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Existing approaches consider crime as either the consequence of antecedent social conditions or the outcome of a rational calculation by a predator who chooses crime as a utility-maximizing career. Consequently, these approaches propose either an improvement in social conditions or an increase in penalties as a means to reduce criminal activity. This article adopts a game-theoretic framework and examines crime as a game between criminals and the police. On realistic grounds, this approach represents an improvement on the conventional economic analysis of crime, concluding that under a wide variety of conditions (described by axioms 1 through 4), an increase in the severity of the penalty has no impact on criminal behavior at equilibrium. In fact, an increase in the penalty reduces the frequency of law enforcement at equilibrium. The major policy implication is that in order to reduce criminal activity at equilibrium, one has to modify the payoff structure of the police. Several variations of the model are examined.

Penalty Has No Impact on Crime:

A GAME-THEORETIC ANALYSIS

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Does the death penalty reduce crime? The question has important policy implications: If capital punishment has no deterrent effect, most people would presumably prefer its elimination. More generally, do severe penalties help reduce crime? Should penalties be increased in order to protect law-abiding citizens from criminals?

The answers to these factual and normative questions divide the experts and the public along different dimensions. The public is divided politically with respect to these questions since there is a high correlation between an

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affirmative answer and conservative ideology, on one hand, and a negative answer and liberal ideology, on the other. In addition to political sensitivities, the experts are divided professionally: Economists usually answer these questions in the affirmative, while sociologists are more reserved. Finally, viewing the subject methodologically, there is a correlation between an affirmative answer and intentional explanations in the social sciences and a negative answer and causal explanations.

This article will be concerned with the last two dimensions of disagreement among the experts, namely, the professional and the methodological. However, as the title indicates, the article has broad policy (and political) implications.

Methodologically, crime (like other social phenomena) can be explained in two ways: First, it can be attributed to previously existing factors which operate on the predator (e.g., socialization) or the prey (e.g., wealth), or both (e.g., social inequalities); and second, it can be a matter of choice by an individual who is more or less rational (the predator) in specific circumstances. I will call the first kind of explanation a *causal* explanation of crime, and the second an *intentional* explanation.[1] The two approaches lead to diametrically opposed policy prescriptions because, as the psychiatrist Gaylin (1982, 253) claimed: "If an act is not a choice but merely the inevitable product of a series of past experiences, a man can be no more guilty of a crime than he is guilty of an abscess." However, these differences in approach are methodological: Both approaches are partially correct and not mutually exclusive since crime is the consequence of choice by rational individuals who are socially influenced. A more complete approach where both causal (social and environmental) and intentional (rational calculation) factors are taken into account is presented by Wilson and Herrnstein (1985).

The intentional approach considers crime like any other career. Such an approach suggests that an increase in the penalty, which supposedly decreases the expected utility from crime, will reduce its frequency. The causal approach considers crime a consequence of antecedent conditions (social background) and suggests the improvement of living conditions (mainly in poor neighborhoods) as the remedy.

Professionally, economists are the prominent advocates of intentional explanations, while most (but by no means all) sociologists advocate causal explanations. Thus, in the remainder of the article, the intentional approach to crime will be termed the economic approach (EA), and the causal approach, the sociological approach (SA). The reader will discover that the

game-theoretic approach that I propose leads to different results from both of these approaches and from the synthetic approach of Wilson and Herrnstein.

Such disagreements could be resolved through empirical analysis: use of the appropriate statistical techniques to test whether the coefficients of the relevant variables have the correct sign and are statistically significant. Unfortunately, reality is very complicated and our tests have not been conclusive. For example, with respect to the deterrent effect of the death penalty, Leamer (1983) examined the empirical evidence in favor of five different theories which he called "right-winger," "rational maximizer," "eye-for-an-eye," "bleeding heart," and "crime of passion," finding that sometimes the effect of the death penalty on crime is positive! The theories differ with respect to which variables are included a priori in the regression as relevant. In another work, Leamer (1982) found that a narrower set of priors still leads to inconclusive inferences. Similarly, Blumstein, Cohen, and Nagin (1978), reviewing the empirical literature on the deterrent effect of penalty on crime for the National Academy of Sciences, concluded:

> Yet, despite the intensity of the research effort, the empirical evidence is still not sufficient for providing a rigorous confirmation of the existence of a deterrent effect. Perhaps more important, the evidence is woefully inadequate for providing a good estimate of the magnitude of whatever effect may exist. . . . Any unequivocal policy conclusion is simply not supported by valid evidence.[2]

This article provides a theoretical answer to the question," Is there a reason why the empirical evidence is inconclusive, besides external noise?" The article is organized in four parts. First, the two different approaches are presented along with an evaluation of the empirical evidence. Both approaches present important problems at the theoretical level, and the empirical evidence in their favor is weak and/or contradictory. These problems will lead to the second part of the article in which a game-theoretic approach will be presented to account for the theoretical and empirical problems of the previous approaches. This approach leads to the conclusion that an increase in the size of penalties (as the EA advocates) or an improvement in societal conditions (as the SA suggests) will not influence the long-run frequency of crime; on the contrary, they will reduce the frequency of law enforcement by the police. The third part of the article discusses different applications and variations of the model. The fourth and final part summarizes the differences between the conventional wisdom and the game-theoretical approach by indicating why they differ. New directions for further research will be offered as well.

I. THE SOCIOLOGICAL AND THE ECONOMIC APPROACHES TO CRIME

THE SOCIOLOGICAL APPROACH

Although there is a methodological similarity among different sociological theories, there is little common ground among them as far as the causes of crime are concerned. For example, in his review of the relevant literature, Schafer (1969, 255) concluded that "hardly any of the thinkers of the causes of criminality omitted poverty or economic conditions from their catalog of crime factors." However, a negative relationship between socioeconomic status and crime has been disputed (Tittle, Villemez, and Smith 1978, 1982).

Without claiming to be exhaustive, one can find the following sociological theories of crime: anomie,[3] socialization/psychodynamic,[4] subcultural,[5] differential association,[6] community ecological,[7] and Marxist.[8] From this summary account, it becomes obvious that although the causes of crime may be of a wide variety (i.e., psychological, social, economic, or other kinds of deprivation), they all exist prior to the criminal behavior, which can be eliminated *if and only if* these causes are removed.

THE ECONOMIC APPROACH

The economic approach is more unified than the sociological one because of the rationality assumption on which it is based. Crime is considered as a career, subject to the same cost/benefit analysis as any other.[9] Individuals maximize their expected utilities from different choices under social, economic, and other constraints. For example, the predator compares his expected gains minus his expected costs from different careers, choosing crime if it maximizes this difference. According to this approach (as a first approximation), the expected loss from crime is the probability of punishment multiplied by the punishment (Becker 1970; Stigler 1973). Hence "increasing the punishment would seem always to increase the deterrence" (Stigler 1973, 526; see also Gibbs 1975). Some empirical studies confirm these expectations (see Ehrlich 1973; Sjoquist 1973).

However, other empirical studies (Logan 1972; Jones 1973) have indicated that while there is a negative relationship between the probability of imprisonment and the crime rate, the relationship between severity of the penalty and the crime rate is more problematic (either curvilinear, nonsignificant, or even of the opposite sign).[10] These findings led to the revision

of the theory in two different directions: The first was risk aversion (Becker 1970) and the second was interactive effects (Greenwood and Wadycki 1973; McPheters and Stronge 1974; Ehrlich and Brower 1987).

