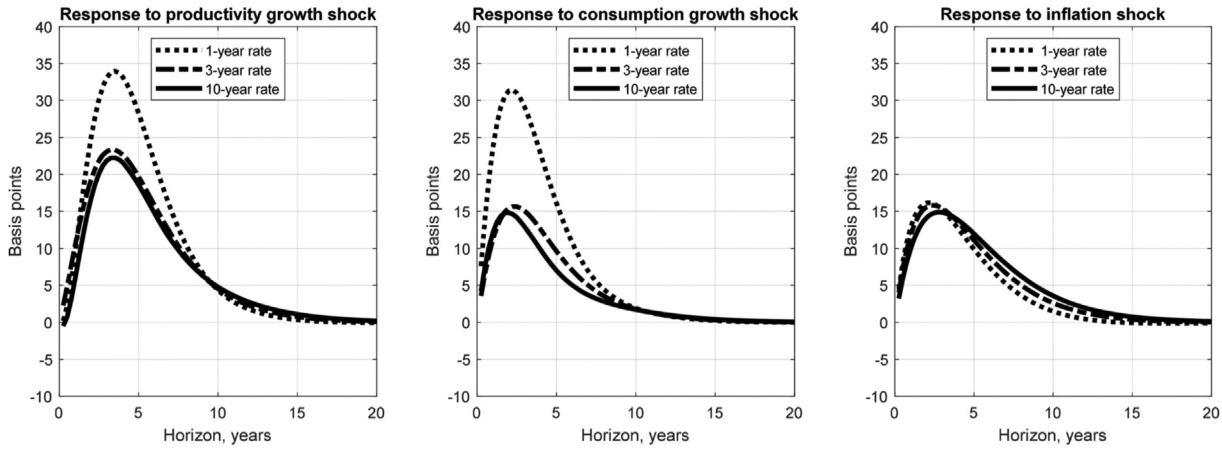

Fig. 10. Impulse responses of yields to F_t^2 indicators.

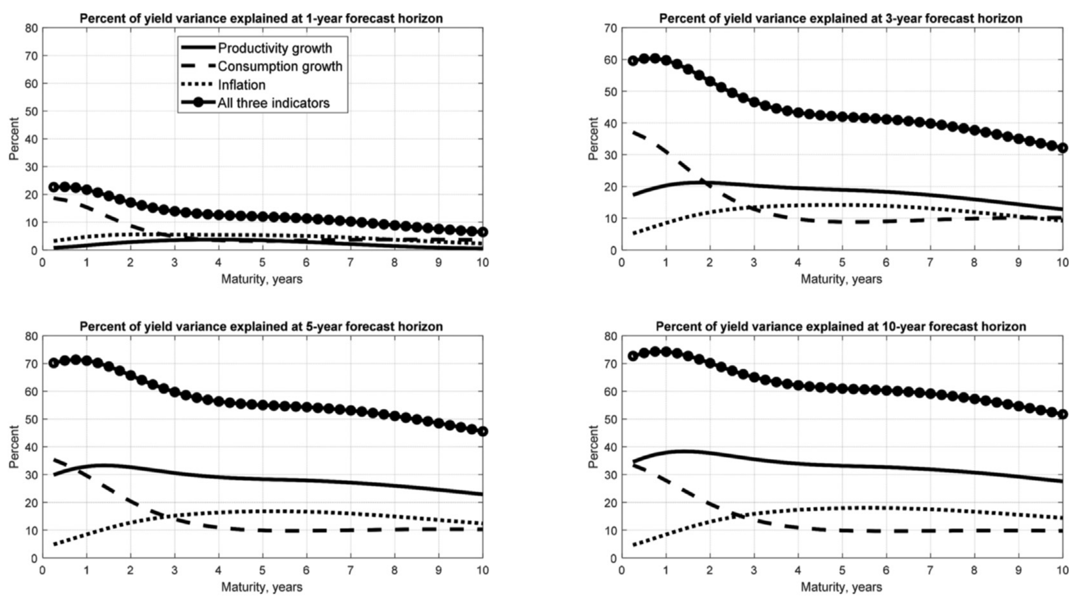

Fig. 11. Variance decomposition for F_t^2 indicators.

