CRIME AND PUNISHMENT: ARE ONE-SHOT, TWO-PERSON GAMES ENOUGH?

George Tsebelis argued in the March 1989 issue of this Review that decision theory is completely appropriate for analyzing games against nature but not appropriate for dissecting games against a rational opponent. Analysts who mistake a rational opponent for nature in constructing models commit what Tsebelis calls "the Robinson Crusoe fallacy." In this controversy, William Bianco and Peter Ordeshook attack components of Tsebelis's argument. Bianco believes the model should be set up as an iterated, rather than a one-shot, game. Ordeshook feels that proper modeling cannot rely merely on two-person games and, in addition, he argues that Tsebelis commits some technical errors. In his reply, Tsebelis joins the issues and buttresses his original analysis.

George Tsebelis (1989) shows that many analyses of crime and criminal behavior assume each actor chooses his strategy given fixed strategy choices for the police. As Tsebelis points out, it is more accurate to assume the players make simultaneous strategy choices. He then presents a one-shot game in which two players—the police and the public—choose their strategies simultaneously. In this game, changes in the severity of punishments have no effect on the incidence of crime but do cause police to become more lax in their enforcement of law. Similarly, increased rewards for not violating the law (welfare measures) reduce the amount of law enforcement but not the crime rate.

I focus on a critical assumption in Tsebelis' analysis. While he discusses the relationship between police and public in terms of repeated interactions, his results and empirical implications are derived from a single-shot game. Tsebelis' model is therefore a simplification of the phenomena he is modeling. A citizen's decision to speed is not made only once; rather, it is a series of decisions made over time, whenever the citizen gets into a car. Similarly, a police officer must decide each day whether to enforce the law or not. Repeated interactions are also characteristic of the other relationships cited by Tsebelis: government regulators and regulated industries, leaders and subordinates in a hierarchy, and congressmen involved in an oversight relationship with an agency.

Results such as the Folk Theorem (Fudenberg and Maskin 1986; for similar results see Axelrod 1984) suggest it may be inappropriate to model iterated situations as one-shot games. In an iterated game, players can coordinate their strategies to produce mutually beneficial outcomes that are not equilibria in a one-shot version of the game. These agreements are sustained by the use of "trigger strategies" (Friedman 1971) whereby a player abides by the agreement as long as all other players do so. This behavior can be seen as the abstract equivalent of phenomena such as "trust," "norms," or "reputation." However, such phenomena cannot occur when players interact only once. Given the effect of iteration and the
fact that real-world interactions between the police and the public are repeated, it seems essential to generalize Tsebelis’ one-shot results to the iterated case.

The changes produced by iterating the police-public game are striking. In the one-shot game, the only equilibrium outcome is one where players use mixed strategies. However, in some iterated versions of the game, pure strategy outcomes can be sustained as subgame perfect equilibria (Selten 1975) if players do not discount future payoffs “too much” (to be made more precise later). In an iterated game analyzed here, the outcome where the public does not break the law and police do not enforce the law can be produced as an equilibrium if players use appropriate strategies. In this game, welfare measures make it easier to sustain this no-crime outcome, rather than having no effect, as in the one-shot game (Tsebelis 1989, Theorem 4).

In sum, I agree with Tsebelis that decision theory should not be used to model game-theoretic situations. However I do not share his view that a one-shot game provides insight into the forces driving real-world crime and law enforcement.

Table 1 gives the one-shot, two-player, police-public game presented in Tsebelis 1989. In this game, the public chooses to speed or not to speed; the police choose to enforce the law or not (i.e., enforce or shirk). By construction, the public prefers to speed if the police shirk but not to speed if the police enforce the law. The police prefer to enforce if the public speeds but to shirk if the public does not speed. These assumptions are specified in terms of restrictions on the player’s payoffs:

Public \( c_1 > a_1, \) or \( 3 > 2 \)
\( b_1 > d_1, \) or \( 7 > 5 \)

Police \( a_2 > b_2, \) or \( 7 > 6 \)
\( d_2 > c_2, \) or \( 7 > 4 \)

Given these restrictions, Tsebelis shows (Theorem 1) that the only equilibrium outcome in this game is a mixed strategy outcome in which

\[ p^* = (d_2 - c_2)/(a_2 - b_2 + d_2 - c_2), \] (1)

where \( p^* \) is the probability that the public speeds; and

\[ q^* = (b_1 - d_1)/(b_1 - d_1 + c_1 - a_1), \] (2)

where \( q^* \) is the probability that the police enforce. The probability that the public does not speed equals \((1 - p^*),\) while the probability that the police shirk equals \((1 - q^*).\) Given the payoffs in Table 1, the equilibrium mixed strategies are \( p^* = .75 \) and \( q^* = .66; \) that is, in equilibrium the public speeds with probability .75 and the police enforce the law with probability .66, yielding expected payoffs of 3.63 for the public and 6.25 for the police. Given this equilibrium, Tsebelis shows that increases in the penalty for being caught speeding (i.e., a decrease in \( a_1 \)) have no effect on the likelihood that the public chooses to speed (Theorem 2). The reason is that the increased penalty increases the likelihood that the police shirk (Theorem 3). Similarly, welfare measures (increases in \( c_1 \) or \( d_1 \)) leave the crime rate unchanged but increase the likelihood of shirking by police (Theorems 4, 5).

Consider the effect of repeated play on police-public interactions. Suppose that the police and the public participate in an infinite series of one-shot games where the payoffs in each iteration \( t \) are given by Table 1. Thus, in the first iteration (itera-
Crime and Punishment

tion 0) the public chooses to speed or not to speed while the police choose to enforce or to shirk. Players then incur payoffs based on their strategy choices. On the second iteration, the players again face the same choices and possible payoffs. The series of one-shot interactions and strategy choices continue infinitely. Each player's payoff for the entire game is the (discounted) sum of its payoffs in each iteration of the game. Payoffs in iteration $t$ are discounted by $w^t$, where $0 < w < 1$. Finally, on any iteration $t$ players know the history of the game—the strategy choices made by each player in all previous iterations. These assumptions are standard features of iterated games (see Friedman 1986, chap. 3; Ordeshook 1986, chap. 3).

