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# COERCION AND REVOLUTION: VARIATIONS ON A PREDATOR–PREY MODEL

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**Abstract**—A series of two-state models are advanced governing the dynamic relationship within a state of (1) revolution and coercion, (2) revolution and relative deprivation and (3) revolution and outside intervention. These models combine, in abstract form, the major thrusts of the work of Tilly (structure and organization without psychology) with the work of Gurr (psychology without organization and structure). They incorporate, again in abstract form, considerations of foreign intervention motivated by the work of Jackson, Russett, Snidal, Sylvan, Duvall and Freeman (thus taking an hypothesis from formalizations of "dependencia"). The two-state models are logically aggregated and both the two-state and multi-state models analyzed for their deductive implications. In particular, a sensitivity analysis of their comparative statics and an analysis of the quality and amplitude of the implied time paths are set forth. Thus, the focus of the analysis is on the contrast between long-run and short-run behavior. It turns out that the two-state models are each equivalent to the logic of predator–prey dynamics and that this property is preserved in the combined multi-state model. The resulting structure provides an especially lucid formulation for the analysis of the dynamics of revolutionary processes, highlighting the role of the component two-state subsystems in the larger system.

## INTRODUCTION

Revolutionary processes have been investigated for a long time and from a variety of theoretical and empirical perspectives. The revolution in France worried and fascinated Edmund Burke, Karl Marx furnished the best known theory of revolutionary transformation, and the analysis of revolution and civil violence continues to the present day, most notably in the work of Tilly and Gurr. These contemporary scholars are concerned to place theories of revolution on a sound, scientifically justifiable, empirical base. The present effort seeks to further that body of work by means of a dynamic model which clarifies the relationship among revolution, coercion, relative deprivation and foreign aid. This leads to a reconciliation of conflicting theories, like those advanced by Tilly and Gurr, with important consequences for empirical investigations of revolutionary phenomena.

For example, it can be argued that the central thrust of Tilly's formulation of the engine of civil violence is an organizational and structural view, while Gurr's construction is intuitively based on the private psychology of the individual which is linearly aggregated to the level of societies. A choice between these theoretical postures has consequences for methods of investigation: cross national data was used by Gurr and within country evolutionary development was employed by Tilly. Our hope is to follow Tilly and formalize the development as a specific dynamic structure within which the logic of Gurr's argument may also be represented as within country. The reconciliation that is effected emphasizes the dangers of cross-sectional approaches when used to infer the structure of revolutionary processes. The main thrust of this paper is, therefore, theoretical or structural—to explain why and to what extent different theories of revolution are conflictual. We also hope to shed light on the complex patterns of rioting response to differing policies, like the ones in Northern Ireland reported by Peroff and Hewitt (1980, especially the summary table on p. 609).

Our point of departure is the observation that revolutions are processes—they unfold in time—and, although they are punctuated by great events, the key to understanding lies in the notion that revolutions exhibit a history of development. Revolutionary events are joined to other (perhaps revolutionary) events in a systematic and connected sequence. The metric of sequencing may not be transparent, but we take the metric to be roughly correlated with real time and use time as the technical device for representing sequencing in all that follows. Time may be thought of as

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continuous or discrete and most of our argument is couched in a way that will fit with either taste in the measurement of time. Such measurement is not without modeling consequences, however (Kohfeld and Salert 1982), and we take some advantage of the continuous formulation in order to use a result from the theory of qualitative stability (Quirk and Ruppert 1965; Kohfeld 1981).

This perspective on revolution as systematically developmental within a specific context certainly includes the continuing struggle of regimes, e.g. in Central America, to deal with revolutionary movements from the left or right which are organized in the form of guerrilla warfare, and also less dramatic forms of civil unrest and state response. A second component of this perspective is that, ultimately, a system of interrelated determinants of revolutionary processes are reciprocally dependent. The consequences of this idea are important for empirical and theoretical work. For empirical work, when combined with the notion that revolutions are dynamic processes, the interdependence of components calls into question the practice of conducting cross-sectional studies of revolutions. For theoretical work, the moral emerges with even greater clarity: theoretical models of revolutionary processes must be non-trivially dynamic. Why? Because the essence of revolutionary activity is a sequenced pattern of response and counter response between state and revolutionaries. Revolutionary activities are not analogous to market behavior (which is typically close to equilibrium and is appropriately analyzed with comparative statics) but rather notable revolutionary processes typically represent societies in profound conditions of social dislocation and social disequilibrium. Moreover, revolutionary processes may exhibit a pattern of escalation which invites theoretical treatment with formal tools which capture the dynamics of interdependent spirals in natural fashion. Put another way, the principal components of theoretical models find their natural representation as endogenous components in a simultaneous dynamic system. In this respect we particularly follow the lead of Jackson *et al.* (1978) in their formalization of dependencia.