The risk aversion modification makes the claim that criminals respond more to changes in the probability of arrest than to changes in penalties. However, for this modification to work, at least one of two conditions must be met: Either a theory of risk-aversion which would predict under what conditions people are expected to be risk-averse or an independent empirical assessment of criminals' risk aversion is required. In the absence of either of these conditions, invoking risk aversion to explain criminal behavior leads to a circular argument.

The interactive effects modification is theoretically less ambitious but more empirically accurate. Although punishment (both in terms of probabilities and payoffs) deters crime as the economic theory predicts, at the same time once crime increases, it becomes likely that punishment will be raised to meet that threat. In Ehrlich's (1977, 771) terms: "The tendency of state courts, juries, and other relevant authorities to retain or impose the death penalty is expected to be responsive to the perceived social cost of capital crimes." We can see that these theories replace a causal relationship (crime is a function of punishment) with an interactive one (crime is a function of punishment, but punishment is also a function of crime). Empirical estimations of models with simultaneous equations lead to the expected results. But the theoretical bases of this theory are not more complicated than the two propositions taken together. In particular, while each one of these propositions makes sense in isolation, they are not derived *together* from any common set of assumptions about human behavior.

One major modification introduced by interactive theories to the standard economic analysis is the introduction of the time dimension. Indeed, the economic approach uses exclusively comparative statics analysis while the interactive effects presuppose (although, because of the lack of theoretical foundations, they never treat explicitly) a dynamic analysis.

II. CRIME AND PUNISHMENT: A GAME-THEORETIC APPROACH

The game-theoretic approach proposed here will help improve on all the previous shortcomings of the economic approach. In particular, it will help treat risk aversion endogenously, use both comparative statics and dynamic analysis, and provide a theoretical basis for a dynamic approach to crime.

In terms of the previous discussion, game-theoretic explanations are intentional explanations, since they explain human behavior not in terms of antecedent conditions but as a rational reaction to a constantly changing (human) environment. Causal factors are taken into account in the payoffs of different actors: Penalties for the deviance of lower classes are more severe than for the middle classes, if we believe the sociological literature. Given these (socially determined) payoffs, the different actors adapt to their environment and try to maximize their rewards, which illustrates the conceptual similarity with the economic approach.

However, while the economic approach considers the problem of crime and crime prevention as derivative of a decision-making problem of the criminals alone, the game-theoretic approach includes one more actor—the police.[11] The situation is then modeled as a game between the criminals and the police in which each one of the players can modify his strategy in response to the other. Therefore, an essential difference surfaces: For each player, the environment is not constant but constantly changing in response to his actions. As we will see, this difference not only increases the realism of the model but accounts for the different conclusions and policy prescriptions between the traditional economic and the game-theoretic approach.

In a simplified game, the criminals have a choice between two strategies: either to violate (V) or not to violate (NV) the law. Similarly, the police have two options: either to enforce (E) or not to enforce (NE) the law. Table 1 represents the strategies and the payoffs in this simplified game.

Assume that the criminals prefer to violate the law if the police do not enforce it but prefer to abide by the law if the police enforce it. We will call these two assumptions 1 and 2, respectively, expressing them in algebraic terms:

Assumption 1: $a_1 > b_1$
Assumption 2: $c_1 > d_1$

We will also assume that the police prefer not to enforce the law if there are no criminals around while preferring to enforce it if criminals violate the law. We will call these two assumptions 3 and 4 respectively, expressing them in algebraic terms:

Assumption 3: $a_2 > b_2$
Assumption 4: $c_2 > d_2$

Several objections may be raised concerning these assumptions. For example, one might object that the actors are not unitary but are each

TABLE 1: Game Between Criminals and Police with Complete Information

	E (enforce)	*NE (not enforce)*
V (violate)	d_1 c_2	a_1 d_2
NV (not violate)	c_1 b_2	b_1 a_2

Assumptions: $c_1 > d_1$, $\alpha_1 > b_1$, $c_2 > d_2$, $a_2 > b_2$.

composed of a large number of individuals; that the actors do not have only two strategies each, but an infinite number; or, with respect to the public's payoffs, that some people would not violate the law (or at least some particular law) even in the absence of police agents; or, concerning the police's payoffs, finally, that reliable police agents do not stop enforcing the law when crime goes down. These are some of the objections that will be examined later in this article. For the time being, this very simple model is required for a clear exposition of my argument.

Before proceeding further, let me clarify one more point: The payoffs in this model should be understood as *expected values* because there is noise caused by factors not included in the model. For example, even if a criminal is arrested, there is a probability that he or she will not be convicted, or if the police make an arrest, there is a probability that they will not get the credit. Therefore, it is possible that assumption 2 will not hold if the probability of conviction is very low. Similarly, it is possible that assumption 3 will not hold for some police agents. Again, examination of such objections will be postponed until subsequent models are analyzed later in this article. For the moment, I want to investigate what happens under assumptions 1 through 4; as a preview of the results to come, this simplified model will lead to exactly the same conclusions as the more sophisticated ones. So, understanding the logic of the bare-bones model is essential for the remainder of the article.

Under assumptions 1 through 4:

If the police enforce the law, the criminals will stop violating it (assumption 2).
If the criminals stop violating the law, the police will stop enforcing it (assumption 3).
If the police stop enforcing the law, the criminals will violate it (assumption 1).
If the criminals violate the law, the police will enforce it (assumption 4).
If the police enforce the law, then the criminals will stop violating it (assumption 2).

In other words, no matter which combination of strategies results, either the police or the criminals will have the incentive to modify their choice.

In game-theoretic terms, the game presented in Table 1 has no pure strategy (Nash) equilibrium: The two players have no pure strategies which are optimal responses to each other. Therefore, the only equilibrium strategies that exist in this game are mixed strategies: probability distributions over the set of pure strategies (Ordeshook 1986, chap. 3). In other words, each player will use a combination of pure strategies so that when both apply a specific pair of strategies, neither will have an incentive to deviate from his or her mixture. In this case, each player's strategy will be the optimal response to the other player, and therefore, both players will retain these (equilibrium) strategies.

In order to compute such strategies, two things must be accomplished: first, assign a probability p to the criminals who choose to violate the law (and $1 - p$ to those who abide by it) and a probability q to the police who enforce the law (and $1 - q$ to those who do not); and second, find a pair (p^*, q^*) of p and q with the quality that the best answer for the criminals, if the police follow the mixed strategy specified by q^*, is to mix their pure strategies (using p^* and $1 - p^*$ as weights), and the best answer for the police, if the criminals follow the mixed strategy specified by p^*, is to mix their pure strategies (using q^* and $1 - q^*$ as weights).

The calculation of p^* and q^* gives (Luce and Raiffa 1957):

$$p^* = (a_2 - b_2) / (a_2 - b_2 + c_2 - d_2) \tag{1}$$

$$q^* = (a_1 - b_1) / (a_1 - b_1 + c_1 - d_1) \tag{2}$$

The specified probabilities p^* and q^* are in the (0, 1) open interval, making them acceptable as solutions because of assumptions 1 through 4. Moreover, since the game has no pure strategy equilibria, the mixed strategies specified by the probabilities p^* and q^* are the *unique* equilibrium strategies of the game.

This last proposition is very important for the subsequent development of the article; therefore, it must be singled out and stated formally:

Theorem 1. Under assumptions 1 through 4, the only equilibrium in the police-criminals game is in mixed strategies as specified by equations 1 and 2.