Tsebelis shows that if the police-public game is played once, the unique equilibrium mixed strategies are $p^* = .75$ and $q^* = .66$. However, the Folk Theorem suggests that if the game is iterated, other outcomes can be supported as equilibria. Consider the outcome where the public does not speed and the police shirk—the lower left-hand cell in Table 1. Both players receive a higher payoff in this outcome than they do in the mixed strategy outcome. For the public, $5 > 3.63$; for the police, $7 > 6.25$. This outcome is not an equilibrium if the game is played once. If the police shirk, the public prefers to speed, thereby increasing its payoff from 5 to 7. But what if the game is iterated? As I will now show, if $w$ (the discount rate) is "high enough," the players can choose strategies that yield the no-speed, no-enforce outcome as a subgame perfect equilibrium. In other words, by coordinating their strategies, the players can make themselves better off.

Suppose the players use the following trigger strategies. Label the police strategy as $e$-trigger ($e$ for enforce):

- **$e$-trigger $t = 0$:** Shirk.
- **$t > 0$:** Shirk if the public has not sped in all previous iterations. Use the mixed strategy $q^*$ otherwise.

That is, the police begin the game by shirking and shirk on subsequent iterations if the public does not speed. However, if the public ever speeds, the police respond by reverting to the mixed strategy where they enforce the law on each iteration with probability .66.7 Label the public's strategy as $n$-trigger ($n$ for not speed).

**$n$-trigger $t = 0$:** Do not speed.

- **$t > 0$:** Do not speed if the police have shirked in all previous iterations. Use the mixed strategy $p^*$ otherwise.

That is, the public begins the game by not speeding and continues to do so as long as the police shirk. If the police ever switch to enforcing the law, the public responds by using the mixed strategy $p^*$ on all subsequent iterations.

If the players use their trigger strategies, the no-speed, no-enforce outcome will be realized on each iteration of the game. These trigger strategies constitute a subgame perfect equilibrium if neither player prefers to switch from its trigger strategy to its maximin strategy (the strategy $p^*$ for the public and $q^*$ for the police), given that the other player responds by switching to its maximin strategy on subsequent iterations.

Consider first the public's incentive to switch. If both players use their trigger strategies, the public receives a payoff of 5 on each iteration, or a total payoff for the game of

$$5 + w^5(5) + w^6(5) + \ldots = 5/(1 - w).$$

If the public deviates from the agreement
with police and speeds on iteration 0 (or any iteration \( t \)) and plays its maximin strategy \( p^* \) thereafter, the police will respond by reverting to their maximin strategy \( q^* \) on the next iteration. In that case, the public receives a payoff of 7 initially and 3.63 on each iteration thereafter, or

\[
7 + w(3.63) + w^2(3.63) + w^3(3.63) + \ldots = 7 + w(3.63)/(1 - w).
\]

Therefore, if the police use their trigger strategy, the public prefers to use its trigger strategy rather than deviate if and only if

\[
5/(1 - w) \geq 7 + w(3.63)/(1 - w).
\]

After some algebra, this condition can be expressed in terms of a minimum value for \( w \): \( w \geq .59 \). Turning to the police, they receive a payoff of 7 on each iteration where they shirk and the public does not speed, or a total payoff of \( 7/(1 - w) \) if both players use their trigger strategies. If the police switch to enforcing the law and the public responds by reverting to \( p^* \) subsequently, the police receive a payoff of 4 initially and 6.25 on subsequent iterations, or a total payoff of \( 4 + w(6.25)/(1 - w) \). By inspection the police never have an incentive to deviate from an agreement to play trigger strategies: \( 7/(1 - w) > 4 + w(6.25)/(1 - w) \) for all \( w \).

Therefore, the no-crime, no-enforce outcome can be realized as a subgame perfect equilibrium in the iterated police-public game with payoffs if players use their trigger strategies and \( w \geq .59 \). In contrast, the only equilibrium to the one-shot game is one where players use mixed strategies. The difference between these results must be stressed. Analysis of the one-shot police-public game finds there is some probability in equilibrium that the public will speed and some probability that police will choose to enforce the law. However, in the iterated game examined here, if \( w \geq .59 \), the players are willing and able to coordinate their strategies to achieve an equilibrium outcome where the public does not speed and the police do not enforce the law.8

Adding iteration also modifies Tsebelis' conclusion about the effect of welfare measures. For example, suppose the public's reward for not speeding is increased: its payoff in the no-enforce, no-speed outcome is increased from 5 (as in Table 1) to 6. With this change, the player's equilibrium mixed strategies in the one-shot game are \( p^* = .75 \) and \( q^* = .5 \). If the players use these mixed strategies, the public receives a payoff of 4.5, while the police again receive a payoff of 6.25 on each iteration. However, if the players use trigger strategies to coordinate on the no-speed, no-enforce outcome, the public receives a payoff of 6/(1 - \( w \)). As a result, the public is better off coordinating with the police rather than deviating if \( 6/(1 - w) \geq 7 + w(4.5)/(1 - w) \), or \( w \geq .4 \). Remember that without the welfare measures, the value of \( w \) needed to sustain the no-speed, no-enforce outcome is .59. Therefore, in the iterated police-public game, welfare measures may reduce the crime rate by reducing the value of \( w \) needed to sustain an outcome where the public does not speed. In contrast, welfare measures have no effect on the equilibrium crime rate in the single-shot game (Tsebelis 1989, Theorem 4).

Tsebelis is exactly right in saying that previous analyses of criminal behavior have erred in using decision theory to model a situation that is game-theoretic in nature. I have drawn on the theory of iterated games to demonstrate an equally important point: if players interact repeatedly, it is inappropriate to make predictions about their behavior by analyzing a single interaction. Tsebelis' results are based on a game where the police and the public interact exactly once. As I have shown, the behavioral
predictions made by Tsebelis may not hold in an iterated version of the game. Therefore, empirical implications drawn from Tsebelis' analysis may not hold in the real world, where the police and the public interact repeatedly over time.

