MULTIPLE VOTE SYSTEM

A Remedy for Political Polarization

Jesse M. Crosson[+]
George Tsebelis[++]

~

[+] Fellow, Center for the Study of Democratic Politics, Princeton University; Assistant Professor, Trinity University
[++] Anatol Rapoport Collegiate Professor of Political Science, University of Michigan

ABSTRACT
We examine the mechanical effect of a multiple vote, proportional representation electoral system on party vote share in n dimensions. In one dimension, Cox (1990) has proven that such a system is centripetal: it drives parties to the center of the political spectrum. However, as populism has swept across Western Europe and the United States, the importance of multiple policy dimensions has grown considerably. We use simulations to examine how a multiple vote system could alter electoral outcomes in all possible parliamentary systems. We find that multiple vote systems act centripetally in multiple dimensions too, except in extreme cases where parties are sorted into ideological clusters at opposite corners of the ideological space. However, even in these cases slight disturbance of the conditions (by the introduction of an additional party even if it is very small) restores the centripetal properties of the multiple vote system.

I. INTRODUCTION

Modern electoral competition has become more complicated with the emergence of new issue dimensions (most notably, immigration and economic inequality, but also environment, globalization, institutional efficiency, etc.). It has also become more unpredictable, as the success of Donald Trump in the U.S. and Emmanuel Macron in France (among other examples) were considered hopeless longshots only a year (and sometimes less) before their victories in two major Western democracies. Probably the most telling case in terms of multidimensionality and unpredictability is the UK, where the Brexit dimension was not at all captured by the exiting party system, and lead to the emergence of new parties, the splitting of old ones, and series of negative votes in Parliament (the only positive one requiring an extension of negotiations after three years without a plan), leading to its "prorogation" (suspension), months before the electoral triumph of Johnson. It becomes clear that our understanding of a series of issues, such as voters' preference formation, relation between electoral systems and voting, as well as analyses of voting on the basis of one-dimensional models (Downs 1957) has to be reevaluated.

The goal of this paper is to examine the effect of one particular electoral system, a multiple vote system that can produce centripetal forces on the party system of a country, and examine the specific conditions under which this result can be obtained. While the formal difference of this electoral system seems small (provide voters with multiple votes, and therefore multiply their choices), the substantive difference is fundamental: it is thought to break down "party identification," since it permits voters (if they want) to use multiple criteria to select among parties, therefore enabling them not to "identify" with any one party in particular. We will show how this lack of identification combined with the fact that, most of the time, one's second or third choice will be as close as possible to their initial preference (single peaked preferences) lead to a party system where centrist parties prevail, and political debate becomes less polarized. We also identify the (very rare) conditions under which this kind of result may not prevail, and demonstrate that a small perturbation of these conditions will lead again to a centripetal party system.

II. MULTIPLE VOTE SYSTEMS IN THEORY AND PRACTICE

Multiple Vote Systems in Theory

Electoral systems have been shown to affect not only the number of parties (Duverger 1951) but also their positions along the political spectrum (Cox 1990). With respect to the latter, Cox (using a one-dimensional policy space) has demonstrated that multiple votes create centripetal forces inside a political system. More specifically, when the number of candidates is small enough relative to the number of voters per voter, and when cumulation (i.e., allowing a voter to cast all of her votes for one candidate) is not allowed, centripetal forces will predominate and candidates and parties will be drawn to the center of the political spectrum.

Though Cox's results are subject to a series of assumptions about voters, candidates, and the policy space, they provide a clear theoretical framework for understanding why a polity might choose to implement a multiple vote system as a means for combatting political polarization. Cox's results are not the only (nor the first) to suggest that multiple vote systems moderate candidates, however. Indeed, the "approval voting" literature has also suggested that multiple vote systems

could advantage moderate candidates, and in the American context,[1] weaken (or even destroy) the two-party system (Brams and Fishburn 1978). Under approval voting, voters receive *n* votes that they may or may not choose to use on different candidates in an election. Under this system, cumulation is prohibited, similar to the centripetal case underscored in Cox's analysis. The practical effect of this system, its proponents (e.g., Brams and Fishburn 2007 [1980], Kellett and Mott 1977) argue, "would probably be to give comparatively more support to moderates" (Brams and Fishburn 1978: 840). Consequently, such proponents have in the past argued that the major parties in America should adopt approval voting as their primary-election voting system, because "most delegates find [moderates] acceptable," while "extremists […] are only acceptable to ideological factions in their party" (840).