COMPARATIVE STATICS AND WRONG POLICY PRESCRIPTIONS

The game-theoretic approach to the problem will help us examine the impact of different policy measures on crime. For example, what happens if

the legislator influenced by the arguments of economic analysis increases the penalty for a crime? The expected size of the penalty is represented in our model by the payoff d_l that the criminals receive when they violate the law while the police enforce it. So, d_l should not be conceptualized as the maximum legal penalty, nor even as the average penalty, but as the average penalty discounted by the probability that the criminal will get convicted once arrested. For example, if jails are full and judges give smaller sentences, d_l declines although the law has not changed. Examination of equations 1 and 2 indicates the following outcomes, which will be singled out for their importance and counterintuitive significance:

Theorem 2. Increasing the expected penalty leaves the frequency of violation of the law at equilibrium (p^*) unchanged.

Theorem 3. Increasing the expected penalty decreases the frequency with which the police enforce the law at equilibrium (q^*).

The proof of these two theorems is straightforward. Inspection of equation 1 indicates that p^* does not depend on d_l, while equation 2 indicates a monotonic relationship between d_l and q^*. While the formal proof is easy, what is perhaps more necessary in this case is an intuitive explanation of the discrepancy between conventional wisdom and outcomes of formal reasoning. Why do economists, policy analysts, and citizens in general believe that the size of the penalty changes the propensity to commit crime? The answer to this question will be postponed for now.

Other policy prescriptions to the problem of crime suggest the development of welfare policies in the hope that crime will cease to be an attractive career. In terms of our model, such policies would represent an increase of c_l and/or b_l, the expected payoffs for not violating the law whether the police enforce the law or not. Note that the logic of these policy prescriptions is very similar to the logic of the economic approach to crime: They reward virtuous behavior instead of punishing deviance. In both cases, the expected utility of crime relative to compliance with the law is expected to decrease.

Examination of equations 1 and 2 indicates the following consequences of increasing welfare policies:

Theorem 4. Welfare measures (increasing either c_l or b_l) leave the frequency of violation of the law at equilibrium (p^*) unchanged.

Theorem 5. Welfare measures (increasing either c_l or b_l), in general, decrease the frequency with which the police enforce the law at equilibrium (q^*).[12]

These results deviate sharply from both dominant approaches to crime prevention; they also deviate from their synthesis as presented by Wilson and Herrnstein (1985). It is, therefore, only reasonable to ask: What are the logical and/or empirical foundations of the validity of this approach? One can consider the absence of clear-cut evidence in favor of any one of these theories as a sufficient empirical indicator. However, more theoretical evidence for my solution must be produced, in particular, answering the question: Even if under assumptions 1 through 4 there is a unique equilibrium, how do we know that players will choose the corresponding strategies?

STABILITY OF THE EQUILIBRIUM

This section provides five different reasons to explain why the equilibrium calculated by equations 1 and 2 is reasonable. Although each of these approaches starts with different substantive assumptions, they all result in the same conclusion: The equilibrium strategies calculated in equations 1 and 2 not only have normative merit (they are mutually best responses) but positive merit (they are likely to be chosen). The first four approaches assume perfect information, that is, that the payoffs of the players are common knowledge.[13] In addition, the first approach, labeled model 1, assumes strategic behavior on the part of both players and discrete choices; the second approach, labeled model 2, relaxes the assumption that each player has only two strategies and assumes infinite strategy spaces; the third approach, labeled model 3, assumes myopic adjustive behavior by both players; while the fourth approach, labeled model 4, is a mixture of the second and the third in which one player is assumed to be fully rational and the other myopic. Finally, the fifth approach, labeled model 5, assumes the rationality of the players but relaxes the assumption of perfect information: Each player will be assumed to know his or her own payoffs but have incomplete knowledge of the opponent's.

Model 1: Rational approach with perfect information. There is a serious objection concerning the mixed strategy nature of the unique Nash equilibrium.[14] Mixed strategy equilibria, the argument goes, lack the self-enforcing properties of pure strategy equilibria. Indeed, on the face of it, if one of the players follows an equilibrium strategy, the other can choose *any* pure or mixed strategy without changing his or her own payoffs. However, in this case, the player who deviates from an equilibrium strategy is left open to a deviation of the other player, which may reduce his or her payoff.

In our game, if the police choose to enforce the law with a probability less than q^*, the criminals can switch to their pure strategy of violating the law, making the police worse off. Similarly, if the criminals violate the law with probability higher than p^*, the police may choose to enforce the law with probability 1, making the criminals worse off.

What happens if the police choose to enforce the law with probability higher than q^* or the criminals choose to violate the law with probability less than p^*? An argument for these deviations can be made easily, since the other player cannot reduce the payoff of the player who deviates from equilibrium. However, the other player has to modify his or her behavior in such a way (the criminals stop committing crimes or the police stop enforcing the law) that the deviant player will be induced to change strategy once more in order to improve his or her payoffs.

It can be shown that the equilibrium of this game presents all the stability requirements proposed by the relevant literature. In particular, it can be shown to be regular (Harsanyi 1973b), perfect (Selten 1975), essential (Wu Wen-Tsun and Jiang Jia-He 1962), and proper (Myerson 1978).[15] Moreover, since it is the unique equilibrium of this game, it is stable in the sense of Kohlberg and Mertens (1986, 1004), that is, the equilibrium satisfies both the backwards induction rationality of the extensive form and the iterated dominance rationality of the normal form. At the same time, it is also independent of irrelevant details in the description of the game. Therefore, the solution described by equations 1 and 2 and the subsequent policy conclusions are as solid as any game-theoretic model can be.

Model 2: Continuous strategy spaces. Now, imagine that each player has an infinity of available strategies, that is, that both the criminals and the police have to decide on the *level* of violation and compliance. In algebraic terms, the public has to decide the level x of violation ($x = 1$ means complete violation and $x = 0$ complete compliance); the police has to decide the level y of enforcement ($y = 1$ means complete enforcement and $y = 0$ means no enforcement). Assume also that each player's payoffs are linear functions of the strategies of both players.[16] Then, each player will have the following payoffs as a function of the strategies chosen by each:

$$u_1 = (b_1 - c_1 - a_1 + d_1)xy + (c_1 - b_1)y + (a_1 - b_1)x + b_1 \tag{3}$$

$$u_2 = (a_2 - b_2 - d_2 + c_2)xy + (b_2 - a_2)y + (d_2 - a_2)x + a_2 \tag{4}$$ [17]

Equations 3 and 4 can be used to calculate the equilibrium pair of strategies of this game, that is, of a pair of strategies that are optimal responses to each other. The only equilibrium pair of the game described by equations 3 and 4 is given by the equations:[18]

$$x^* = (a_2 - b_2) / (a_2 - b_2 + c_2 - d_2) \tag{5}$$

$$y = (a_1 - b_1) / (a_1 - b_1 + c_1 - d_1) \tag{6}$$

Comparison of the equilibrium frequencies of model 1 (equations 1 and 2) with the equilibrium levels of this game (equations 5 and 6) indicates that the two equilibrium pairs are identical.

Model 3: Evolutionary approach. Suppose that the total population is composed of two different categories of people: law-abiding citizens (labeled NV for nonviolators) and criminals (labeled V for violators). In addition, suppose that the police are composed of two subpopulations: the conscientious law-enforcing agents (labeled E for enforcing) and the non-law-enforcing police officers (labeled NE for not enforcing). The reader can verify that the two players of the previous approach have been divided into subpopulations which apply only one strategy. Whenever two subpopulations meet, the payoffs of Table 1 apply. The expected utilities of these random encounters for each subpopulation (assuming random mixing) are:

$$EU_V = p_E d_1 + (1 - p_E) a_1 \tag{7}$$

$$EU_{NV} = p_E c_1 + (1 - p_E) b_1 \tag{8}$$

$$EU_E = p_V c_2 + (1 - p_V) b_2 \tag{9}$$

$$EU_{NE} = p_V d_2 + (1 - p_V) a_2 \tag{10}$$

in which p_E and p_V are the probabilities, respectively, that a police officer is enforcing the law and that the person from the general population is a criminal.