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George Tsebelis offers two results about bimatrix games (Theorems 6 and 7), both of which are false. Moreover, by focusing on two-person games, he fosters an incorrect view of game theory's implications. It is well known of course, that assessments of public policy based on simple decision-theoretic perspectives that fail to accommodate the interdependence of individual decisions can be misleading. For example, consider this event: a first-string star quarterback is injured; but his replacement, after calling a few perfunctory running plays, proceeds to pass more frequently than the injured star. Such an event might seem paradoxical, but only if we assume that people maximize expected payoffs against opponents whose actions (defensive tactics) are invariant with changing circumstances (such as quarterback injuries). If we suppose that the opponent also adjusts to such change and that everyone takes everyone else's adjustments into account, game theory resolves the paradox. In our example, passing more frequently with a less proficient player at the helm becomes understandable if we learn that the best response of the defense is to strengthen the line against the run so as to leave the offense with little option other than to pass.

Accommodating interdependent choice in two-person contexts, though, yields another apparent paradox that is the centerpiece of Tsebelis' analysis. Specifically, we find that the calculation of one player's equilibrium mixed strategy depends only on the opponent's payoffs and vice versa, which suggests that if we wish to influence a person's mixed strategy, we should change the opponent's, rather than that person's, payoffs. If we are accustomed to thinking in decision-theoretic terms, this argument seems a strange disjuncture in reasoning. This disjuncture, though, is more apparent than real. First, the possibility of expressing a solution without reference to one's own payoffs arises only in two-person games. For games with $n > 2$ players, an equation stating the mixed equilibrium solution for one player must be formulated in terms of the remaining $n - 1$ players' strategies (thereby formalizing the game-theoretic reasoning that conditional on beliefs, each player chooses a strategy that is a best response to the strategies of everyone else); and, except for special cases, no equation can be solved without simultaneously solving for the strategies of all other players. In such simultaneous equation systems, all strategies are functions of each other. Thus, each strategy is a function of every player's payoffs. Only for two-player games are we guaranteed that the algebra of solutions reduces to the simple form upon which Tsebelis bases much of his analysis. Perhaps it is for this reason that game theorists find the algebra of two-person games to be little more than a curiosity rather than a basis on which to formulate public policy.

Second, the disjuncture identified by the algebra of two-person games is misleading even for that case because the connection with one's own payoffs is made through the concept of an equilibrium and the rationalizations for this concept's relevance. Thus, a particular strategy, identified by some algebraic identity, has behavioral meaning only because it is part of an equilibrium strategy pair and because the concept of an equilibrium is hypothesized to have meaning—and equilibrium pairs are a function of both persons' payoffs. Hence, it is only when we
Table 2. Mixed Equilibrium Strategy
Zero-Sum Game I

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<thead>
<tr>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$b_2$</th>
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</thead>
<tbody>
<tr>
<td>1, -1</td>
<td>-1, 1</td>
<td></td>
</tr>
<tr>
<td>-1, 1</td>
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<td>-2, 2</td>
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pursue the shorthand calculation of mixed strategy equilibria and disregard the rationale of our calculations that an apparent disjunction appears. Put simply, game theory is more than a set of algebraic relationships, and only when we take that theory as a whole can we give substantive interpretation to any piece of algebra. To suppose otherwise is to abuse that theory.

With respect, now, to technical errors, the first arises with respect to Theorem 6, which asserts, "If a two-player game has no pure strategy equilibria, it has a unique mixed strategy equilibrium." Aside from the fact that the essay, at this point, deals with $(n + 1) \times (n + 1)$ games, preconditions are unstated. However, the game's size is irrelevant, since we can expand a game without altering its strategic character by adding strategies that are strictly dominated by a preexisting strategy. Counterexamples to the asserted result, then, include the zero-sum game of Table 2. Row chooser has a mixed equilibrium strategy that gives positive probability only to $a_1$ and $a_2$, an equilibrium strategy that gives positive probability only to $a_3$ and $a_4$, a class of equilibrium strategies that gives positive weight to any three pure strategies, and another class that gives positive weight to all four of row chooser's pure strategies. Moreover, although column chooser has a unique mixed equilibrium strategy, $(1/2, 1/2)$, we can increase the number of equilibria for this player merely by reproducing columns $b_1$ and $b_2$ under the labels $b_3$ and $b_4$.

Tsebelis (p. 88) also asserts that games can be neatly subdivided into three classes: "(1) dominance solvable games and those with unique pure strategy equilibria, (2) games with multiple pure strategy equilibria, and (3) games without pure strategy equilibria (that have one, mixed strategy equilibrium)." However, in the following game, column chooser has an infinity of mixed equilibrium strategies $(1/2 \leq q^* \leq 4/5$, where $q^*$ is the equilibrium probability that column chooser selects $b_1$); whereas row chooser selects $a_2$ with certainty, so an equilibrium can be a mixture of pure and mixed strategies, as in Table 3.

So that we should not suppose that counterexamples are limited to zero-sum games, consider the four-by-four non-zero sum game in Table 4. This game has at least two disjoint mixed equilibria, $[(1/2, 1/2, 0, 0), (1/2, 1/2, 0, 0)]$ and $[(0, 0, 1/2, 1/2), (0, 0, 1/2, 1/2)]$ that yield distinct expected payoffs as outcomes.