60 percent of the variance of short-term yields, even at a 10-year forecast horizon. The EP_t and π_t indicators explain almost none of the variance, an indication that neither of these variables adds information for forecasting beyond what is already contained in the cross-section of yields and summarized by the three principal components in X_t.

We next consider a triple of indicators F_t^2 consisting of US productivity growth, consumption growth and inflation. Productivity growth (now in place of the *EP* ratio) has a significant positive effect on yields at

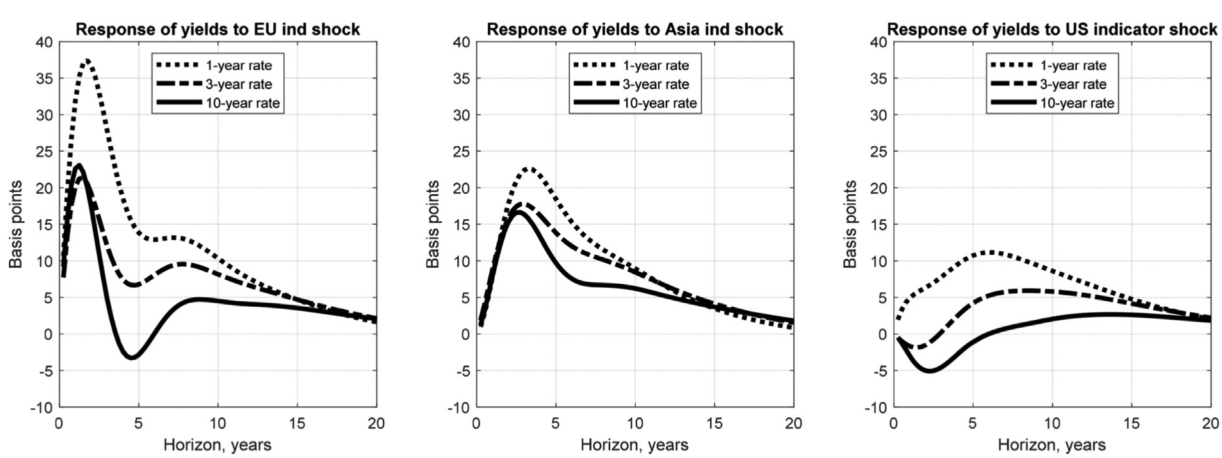

Fig. 12. Impulse responses of yields to F_t^3 indicators.

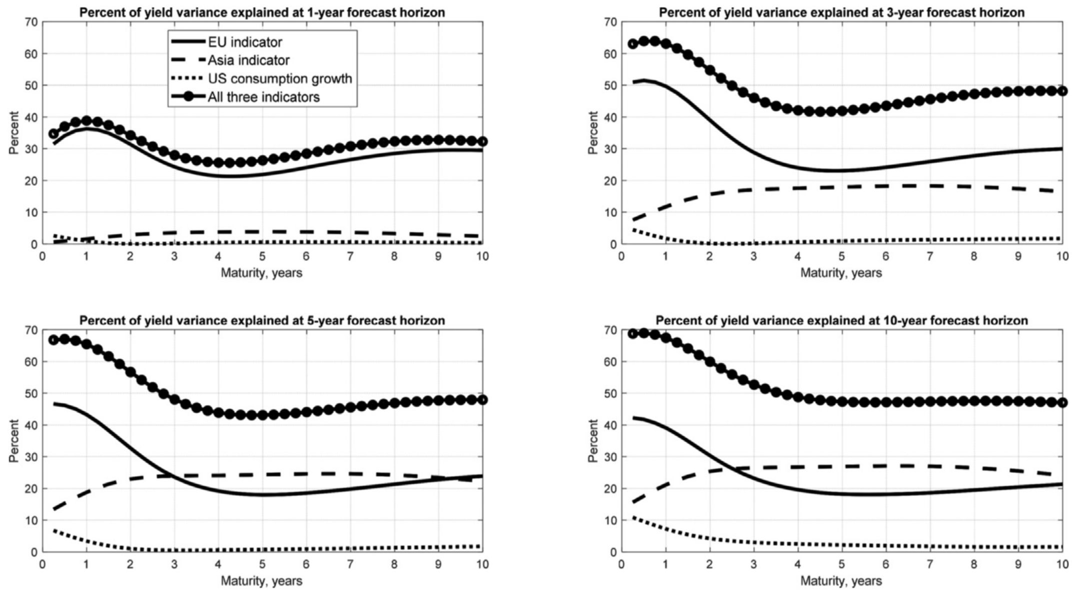

Fig. 13. Variance decomposition for F_t^3 indicators.

all maturities at the one- to 10-year forecasting horizon. The impulse responses to consumption growth shown in Fig. 10 are little changed relative to the previous VAR specification, although the magnitude of the effects are slightly smaller. The inflation indicator now comes in positive, moving all three yields and peaking at the two – to three-year horizon.