Consider that each member of each subpopulation, after each interaction with a member of the other population, looks around and checks whether the

members of the other subpopulation are doing better or worse than he or she is.[19] A criminal, for example, considers the results and asks whether crime "pays." An honest police officer asks whether being honest "pays." In all of these cases, people emigrate from the less profitable to the most profitable subpopulation. Consider that the frequencies (or the probabilities) of these movements of police officers from enforcing to nonenforcing and vice versa, and of the public between violating and not violating the law, are proportional to the difference in the rewards of the two subpopulations. This means that the more pronounced and visible the difference, the more people will switch from one category to the other. In simple terms, when crime pays, more people will pursue a life of crime. The formal expression of these assumptions[20] is given by the following (differential) equations:

$$dp_V/dt = k[(p_E d_1 + (1 - p_E)a_1) - (p_E c_1 + (1 - p_E)b_1)] \quad (11)$$

$$dp_E/dt = m[(p_V c_2 + (1 - p_V)b_2) - (p_V d_2 + (1 - p_V)a_2)] \quad (12)$$

These two equations describe the composition of the two populations at any point in time. Criminals (V) for example, might think that the police enforce the law so frequently (high E) that crime does not pay. In this case, they will switch to being law-abiding citizens (NV), rendering law enforcement (E) nonprofitable: Some E will become NE.

Will this process ever equilibrate, and if so, what is the composition of the two populations at equilibrium? The answer to both questions is within reach if we set the left-hand side of equations 11 and 12 to zero and search for the values of p_V and p_E that satisfy the system. Simple calculations indicate that such values are given by the equations 1 and 2.

There are three important contributions of the evolutionary approach to the argument of this article. First, the objection can be made that the rational approach assumes unified actors, while neither the police nor the criminals have such capacities of centralization and overall strategic thinking. The evolutionary approach indicates that the centralization and rationality assumptions are not essential for the argument.

Second, the evolutionary approach is the natural theoretical framework for the interactive models of crime. The assumption of adaptive behavior by both players leads to an action-reaction model which is the conceptual basis of simultaneous equation estimations.

Third, the evolutionary approach can help understand the reasons of the discrepancy between the game-theoretic and the conventional economic

approach. Indeed, it is very plausible that in the short run, criminals will reduce or shift their activities when penalties increase. However, once the police realize this change in criminal behavior, they will modify their own strategy and reduce the frequency of law enforcement. The criminals will then modify again, and again . . . and the new equilibrium will be described by equations 1 and 2, in which the increase in the penalty has no impact on criminal behavior. So, the reason for the discrepancy between the game-theoretic and the traditional economic analysis of crime is that the latter has committed two related simplifications: (a) ignoring one of the relevant players (the police) and (b) confining itself to the short-run consequences of the policy. The simplification of the sociological approach is similar: In the short run, welfare measures may have the desired impact, but this will lead the police to modify their strategy and enforce the law less frequently. In turn, the criminals will increase their frequency of violating the law, and finally the process will equilibrate at the strategies described by equations 1 and 2.

Model 4: Mixed approach.[21] In the first two models, very high standards of rationality were set and met by both players, while in the third, only a simple adaptive behavior was required. It is reasonable to believe that the police (as a more or less unitary actor) approximate the first kind of rationality, while the criminals are more likely to demonstrate the adaptive behavior required by model 3. Is the equilibrium of the police-criminals game robust for this more realistic set of assumptions?

Assuming full rationality for the police and myopic behavior for the criminals translates into the following game: The police have to choose one of the infinite number of strategies, knowing that the criminals are going to adopt their best response to any strategy chosen. This description is technically called a Stackelberg equilibrium, with the police playing the role of the leader (Moulin 1981). To investigate the dynamics, assume that the police choose to enforce the law with full force. In this case, the criminals will never commit crime and the absence of crime will induce the police to reduce law-enforcement activities. If, on the other hand, the police choose not to enforce the law at all, crime will thrive; in this case, the police will be forced to increase law-enforcement activities. None of these police strategies is, therefore, part of an equilibrium pair.

To calculate the Stackelberg equilibrium, let us investigate the two logical possibilities. First, what happens if the police adopt a strategy in which the level of law enforcement is greater or equal to the one indicated by equation 6? In this case, straightforward calculation of the criminals' maximizing behavior indicates that they will always comply with the law, creating

incentives for the police to reduce the level of law enforcement. The lowest (according to our assumptions) possible level is y^*. Thus, y^* is a Stackelberg equilibrium strategy, a level which, if adopted, will remain stable.

Let us now investigate the other alternative, that is, what happens if the level chosen by the police is strictly less than, y^*. In this case, the criminals will violate the law, creating incentives for the police to step up enforcement. Consequently, the police have to find the highest possible level of law enforcement which remains strictly less than y^*. No such level exists, and therefore, there is no other Stackelberg equilibrium for the game.

Another interpretation of the Stackelberg equilibrium solution is that the two players move sequentially, the police first and the criminals second, and that the police know that the criminals will find their best response strategy to the police's initial choice. Again, the equilibrium in this sequential game prescribes level of law enforcement y^* (equation 6). Note that at level y^*, the criminals are indifferent between violating and not violating the law, so they can choose any strategy.

Model 5: Rational approach with incomplete information and infinite number of possible "types." Suppose, now, that each of the two players is rational, but while each knows one's own payoffs, he or she knows the opponent's payoffs only approximately. In a more technical way, assume that there is a random component in each player's payoffs and that each player knows the value of the random variable for oneself but not for the opponent. So, the criminals know exactly their own payoffs, but they can only guess the payoffs of the police, and vice versa. Table 2 presents such a game with incomplete information, where x and y are random variables and e_1 and e_2 are arbitrarily small numbers. The appendix shows that in this case, too, when the random component tends to zero (when e_1 and e_2 tend toward zero), that is, as the players learn more about each other, the frequency of the strategies they select tends again toward the right-hand side of equations 1 and 2.

More generally, Harsanyi (1973a) has proven that a mixed strategy equilibrium can be approximated by two players with incomplete information (knowing only their own payoffs), when the random disturbances of the other player's payoffs decrease (tend toward zero). In Harsanyi's own words:

> Equilibrium points will be stable, not because the players will make any deliberate effort to use their pure strategies with probabilities prescribed by their mixed equilibrium strategies, but rather because the random fluctuations in their payoffs will *make* them use their pure strategies approximately with the prescribed probabilities. (p. 23)

TABLE 2: Game Between Criminals and Police with Incomplete Information (infinity of types)

	E (enforce)		*NE (not enforce)*	
V (violate)	$d_1 + ex$	$c_2 + ey$	$a_1 + ex$	d_2
NV (not violate)	c_1	$b_2 + ey$	b_1	a_2

Assumptions: $c_1 > d_1$, $\alpha_1 > b_1$, $c_2 > d_2$, $a_2 > b_2$; $0 < e < 1$; x and y random variables with uniform distribution in the [0, 1] interval. Player 1 knows x but not y and player 2 knows y but not x.