## BASIC CONCEPTS

The models developed below relate four concepts which recur in discussions of revolutionary processes by means of two other concepts which emerge from the logic of the modeling development, and occur, as it were, as *sub silentio* model components. These latter two components are the notion of structure, which is imposed by the equation systems, and the notion of time dependence, which is imposed by the modeling language employed—that of differential equation systems. Thus the time derivative of revolution, where revolution is a system state that may be characterized quantitatively at a ratio scale measurement level, is denoted by

$$\partial \mathbf{R}/\partial \mathbf{t}.$$

It is known that both the stability of equilibria and the qualitative behavior of time paths may differ for continuous and discrete time (May 1974; Kohfeld and Salert 1982; Samuelson 1947). However, our main results are little affected by this distinction and thus, as a matter of convenience and occasional technical advantage, we model in continuous time.

The substantive concepts chosen for manipulation are four in number: relative deprivation, revolution, state coercion and foreign intervention. To keep mnemonic confusion to a minimum we designate each of these concepts by a bold capital Roman letter, relevant fixed parameters by bold lower-case roman letters and general exogenous system inputs by lower-case Greek letters. The essential conventions for the substantive concepts are as follows: **D** for (relative) deprivation; **R** for revolution; **C** for coercion; and **A** for foreign intervention (foreign aid). Each of these fundamental components requires brief comment.

The concept of relative deprivation, thrust into prominence by the work of Gurr (1970a), stands in sharp contrast to the other three, in that it is not directly measurable. Of course, measurement theories can be and have been advanced purporting to provide a strategy of measurement for relative deprivation (Gurr 1968, 1970b) but it is at bottom a psychological state based on a comparison operation either with other individuals or with prior perceived personal states (subjectively remembered) and hence takes on a different character than revolution, coercion or foreign intervention. The touchstone of measurement of revolution, state coercion and foreign intervention is that they are in principle objective and public, rather than subjective and idiosyncratic. Revolution may be measured in many ways, perhaps none of them wholly satisfactory, yet these variations do not present metric problems in principle. One might use territory controlled, or number of villages and cities controlled or proportion of the population supporting (or not supporting) the regime as measures of revolutionary activity. Similar quantities could be used for civil violence that fell short of revolutionary character. State coercion could be measured by punishments meted out or by the budget amounts devoted to the control of revolutionary activity and foreign intervention could be measured in dollars shipped, or guns shipped or military trainers sent or some similar units. It appears that each of these more objective concepts presents no insurmountable obstacle in principle [but see Snyder (1978) for a catalog of some difficulties]. As we hope to show below, the value of retaining relative deprivation is that it provides a psychological and motivational bridge between the more objectively measurable basic concepts and the behavior of the individuals who act out revolutionary events.

## DEPRIVATION, REVOLUTION, PREDATORS AND PREY

The relative deprivation hypothesis asserts that, as relative deprivation increases, the potential for collective violence increases. Under certain conditions (normative and utilitarian justifications of violence, balance of forces) this potential is transformed into actual political violence. According to this theory, the connection between relative deprivation and revolution is causal. This reasoning is correct, but captures only a portion of the relationship between relative deprivation and revolution. Whether, or not, revolutionary activity is considered rational (DeNardo 1985) or psychological (Gurr 1970a), revolutionary activity releases the frustrations resulting from being relatively deprived and has the additional effect of reducing relative deprivation. This second effect, that revolutionary activity reduces relative deprivation, is omitted from Gurr's model. It turns out that this second effect transforms the original model in important ways.

In short, the relationship between revolutionary activity and deprivation is what is called a predator–prey relationship in the biological literature (May 1974, pp. 79–84). In the original biological model the multiplication of rabbits is controlled by predation by wolves and the food supply problem for the wolves is solved by eating rabbits. In our version, as revolutionary activity increases as a function of increases in the supply of relatively deprived persons, simultaneously relative deprivation is adversely affected (psychological release) by revolutionary activity. Relative deprivation is prey. The same predator–prey relationship holds between other components of our model. Thus, in the two-state models to be introduced below, when one state enhances a second, the reverse effect is assumed to hold when viewed from the perspective of the second.

Consider the dynamic interaction between (observable) revolutionary activity and (non-observable) relative deprivation:

$$\partial \mathbf{D}/\partial \mathbf{t} = -\mathbf{aD} - \mathbf{bR} + \alpha \tag{1}$$

and

$$\partial \mathbf{R}/\partial \mathbf{t} = +\mathbf{fD} - \mathbf{dR} + \beta. \tag{2}$$

The rate of change of relative deprivation is increased by some external causes (represented by the term $\alpha$) and is decreased, (1) proportionally to the current level of deprivation (an inertial effect—if the cause, **a**, goes away then the level of deprivation decreases) and (2) proportionally to the level of the state of revolution (a revolutionary activity produces psychological release of frustration). The constants of proportionality are **a** and **b**, respectively. The equation is intended to capture the idea that revolutionary activity moves the population level of relative deprivation downward and thus the critical term is $-\mathbf{bR}$. We of course assume that all parameters are positive and the sense of the argument is expressed in attached signs. As (relative) deprivation increases so does the rate of change of revolution. There may be other causes (**b**) and unstimulated (uncaused) revolution dies out ($-\mathbf{dR}$).