Table 3. Mixed Equilibrium Strategy
Zero-Sum Game II

<table>
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<tr>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$b_2$</th>
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<tbody>
<tr>
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<td>4, -4</td>
<td></td>
</tr>
<tr>
<td>2, -2</td>
<td>7, -7</td>
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<td>6, -6</td>
<td>6, -6</td>
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Table 4. Mixed Equilibrium Strategy
Non-Zero Sum Game

<table>
<thead>
<tr>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
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<td>2, 4</td>
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<tr>
<td>-5, -5</td>
<td>-5, -5</td>
<td>4, 2</td>
<td>1, 8</td>
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<td>-5, -5</td>
<td>-5, -5</td>
<td>1, 8</td>
<td>4, 2</td>
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This example, moreover, also has a totally mixed equilibrium—a mixed strategy equilibrium that spans each player's entire strategy space—[(5/18,5/18,4/18,4/18), (15/62,15/62,16/62,16/62)]. (Notice that although this type of equilibrium is unique in this particular example, it is not unique in our first example.) In general, then, games can have a variety of mixed equilibria with different properties, and much of Tsebelis' discussion referring to the uniqueness of such equilibria requires restatement.

Problems also arise with respect to Theorem 7, which reads, "In a two-player game with no pure strategy equilibria, modification of the payoffs of one player will lead that player either to change the pure strategies that he or she mixed or to leave the mixed equilibrium strategy of the opponent." The clause "... will lead ... he or she mixed" is confusing; but a reasonable restatement is, "... will lead that player to change the set of pure strategies receiving nonzero probability from his or her mixed equilibrium strategy." This restatement presumes that we should first make certain that we do not change a player's payoffs in such a way as to render one or more strategies dominated, since then the successive elimination of dominated strategies can reveal a radical change in the game's strategic character. And although I do not otherwise quarrel with the assertion that one player's strategy will be unchanged by changes in the opponent's payoffs, the assertion that the opponent's strategy necessarily changes is false. For example, the theorem is false if we allow payoff changes in cells that occur with zero probability in equilibrium. And even if we suppose that mixed strategies span each player's strategy set, we must qualify matters by supposing that the theorem presupposes something other than linear monotonic changes, since solutions are invariant with such transformations (otherwise the rationalization for expected utility calculations is invalid). Nevertheless, the theorem remains untrue without additional refinement.

Consider Tsebelis' equation 1: \( p^* = (d - c)/(a - b + d - c), \) where \( p^* \) is the probability that player 1's first strategy is chosen in equilibrium and \( a, b, c, \) and \( d \) refer to player 2's payoffs. Simple manipulation yields \( k = (d - c)/(a - b), \) where \( k = p^*/(1 - p^*). \) Letting \( k \) be a constant, the equality is maintained if \( (a - b) \) and \( (d - c) \) are constant but we can change \( a \) and \( b \) or \( d \) and \( c. \) For example, if no pure strategy equilibrium exists, the mixed strategy solution for row chooser, \( (5/7,2/7), \) is the same in both of the games in Table 5, where the numbers in the cells correspond to column chooser's payoffs. We cannot keep \( k \) constant, of course, if we change only one variable; but the fact remains that a valid result requires additional constraints on allowed payoff changes. However, at this point formulating and rigorously proving a correct theorem seems more effort than it is worth; and limiting discussion to two-person games not only precludes general relevance but also can yield misleading inferences. In any event, once we acknowledge the importance of accommodating interdependent choices into our analyses, we can refer to the considerable literature that treats the mathematical properties of solutions to games.

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In my article on the “Robinson Crusoe Fallacy” I made the point that using decision theory in the place of game theory may lead to incorrect conclusions, such as the belief that modifying a player’s payoffs will modify its behavior. I used a two-by-two game to demonstrate that under certain conditions (the absence of pure strategy equilibrium), modification of one player’s payoffs will lead to a modification of its opponent’s strategy. The supporting game is not original and can be found in game theory textbooks (Luce and Raiffa 1957; Moulin 1981; Rasmusen 1989). The article’s originality lies with the following three characteristics of its main result:

1. The main result holds under a wide variety of model specifications. In particular, it holds in models that assume parametric as well as strategic rationality, models that assume sequential as well as simultaneous moves by the players, and models that assume complete as well as incomplete information (one- or two-sided). Given that game theoretic models are usually highly sensitive to institutional or informational modifications, this stability of the equilibrium solution is remarkable indeed.

Of all these modeling variations, I consider the one in which the players are assumed to possess parametric rationality to be the most realistic because it does not assume that the two players—the police and the public in my example—are unified or that they have capabilities of coordination and strategic calculation. Each member of the police and the public maximizes his or her own payoffs given what the others do, and the equilibrium emerges as the outcome of all these independent decisions.

2. The main result is not an artifact of the two-by-two game. It holds for any finite number of strategies used by two players.

3. The main result is relevant to a wide variety of situations, including crime, international economic sanctions, hierarchies, regulation, and, in general, situations involving compliance to authority and monitoring.

Bianco’s contribution to this controversy takes issue with item 1 by proposing another reasonable modification, leading to a different equilibrium. I find Bianco’s underlying assumptions highly restrictive; and I will demonstrate that even if these assumptions are followed to the letter, an additional game-theoretic refinement leads to my solution. Ordeshook challenges items 2–3. He first makes some technical remarks concerning the bimatrix games in my article. This part of his argument focuses on item 2. I welcome his finding of technical errors, but they are minor and easily corrected by small verbal modifications. Ordeshook then makes a more general and substantive argument concerning item 3. He claims that “by focusing on two-person games, that essay fosters an incorrect view of game theory’s implications.” The major part of my answer responds to this contention, and I show why the arguments I made in the Robinson Crusoe fallacy article neither are mathematical artifacts nor lead to partial and incorrect conclusions but instead exhibit remarkable stability even in models where one introduces third players or incomplete information.

Bianco claims that if one assumes that the game between criminals and the police is iterated instead of single-shot, application of the Folk Theorem of iterated games suggests that the players can arrive at equilibria other than the one I indicated, and with different properties. Bianco focuses on a particular equilibrium, one where there is no crime and the police do not monitor. Investigation of the robustness of my results for iterated games is intrinsically interesting and empirically important because in certain
situations (for example, small towns) the interaction between criminals and the police may resemble an iterated more than a single-shot game. I have no disagreement with Bianco’s findings. However, I do find his extension of my model less relevant as a realistic model of crime than it may appear.