A similar logic has evolved in support of ranked choice voting, particularly in state-level elections in the United States. According to one prominent version of ranked-choice voting., voters rank candidates on their ballots according to preference, ranging from least to most favorable. Should a single candidate not receive a majority of first-place votes following the election, the ballots of the last-place candidate flow to the respective remaining candidates—a process that is repeated until the final winner is selected. Proponents of this system argue the procedure generates a winner that is more centrist, more widely acceptable, or both for a larger portion of the electorate than does majority rule or, especially, plurality elections (Santucci 2018, Fromuth 2019).

Multiple Vote Systems in Practice: Past and Present

The centripetal forces inherent to multiple vote systems help to explain why variants of the multiple vote system have been adopted in a wide variety of settings over thousands of years. Most of the times such systems have been adopted in single member districts (with one winner). In ancient Greece, the Spartans' "acclamation vote" served as an early form of approval voting, as voters were allowed to shout in favor of more than one candidate for the *Gerousia* (Girard 2010, Tsebelis 2018). Though certainly quieter than the Spartan vote, papal elections from the late thirteenth to the early seventeenth century also took a form that resembled approval voting. According to this voting system, cardinals had the option of voting for more than one papal candidate The pairing of this system of voting with a 2/3rds qualified majority threshold created long vacancies in the papacy, ultimately leading to the voting system's demise. However, as Colomer and McClean (1998) argue, the system did encourage the election of largely unobjectionable popes, which helped to address longstanding tension (and even violence) within the Church.

Political entities today have also adopted versions similar to the multiple vote system. Most notably in the American context, the states of Washington and California have adopted the so-called "top-two" primary system, in order to combat polarization. Under the top-two primary, candidates from all parties are consolidated into a single party, wherein voters select their most-preferred candidate. The top two vote-getters from this primary round then advance to the general election—regardless of their partisan affiliations. Consequently, candidates from the *same political party* may compete against one another in the general election. This novel system first arose in Washington in the mid 2000s, as a response to a court decision to dismantle Washington's long-standing "blanket

[1] For which the early approval voting proponents were writing.

primary." The blanket primary arose in the 1930s as a response to political parties exerting undue influence over farmers and other private citizens (Shea 1984). It functioned similarly to the top-two primary, in that voters could cast a primary election vote for candidates of any party they pleased. However, same-party competition in the general election was impossible, because the top vote-getters from *each* party (and not overall) were the candidates to advance to the general election.[2] When California copied this primary election system in 1996, the Supreme Court struck down the system (*California v. Jones,* 2000). Washington responded by creating and implementing the top-two primary a few years later. While the aims of the system were many, one aim was to advantage centrist candidates for office. The need for such a system arose in part from the states' similar political geographies: both states both possess relatively large, liberal cities along the western cost and rural conservative territory inland. Given that all legislatures in America are elected via geographic districts in first-past-the-post elections, this sort of geographical polarization carries with it the distinct possibility of heightened political polarization in the legislature.

The top-two primary addresses the challenge of partisan-homogenous districts by leveraging the votes of the "out-party" in the district. In particular partisan-imbalanced districts, such a system is expected to allow for same-party competition in the general election, allowing "out-party" voters to elect the more moderate of the two candidates. For instance, in the Republican-district example highlighted above, Democrats would no longer "waste" their votes on a Democratic candidate who could not possibly win. Rather, they can thrust their support behind the more moderate Republican, joining forces with moderate Republican voters to elect this candidate. Though not a multiple vote system in the most traditional sense, the top-two primary does indirectly grant each voter two votes. It also releases voters from official attachments to a particular party's candidates. Some evidence suggests that the top-two primary moderates a legislature (Grose 2014, Crosson 2020).