The model defined by equations (1) and (2), call it the deprivation and revolution model, can be put to useful work. We formulate Gurr's argument as interdependent and dynamic instead of static and causal, and, by this strategy, reconcile the two competing conceptions of revolutionary activity. Coefficients in these dynamic models are contextually dependent on the nature of specific regimes as Tilly assumes and the states of the system covary in a way partially determined by Gurr's theory. We hope to persuade the reader in the sequel that this is a sensible and fruitful strategy.

If (relative) deprivation ultimately, through perhaps a number of psychological steps, leads to revolution, then this relationship must occur in time according to some rule approximated by equation (2). That equation gives the simplest representation that must be obtained, given Gurr's argument, between revolution and deprivation described dynamically. And a dynamic representation is the only plausible construction for real systems of behavior. But does this dynamic representation have any consequences since it appears to have the form Gurr intended? The answer is not immediately obvious but affirmative.

The not so obvious consequence is that the practice of evaluating Gurr's argument by appeal to cross-sectional data is supportable only under conditions which are very unlikely to be met in practice. This is a double-edged argument in the present context, for not only are such processes not likely to be at equilibrium, but also the inference to the shape of the relationship is obscured if they are not at equilibrium. This latter point becomes evident in the discussion of coercion below.

The more obvious point, that the process is not likely at equilibrium which has substantial consequences for ordinary statistical inference from cross sections, is easily seen by comparing the two model estimation forms appropriate for the two situations. Note that the stability of the equilibria, should the processes be close to them, is not problematic for this model (May 1974, p. 71). The stability is guaranteed by the signs of **a** and **d**, which guarantee that the real part of any roots are negative and hence the solution always converges. But that is not the problem in any case. What is worth seeing is how discrepant the equilibrium analysis model is from the disequilibrium analysis model. We write the appropriate comparisons out for the system state revolution, **R**.

To estimate such systems the usual strategy is to represent the model in discrete time when a time series is available, which yields a second-order difference equation:

$$\mathbf{R}[\mathbf{t}+2] = (2 - \mathbf{a} - \mathbf{d})\mathbf{R}[\mathbf{t}+1] + (-\mathbf{a} - \mathbf{bf} - 1 + \mathbf{a} + \mathbf{d})\mathbf{R}[\mathbf{t}] + \mathbf{g}(\alpha, \beta) \tag{3}$$

which may be compared with the equilibrium form for the same process of

$$\mathbf{R}^* = (\mathbf{a}\beta + \alpha\mathbf{f})/(\mathbf{ad} + \mathbf{bf}). \tag{4}$$

Details are given by Goldberg (1958). If the process is at equilibrium then equation (3) becomes (degenerates into?) equation (4). It is easy to see that any model estimation based on the two models differs by the absence of the lagged values for revolution. It should also be immediately clear that the coefficients in equation (4) are not identified and this problem cannot be made to go away. If revolutionary processes everwhere are at equilibrium then use of equation (4) is certainly justified, even though the coefficients for the causes will always be unidentified up to multiplication by the factor $1/(\mathbf{ad} + \mathbf{bf})$. But such an assumption is assuredly false empirically.

Similarly, the equilibrium value for deprivation may be written as

$$\mathbf{D}^* = (\alpha\mathbf{d} - \mathbf{b}\beta)/(\mathbf{ad} + \mathbf{bf}). \tag{5}$$

Combining equations (4) and (5) by dividing one by the other we obtain

$$\mathbf{R}^* = \mu\mathbf{D}^* + \text{constant}, \tag{6}$$

where

$$\mu = (\mathbf{a}\beta + \alpha\mathbf{f})/(\alpha\mathbf{d} - \mathbf{b}\beta),$$

and we note $\mu$ is system specific. The conclusion seems inescapable that serious attempts to assess the empirical usefulness of Gurr's approach must follow some formulation such as equation (3) and employ data over time (cf. Snyder 1978). It appears to us that the cross-sectional analysis of data for such processes in only justified by two strong hypotheses. First, there is equilibrium across all observed states (never mind cross country differences) and second, that there are no cross country differences. Such simplifications are not substantively tenable.

The moral which remains points in the direction of always employing data over time and, when that fails to be possible, always form data analysis expectations based on a dynamic representation looking for implications in the cross-sectional data [an extraordinary example of this strategy is found in MacKuen (1981)]. In the remainder we pursue only dynamic formulations. The essential arguments of both Tilly and Gurr are dynamic and probably so are the arguments of most others who have addressed this or similar phenomena even in quite disparate fields (Granovetter 1978; Burbeck, Raine and Stark 1978). We see no reason to duck the inherently dynamic character of revolutionary processes in modeling given this common judgment of investigators.

Now reconsider the model of equations (1) and (2). It is clear that revolutionary activity is stimulated by factors other than relative deprivation and a persuasive representation ought to include such additional causes. The most obvious omission is the activity of the state in attempting to suppress or control revolutionary activity. Hence, the model should be elaborated by adding a system state to the model describing governmental activity which we earlier defined as coercion, **C**. The elaboration will be more instructive, however, if we make the elaboration in stages. Thus, instead of tacking the system state **C** onto the system specified by equations (1) and (2), which we will do ultimately, we first specify a model for the relationship between revolutionary activity and coercion absent other influential factors.