It is very likely that in the police-criminals game, each one of the two players is uncertain about the other's payoffs (or at least some of them), therefore, the incomplete information approach answers the question of the stability of the unique Nash equilibrium of the game.[22] The natural interpretation of this approach is that the game is played several times by different policemen and different criminals, each one of them being uncertain about the other's payoffs. In this case, as the uncertainty about the other player's payoffs decreases, the frequency with which each group will choose each one of its strategies is given by equations 1 and 2.

The behavioral assumptions of each of these approaches are completely different. The players have been assumed to (a) operate under complete or incomplete information, (b) be fully rational on both sides, (c) take up adaptive behavior, (d) be rational on one side and adaptive on the other, (e) each be unitary, each composite, or one unitary and the other not, and (f) move simultaneously or sequentially. In all these cases, we were led to the same equilibrium strategies.[23]

The reason why such different models converge to the same equilibrium outcome is that the Nash equilibrium concept is the only rational solution to simultaneous games.[24] Moreover, "almost all normal form games possess equilibria which are all regular" (Van Damme 1987, 44). Therefore, if the only equilibria of a game are in mixed strategies, (in general) these equilibria will exhibit all the desirable properties of stability. For example, since they are regular, they can be shown to be the limit equilibria in games with incomplete information and perturbed payoffs (model 5); similarly, they are likely to be evolutionary-stable (model 3). And since assumptions 1 through 4 lead to a unique equilibrium, this equilibrium is the outcome of approaches with very different behavioral assumptions as well.

Table 3 summarizes the assumptions of the five different models presented thus far and one, more complicated model with incomplete information

TABLE 3: Assumptions of Different Models Leading to the Same Equilibrium (Equation 1 and 2)

Model	*Unified Players*	*Behavior*	*Information*	*No. of Strategies*
1	Yes	Strategic	Complete	2
2	Yes	Strategic	Complete	Infinite
3	No	Adaptive	Complete[b]	2
4	Police, Yes	Mixed[a]	Complete	Infinite
5	No	Strategic	Incomplete	2
6[c]	No	Strategic	Incomplete	4

a. The police are strategic; the criminals are adaptive.
b. In fact, each player uses only his or her own payoffs in order to select a particular strategy.
c. The conditions under which model 6 leads to the equilibrium prescribed by equations 1 and 2 are presented in part 3 of the article.

(model 6) which will be presented later. In general, the equilibria of game-theoretic models are extremely sensitive to modifications of informational assumptions or to modifications in the sequence of moves. The fact that all these models lead to the same equilibrium is an indication of its remarkable stability.

The results of this model may be surprising and doubtful at first, though eventually more believable after a closer examination; however, they leave the policymaker in a quandary: If modification of penalties and rewards does not work, how can crime be reduced? The answer provided by this model is simple. Any modification of the payoffs of the criminals will result in a change in the strategy of the *police* (equation 1). Conversely, any modification of the payoffs of the police will result in a change in criminal behavior (equation 2). As equation 2 indicates, an increase of c_2 (the prize that the police receive when they enforce the law in the presence of crime) or a decrease of d_2 (increase in the blame if they fail to enforce the law in the presence of crime) will decrease the proportion of crime commitment at equilibrium. Similarly, any increase of b_2 (the prize that the police receive when they enforce the law even in the absence of crime) or a decrease of a_2 (increase in the blame if they fail to enforce the law even in the absence of crime) will decrease the proportion of criminal activity at equilibrium.[25]

These payoffs of the police can be altered in several different ways. First, by increasing the number of law-enforcement agents, their task becomes easier, decreasing the payoffs d_2 and a_2 even though such a solution may present important economic costs for society. Second, payoffs can be altered by increasing police incentives to capture criminals: rewarding the agents

proportionally to the number of arrests. Such solutions have already been applied for some kinds of crime (traffic violations, for example), however, a more generalized application of similar methods may present very significant social costs. Indeed, the public image of the police may suffer if they are perceived as attempting to increase their personal profits by persecuting citizens and/or criminals indiscriminantly. Third, payoffs can be altered by increasing social pressure on the police if they fail to fulfill their functions. It seems that this is the most economic and socially acceptable solution, but for this reason, its potential may already have been exhausted. In this case, use of methods 1 and 2 will be required for long-term reduction in crime rates.

III. VARIATIONS OF THE MODEL

Thus far, we have considered a 2 × 2 game without pure strategy equilibria (guaranteed by assumptions 1 through 4) which had a unique mixed strategy equilibrium. What happens if we relax some of the assumptions 1 through 4? This part of the article first examines four simple variations which violate assumptions 1 through 4, and then presents a more complicated model where some of the players have the preference-ordering indicated by assumptions 1 through 4 and some do not.

FOUR SIMPLE MODELS WITH DIFFERENT OUTCOMES

It can be shown that there are three categories of 2 × 2 games: (a) those without a pure strategy equilibrium (with a unique mixed strategy equilibrium); (b) those with a unique pure strategy equilibrium; and (c) those with two pure strategy equilibria and a mixed strategy equilibrium (three equilibria; Moulin 1981).

The game without pure strategy equilibrium has been examined already. The only game that needs examination is one in which a unique pure strategy equilibrium exists. There are four possible cases for such payoff matrices.

The criminals prefer to violate the law, no matter what action the police take. In this case (under assumptions 3 and 4, the police will always enforce the law to the limits of their capacity), the unique outcome will be that criminals always violate the law and the police always enforce it. The closest real-life situation would be foreign immigrants illegally crossing the southern U.S. border. As long as crossing the border continues to be the dominant

strategy for illegal immigrants, there is no other possible outcome. In order to modify the payoff structure of this game and transform it to the game examined in part 2, the strategy "violate the law" must cease to be dominant. There are two different cures: the stick approach (impose penalties for violation of the law) or the carrot approach (improve the initial conditions of life of potential illegal immigrants).

Note how similar these solutions are to the propositions of the economic and the sociological approaches. We can now go one step further in our evaluation of these theories and indicate that they implicitly assume that the strategy "violate the law" is dominant for the criminals. Indeed, only under this condition will their prescriptions be correct in the long run. Assume, for a moment, the existence of a dominant strategy for the criminals. In looking at the impact of either the economic or the sociological prescription on this strategy, we find that both prescriptions eliminate dominance, transforming the unique equilibrium of the game from pure strategy into mixed strategy which reduces substantially the frequency (from 1 to q^* specified by equation 2) of crime commitment. From this point on, however, any further application of these prescriptions is doomed to have only a short-term impact at best, while their long-term consequences do not influence the frequency of crime commitment, as we have already noted.

The criminals prefer not to violate the law, even if the police are not present. Such an assumption appears unrealistic. However, it might be valid for certain subpopulations and for certain kinds of crime. For example, most people would not commit murder regardless of whether the police were around or not; however, most people would violate the speed limit on a deserted highway at night. We could take such variations into account simply by varying the potential population of "criminals." Thus, in the first case, only a very small percentage of the population would be included (only the ones likely to commit murder), while in the second, we would call practically the whole population "criminals."

In the counterfactual case (no one wants to violate the law regardless of whether or not the police are around), the unique equilibrium of the game is no crime and no law enforcement—a true paradise.

The police prefer to enforce the law, independent of criminal action. Again, there is no crime; however, in this case, it is for fear of the police. Note that this assumption eliminates the strategic possibilities of the police. And, as has been stated, elimination of the strategic capacities of one actor transforms the problem into a simple decision-making case. However, this

objection must be taken seriously on empirical grounds, instead of dismissing it because of methodological tastes, that is, in order to apply game theory.