Outside of the top-two primary, ranked choice voting has also developed as one of the most popular variants of multiple vote systems among election reform advocates today. In the most prominent victory for ranked-choice advocates to date, the U.S. state of Maine adopted a system of ranked-choice voting for its legislative and gubernatorial elections. As noted above, proponents of the system tout its majority-friendly and centripetal features, though the recency of the reforms have precluded direct empirical tests of these assertions. Nevertheless, reformers ultimately succeeded in Maine due in part to the election and reelection of a widely unpopular Republican governor who never succeeded in securing absolute majority support from voters in the state (Santucci 2018).

Other modern entities have also either proposed or adopted versions of multiple-vote systems. According to a review on approval voting success and failure by Brams and Fishburn (2010), some five professional societies have adopted some version of multiple votes system. These include the Mathematical Association of America, the American Mathematical Society, the Institute for Operations Research and Management Sciences, the American Statistical Association, the Institute of Electrical and Electronics Engineers, the Society for Judgment and Decision Making, the Social Choice and Welfare Society, the International Joint Conference on Artificial Intelligence, and the European Association for Logic, Language and Information, the Econometric Society, and the National Academy of Sciences. While elections in these societies may not be exactly ideological or

[22] See https://www.sos.wa.gov/elections/blanket_primary.aspx for more information.

high stakes, Brams and Fishburn find that the multiple vote systems appear to advantage candidates who enjoy support from a large cross-section of the societies' memberships. Similarly, in highly multidimensional contests like gymnastics and diving, Olympians are judged using either multiple rankings or ratings. In doing so, officials hope that the athlete agreed upon as best by the largest group of judges will be selected for a medal. Finally, two Nobel Prize winners in Economics (Maskin and Sen 2016) have argued that a majority requirement in American primaries would have eliminated Trump as the candidate for the Republican party. The same would have happened with an STV system whether at the Republican primaries, or at the general election.

In multimember districts the empirical analysis comes from the STV systems in Ireland and Malta (and the Australian Senate). The general agreement from these analyses is that "Although the system provides representation to minor parties, results in single transferable (STV) elections generally have shown that minor centrist parties benefit from the system and minor radical parties are penalized". (Encyclopaedia Britannica). More to the point, researchers have found that voters sometimes transcend party or group barriers, and vote invidual candiates of their liking. For example, Mitchell (2014) comparing the electoral results before and after the 1998 Belfast Agreement in Ireland notes that "prior to the 1998 Agreement inter-ethnic vote-pooling in Northern Ireland was very close to zero. Afterwards (1998-2007) terminal transfers from the moderate unionist UUP to the moderate nationalist SDLP averaged 32 per cent (and 13 per cent in the opposite direction). Although most transfers clearly remain within ethnic blocs, these inter-ethnic terminal transfers are a change with the past and suggest that SW may be an appropriate electoral system choice for some divided societies."

Theoretical and Practical Challenges for Understanding Multiple Vote Systems

While previous implementations of multiple vote systems have encountered some success in terms of electing broadly supported, moderate candidates, they nevertheless face limitations in both theory and practice. First, while studies like Cox's prove in one dimension that multiple vote systems can draw candidates to the center of the political spectrum, he (nor anyone else to date) does not offer a proof in *n* dimensions. This stands as a challenge to the multiple vote system, as a second or third dimension can change the definition of "moderate" in a political system and create possibilities for candidates to be close to one another in one dimension while remaining disperse in another.

Perhaps the most relevant example in contemporary politics lies in the current populist movements across the United States and Western Europe. Indeed, while populists lie to the far right of the political spectrum on cultural issues, they nevertheless often support interventionist policies in economy. In the United States, for example, top advisers to President Donald Trump have expressed strong support for spending on infrastructure, while nevertheless maintaining culturally rightist positions on religion and immigration. Similarly, in Great Britain, populists have maintained an anti-Muslim posture while at the same time driving the Brexit campaign and a swell of anti-free-trade rhetoric. In short, the rise in popularity of populist ideas has muddied the neat left-right distinction implied by unidimensional models. As we find, this can present challenges for the effectiveness of multiple votes systems.