## REVOLUTION AND COERCION

Relative deprivation is a subjective individual state and difficult if not impossible to measure. Coercion by the agents of established order is probably easier to measure and certainly palpably observable. However, the consequence of the use of state coercion on revolutionary processes is ambiguous. It has been argued [Jackson *et al.* (1978) following Gurr (1970)] that the relation between revolutionary activity and state coercion is roughly curvilinear and not necessarily symmetric [see figures in Jackson *et al.* (1978, pp. 633, 635)]. The interesting question which is presented by this hypothesis is the following: how can we know this? Put another way, given that cross-sectional analyses with static models are almost certainly misleading in assessing this relationship what are the potential consequences of a (pure) dynamic approach in empirical applications? We pursue this issue by formulating a second elementary model specifying idealized interdependencies between the system states of revolution, **R**, and coercion, **C**.

Concern with the shape of the empirical phenomenon has long been associated with the notion of relative deprivation (Davies 1969; Miller, Bolce and Halligan 1977; Crosby 1983). We propose to avoid reviewing those issues by pursuing the lead of Jackson *et al.* (1978) and modeling in terms of state coercion. What is problematic for purposes of modeling is the direction of dynamic dependence for the off-diagonal elements in the modified predator–prey formulation. The signs of these coefficients must be opposite but which should be positive and which negative? Consider the model given by

$$\partial \mathbf{R}/\partial \mathbf{t} = -\mathbf{dR} + \mathbf{gC} + \beta \tag{7}$$

and

$$\partial \mathbf{C}/\partial \mathbf{t} = +\mathbf{hR} - \mathbf{kC} + \gamma. \tag{8}$$

One might argue that as revolutionary activity increases it evokes coercive response up to a point and then, as higher levels of revolutionary activity are achieved, the regime exhausts its resources and its repressive capabilities shrink. On the other hand, it seems equally plausible to argue that as coercion increases it exacerbates revolutionary activity up to a point and then, above some threshold value, coercion successfully suppresses revolutionary activity. Now the choice of signs, given these two arguments is doubly complicated. Which hypothesis is correct? Where in the time line of the process is the observer (or the observations) located? We suspect that the problem is not well formulated in these terms and that the matter is greatly clarified by appealing to the dynamics of the system specified by equations (7) and (8).

The way to a clearer vision of the relationship is to exploit the dynamics of the model of equations (7) and (8) under the hypothesis that the motion is cyclical. Under this hypothesis the motion of the system is periodic and the appropriate display device is the phase plane. Figure 1 exhibits the phase plane for the parameter combination **g** negative and **h** positive and a similar figure results, with opposite rotation, if the signs are reversed. The figure traces out a hypothetical trajectory typical of the motion engendered by the model.

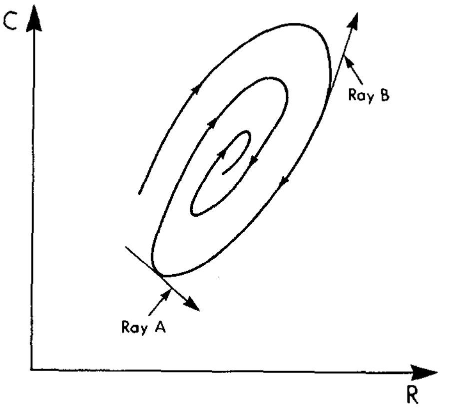


Fig. 1. Phase plane for revolution and coercion under the hypothesis that motion is cyclical with **g** negative and **h** positive.

Either sign combination can produce appropriate curvature and the sign combination determines the direction of rotation. The major implication is that the determination of the signs of the critical parameters **g** and **h** may be left to empirical determination from appropriate data (some time series) and the apparent contradiction between the intuitive arguments is seen to be an artifact of employing static reasoning for an inherently dynamic process. This result may be expressed in the proposition that the dynamics of the process determine the shape of the relationship between coercion and revolution rather than conversely.

Substantively, it is as unrealistic to suppose that state coercion is unconstrained (depends only on revolutionary activity) as it is to suppose that revolutionary activity depends solely on relative deprivation. The classic form of resource support is foreign aid to repressive regimes. The prototypical case is probably military aid from the United States or the Soviet Union to smaller nations, e.g. Vietnam, Afghanistan or El Salvador. Thus the next model we elaborate represents the interaction between coercion, **C**, and foreign aid, **A**.

## COERCION AND FOREIGN AID

Empirically, we believe that foreign aid supports the repressive or anti-revolutionary activities of the regime, while coercive activity induces the supplier of aid to become reluctant to maintain supply levels. Thus, the consequence of foreign aid is to increase the objective ability of a regime to be coercive. On the other hand, the supplier responds to increased levels of coercion by reducing aid since the supplier reasons that the regime is becoming an increasingly bad bet for survival. Alternatively, the identical sign pattern for parameters can be achieved if it is recognized that resources are limited for many reasons both objective and subjective. In particular, as the debate over aid to El Salvador (or the aid to the Nicaraguan Contras) illustrates, the supplier's ability to offer aid can be politically limited. The political constraints furnish continuing basic fare on the front pages of our newspapers. Whether domestic politics (isolationism) or domestic suspicion of foreign regimes that continually escalate internal coercion (human rights concerns) is the cause, the consequences for the signs of model parameters are the same.