Fig. 11 (analogous to Fig. 9) shows the variance decomposition for the indicators in F_t^2. Relative to the previous VAR specification, we see that all three variables contain some information beyond the principal components. Together, the three indicators explain between 60 and 75 percent of yield variance at the short end of the yield curve, and between 30 and 50 percent of variance at the long end. Productivity growth now explains as much as 40 percent of the variance of yields, particularly at the longer forecast horizons. Consumption growth continues to pick up variance at the short end of the yield curve. The inflation indicator π_t now explains up to 20 percent of the variance.

To summarize, there is little information in the *EP* ratio beyond what is already captured by the principal components. In contrast, consumption growth and productivity growth play distinct roles in explaining the variance of yields, with consumption growth being most important for short-term rates, and productivity growth for both short- and long-term rates.

4.3.2. Global economic indicators

As discussed in Section 2, global factors play an increasingly important role in U.S. financial markets, at least in theory. Recent literature on global interest rate spillovers (e.g. Obstfeld (2019) and Iacovello and Navarro (2019)) and our review of forecasting methodologies in Section 3.1.4. point at global forward-looking variables as potential indicators. We now change the vector F_t to include a set of leading indicators for global economic activity to test this view in practice. In the previous section we found that consumption growth was an important factor for explaining U.S. interest rates. We add to the vector OECD-constructed composites of leading economic indicators[11] for the 19 Euro-area economies (g_{EU}^e) and the major five Asian economies (g_{A5}^e) (China, India, Indonesia, Japan and Korea).

Our sample period is 1990:Q2-2019:Q1, starting with the earliest quarter when the Asian growth indicator is available. The constituent series in the composite indicators are selected based on broad coverage of current economic activity and validity as leading indicators of future real activity. The series include, for instance, activity at the early stages of production, factory orders, construction permits, measures of business confidence and the like. The vector F_t is defined as follows:

$$F_t^3 = \begin{bmatrix} g_{EU,t}^e \\ g_{A5,t}^e \\ g_{C,t} \end{bmatrix}.$$

Fig. 12 shows the impulse responses of 1-, 3- and 10-year nominal yields to a one standard deviation structural shock to each of the three indicators.

The impulse responses indicate that higher expected growth in the EU and Asia raise U.S. nominal interest rates and that the effects are quite persistent. The EU indicator is especially important for shifting short rates. Higher expected consumption in the U.S. raises short rates more than long rates, but the impulse response is small overall. The indicators are jointly significant with a $P-$ value of $1.4 \cdot 10^{-5}$.

Fig. 13 shows the variance decomposition. The EU indicator is dominant, especially at the short end of the yield curve, where it explains about 40 percent of the variance; the indicator for Asian economic activity explains about 20 percent of the variance at maturities of four to ten years. Overall, the set F_t^3 with international variables explains a bit less variance in yields than the set F_t^2. The presence of $g_{EU,t}^e$ seems to reduce the ability of $g_{C,t}$ to explain yield variance. This is, perhaps, because the correlation between the two is 0.41.

To summarize, three indicators, $g_{EU,t}^e$ $g_{A5,t}^e$ and $g_{C,t}$ – all related to expected growth in real activity – emerge as reasonable candidates for explaining the variance of interest rates. The triple of economic indicators F_t^2 accounts for between 60 and 75 percent of yield variance at the short end of the yield curve, and between 30 and 50 percent of variance at the long end. These results are broadly consistent with Ang and Piazzesi (2003) who find that macroeconomic variables can account for a large portion of variance in yields, especially at the short end of the yield curve.