In addition, though the STV system asks voters to rank candidates, the multiple votes it provides are *conditional* choices only. That is, the voters' second or third choices enter influence the election only after the first vote is invalidated. This feature makes each additional choice less important than the prior, *and* complicates the system.[3]

Given these challenges to the current theory and practice of multiple vote systems, we focus in this paper on the mechanical consequences of adding *n* issue dimensions and a proportional voting mechanism to a multiple vote system. In doing so, we find that multidimensionality and proportional representation create centripetal party systems, just as current literature predicts. We also find that under certain extreme centrifugal conditions that the electoral system cannot overcome in the first election, but is likely to modify in the medium and long run. We conclude by discussing some scope conditions of this effect.

III. A MULTIDIMENSIONAL, MULTIPLE VOTE MODEL

Voting System Design

Tsebelis (2014) has proposed a multiple vote electoral system, which combines multiple votes with proportional distribution of the results.[4] In this study, we examine this system: a rank-list style proportional voting system with *m* votes per voter. Under this system, voters may cast up to *m* votes in total, with a maximum of one vote per party In other words, even if a voter strongly prefers one party to the next best option, she may *not* cast a second or third vote for that preferred party (no cumulation of votes). However, if she strongly dislikes all other options besides her most preferred party, she can opt against casting more than one ballot. Thus, voters may cast any number of votes they desire, with a maximum of *m* and a minimum of 1 (all abstainers are assumed to have already been removed). In our implementation of this system, the ballot entities are conceived of as parties (although one could imagine implementing a similar system with actual candidates, instead of parties).

According to this model, parties receive the same proportion of representation in the legislature as a proportion all votes cast. In our multiple vote case, this proportion is not as straightforward as the single-vote case. In our system, representation is allotted by

$$P_i = \frac{V_i}{m * N - A}$$

where P_i is the legislative proportion earned by party *i*. V_i is the total number of votes cast for party *i*, *m* is the number of votes allotted to each voter in the system, and *N* is the total number of voters.

[3] Results in STV in multimember districts depend on the way the "quota" (that is the cutoff point between the (n)th or the (n+1)th choice of a voter) will be activated, making voter choices complicated and sometimes leading either to sincere preferences (Bowler and Grofman 2000: 268) or to the "donkey vote" (Australian term), that is, vote for the different candidates in the order they appear on the ballot. In addition, given the calculations required, the results take a long time to be announced.

[4] The proportional distribution is used in multimember constituencies. In single member constituencies the electoral system presented here is identical with approval voting.

A is an important term in this fraction, as it signifies the total number of abstentions present in an election. As noted above, voters can choose against casting all their multiple votes, if they deem some unacceptable. Thus, the inclusion of this term is necessary for calculating the actual proportion of total votes cast.

Our proportion P_i differs from ranked choice voting in that it all votes count equally in the final tabulation of V_i. This is a significant difference from the STV system, where the multiple votes count only conditionally, that is, only *after* incapacitation of the previous vote. This difference has the double effect of 1. Simplifying the system for both voters and authorities alike (two of the main criticisms of STV), and 2. Providing more motivation for voters to use many of their votes, since such a behavior increases their contribution to the electoral result. Casting multiple votes, we believe, is less cognitively demanding for voters, as it requires them only to inquire whether a given candidate is sufficiently acceptable to warrant one of their votes. Given that some literature has demonstrated that voter exhaustion leads ranked-choice voting to rely on only a fraction of total ballots in the final vote distribution used to select a winner (Burnett and Kogan 2015), we believe the equally weighted and singularly tabulated votes in our system improve upon this particular weakness of ranked-choice voting.

Modeling Assumptions and Mode of Analysis

To examine how multidimensionality and proportional representation impact the centripetal nature of multi-vote systems, we create a voting simulation in **R,** using the electoral system defined above**.** Simulation is necessary in this context, because of our interest in multidimensional issue spaces (analytic proofs in n dimensions are impossible). Instead, we run simulations and examine the results that obtain for various parameter specifications.