The formalism that we are about to specify does not capture that distinction in the possible causes of the behavior of a supplier of foreign aid. This does not mean that we believe the distinction theoretically or substantively unimportant. The consequences for the interdependent dynamics between foreign aid and coercion are behaviorally the same. Regardless of the underlying story, it does no great harm to model that interdependency with one representation, even when the cause remains a distinction with a difference from other theoretical perspectives. Our interest is in the dynamics, and for these purposes the same representation will do.

The two-state model capturing these arguments may be written as

$$\partial \mathbf{C}/\partial \mathbf{t} = -\mathbf{k}\mathbf{C} + \mathbf{l}\mathbf{A} + \gamma \tag{9}$$

and

$$\partial \mathbf{A}/\partial \mathbf{t} = -\mathbf{mC} - \mathbf{nA} + \partial. \tag{10}$$

The system state **A**, foreign aid, in equation (9) can be thought of as an elaboration of the fixed parameter **h** in equation (8). Tractable linearity is thereby achieved but at the price of increasing the order of the resulting system. Similarly, the system state **C**, coercion, is introduced in order to correct the simplification of the fixed parameter **d** in equation (2). Compare the three simple systems we have set forth—equations (1) and (2); equations (7) and (8); and equations (9) and (10). In each of these two-state systems the structure of the model is similar. The main diagonal carries a negative sign for each state. The off-diagonal parameters carry opposite signs depending on the logic of the argument underlying the model. In the case of coercion and resolution it was argued that this asymmetry in signs was the crucial logical feature. Finally, all state equations have been written with generalized inputs or forcing functions represented by Greek letters. We now turn to the expanded and more complex model obtained by combining these three submodels.

## THE ELABORATED MODEL

The natural question which now arises is this: by what rule of combination should the elaborated model be constructed? We propose a first approximation solution to this problem by appealing to the structure of interdependent systems and to the fixed parameter argument used above to justify the introduction of interdependencies between states. The former appeal confronts the problem of what sort of coefficients to introduce between the states not yet connected by an explicit argument specifying the interdependency. The simplest solution is to specify these parameters as zero because all states are connected to all other states in the completed system. Thus deprivation, **D**, is connected to foreign aid, **A**, by means of the intermediate states of revolution and coercion. Effects can travel, as it were, from one state to another by virtue of paired state connections.

Following this procedure produces the system exhibited in Table 1. The table has lines imposed upon it which mark out the most interesting model.

The submodel which excludes deprivation, **D**, is the most interesting since it specifies a third-order system among the system states most susceptible to measurement, i.e. with (relative) deprivation eliminated. The four-state system or any three-state or two-state subsystem exhibit some interesting logical properties. The most important of these is that the implied model equilibrium is stable and independent of the magnitude of the parameters, i.e. the model is qualitatively stable. Other properties emerge from the analytics.

The analytics for revolution and coercion arise from the characteristic equation for that subsystem. Comparing equations (5) and (6) with the appropriate portion of Table 1 shows that the systems are identical. The characteristic equation for the system is

$$\lambda^2 + (\mathbf{d} + \mathbf{k})\lambda + (\mathbf{dk} + \mathbf{gh}) = 0. \tag{11}$$

The qualitative behavior of this subsystem is determined by equation (11) and all other two-state systems have a similar structure. The three-state and four-state systems have more complicated motions, but that motion is obtained by additive combinations arising from components no more complicated than equation (11). Thus, the key to taking apart the behavior of larger systems is a repetition of the analyses for coercion and revolution.

Because of the negative entries on the main diagonal of the system in Table 1, coupled with the strategic occurrence of zeros in some off-diagonal positions, the systems that can be constructed from Table 1 by selecting adjacent system states of order two, three or four are all stable. This arises from the pattern of signs and zeros alone and does not depend on coefficient magnitudes.

The equilibrium for the system states **R** and **C** is the point in the $(\mathbf{C}, \mathbf{R})$ phase space with coordinates

$$(\mathbf{C}^*, \mathbf{R}^*) = (((\gamma \mathbf{d} - \beta \mathbf{h})/(\mathbf{dk} + \mathbf{gh})), ((\beta \mathbf{k} + \gamma \mathbf{g})/(\mathbf{dk} + \mathbf{gh}))). \tag{12}$$

Note that all analytics essentially depend on equations (9) and (10). The question arises: what is the qualitative behavior of the time path of the system states? This is answered by manipulating the characteristic equation and it is worth emphasizing that, in this linear system, all states have the same motion. Variations in initial amplitude and phase are determined by initial conditions,

Table 1. Elaborated model for the joint dynamics of deprivation, revolution, coercion and foreign aid

| | | | | | |
|---|---|---|---|---|---|
| $\partial \mathbf{D}/\partial t =$ | $-\mathbf{aD}$ | $-\mathbf{bR}$ | $+0$ | $+0$ | $+\alpha$ |
| $\partial \mathbf{R}/\partial t =$ | $+\mathbf{fD}$ | $-\mathbf{dR}$ | $-\mathbf{gC}$ | $+0$ | $+\beta$ |
| $\partial \mathbf{C}/\partial t =$ | $+0$ | $+\mathbf{hr}$ | $-\mathbf{kC}$ | $+\mathbf{lA}$ | $+\gamma$ |
| $\partial \mathbf{A}/\partial t =$ | $+0$ | $+0$ | $-\mathbf{mC}$ | $-\mathbf{nA}$ | $+\delta$ |

Hence an analysis of the motion for say, coercion, is logically equivalent to an analysis of the motion for revolution.