4.3.3. Comparisons of forecasting performance

Previous research demonstrated that changes in the level component of yields are not forecastable in a yields-only model (e.g. Duffee, 2013). As our discussion of the literature in Section 3 and results in Table 3

[11] See the Appendix for details.

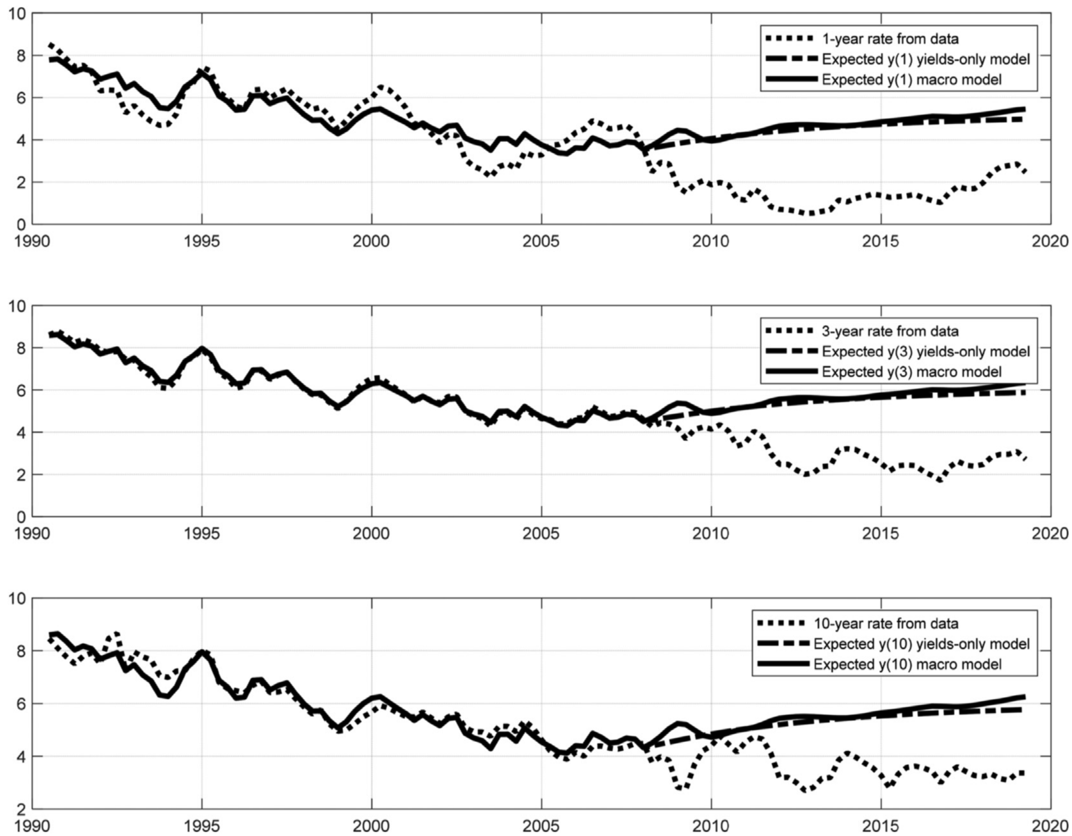

Fig. 14. Out-of-sample forecasting with F_t^2 indicators.

suggest, it remains unclear whether the U.S. macroeconomic variables improve forecasts of interest rates once the information from yields is fully taken into account. We have found that there is a connection between macroeconomic variables and interest rate dynamics in sample. We now ask whether the addition of domestic and international variables improves the forecast out of sample. Accordingly, we use a 6-factor macro-finance model that includes three principal components of yields and three additional macroeconomic indicators. We compare out-of-sample forecasting properties among the yields-only model (that is, a special case of (6) with $\Lambda = 0$), the macro-finance model with domestic macroeconomic indicators (F_t^2) and the macro-finance model with both global and domestic indicators (F_t^3). Because most of the trend in yields is captured by the first (level) principal component, our comparisons focus on the models' ability to predict the level component of yields, $X_{1,t}$. For consistency, we estimate each of the three models using the 1990:Q2-2007:Q4 period, a subsample that includes the period prior to the Great Financial Crisis. We show that the extended model can sometimes track the falling trajectory of interest rate much better than a yields-only model.