To proceed with the simulation, we created a customizable function in **R** that implements the aforementioned multiple-vote PR system. The function proceeds as follows. First, the user specifies several system-wide parameters of interest. These include both the number of votes m allotted per voter and the total number of voters N in the political system. The user must also define the ideal points of each party i in each issue dimension d. The function generalizes to any number of parties and dimensions, on the condition that the user provides an ideal point estimate for every party in each dimension. In addition, parties may decide *not* to take position in some dimension.[5]

Beyond these parameters, there are several other user-defined parameters of note, including one related to abstentions (which are incorporated directly to the voting decision rules programmed into the model). In the model, voters have *single peaked preferences*, that is, vote on basis of ideological proximity: voter $n \in N$ casts each vote on the basis of the following decision rule:

$$argmin_i \left(\left| |n - i| \right|^d \right)$$

[5] In this case this issue dimension does not influence the calculations of voters with respect to this party.

where $||*||^d$ represents the Euclidean distance in *d* dimensions between voter *n*'s and party *i*'s ideal point. As noted earlier, voters may vote for each party only once, (like in approval or transferable voting).

Because voters are prohibited from casting multiple voters for their top choice, they are not obligated to make use of all their vote choices *m*. Instead, voters will only cast a vote for a party if and only if the following condition obtains:

$$||n - i^{-}||^{d} < a$$

where i^{-} refers to the nearest available party and *a* refers to a user-defined range of acceptability. In other words, once the distance between voter *n* and the remaining parties exceeds the user-defined range of acceptability *a*, voter *n* will stop casting votes. If the user is not interested in restricting voter behavior in this way, *a* can be easily set to a very large number.

Finally, to render our model more "realistic," we incorporate an error term in the voter's calculations. Voters make their choices on the basis of distance between their preferences and the parties' programs; however, with a probability 1-r they may not select the party closer to them. In this case, the voter casts her vote randomly to one of the available parties. This behavior can also be generated if a voter attributes higher significance to a particular issue and is willing to vote for a party that is closer to her in a particular dimension, despite the fact that the overall distance (taking into account all dimensions) is large. While this parameter adds noise to our results, we believe it is a useful way to relax the strictly single peaked preference account of voting inherent to the model's implementation. Like *a, m, N, d,* and *i*, *r* is a user-specified parameter that represents the probability that voter *n* selects the party closest to her.

Simulation Procedure

The simulation proceeds by first transforming a matrix of party shares into a society of voters. Because multidimensional ideological estimates do not exist for entire citizenries, we begin first with a user-specified list of proportions of the legislature held by each party. From these proportions, the simulation creates a vector of length *N* with voter identities and ideal points equal the proportions and ideological locations of the legislative parties. In other words, if Parties X, Y, and Z occupied 20, 30, and 50 percent of the legislature, respectively, then a 10-person society would include 2 citizens who identify with Z, 3 who identify with Y, and 5 who identify with Z. The first vote vector is always equal to the actual electoral outcomes from the year in question-20 percent X, 30 percent Y, and 50 percent Z in the example above.

After generating this initial vector of voters, the algorithm then calculates the Euclidean distance between all voters and parties and determines which party lies second-closest to each party's voters. If this distance is greater than the acceptability parameter *a*, the voter refrains from casting any more ballots. If the distance is less than *a*, the voter (with probability *r*) casts a vote for the most proximate party. With probability 1-*r*, however, she casts her vote randomly. Once this process occurs for all voters, votes are tabulated for each party and representation is allotted accordingly.