Probably the most important qualitative property of a time path is its oscillatory behavior. If movement of system states is smooth and monotonic, without oscillation, one consequence is immediate and very important. Under conditions of smooth growth or decay, linear approximation based on simple statistical models will not necessarily be misleading. However, if the system is oscillatory such simple models will be grossly misleading—a difficulty outlined earlier in the discussion of problems in cross-sectional approaches to dynamic processes. The necessary and sufficient condition for oscillatory behavior may be written as

$$(\mathbf{d} - \mathbf{k})^2 < 4\mathbf{gh}. \tag{13}$$

Inspection of inequality (13) shows that if the magnitudes of the parameters **d** and **k** are very small, or if the parameters are approximately equal in value, then oscillations are likely. Are these two conditions otherwise distinguishable? The answer is yes. Although either condition leads towards oscillation, the two parameters on the main diagonal also determine the responsiveness (inertia) of the system. This is reflected in the rate of damping of the motion of the system. If the parameters approach zero, the system is very unresponsive. In contrast, if they are large in magnitude then the system response is rapid, i.e. damping of the effect of any disturbance or input to the system occurs quickly.

If oscillation is to be avoided two conditions must be satisfied. First, the system should exhibit low interdependence between coercion and revolution. This means the parameters **g** and **h** are small in magnitude. Second, the system should exhibit asymmetry in the natural rate of decay (the system inertia) for the states. Thus, the magnitudes of **d** and **k** should be inversely related. These two conditions guarantee smooth rather than oscillatory motion. In substantive terms, loosely coupled systems of coercion and revolution exhibit smooth behavior. Conversely, high interdependence between coercion and revolution leads to periodic motion. In the limiting case where $\mathbf{d} = \mathbf{k} = 0$ the roots of the characteristic equation are pure imaginary numbers (no real part) and the system does not converge to the equilibrium. This pathological case provides endlessly repeated and undamped cyclic behavior.

As asymmetry between the inertial parameters **d** and **k** increases not only does the likelihood of oscillation decrease, but also the period of the motion increases. At some point, provided that **d** and **k** are sufficiently asymmetric and one of them is sufficiently large, the discrepancy passes beyond a point at which the motion becomes smooth. Thus, if the state, through its coercive activity, exhibits strong tendencies to continue coercion at attained levels, while revolutionary activity tends to decay rapidly spontaneously (both plausible conditions), then the period of motion will either be very long or the period will be infinite, i.e. the motion becomes monotonic.

If the state and revolutionary activists have comparable decay rates, comparable inertial coefficients, then the periodicity of the motion is determined by the product of **g** and **h**. As the joint product of these two parameters decreases the period of the motion increases. Interpreted in substantive terms this asserts that as interdependence between coercive and revolutionary activity decreases the motion of the system becomes smoother—more long-term. Conversely, greater interdependence leads to oscillations with short periods which to an observer of revolutionary processes would appear as volatile patterns of behavior. This is intuitively pleasing and marches with general results argued extensively by May (1974).

## TIME PATHS AND EQUILIBRIA

What is of great empirical interest, however, is the relationship between time paths and equilibrium states because external constraints imposed on the process might prevent it from

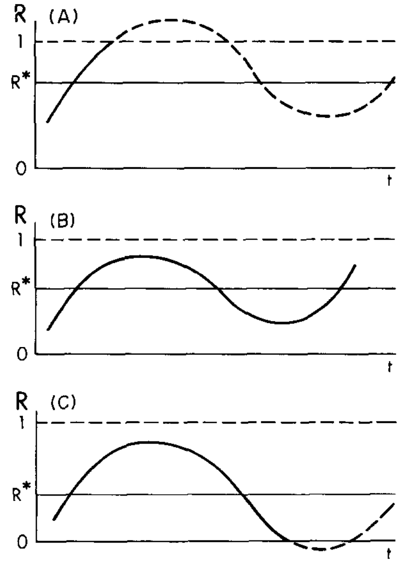


Fig. 2. Revolutionary time paths and constraints.

reaching its equilibrium values. In order to investigate this issue, we make use of an important property of linear systems of differential equations: the characteristic equation remains the same for any linear transformation of the state variables of the system. In the Appendix we show this property for a system of two linear differential equations. We can therefore choose linear transformations of the units of measurement in a convenient and politically meaningful way.

Choose revolution, **R**, to be measured as the proportion of territory occupied by guerrilla forces or as the proportion of the population living in territories occupied by guerrillas, or any other such normalized unit. Such a choice of units imposes some logical constraints on the model. In particular, the time paths of the values for the state **R** as well as its equilibrium value may not lie outside the [0, 1] interval.