First, Bianco’s argument follows. Consider the police and the public as unified players. By playing their mixed strategy equilibrium they forgo the gains of coordination. For example, they could agree not to violate the law and not to patrol unless one of them reneges, whereupon both would follow the strategies prescribed by the calculations of my article.

What are the behavioral consequences of such strategies? If coordination is possible, there is no crime and no enforcement. But if one player believes that the other will not forgo the gains of cooperation to punish a single deviation, it may deviate once to test his opponent. According to Bianco, crime will then have a frequency \( p^* \) and law enforcement \( q^* \), exactly the values I gave in my article. Or if one player makes a mistake, the outcome that Bianco prescribes is the same as mine. Similarly, if a stranger comes into town and breaks the speed limit, the frequencies of crime and law enforcement presented in my initial article will prevail as a result of Bianco’s argument, not mine.

The point is that if coordination among all the individuals comprising the two collective players fails, my equilibrium is likely to prevail. His solution (no crime and no enforcement) requires coordination among all individual players. How likely is this coordination? Is it a reasonable approximation of reality? The question of the realism of Bianco’s argument seems to me very serious. It cannot, however, be raised about my model, because, as I show, evolutionary arguments (which make no assumption of strategic calculation or coordination) lead to the same equilibrium as the one-shot game where the players are unified and rational.

Second, even if one makes the heroic assumption that coordination is possible, there are additional (more technical) problems with Bianco’s argument. Bianco presents a numerical example leading to the equilibrium no crime and no monitoring \( (d_1,d_2) \) in the notation of my article). He notes that his result “should not be read too closely” (n. 8) and that different numerical examples would lead to different equilibria.

The reason that this equilibrium is possible in the iterated game is that it Pareto dominates the single-shot game equilibrium; that is, it is to the mutual advantage of both (collective) players. Besides assumptions A1–A4 of my article, Bianco makes two auxiliary, implicit, and crucial assumptions in his numerical example: \( d_1 > c_1 \) and \( d_2 > b_2 \). By modifying these new assumptions, one can present any particular outcome of the initial game as an equilibrium in the iterated game. For example, if one makes the auxiliary assumptions that \( b_1 > a_1 \) and \( b_2 > d_2 \), \((b_1,b_2)\) results as the equilibrium. If \( c_1 > d_1 \) and \( c_2 > a_2 \) are the auxiliary assumptions, \((c_1,c_2)\) results as the equilibrium. Even the result of permanent violations and permanent enforcement \((a_1,a_2)\) can emerge as an equilibrium outcome of an iterated game under the (implausible) auxiliary assumptions \( a_1 > b_1 \) and \( a_2 > c_2 \).

But regardless of auxiliary assumptions, the Folk Theorem leads us to expect an infinity of equilibria (Fudenberg and Maskin 1986). For example, if no crime and no enforcement is an equilibrium in the iterated game, so are all the outcomes where the public violates the law and the police enforce the law with any frequency smaller than the equilibrium frequencies in the one-shot game \((p^*,q^*)\). The Folk Theorem is a possibility theorem. It says that practically any outcome is possible. As a result, it is a very poor guide for prediction when used alone.
This is why game theorists have offered additional refinements of the concept of (perfect) equilibria in iterated games. One such refinement is the concept of stationary equilibrium, which leads to the same behavior in each iteration of the game. As a result, stationary equilibria are the natural focal points when new players participate in the game. But in the police-public game, the equilibrium strategy pair in the single-shot game is the unique stationary equilibrium in the iterated game.

Ordeshook's technical points dispute item 2, that the analysis of the two-by-two game can be generalized for any finite number of strategies of the two players. I agree with Ordeshook's numerical examples and with their conclusions but question their relevance to his main point.

Let me start with an example. Consider the statement that a change in distance or time yields a change in velocity, velocity being distance divided by time. This statement is technically false because one can double both distance and time, thereby changing both, without changing velocity. However, the statement is true the rest of the time (that is, almost always), and any argument including it needs very little substantive modification to prevent falsification along these lines. Ordeshook's last example is exactly of this kind (it holds only because of the specific numerical values he uses); and the rest of his numerical examples share similar features, that is, technically they falsify some of my statements but leave them largely intact substantively.

Ordeshook is correct that a game without pure strategy equilibrium has, in general, multiple mixed strategy equilibria (see Harsanyi 1973b). However, all these mixed strategy equilibria, contrary to Ordeshook's assertion that they "have different properties," share a common property that lies at the heart of my argument; namely, each player's equilibrium strategies depend on its opponent's payoffs. As a result, a change in one player's payoffs will in general either destroy the mixed strategy equilibrium or lead the opponent to modify its strategy (as I demonstrated in my article). It is also possible on occasion that changes in the payoffs may be of a special numerical value such that even the opponent will not have to modify its strategy (changes in the payoffs of strategies mixed with probability zero or linear transformation of all payoffs of one player, to use Ordeshook's examples). However, Ordeshook's examples rely on particular numerical modifications and do not affect the general validity of my argument.

Concerning the partition of two-person games, Ordeshook presents an interesting example with an infinity of mixed strategy equilibria. This example is, again, exceptional. In game-theoretic terms, the equilibria Ordeshook presents are irregular. However, it can be shown that in "almost all normal form games all equilibria are regular (hence, quasi-strict, strongly stable, essential, isolated, strictly proper, proper and perfect)" (Damme 1987, 41). Damme explains why irregular equilibria are exceptional: "The existence of an irregular equilibrium entails a special numerical relationship among the payoffs of the game, and this relationship can be disturbed by perturbing the payoffs of the game slightly."

Consequently, my classification holds for "almost all" normal form games. "Almost all" normal form games can be partitioned into games with (1) multiple pure strategy equilibria, (2) one pure strategy equilibrium, or (3) no pure strategy equilibrium (which, as Ordeshook points out, will have one or more mixed strategy equilibria).