For multiple vote systems that feature more than 2 ballots, the algorithm proceeds as follows. First, rather than assuming that all voters in a given party share exactly the same preferences, the algorithm instead assumes that each voter *j* is likely located somewhere *between* her

first-choice and second-choice parties. To capture this dynamic, then, the algorithm *recalculates* the vote proportions occupied by each party *on the basis of the two-vote (rather than one-vote) outcome*. This relaxes the assumption that all party members share their party's ideal point, rendering the model more realistic in the votes it assigns to voters.[6] For all ballots cast beyond this point, this procedure repeats: at the beginning of each round of voting, the *total* vote count is used to reallocate voter locations, and votes are cast on the basis of proximity.[7] The procedure stops when the algorithm reaches the maximum number of ballots supplied by the user.

In this paper, we base our simulations on actual countries, using the classification of party systems generated by Laver and Benoit (2015). They present a mutually exclusive and collectively exhaustive classification of party systems into 5 basic categories. According to this system of classification, Category "A" countries exhibit a single "winning" party that controls all legislative decisions; Category "B" countries are led by a single, dominant party that governs in coalition with a smaller party;[8] in Category "C", the legislature is led primarily by three parties—any *two* of which are large enough to form a coalition government; Category "D" countries, on the other hand, are dominated by two "top" parties; Category "E" countries exhibit a party system that is truly "open," in that no winning two-party coalition is possible (based on the sizes of the parties in the system).

For our purposes, the most interesting countries for our model are of types C and E, because the mechanics of the proposed system are straightforward in the other three cases: the dominant party or one of the two competing coalitions will have a majority. Countries of type "C" or "E" however, provide unpredictability of the results which, provides a fertile ground for voters to use their preferences to influence the electoral results in a more significant way. If new parties emerge in the other systems, the policy space dimensions will increase and the party system will move to one of the two categories we examine. Interested readers may use our appendix to apply our model to analyze any system or particular country they want. In this application, we select Germany and Romania to serve as examples of Type C, and the Netherlands and Belgium as examples of Type E. We consider these four cases as sufficient to investigate the properties of the multiple vote system we propose.

To generate ideological positions for each party, we rely upon data from the Manifesto Project (manifestoproject.wzb.eu). The Manifesto Project provides high-dimension data on party ideology, based on the contents of party manifestos. For our four countries, the Manifesto Project provides ideological positions on 21 dimensions, all of which were used in our simulations. Once the parties of the selected countries were matched to the Manifesto Project data, our algorithm could measure *n* dimensional Euclidean distances between our generated voter populations and the locations of each of the parties. Given the high dimensionality of the data, providing visual representation of the parties' ideal points is impossible. However, as we present our results, we

[6] While the results presented below feature this more realistic depiction of voters and their locations, the results do not differ substantively compared to results assuming all voters share their initial party's ideal point.
[7] On the condition that the same individual voter cannot cast a vote for the same party twice.
[8] Laver and Benoit develop subcategories within the B-type, based on whether the dominant party can form a winning coalition with *any* other party in the system, or only with a particular subset of parties.

ultimately present ideological centrism as each party's distance from the "center of gravity"[9] of the ideological distribution of voters. We define these measures more precisely in the results section.

Taken together, our expectations are as follows:

> Proposition 1 (centripetal effect): *Multiple votes will increase the shares held by centrist parties (and reduce extremist ones).*

The logic underlying this proposition is simply that centrist parties will receive votes from all directions, while extremist ones only from their own area (if there are neighbor parties).

> Proposition 2 (redistributive effect): *Multiple votes will have a negative effect on the initial size of parties.*

Indeed, smaller parties will get higher number of "transfer" votes than larger ones and vice versa. Beyond these two propositions, however, we do not anticipate that the other variables will have a systematic effect on party shares, but will depend on the distribution of parties in space.

IV. RESULTS

In order to show the centripetal effects of the multiple voting system we present our results as a comparison between the *n* vote cases and the classic, one-vote system of proportional representation, asking which parties gain (and lose) as a result of the *n* vote system? In general, we present results using the multidimensional *center of gravity* as our measure of moderation.[10]

As noted above, our simulation features parameters that may affect our findings. These include the probability 1-*r* that voters will fail to vote on the basis of ideological proximity, , the range of "acceptability" (ideological distances within which an individual is willing to actually cast a vote), and the total number of ballots. Thus, in presenting our results, we regress the gains from the *n*-vote system (relative to the one-vote) on each of these parameters: the propensity of voting based on propensity (1-*r*), acceptability (*a*), and the number of ballots (*b*). Inclusion of each of these covariates ensures that we hold factors besides ideological centrism constant when examining the centripetal forces present in the n-vote system.