Without loss of generality, let $t = 0$ denote the end of our sample period. To forecast the time path for X_t using solely the information on the history of realizations of economic indicators $F_1, \ldots, F_t$, set $v_t = 0$ for all $t \geq 0$ and iterate equation (6) forward from the initial condition X_0 taken from the data. In other words, we calculate a conditional expectation

$$\widehat{X}_t = \mathbb{E}(X_t | X_0, F_1, \ldots, F_t). \tag{7}$$

In the yields-only model, the forecast of X_t is $\mathbb{E}(X_t|X_0)$. In the macro-finance model, by contrast, the forecast of X_t is updated each period, using the most recent observation in F_t.

Figs. 14 and 15 compare the resulting trajectories for predicted yields. To isolate the impact of model specification on the level component of yields, the figures depict yields predicted with just the first principal component, according to

$$\widehat{y}_{n,t} = a_n + b_{1,n}\widehat{X}_{1,t} \tag{8}$$

instead of using all three components, as in equation (5).[12]

The interest rate forecast from the yields-only model (depicted by the dashed line on the figures) based on information available at the end of 2007 shows the 10-year nominal rate gradually rising from 4.3 percent in 2008 to about 5.8 percent by 2019. The rising forecast trajectory looks similar to the Blue Chip interest rate forecasts depicted in Obstfeld and Tesar (2015, Fig. 5).

Comparing Figs. 14 and 15, we can see that forecasting performance changes substantially depending on the set of macroeconomic indicators.[13] The specification of the macro-finance model with economic indicators, F_t^2, that includes US labor productivity growth and the weighted averages of forecasted and past year's consumption growth and

[12] Given our regression results in Table 3, one should not expect information contained in F_t to produce high-quality forecasts of the second and third principal components. The estimates of the dynamic model bear this out. Forecasts of $\widehat{X}_{2,t}$, $\widehat{X}_{3,t}$ from (7) are, in fact, volatile, and so are predicted yields calculated from (5).

[13] Our estimates of the model with macro variables show that the level component of yields depends on both productivity growth and expected inflation, and the component's estimated persistence is substantially reduced compared to that in the yields-only model. These findings contrast with Ang and Piazzesi (2003), however, and they appear to be sensitive to both the sample period and the set of macroeconomic indicators included in the model.