Thus, Ordeshook's technical points are correct. However, because of their exceptional character, they do not modify the substance of item 2, that the Robinson Crusoe fallacy is not an artifact of a two-by-two game. His major point, however, is that because my results are derived from two-person games they are "little
Crime and Punishment

more than a curiosity rather than a basis on which to formulate public policy." If correct, this assertion would be fatal for my argument.16

How reasonable is a two-person game as a modeling tool to understand social or political situations? It obviously depends on the situation one wants to model. It also depends on the state of the art in a particular field, that is, on the other tools available. In my article I claimed that the Robinson Crusoe fallacy was relevant to cases such as international economic sanctions or regulation, where the number of players is either actually two or very restricted and where, as a result, such an abstraction is legitimate. Ordeshook does not address this point. Instead, he focuses on the model's application to the problem of crime, where the large number of actors involved makes the assumption of a two-person game more problematic. Though partial (that is, addressing only one of the applications of my argument), his question is justified by the social importance of the crime issue. Before proceeding, however, let me clarify that the state of the art in the field is such that the alternative to my two-person game approach to crime is not Ordeshook's multiperson game approach but a one-actor decision-making model, which arguably is not a step forward but a step backward.

By way of a positive defense of my model, I shall introduce some additional actors, such as the legislature and the courts. I shall show that under reasonable modeling assumptions in this multiperson game the logic of my model remains unchanged, contrary to Ordeshook's expectations. In particular, I show that the modification of the payoffs of one player does not affect its behavior but (in general) induces changes in the equilibrium behavior of his opponent. I shall then complicate the model further and introduce even more actors under the form of different types of police and of publics. I shall use games with incomplete information to show that the conclusions of my initial article hold not only qualitatively (the equilibrium has the same properties as the equilibrium of the initial article) but also quantitatively for the particular model I present here; that is, the equilibrium is exactly the same as in the initial model. Finally, I shall use this incomplete information model to show how the two-person game approach can generate further interesting insights into political and social situations. I hope that this evidence will put Ordeshook's objections to rest.

Introducing Third Players: The Legislature or the Courts

The police-public game is part of a more complex game where a third player, the legislature, makes the rules. I will introduce the legislature into my model as an explicit third player and show that contrary to Ordeshook's expectations, both its logic and conclusions remain intact.

The role of the legislature is to vote on different laws regulating penalties for illegal activities (and possibly the rewards of legal activities) and to create an incentive system for the police to enforce the law. In game-theoretic terms, the legislature ex ante decides the payoffs of the police and the public. Figure 1 represents a more complicated game where the legislature moves first and selects between two different sets of payoffs for the police and the public or sets the rules of the police-public game. In normal form this game can be represented by a two-by-four-by-four table, where the legislature has to choose between two strategies while the police and the public have four strategies each.17

This is a three-player game and, in general, as Ordeshook says, one should expect that modification of the payoffs of one player affects its behavior as well as that of the other players in the game. However, additional reflection suggests that this game has a particular form due
to the fact that the legislature moves first, setting the rules for the remainder of the game, whereas the public and the police move only after they know these rules. For this game, application of standard game-theoretic rules leads to the following solution (following Selten 1975). The legislature knows that whatever its choices of rules, the police and the public will play the equilibrium strategy pair calculated in my Robinson Crusoe article. It can therefore calculate its costs and benefits from each set of rules and select whatever rules are optimal for itself. Subsequently, the police and the public will play whichever game the legislature selected, as prescribed in my article. Therefore, in this three-person game, modification of the public’s payoffs will (in general) modify the equilibrium behavior of the police, and vice versa.

Note that this is not the only equilibrium of the game; and for the remaining mixed strategy equilibria Ordeshook is right—modifying the payoffs of the public or of the police will (in general) have consequences for the behavior of both players. However, in the presence of multiple equilibria standard game-theoretic techniques require one to search for an equilibrium with additional properties of stability. The equilibrium I have calculated is the only subgame perfect equilibrium of the three-player game, and it has exactly the properties described by the main result in my article. What drives this more complicated three-person game to the same outcome as the simpler two-person game is the fact that the third player moves before the other two.¹⁸

The same arguments can be replicated if one brings in other players who move after the police and the public have selected their strategies: lawyers, courts,
Crime and Punishment

Figure 2. Game between Public and Police: Incomplete Information (each player has two possible types)

Assumptions: \( c_1 > \alpha_1, \ v_1 > b_1, \ d_1 > c_1 > e_2 \)

Dominant strategies are indicated by heavy lines

the prison system, and so on. Again, application of the appropriate game-theoretic concepts leads to the results presented in my article. As long as the dyadic interaction of the police and the public remains intact and the other players move either before or after those two players, my results hold. The next question is, of course, what happens if other actors move simultaneously with the two main players?

Introducing Different Types of Police and Publics:
The “Naive” and the “Sophisticated”

I cannot see how any other distinct player could participate simultaneously in the police-public game. However, a more realistic representation would make use of different kinds (or “types”) of police agents or members of the public participating in the game. These different members of the public and the police have different payoffs and when they interact with each other they receive the payoffs corresponding to their mutual choice of strategy and their type. As a first step in this approach, imagine that there are two types of public: good citizens, who never violate the law, and bad ones, who prefer to violate the law when the police are not around. The first have payoffs making compliance with the law the dominant strategy; I will call them “naive.” The second have payoffs similar to those in my Robinson Crusoe article; I will call them “sophisticated.” Similarly, imagine that there are two types of police agents: “good” ones, who always monitor, and “bad” ones, who monitor only when the public violates the law. The first have payoffs making monitoring the dominant strategy; I will call them “naive.” The second have payoffs like the police in my Robinson Crusoe article; I will call them “sophisticated.” Imagine also that each member of the public and each police agent knows his or her own payoffs (his or her “type”) and the proportion of naive.
and sophisticated players in the general population but does not know the type of opponent being faced.