Two arguments can be made for this case:

1. The police are not a strategic actor since they have to obey orders from some other authority (elected politicians, or the Ministry of the Interior). Thus choices are made *for* the police at a different level and police response to criminals is not optimal. The argument appears sound and would be valid if the behavior of every level in the hierarchy of law-enforcement agencies could be monitored perfectly. However, is it reasonable to assume that a police chief who cracks down on a specific kind of crime in a specific area would not use forces for another type of crime in another area, reducing the frequency of law enforcement for the specific crime in the specific area? And, even if that police chief does not apply such a strategy, would it be reasonable to assume that agents would obey the chief's orders and continue patrolling if there is no obvious reason, even when they are not being monitored?

2. Assumption 3, which regulates police preferences in the absence of crime, is true at the limit but not at the margin: Police officers prefer not to patrol when there is no crime at all; nevertheless, they still prefer to continue patrolling as long as there is some probability of crime. This argument is equivalent to the statement that the police have more than two strategies available. This is the most difficult argument to dispute.[26] However, consider the case where both players have several strategies available. The criminals can commit several types of crime while the police enforce the law with a different intensity for each type of crime. Generalization of the argument presented in part 2 indicates that if there is no pure strategy equilibrium and as long as each player mixes the same pure strategies, *modification of the payoffs of one player will influence the choice of the mixed strategy of the other*: the essential result of this article.[27] Consequently, the robustness of the results survives these objections.

In the counterfactual case, however, if police prefer to enforce the law no matter what the behavior of the criminals, the unique (pure strategy) equilibrium of the model is that police enforce the law and there is no crime. This counterfactual case can be used as indirect evidence against the assumption that produced it (that is, violation of assumption 3).

The police prefer not to enforce the law even if it is violated. This case involves an extremely timid or ineffective police. One can find real cases approximating this situation: for example, police reaction to organized crime in certain areas of the United States or Italy. The outcome is that the law will always be violated. However, reality is more complicated than such simple

TABLE 4: Game Between Criminals and Police with Incomplete Information (two types for each player)

	E (enforce)		*NE (not enforce)*	
V (violate)	d_1	c_2	a_1	d_2
NV (not violate)	c_1	b_2	b_1	a_2
			1_1	g_2

Assumptions: $c_1 > d_1$, $1_1 > \alpha_1 > b_1$, $c_2 > d_2$, $a_2 > b_2 > g_2$
Probability of good police, p_g
Probability of law-abiding citizens, p_l

games with or without pure strategy equilibrium. A more complicated game is now examined.

A MORE COMPLICATED GAME WITH INCOMPLETE INFORMATION

One particularly interesting complication is if the public is composed of some law-abiding citizens and some potential criminals, who would not hesitate to violate the law if the opportunity were presented. A similar argument can be made for police agents: Although the payoff matrix shown in Table 1 seems a reasonable representation of the incentive structure of some police agents, there is certainly a big proportion of police agents who prefer to enforce the law all of the time (for whom enforcement is the dominant strategy). In this model, some of the citizens are law-abiding and some are not, and some of the police always enforce the law while others do so only when crime is high. This more realistic model of society, labeled *model 6*, will be proven to lead to the same equilibrium found in models 1 through 5.

Table 4 presents the payoffs of such a mixed population game. Law-abiding citizens have the dominant strategy not to violate the law ($c_1 > d_1$, and $1_1 > \alpha_1$), while the payoffs of the rest of the public follow the pattern of assumptions 1 and 2. Good police agents have a dominant strategy of enforcing the law ($c_2 > d_2$, and $b_2 > g_2$), while the payoffs of the rest of the police force are given by assumptions 3 and 4.

Figure 1 presents such a game, where the public is composed of law-abiding citizens (with probability p_l) and potential criminals (with probability $1 - p_l$), and the police force is composed of good and bad agents (with probability p_g and $1 - p_g$, respectively). When a member of the police and a member of the public interact, they do not know what type of opponent they meet. The

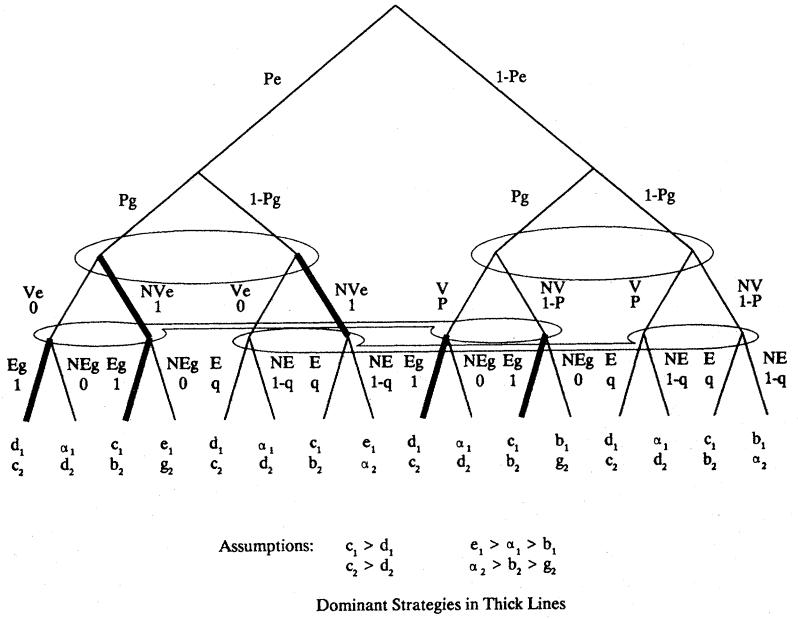

Figure 1: Game between public and police (extensive form) with incomplete information.

incompleteness in the information structure of the police and the public is expressed by the information sets in Figure 1.

To recapitulate, in the game shown in Figure 1, players operate with incomplete information: They know their own type (payoffs), but they do not know the type (the payoffs) of the opponent they face. Everyone knows that the police is composed of p_g good agents (with g_2 in their payoff matrix) and $1 - p_g$ bad agents (with α_2 in their payoff matrix). Moreover, everyone knows that the public is composed of p_l law-abiding citizens (with l_l in their payoff matrix) and $1 - p_l$ potential criminals (with b_l in their payoff matrix). The following inequalities hold among payoffs:

$$c_l > d_l \tag{13}$$

$$e_l > \alpha_l > b_l \tag{14}$$

$$c_2 > d_2 \tag{15}$$

$$\alpha_2 > b_2 > g_2 \tag{16}$$

In this more complicated and more realistic game, the players with dominant strategy always play these strategies, while the remainder of the players use mixed strategies. The calculation of the frequencies that the potential criminals violate the law (p^*) and that bad police agents enforce it (q^*) gives the following results:

$$p^* = (\alpha_2 - b_2) / ((1 - p_1)(\alpha_2 - b_2 + c_2 - d_2)) \tag{17}$$

$$q^* = ((\alpha_1 - b_1) - p_g(\alpha_1 - b_1 + c_1 - d_1)) / ((1 - p_g)(\alpha_1 - b_1 + c_1 - d_1)) \tag{18}$$

Closer examination of equations 17 and 18 and comparison with equations 1 and 2 reveal that all the theorems presented in this article hold also for this more complicated game with incomplete information. In particular, modification of the payoffs of the police does not affect the behavior of police agents but of criminals, and modification of the payoffs of the public does not affect the frequency of crime but of law enforcement. Moreover, the sophisticated agents (those with nondominant strategies) compensate for the existence of agents with dominant strategies. For example, if there are no law-abiding citizens (if $p_1 = 0$), then, as one would expect, equation 17 reduces to equation 1; and when the percentage of law-abiding citizens increases, the potential criminals increase their crime rates, so that the overall crime rate in society is still given by equation 1.[28] Similar observations can be made with respect to the behavior of strategic police agents.