[FIGURE 1 HERE]

[9] More specifically, we measure the marginal median; see Puri and Sen (1971).

[10] For robustness, however, we also calculated results using the vector of marginal medians (Puri and Sen 1971), as well as the Euclidean or geometric median, as our measures of centrality. The former measure is simple a vector of median values in each dimension, while the latter selects a point in *n* space that minimizes average *n* dimensional Euclidean distance from each point in the distribution. While we do not present those results here, they are substantively quite similar to the presented results.

Figure 1 presents a coefficient plot of the pooled fixed-effect regression described above. Taken together, the results depicted in the figure provide support for the centripetal (Proposition 1) and redistributive (Proposition 2) effects of the multiple vote system. Indeed, as a party's distance from the ideological center increases, vote share decreases. Also as predicted, initial party size is an important variable to control for, as smaller parties clearly benefit most from n-vote alterations. The other variables are overall close to zero.

[TABLE 1 HERE]

Although these results are suggestive, they tell only part of the story—as demonstrated by the country-specific results display in Table 1: the centripetal and redistributive properties of the system exist in all countries when pooled together. But while the coefficients of the parameters used in the model are overall as expected (higher acceptability, higher number of votes, and lower error term in the single peaked preferences lead to more vote gains for the average party) they also show low significance or variability in different countries. This finding indicates that the significance of these parameters depends on the party distribution in each country. Similarly, in the Netherlands, while the signs of the coefficients of centripetal and redistributive effects are the "correct" ones and statistical significance is high, the relative size of the coefficients indicates that the redistributive effect is much more significant than the centripetal one.

Given these significant across country differences, we therefore examine the results more closely by country, collapsing the distribution into a two-dimensional space in Figure 2. To do so, we factor analyzed the 21-dimension space captured by the Manifesto Project and report the two primary factors we recovered. Given this procedure, we remind the reader that the graphs may exhibit some differences from a figure that plots any pair of the "raw" dimensions found in the Manifesto Project.

[FIGURE 2 HERE]

Among the countries that do experience centripetal results, this visualization provides some context as to which parties tend to drive the result. In Germany, for example, gains by the Free Democratic Party demonstrate how proximity to the multidimensional median or centers of gravity can benefit a party in our multiple-vote system. The same is true among one of the E-type countries. In Belgium, sizeable gains by Centre Demecrat Humaniste and ECOLO (as well as losses by Mouvement Reformateur and Libertair, Direct, Democratisch) generate rather strong centripetal results.

Nevertheless, as is plainly depicted for the Netherlands, a different set of dynamics is at play. In the Netherlands, we observe a party system that is highly polarized, where parties are located in clusters that lie far apart from one another. Under these conditions, the multiple vote system lacks a party to attract votes from these extremes. Instead, parties exchange votes within ideological clusters, exacerbating—not alleviating—preference polarization in the legislature. Under such a preference configuration, a different sort of centrality is rewarded: cluster-specific centrality.

As Figure 2 demonstrates, the Reformed Political Party, Green Left, Christian Union, Democrats 66, Christian Democratic Appeal, People's Party, and Labour Party are clustered in the bottom right, while the Party for Freedom, Party for the Animals, and Socialist Party are clustered at the upper left. Under the simplest multiple-vote system (two votes with unlimited acceptability), such a configuration implies that the most centrist parties are too far away from their respective cluster centers—and too far away from the other cluster of parties—to attract many additional votes. Indeed, while the Christian Union does not benefit from its overall centrist position, the Party for the Animals—a small, fringe party—benefits heftily from its central location within the upper-

left cluster. This dynamic persists because of the lack of a party directly *between* the cluster to siphon votes from the edges of the respective clusters.