Figure 2 represents a game of incomplete information modeling the situation I have just described. A fictitious player ("nature") selects whether a particular member of the public will be naive (with probability \( p_{g1} \)) or sophisticated (with probability \( 1 - p_{g1} \)). Similarly, nature chooses whether a particular police agent will be naive (with probability \( p_{g2} \)) or sophisticated (with probability \( 1 - p_{g2} \)). Each player then chooses one of two strategies.19 The payoff for each player associated with each outcome is found at the final nodes of the game tree, and the order of these payoffs is presented in the margin of Figure 2.

Although there are two different police agents and two different members of the public, the use of incomplete information makes the game in Figure 2 technically a two-person game with the same property as the simple two-by-two game presented in my article: modification of one player's payoffs induces the other to change equilibrium strategies (in general).

In the two-person game in Figure 2 each player has four strategies, so that the game has multiple mixed strategy equilibria. However, in only one of these equilibria the players do not use dominated strategies. For this equilibrium, the naive players (police and public) always use their dominant strategies, while the sophisticated members of the public violate the law with frequency

\[
p^* = \frac{(d_2 - c_2)}{[1 - p_{g1}] - (d_2 - c_2 + a_2 - b_2)}
\]

(3)

and the sophisticated members of the police monitor with frequency

\[
q^* = \frac{(b_1 - d_1) - [p_{g2}(b_1 - d_1 + c_1 - a_1)]}{[1 - p_{g2}] - (b_1 - d_1 + c_1 - d_1)}.
\]

(4)

Observation of equations (3) and (4) leads to the conclusion that in this (more complicated and more realistic) game, too, modification of the payoffs of one player leads the opponent to modify strategy (in general). This is a general property: as long as the number of police agents and members of the public is finite, the equilibrium strategy pairs will exhibit this property.21

Comparison of equations (3) and (4) with the equilibrium strategy pair of the simple two-by-two game in my article (see Bianco's equations 1-2) leads to the conclusion that the two games have exactly the same equilibria. In other words, as long as sophisticated players use mixed strategies, the frequency of crime and of law enforcement that the two models predict is exactly the same!22 Although this identity of equilibria is not a general property, it generates some interesting insights.

The Congestion Effect

The reason that the equilibria of the two models (that with incomplete information and that presented in the initial article) are identical follows. Since by assumption the naive players have a dominant strategy, they will select this strategy regardless of other considerations. The sophisticated types, however, calculate their optimal strategy as a function of the optimal (equilibrium) strategy of their opponent, as well as the frequency of their own and their opponent's types. So, at equilibrium, the frequency of violation of the law is a function not only of the payoffs of the opponent but also of the frequencies of the different types of players. Simple observation of equations (3) and (4) suggests that the more law-abiding citizens there are, the more the non-law-abiding feel free to violate the law. Similarly, the more reliable police agents there are, the more the other police agents will free ride.

The conclusion of this brief discussion
is that the existence of identical police agents and members of the public (i.e., having identical payoffs) is not a necessary condition for the results I presented in my article. Even if parts of the public and the police have a dominant strategy and only one part of the players have the order of payoffs I presented there, the equilibrium behavior of the players will approximate the equilibrium calculated in my article.

I have often heard the following objection to the supposed lack of realism of the Robinson Crusoe fallacy assumptions: "I would not commit robbery even if I knew that the police were not around" or "Not all police agents stop patrolling if there is no crime." Such objections do not directly address the arguments made in the article, since I do not claim that all members of the public or all police agents have the preferences described by inequalities A1–A4. However, the incomplete information model presented here answers such objections directly. It does not matter whether all members of the public or all police agents can be described by A1–A4. As long as there is a sufficient number of them, the equilibrium behavior will be the same.

In a seminal article, Haltiwanger and Waldman (1985) examine what they call congestion effects, that is, situations where each person is made worse off, the greater the number of others who make the same choice. They prove that in such cases the existence of some sophisticated participants (that is, those who can on average anticipate outcomes) will produce the same outcome as if all participants were sophisticated. Haltiwanger and Waldman use the job market as an example of their argument: if some agents can correctly anticipate that there will be an excess of doctors in the years to come, they will choose other professions, generating an equilibrium approximating the situation where all agents could guess the future correctly.

Crime exhibits properties similar to those characteristic of congestion effects. The two types of police and public in my example do not differ in their predictive capacities but rather in their capacities to adopt different strategies: one type of each player can adopt only one strategy, while the other can make a choice according to prevailing conditions. And it is this ability to engage in strategic choice that drives the more complicated model with incomplete information to the same equilibrium outcome as the simple two-by-two model.

The previous discussion suggests that two-person games, if used appropriately, are not mathematical curiosities irrelevant for policy, as Ordeshook claims, but flexible and productive intellectual tools. Moreover, Ordeshook's claim that "limiting discussion to two-person games not only precludes general relevance but also can yield misleading inferences" is not relevant to my analysis. First, it is not relevant because two-person games may be subgames of more complicated multi-person games; in this case, any subgame perfect equilibrium by definition prescribes equilibrium behavior in the subgame. Second, it is not relevant because by using incomplete information games with more than two physical players can sometimes be represented as two-person games.

These are some of the reasons why, contrary to Ordeshook's assertions, two-person games have been used by game theorists for analysis of complicated situations and policy prescriptions. Examples include tax policies (Graetz, Reinganum, and Wilde 1986), regulation (Baron and Myerson 1982) and government policy (Kydland and Prescott 1977).

To conclude, Bianco's and Ordeshook's helpful criticisms and thoughtful objections suggest some additional modeling possibilities that I had not considered and highlight some new implications of the Robinson Crusoe fallacy article. In particular, Bianco's comments led me to see
that the equilibrium I had calculated is also the only stationary equilibrium of an iterated game; and Ordeshoke's remarks, along with pointing out some mistakes and exceptions to some of my claims, helped me realize that the initial simple model had more applications than I had anticipated.

However, it would be a mistake to conclude that all game-theoretic models of crime would necessarily lead to the same result as that presented in the Robinson Crusoe article. Additional modifications and complications that increased the realism of the model could alter its conclusions, just as my argument modifies the conclusions of the economic approach to crime.