Such constraints are more important for time paths than for equilibrium values, as Fig. 2 demonstrates. In fact, although the equilibrium value **R*** always lies in the [0, 1] interval there are additional consequences for the time path of **R**. In the first case (Fig. 2A), a successful revolutionary process is shown—the time path intersects the point where 100% of the population has come under revolutionary control. In the second case (Fig. 2B), a chronic conflict without definite resolution is displayed. This process never terminates unless the structure of the situation changes altering the parameters governing the process or, of course, unless the constraints are altered. In the third case (Fig. 2C), the revolutionary process is initially successful but then is aborted.

Clearly these patterns become much more complicated in the case of a three-state model and linear or monotonic extrapolations may be completely erroneous when dealing with revolutionary processes. Furthermore, from the point of view of the model proposed here there is no fundamental difference between a new coercive law produced by the same government or several coercive laws produced by a new and more severe government. The choice of actor to implement a policy may be irrelevant with respect to the outcome. Hence, substituting one set of generals for a set of admirals or conversely may not be consequential for the revolutionary process under way even though it would be highly newsworthy, visible and unusual. More plausibly the event itself could be explained or interpreted by the model results which, for example, might dictate a substantial or moderate change in the level of coercion. Agency is immaterial to the formalism. A similar argument holds for revolutionary processes and changes in revolutionary leadership. On the other hand, if such changes are important enough they will be reflected in changes in the values of parameters in the model. Estimation techniques can detect such changes if they are meaningful (Johnston 1984, pp. 392–396).

Such a thesis may appear mechanistic at first reading since it alienates strategic choices taken by historical actors from the process in which they participate in favor of some impersonal logic of revolutionary processes. But the actors may learn and hence maintain their leadership role precisely because they correctly read the current situation as it has historically developed. And if they do not learn then why is change not appropriate? To the extent that the model captures elements of this learning process the description of the future as a composition of past history and behavioral rules becomes a plausible description. It is more believable to suppose that nations are not lost for want of a horseshoe nail than conversely.

What more can be expected of the elaborated models constructable from the exhibit in Table 1? The linearity of the system produces some immediate consequences for system dynamics for the motions that are possible. As each state is added to the system a term is added to the general analytic solution. For the subsystems of order three the possibilities include a smooth component plus oscillation. Subsystems of order four, i.e. the elaborated model entire, are capable of two trends plus oscillation or even two different trigonometric motion sources. The general theory of qualitative stability assures us, in the case of this particular model, that all equilibria are stable and thus no matter what the transient motion—smooth or oscillatory or some combination—the movement is always progressively toward the equilibrium value if time is extended long enough and if exogenous shocks to the system are absent.

## WHICH CURVE IS THE CURVE OF REVOLUTION?

The canonical model can be used to examine an important theoretical problem: when are revolutions likely to occur? There is considerable controversy among workers in the field.

Zimmerman (1983, p. 310) quotes deTocqueville:

> "Revolution does not always come when things are going from bad to worse. It occurs most often when a nation that has accepted, and indeed has given no sign of even having noticed the most crushing laws, rejects them at the very moment when their weight is being lightened. The regime that is destroyed by a revolution is almost always better than the one preceding it, and experience teaches us that usually the most dangerous time for a bad government is when it attempts to reform itself."

This thesis may be compared with an argument offered by Davies (1969, p. 6), who offers a different hypothesis for the development of revolutions combining deTocqueville and Karl Marx, to wit, the J-curve hypothesis:

> "The J-curve is this: revolution is most likely to take place when a prolonged period of rising expectations and rising gratifications is followed by a short period of sharp reversal, during which the gap between expectations and gratifications quickly widens and becomes intolerable. The frustration that develops, when it is intense and widespread in the society, seeks outlets in violent action."

Grofman and Muller (1973) examine these two theses and come to the conclusion that when relative gratification increases or decreases the potential for political violence increases. They term this result the U-curve hypothesis thus adding one more explanation for the likelihood of revolutions.

This confusion led Tilly (1975, p. 529) to the following summary:

> "A proper verification that the phenomenon exists will require comparisons of periods with J-curve, U-curve, M-curve, and no curve, as well as between revolutions and nonrevolutions, in order to see whether there is in fact an affinity of one for the other."

In terms of the model developed above, deTocqueville's model should be interpreted in terms of the coercion–revolution submodel while the subsequent J-curve and U-curve arguments refer indirectly to the deprivation–revolution submodel. Each one of these hypotheses is much more elaborated than our crude model yet our model can produce patterns congruent with each of these theses. Moreover, if the relationship is the one we hypothesize, observations expanded in time may leave the (false) impression of random patterns, or, to use Tilly's phrase, "no curve at all".

These considerations lead to two conclusions. The first is the parsimony of our formation. Indeed, with simple assumptions our model can produce very complicated and often controversial (at least in the empirical literature) patterns. The second conclusion is the comparative advantage of our modeling language for detecting and interpreting the structure of phenomena that otherwise seem contradictory, incomprehensible, or random. The following section elaborates this conclusion with an empirical example.