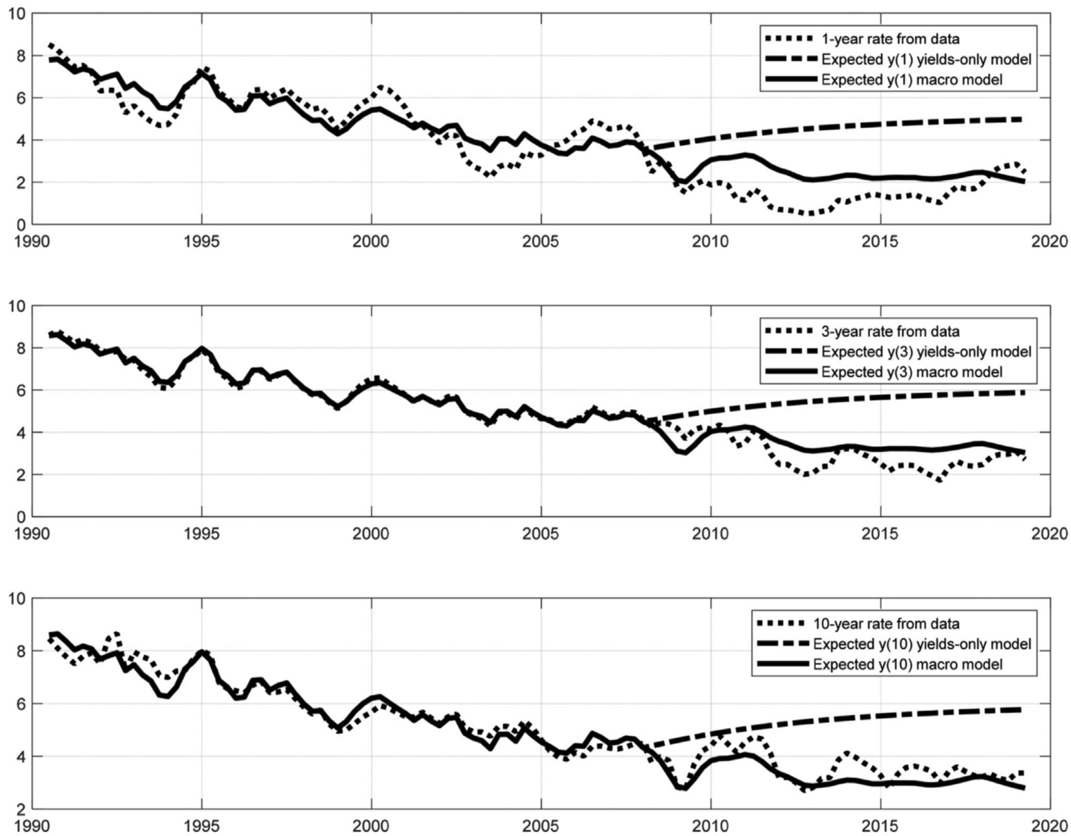

Fig. 15. Out-of-sample forecasting with F_t^3 indicators.

inflation not only fails to capture the falling trajectory for the interest rate after 2008 but also does *worse* than the yields-only model.[14] We think that this may have to do with the (wrong) sign for the coefficient on inflation in the first row of the estimated Λ matrix in (6). Since inflation after the global financial crisis was generally below average, the model's forecast for the interest rate would tend to be high. By contrast, the macro-finance model with EU and Asia-5 leading indicators tracks the interest rate levels much better out-of-sample. It seems that information contained in the OECD regional leading indicators is more relevant for predicting interest rate levels.

It is not entirely clear why the macro-finance model with global factors tracks the level component of yields better than the model with domestic factors. One hypothesis may be that domestic macroeconomic conditions are priced into yields to a larger extent than global conditions. If true, the coefficients of the matrix Λ will not fully capture the links between domestic macroeconomic conditions and yields, as some of that information could already be encoded into the principal components X_t. Our out-of-sample forecasting exercise does not use out-of-sample information on X_t but it does use the out-of-sample information on F_t. By contrast, if global macroeconomic indicators are priced into yields to a smaller extent, the model would capture less feedback between the global indicators in F_t and the principal components.

Another hypothesis may be a regime change with respect to inflation after the global financial crisis, which would make a stationary VAR an inappropriate model of yield dynamics, especially when inflation-related variables are included in the state vector F_t. To summarize, the comparison exercise in this section illustrates that global macroeconomic indicators hold some promise in forecasting interest rates out of sample. Our analysis suggests that information about expected growth in other large economies has relevance of explaining shifts in U.S. interest rates.

5. Conclusion

The goal of this paper is to examine the role of domestic and global macroeconomic variables in forecasting U.S. interest rates.

This paper reviewed a number of methodologies for constructing long-range interest rate projections. Traditional statistical forecasting models based on stationary VAR dynamics face some limitations in their ability to capture low-frequency movements in interest rates, at least with available sample lengths of 150 quarters or so. Several other methodologies for long-range interest rate projections – particularly, those that can make use of long time series – may complement the insights from forecasting models.

Two methodologies are potentially useful for constructing long-range interest rate projections: semi-structural methods of interest rate trend decomposition and standard statistical forecasting models with an extended set of explanatory variables, including forward-looking economic indicators. These methodologies use different data and samples, and they provide complementary pieces of information. Moreover, interest rate trend decompositions are potentially informative on the set of state variables that may be included in a VAR-based forecasting model.

We perform a decomposition of the long-run nominal interest rate over the period 1981 to 2019 under the assumption that in the long-run inflation has no effect on the real interest rate. Three variables, the earnings-price ratio of the stock market, the weighted average of past and

[14] The interest rate level component forecasted with F_t^1 indicators is qualitatively similar to that depicted on Fig. 14 – the expected interest rate trajectory is higher than that from the yields-only model.

forecasted consumption growth and year-on-year productivity growth explain 87% of variation in the 10-year real rate. The relative importance of the various macroeconomic determinants changes over time, with the earnings-price ratio mattering most in the 1981–1988 period and consumption growth most significant following recessions.