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Notes

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1. For a notable exception, see Hamburger 1979, 71–74.

2. See especially his discussion of how the players will respond in the short run and the long run to changes in their payoffs (Tsebelis 1989, 82).

3. According to Friedman (1986, 103), the Folk Theorem is due to Robert Aumann. Aumann (1981) suggested that since the late 1960s, game theorists have believed that any individually rational outcome could be supported as an equilibrium, given no discounting of future payoffs. Aumann labeled this result as the Folk Theorem because its source was not known. Subsequent work (summarized in Fudenberg and Maskin 1986) has proved the result and generalized it to iterated games with discounting.

4. For a discussion of whether this claim is generally true for bimatrix games with no pure strategy equilibria, see Ordeshock's contribution to this controversy.

5. Games of infinite iterations with discounting are mathematically identical but analytically more tractable than games where players interact for an indefinite number of iterations. It is the latter case that corresponds to real-world interactions between police and public.

6. This information requirement is nontrivial but does not seem terribly severe. In real-world terms, one might imagine that the public can see whether the police are enforcing the law or shirking by observing whether the police are parked beside the road, radar guns in hand, or outside the local coffee shop. Similarly, shirking police can determine whether citizens are breaking the law (at least in qualitative terms) by noting the speed of cars as they drive by the coffee shop.

7. The Folk Theorem suggests a focus on the trigger strategy where any deviation from coordination triggers "permanent retaliation" against the player who deviates. Intuitively, if this threat does not deter defection, no other threat can do so (for a further discussion, see Friedman 1971).

8. This result should not be read too closely. Player's single-iteration payoffs can be specified such that no outcome dominates the outcome where players use their mixed strategies p* and q* on each iteration. Some configurations of payoffs allow additional outcomes to be realized as equilibria in the iterated game: if $a_1 = 2$, $a_2 = 8$, $b_1 = b_2 = 7$, $c_1 = 3$, $c_2 = 2$, $d_1 = 6$, and $d_2 = 3$, the outcome where the citizen speeds and the policeman shirks on each iteration is an equilibrium, given appropriate strategies and a high enough $w$. Under incomplete information, other outcomes might be realized as equilibria in the iterated game given "reputation effects" (Kreps and Wilson 1982).

9. In the single-shot game, the public gets $EU_p = (b_1c_1 - a_1d_1)/(b_1 - d_1 + c_1 - a_1)$, and the police get $EU_p = (a_2d_2 - b_2c_2)/(a_2 - b_2 + d_2 - c_2)$.

10. The reader can verify that under these assumptions and the assumptions A1–A4 of my article, $d_1$ and $d_2$ are greater than the equilibrium payoffs for the single-shot game (see n. 8).

11. The first equilibrium prescribes permanent crime and no enforcement and might be reasonable for crimes like jaywalking: the second equilibrium prescribes the permanent enforcement and no crime and might be reasonable for crimes like murder.

12. More precisely, in "every structurally equivalent subgame" (see Baron and Ferejohn n.d.). For a general discussion and dispute of Folk theorem arguments, see Guth, Leininger, and Stephan 1988. Their arguments lead to the adoption of stationary equilibria as the appropriate solution concept in iterated games.

13. The equilibria are irregular because they are not isolated; that is, one can find other equilibria in the neighborhood of each.

14. The definitions of these terms can be found in Damme 1987 and are not essential here. However, in a deeper sense, these properties of stability of Nash equilibria in normal form games are the reason why the different models I presented in my article lead to the same equilibrium. It has been shown that Nash equilibria are the only rational solutions to simultaneous games (Bacharach 1987; for further discussion see Tsebelis 1990b, chap. 2). Therefore, if the
Crime and Punishment

only equilibria of a game are in mixed strategies, they will (in general) exhibit all the desirable properties of stability. For example, since they are regular, they can be shown to be the limit equilibria in games with incomplete information and perturbed payoffs (Harsanyi 1973a). Similarly, they are likely to be evolutionary stable.

15. In more precise terms, from all games with pure strategy spaces $S_1, \ldots, S_n$, the set with an irregular equilibrium is closed with Lebesgue measure zero (see Harsanyi 1973b or Damme 1987, 42-43 for the proof). In plain English, the probability that a two-by-three game chosen at random will have an equilibrium like the one in Ordeshook’s example is zero.

16. Ordeshook makes one additional point concerning the necessity to “take [game theory] as a whole” in order to “give substantive interpretation to any piece of algebra.” I assume he means that equilibrium strategies have to be considered in pairs. I agree with him, but I cannot see what he finds objectionable in my treatment. I compare two games with the same payoffs except for the size of the penalty ($a_i$) and observe that the difference between their equilibria is the frequency of police enforcement, not the frequency of crime. This is standard comparative statics analysis.

17. The public can choose to violate the law regardless of the choice of the legislature, can violate the first set of rules but respect the second, respect the first set of rules but violate the second, or always respect the law. Similarly, the police have the choice among four strategies.

18. See Selten 1975 and Myerson 1978. As a result, each of the initial two-person games technically a subgame of the three-person game, and the subgame perfect equilibrium concept is applicable.

19. “Good” police agents always choose to monitor and “good” citizens always choose to abide by the law.

20. To derive these two equations one has to write down the expected utility of each player as a function of the payoffs and the probabilities $p$ (that the sophisticated member of the public violates the law) and $q$ (that the sophisticated police agent monitors) and set the first derivatives with respect to $p$ (for the public) and $q$ (for the police) to zero (Tsebelis, n.d.a).

21. For an infinite number of types of players, this proposition is not true. For the solution of a similar problem where the number of types is infinite and the different types follow a uniform distribution, see Tsebelis 1990a.

22. The reader can verify this statement by calculating the overall frequency of violation of the law as the product of the frequency of sophisticated members of the public and the frequency that this sophisticated public violates the law (equation 3). Similarly, the frequency of monitoring is equal to the proportion of naive police agents and the proportion of sophisticated police and the frequency of monitoring (equation 4).

References