In conclusion, the equilibrium of this more realistic game where some of the players are strategic and some are not is exactly the same as the equilibrium of the game where all agents are strategic, because the strategic agents compensate for the existence of nonstrategic agents.[29]

IV. DISCUSSION AND CONCLUSIONS

The results suggest a divergence from the conventional wisdom concerning the question of how to deter crime. The common approaches suggest

either the increase of penalties or the creation of welfare programs. This article has suggested the modification of police incentives.

There are two reasons for this discrepancy: First, the conventional approach ignores the police as a strategic actor in the law enforcement operation; and the second is a consequence of the first. The results of the conventional approaches are only short-term since the police will adapt their behavior to the new situation. The final outcome of an increase in the size of the penalty would be counterfinal, that is, it will defeat the purpose of the lawmaker in that it would decrease the frequency with which the police enforce the law instead of decreasing the frequency of crime.

The results have been justified in six different ways: The first assumes complete rationality and perfect information of both players, the second introduces an infinite number of choices, the third assumes simple adaptive behavior, the fourth is a mixture in which one of the players is rational and the other myopic, the fifth assumes an infinity of rational players with incomplete information, while the sixth assumes two different types of each player and shows that assumptions 1 through 4 are not required to hold for all players in order to reach the same equilibrium. Of these approaches, the third provides the possibility of investigating the dynamics of the interaction.

Numerous different crimes can be studied in this framework: traffic violations, tax evasion, and crimes resulting in the death penalty. The only difference would be that the population of susceptible criminals would change accordingly. In all these cases, the size of the penalty would not modify the frequency of crime commitment at equilibrium.

The realism of the model can be improved in different directions. Additional actors can be introduced so that the police would have to enforce the law while playing against several different groups of criminals. These criminals would differ in type of crime (e.g., thieves and traffic violators) or in social class (e.g., blue- and white-collar crime), or in neighborhood (e.g., or poor). In all these cases, the most interesting outcome would be the equilibrium strategy of the police. How would the police divide their labor to enforce the law? Several sociological theories about differential rates of law enforcement or differential (class-related) rates of crime could be tested.

Additional strategies for each one of the actors can be introduced as well. For example, one of the actors could choose between different kinds of crime commitment and the police could vary the severity of law enforcement according to the crime. This generalization would not change the qualitative results. As long as there is no pure strategy Nash equilibrium in the game, and as long as the pure strategies mixed by each player are the same, the strategy mixtures of each player will be a function of the payoffs of the other.

Additionally, the study of equilibrium strategies would improve our understanding of crime-related phenomena and policies of crime prevention.

Another empirically interesting modification of the model would be the inclusion of side-payments between players. For example, the criminals (when caught) could bribe the police to let them free. How would such a possibility be captured by our model? In case of arrest, the payoff of the criminal would have to be linked with the payoff of the police (say, through a linear function). In this case, an increase of the penalty would simultaneously increase the payoffs of the police for every arrest but decrease the actual consequences of each (informal) arrest. The consequence of this perverse situation is that the police patrol more often with fewer arrests, and therefore, no public good is produced from the existence of the police.

Finally, an additional modification would be to introduce the legislature or the public as a third player into the game. This third player must set the rules of the game: the payoffs of the criminals and the police. This rule-making capacity is equivalent to the selection of different possible games. The analysis presented here can help legislators select the most appropriate combination of payoffs[30] according to their utilities.

APPENDIX

Consider the game of Table 2, where $0 < e_1 < 1$, $0 < e_2 < 1$ and x and y are independent and identically distributed, each with uniform distribution over the interval from 0 to 1. When the game is played, player 1 (the public) knows the value of x but not of y, and player 2 (the police) knows the value of y but not of x. For every pair of e_1 and e_2, there is a unique equilibrium of the game, which is given by inequalities A1 and A2:

$$\text{If } x > [e_1 (c_1 - d_1) - (a_1 - b_1 + c_1 - d_1)(c_2 - d_2)] / [e_1^2 + (a_1 - b_1 + c_1 - d_1)(a_2 - b_2 + c_2 - d_2)], \text{ play "violate," otherwise, play "not violate."} \quad \text{(A1)}$$

$$\text{If } y > [e_2 (c_2 - d_2) - (c_2 - d_2 + a_2 - b_2)(c_1 - d_1)]/[e_2^2 + (a_1 - b_1 + c_1 - d_1)(a_2 - b_2 + c_2 - d_2)], \text{ play "enforce," otherwise, play "not enforce."} \quad \text{(A2)}$$

Proof. In order to calculate these inequalities, we reason as follows: Each player has to maximize one's own payoffs, given that one observes the random variable affecting his or her payoffs but not the payoffs of the opponent. Call $p(x)$ the probability that player 1 (the public) will play "violate" and $q(y)$ the probability that player 2 (the police) will play "enforce." Player 1 has to choose "violate" when this

is the action maximizing his or her payoffs, no matter what the payoff of the opponent. So, for a value of x greater than some specified value x_0, player 1 will choose "violate." Similarly, player 2 will choose "enforce" for values of y greater than some specified value y_0.

The expected utility of player 1 is given by

$$EU_1 = (d_1 + e_1x)\,pQ + (a_1 + e_1x)\,p\,(1 - Q) + c_1\,(1 - p)\,Q + b_1\,(1 - p)\,(1 - Q) \qquad \text{(A3)}$$

where

$$Q = \int_0^1 q(y)\,dy. \qquad \text{(A4)}$$

EU_1 becomes maximum when $\partial EU_1/\partial p$ is zero, or equivalently when

$$a_1 - b_1 + e_1x - (a_1 - b_1 + c_1 - d_1)\,Q = 0. \qquad \text{(A5)}$$

So, the strategy for player 1 is to play "violate" when x is such that the left-hand side of equation A5 is greater than zero, and "not violate" otherwise.

The expected utility of player 2 is given by

$$EU_2 = (c_2 + e_2y)\,Pq + (b_2 + e_2y)\,q\,(1 - P) + d_2\,(1 - q)\,P + a_2\,(1 - P)\,(1 - q) \qquad \text{(A6)}$$

where

$$P = \int_0^1 p\,(x)\,dx. \qquad \text{(A7)}$$

EU_2 becomes maximum when $\partial EU_2/\partial q$ is zero, or equivalently when

$$d_2 - a_2 + e_2y + (c_2 - b_2 + d_2 - a_2)\,P = 0. \qquad \text{(A8)}$$

So, the strategy for player 2 is to play "enforce" when y is such that the left-hand side of equation A8 is greater than zero, and "not enforce" otherwise.

However, since $p(x) = 1$ when x is greater than the value x_0 calculated from equation A5 and zero otherwise, the integral

$$P = \int_0^1 p(x)\,dx = 1 - x_0. \qquad \text{(A9)}$$

Similarly, since $q(y) = 1$ when y is greater than the value y_0 calculated from equation A8 and zero otherwise, the integral

$$Q = \int_0^1 q(y)\, dy = 1 - y_0. \tag{A10}$$

Substitution of P and Q from equations A9 and A10 to equations A5 and A8 gives a linear system of two equations and two unknowns (x_0 and y_0). The solution of this system is presented by equations A1 and A2. It is easy to verify that when $e_1 \rightarrow 0$, $e_2 \rightarrow 0$ the x_0 and y_0 tend toward the following values:

$$x_0 \rightarrow (c_2 - d_2)/(a_2 - b_2 + c_2 - d_2) \tag{A11}$$

$$y_0 \rightarrow (c_1 - d_1)/(a_1 - b_1 + c_1 - d_1) \tag{A12}$$

Equations A11 and A12 indicate the limit frequency that each player chooses each one of his pure strategies. Since the distribution of x and y is uniform, x will be greater than x_0 exactly $(1 - x_0)$ of the time. Similarly, y will be greater than y_0 exactly $(1 - y_0)$ of the time. Therefore, "violate" will be chosen with frequency $(1 - x_0)$ and "enforce" with frequency $(1 - y_0)$. The reader can verify that these are the same frequencies as those calculated in equations 1 and 2 shown earlier in the article.