Our reduced-form decomposition and interpretation of results in Del Negro et al. (2018) suggest global macroeconomic variables as well as forward-looking indicators as good candidates for the expanded set of explanatory variables in a forecasting model. An important and unresolved question in the literature is whether information about macroeconomic conditions improves the forecast for interest rates, or whether past information about yields is sufficient to fully characterize interest rate dynamics. We add to this debate by exploring the role of domestic and international macroeconomic variables for interest rate forecasting. We find that international variables are increasingly important for understanding and predicting U.S. interest rates.

To assess the forecasting performance of global and domestic indicators, we estimate a macro-finance affine term structure model. This method allows us to ask whether macroeconomic variables add information after conditioning on past information about yields captured by the principal components. Our estimates show there is little information in the earnings-price ratio beyond what is already encoded in the principal components of yields. In contrast, US consumption growth and productivity growth play distinct roles in explaining the variance of yields, with consumption growth being most important for short-term rates, and productivity growth for both short- and long-term rates. We find that growth indicators for Europe and Asia are strongly significant, suggesting that international factors are increasingly important for U.S. interest rate determination.

An important contribution of our work is the addition of international forward-looking indicators to the forecast. We compare the out-of-sample forecasting properties of the dynamic model under three specifications: a yields-only model, a macro-finance model with domestic macroeconomic indicators, and a macro-finance model with both domestic and international indicators. We find that the model with international factors can outperform the other models by better tracking the falling trajectory of U.S. interest rates in the post-2008 period, a trend that is missed by domestic variables.

This paper focused on selecting forecasting indicators based on standard economic theory. Future research can take an alternative selection approach that complements ours. To construct new composite indicators, one might start from a rich set of macroeconomic time series and apply shrinkage methods described in Li and Chen (2014) to select a few composites with the best forecasting performance. Beyond making forecasts more precise, the set of composite indicators may provide new information on the quantitative relationships between macroeconomic variables and interest rates.

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Data Appendix

The data sources used in the paper are as follows.

Data for the Figures unless the source is stated in the caption.

Fig. 1

Real 10-year interest rate – Robert Shiller online data, http://www.econ.yale.edu/~shiller/data.htm. End-of-quarter interest rate was extracted from monthly data. Labor productivity – Bureau of Labor Statistics, FRED series OPHNFB.

Fig. 4

Real 10-year interest rate and EP ratio – Robert Shiller online data, http://www.econ.yale.edu/~shiller/data.htm. End-of-quarter interest rate was taken from monthly data.

Population by age, 1900–2000: Historical Statistics of the United States (Haines, 2006). Population by age, 2001–2018: National population by characteristics, US Census Bureau.

Fig. 6 and equations (5) and (6): yield curves.

The yield curves are constructed according to the methodology in Gürkaynak et al. (2007). The forward rates on the yield curve are constructed from the six Svensson formula (i.e. equation (9) in Gürkaynak et al. (2007)) coefficients. The data set http://www.federalreserve.gov/econresdata/researchdata/feds200628.xls reports Svensson coefficients at daily frequency. Quarterly forward rates are geometric averages of daily forward rates within a quarter.

Data used in constructing economic indicators.

Domestic indicators.

CPI Inflation: NIPA Table 1.1.4, line 2. Inflation forecast (starts in 1981:Q3): Data Files - Survey of Professional Forecasters (CPI). The data file reports the median (across individual respondents) forecast for 4 different survey dates. We take the geometric average of forecasts across the 4 survey dates within the same year.

Real PCE growth: NIPA Table 1.1.3, line 2.

Real PCE growth forecast (starts in 1981:Q3): Data Files - Survey of Professional Forecasters (Real consumption expenditure growth). The data file reports the median (across individual respondents) forecast for 4 different survey dates. We take the geometric average of forecasts across the 4 survey dates within the same year.

EP ratio – Robert Shiller online data, http://www.econ.yale.edu/~shiller/data.htm. End-of-quarter *EP* ratio was extracted from monthly data.

Labor productivity – Bureau of Labor Statistics, FRED series OPHNFB.

Global indicators g^e_{EU}, g^e_{A5}

Data source: OECD composite leading indicators, (CLI), https://stats.oecd.org/Index.aspx?datasetcode=MEI_CLI, annualized growth rate of the trend-restored composite leading indicator, monthly series LOLITOTR_GYSA for major five Asian economies (starts in 1990:Q2) and 19 Euro-area countries. End-of-quarter growth rates were extracted from monthly data.

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