## PUTTING DYNAMICS TO USE IN IRELAND

Can some good be obtained from this formulation of the dynamics of revolution? We return once more to the logic of the two-state subsystem of coercion and revolution and apply it to some results of Peroff and Hewitt (1980) on riot behavior and attempts at control in Northern Ireland. They conclude about all policies, including positive incentive as well as coercive strategies, that "most of the policy decisions or actions were not successful . . ." (pp. 608–609). In support of this claim they adduce some carefully assembled quantitative evidence, reported in their Tables 3 and 4 and summarized in their Table 5. We focus attention on two statistically significant predictors of riot activity reported by Peroff and Hewitt which may plausibly be construed as aspects of a policy of state coercion of revolutionary activity on the part of the British. These two policies were the level of armed forces maintained and the internment of Catholics. What distinguishes these two coercive policies is that, viewed in the monthly perspective of Peroff and Hewitt, one policy worked (Catholic internment) and one failed (increasing military presence). But in light of the hypothesis that the relationship between riot activity and coercive activity follows the logic argued earlier, and, let us suppose in particular is such that oscillations are characteristic, then the conclusions of Peroff and Hewitt are indeed problematic. It may be that army level increases ultimately will lead to riot reduction and that internment of Catholics will ultimately lead to riot increase. More properly, it may be that both their reported outcomes and the opposites are consistent. A reconstruction of their results from a dynamic perspective is characterized by rays A and B in Fig. 1.

In Fig. 1 a general convergent trajectory is displayed for the phase plane of coercion and revolution. Also superimposed on that possible trajectory are two vectors indicating the way in which Catholic internment (ray A) and levels of armed forces (ray B) might fit to that trajectory in a consistent fashion. On the view of the dynamics of revolution and coercion displayed in Fig. 1 it may be incorrect to allow oneself the habit of thought that a policy leads anywhere. The policies displayed in the geometry of that trajectory lead only back on themselves in an endless spiral. The point of the illustration is not that the results of Peroff and Hewitt are wrong but rather that the habit of thought which connects policies and results without any thorough consideration of the form of the dynamic relationship may be misleading. On this view it is simply not fruitful to attempt to answer the policy question without a deeper knowledge of both the dynamics of the process and some ability to consider time horizons systematically in that context.

## CONCLUSION

A canonical model of critical elements governing revolutionary processes has been set forth. The model was used to argue against estimating static models of these processes with cross-sectional data. The analytics of the two-state subsystem were developed and general properties of the four-state system briefly discussed. Finally, the dynamic perspective of the argument was invoked to reinterpret some results concerning riot behavior in Northern Ireland. The model was not evaluated empirically in any crucial single test, and fit has not been a primary concern. Our focus has been rather on several theoretical problems in the relevant literature as well as on some empirical puzzles which appeared incongruent with the models in the literature.

From this point of view all that has been done here is to supplement Gurr's fundamental intuition (relative deprivation leads to revolution) with a complementary statement (revolution decreases relative deprivation). The consequences of this apparently innocuous modification were quite telling: (1) causal reasoning was replaced by interdependent dynamic reasoning; (2) Tilly's conception about the role of organizations and mobilization in a revolutionary process emerged as a specific case of the model, since the major methodological implication of the model is that parameters must be estimated with data over time within a single context; and (3) cross national analyses appear as misleading since they rest on untenable assumptions (revolutionary processes are at equilibrium everywhere and cross country differences may be safely ignored).

The same logic was then developed for different but related concepts—coercion and foreign aid—and the separate models were combined in a more general framework which exhibits some important mathematical and descriptive properties. With respect to descriptive properties of the model a brief demonstration of flexibility in use was given in the case of Northern Ireland where, by hypothesis, the same level of relative deprivation and the same organizational environment produced different effects on revolutionary activity for different levels of state coercion.

To the extent that our reasoning is convincing it dictates that the model parameters are endogenous and hence estimates of their signs and magnitudes can be obtained in application to concrete cases. In such applications revolutionary activities, policy responses and the joint consequences of each become intelligible in a new light according to the specific logic of dynamic interdependence.

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## APPENDIX

Consider the linear differential equation system described in operator notation by

$$\mathbf{DX} = \mathbf{aX} + \mathbf{bY} + \mathbf{k} \tag{A.1}$$

and

$$\mathbf{DY} = \mathbf{cX} + \mathbf{dU} + \mathbf{h}. \tag{A.2}$$

The characteristic equation of the system is given by

$$\mathbf{D2} - (\mathbf{a} + \mathbf{d})\mathbf{D} + \mathbf{ad} - \mathbf{bc} = 0. \tag{A.3}$$

In the case of a linear transformation of one of the variables, e.g.

$$\mathbf{X}' = \mathbf{mX} + \mathbf{n}, \tag{A.4}$$

the resulting system may be written as

$$\mathbf{DX}' = \mathbf{aX}' + \mathbf{bY} + \mathbf{km} - \mathbf{an} \tag{A.5}$$

and

$$\mathbf{DY} = \mathbf{cX}' + \mathbf{dY} + \mathbf{h} - (\mathbf{c}/\mathbf{m})\mathbf{n}. \tag{A.6}$$

But if definition (A.4) is used to substitute into the system specified by equations (A.5) and (A.6), then the characteristic equation turns out to be identical to equation (A.3). Q.E.D.