NOTES

1. For a discussion of the distinction between causal and intentional explanations in the social sciences, see Boudon (1977, 1978) and Elster (1983).

2. See Blumstein et al. (1978, 135-36). I am concerned with what the literature calls "general deterrence," that is, the deterrent effect of penalties on the population in general. Wilson (1983) and Cooter and Ulen (1988) examined also the deterrent effect on previous criminals and concluded that overall deterrence works.

3. Based on Durkheim's ideas and extended to crime by Merton (1968), this theory posits a discrepancy between culturally defined goals and culturally defined means in a social system: Individuals may "innovate," that is, they may continue to accept the goals but replace the legitimate means-to-ends relationship with nonlegitimate means.

4. According to this theory, criminal behavior stems from defective psychological development in the family environment. Certain family characteristics, such as marital tensions, absence of a male figure, child rejection, absence of moral lessons, and others, produce individuals who are immoral, impulsive, and lack a sense of guilt. As a result, these individuals are likely to turn to crime. The most prominent representatives of this school of thought are Alexander and Healy (1935), Glueck and Glueck (1950, 1968), Hewitt (1970), and Nettler (1978).

5. This theory postulates that different groups generate different crime-related values and that lower classes, in particular, share values and norms which are crime-generating. The most known representatives of this tradition are Miller (1958), Wolfgang and Ferracuti (1967), and Banfield (1974).

6. Proposed by Sutherland and Cressey (1978), this theory is summarized as follows:

> Overt criminal behavior has as its necessary and sufficient conditions a set of criminal motivations, attitudes, and techniques, the learning of which takes place where there is an exposure to criminal norms in excess of exposure to corresponding anti-criminal norms during symbolic interaction in primary groups. (DeFleur and Quinney 1966, quoted in Tittle 1983, 342)

The theory, however, does not specify which groups are criminogenic or which individuals will fall under their influence.

7. Shaw and McKay (1969) claimed that social and environmental characteristics of low-income inner-city areas produce high crime rates. Physical deterioration and economic depression lead to the deprivation of residents, population instability, and the absence of social control.

8. Although Marx himself said little about crime, Bonger (1969), Gordon (1973), Greenberg (1981), Quinney (1977), and Spitzer (1975) have written Marxist interpretations of crime. The common theme of this literature is that deprivation of material goods and access to means of production will lead to criminal behavior. Criminal behavior, however, is a characteristic of lower classes for one additional important reason: Egoistic elite behavior is not defined as "crime." For more theories and a more detailed account, see Tittle (1983) from whom I borrow the classification.

9. For example, in *Law and Economics*, Cooter and Ulen (1988, 519) claimed: "The rationally self-interested criminal chooses the seriousness of his crime x to maximize his net payoff, which equals the payoff minus the expected punishment."

10. See also Tittle and Rowe (1974) and the reference to the study of the National Academy of Sciences in the introductory portion of this article.

11. For a similar game-theoretic approach concerning tax evasion, see Reinganum and Wilde (1986) and Graetz, Reinganum, and Wilde (1986). Their approach is similar because they model tax compliance as a game between the public and the IRS but differs from mine because they focus on a sequential game, where the public moves first by filing a tax return.

12. I say "in general" because there are particular numerical combinations of c_l and b_l that leave q^* unchanged (see Tsebelis forthcoming a).

13. Common knowledge is the technical term describing the following situation: Each player knows one's own payoffs as well as those of the opponent; each player knows that the opponent knows that he or she knows, and so on.

14. This is not the only possible objection. The most radical question concerns the very concept of Nash equilibrium: Even if these strategies are optimal responses to each other, there is no guarantee that the players will choose them. This argument was, until recently, answered by pointing out that no pair of strategies other than that of the equilibrium strategies present the characteristic of being optimal responses to each other; therefore, any other pair of strategies will be unstable. However, recently (Bernheim 1984; Pearce 1984), the question of whether the Nash equilibrium is a reasonable solution concept has been reintroduced more forcefully and without (to my knowledge) any satisfactory answer. The arguments of Bernheim and Pearce are incompatible with the previous analysis, and if correct, undermine the validity of not only this article but all existing game theory.

15. See Van Damme (1987).

16. Linearity of the payoffs with respect to both strategies may seem a highly restrictive assumption. It is necessary for this model to lead to the same equilibrium as the previous one.

However, all the subsequent arguments (theorems 2, 3, 4, and 5) hold if one replaces this assumption with the following weaker one: Each player's payoffs are linear functions of one's *own* strategies. That is, any functional form of the payoffs with respect to the strategies of the opponent produces the same qualitative results.

17. The reader can verify that for the extreme strategies where x and y are either 0 or 1, equations 3 and 4 produce the outcomes shown in Table 1. Since functions 3 and 4 are linear, they are the *only* linear functions with this property.

18. For the derivation, see Ordeshook (1986, 131). Technically, these equilibrium strategies are computed by setting $\partial u_1/\partial x = 0$ and $\partial u_2/\partial y = 0$.

19. In a similar model with several subpopulations Friedman and Rosenthal (1986) made the assumption that each subpopulation compares its own results with the results of the population mean. This assumption would modify the form of our equations but not their equilibrium values. In fact, Friedman and Rosenthal's assumption led to what is known in the biology literature as the Lotka-Volterra equations, still remaining unsolved (May 1973; Hirsch and Smale 1974), while my own assumptions lead to a solvable linear system. It is important to stress that equilibrium values and, therefore, all the subsequent results of this article remain the same.

20. Assuming continuous time. If time is considered to be discrete, then equations 7 and 8 will be difference equations (Kohfeld and Salert 1982).

21. I am grateful to M. Wallerstein for suggesting to me the idea of the mixed approach.

22. The approach does not model the game as iterated and therefore neglects questions of reputation building: one player deviating deliberately from an equilibrium strategy to mislead the other about the true value of his or her payoffs.

23. One could add another to these models: an iterated game between the police and the public under assumptions 1 through 4. It can be shown that the stationary equilibrium of such a game is exactly the same (Tsebelis forthcoming a.)

24. See Bacharach (1987), and for further discussion, see Tsebelis (1990 chap. 2).

25. These propositions can be tested by looking at the sign of the first derivatives of equation 1.

26. I am grateful to D. Canon and M. Meurer for pointing out this argument to me.

27. See Moulin (1981, 160-61). For further discussion of relevant points, see Tsebelis (1989, forthcoming a).

28. The reader can verify this statement by multiplying p^* of equation 13 by the number of potential criminals $(1 - p_1)$. However, when the frequency of law-abiding citizens increases, the probability p^* calculated from equation 13 exceeds 1, which means that the potential criminals will always violate the law.

29. In other words, the crime game exhibits what Haltiwanger and Waldman (1985) called "congestion effect."

30. One usually thinks of minimizing crime subject to some budget constraint. But any possible utility function of the public can be maximized in this game. For a more extended discussion of this issue, see Tsebelis (forthcoming b).

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