This spatial distribution of parties explains why in Table the Netherlands demonstrate a low coefficient of centripetalism. In order to corroborate this argument we perform an additional test below, wherein we perturb the Netherlands' party distribution. That is, we show that when a trivially small party is added between the clusters of a polarized system (such as the Netherlands), centripetal forces become more pronounced.

[FIGURE 3 HERE]

Reinforcing Moderation in Polarized Systems

To test our claim that a small spatial deviation can transition a polarized country like the Netherlands into a more centripetal case like Belgium, we introduce a small centrist party—claiming less than 1 percent of the initial vote share—into our Netherlands simulation. We locate the party near the multidimensional median,[11] though our results do not qualitatively change if this location is shifted around the neighborhood of the center of gravity, as we highlight below. The results of this test are depicted in Figure 3, where we have again collapsed the parties into two dimensions.

When we introduce such a small but centrist party, we find that even the polarized Netherlands experiences the expected association between overall ideological centrality and a party's final vote share. In Figure 3, our added party, labeled as "Center," makes sizeable gains as a result of the *n*-vote system. Indeed, because the party draws votes from voters within both extreme clusters, it experiences far greater gains than do parties located in the center of the clusters. As noted above his trend does not apply only to perfectly centrist parties, however. Indeed, so long as the centrist party lies within the acceptability range of the inner-most parties from each cluster, we observe a significant coefficient on *Distance to Center* variable in the above models. This is particularly true given that the most "centrist" existing parties still retain their n-vote gains: that is, according to Figure 3, the Christian Union and Reformed Political Party each make gains in the perturbed n-vote simulation(s). In fact, by rearranging the proximity orderings between existing parties, the introduction of this the central party helped to augment the gains made by existing centrist parties.

The magnitude of these results is more fully captured in Table 2. Here, where we report the regression results for the small deviation from the median depicted in Figure 3 (Model 1), as well as results for larger deviations (Models 2 and 3). In Model 1, we locate the centrist party at the multidimensional median of the existing parties, plus a constant of 0.5 ideological unites. In Models 2 and 3, we introduce larger deviations, perturbing each of the 21 dimensions away from the multidimensional median by random draws from a uniform distribution between -1 and 1 (Model 1) and -1.5 and 1.5. In each case, the introduction of the small, centrist party generates strong centripetal forces, even in a polarized party system like that in the Netherlands.

[TABLE 2 HERE]

CONCLUSIONS

As noted throughout, this paper examines the mechanical effect of a multiple vote system, using actual countries' party distributions as a means for examining how and when our system should encourage the election of centrist parties or candidates. As we caution throughout, our analysis does

[11] Perturbed by a constant of 0.5 ideological units in each of the 21 dimensions.

not examine the strategic effects of such a system. Nevertheless, we believe that examining the mechanical effect of multiple vote systems is important for a number of reasons. First, it demonstrates that such systems do not behave identically in all countries, but nevertheless that countries have broad similarities as Table 1 indicates. Second, it is upon these broad similarities that strategic calculations of voters and parties will be *based*. One may argue that voters have personal, social, or cultural misgivings about voting for an extreme party (like, say, a fascist or communist party). In addition, parties are constrained in their ability to adjust their ideological positions in a rapid fashion; indeed, activists within the party would likely resist such changes, and voters may respond poorly to drastic changes in the ideological "brand" associated with the party. Thus, while future research may account for important strategic considerations faced by voters and parties, these considerations have to be based on the mechanical effects of the multiple vote system in the same way as Duverger's "psychological effect" was grafted upon the "mechanical effect" of the plurality system.

In this paper, we have used simulations in order to calculate the mechanical effects of multiple voting systems (whether they are applied to single or multiple member constituencies). Our findings confirm the ones of Cox in a single dimension. He was able to prove his results, because the combination of single peaked preferences with a single dimension leads to an equilibrium (the median voter). However, in multiple dimensions the equilibrium disappears (and so do the formal proofs), generating the need for simulations. The lack of equilibrium in multiple dimensions leads us to a different logic for our investigation. While Cox's model leaves the parties free to move in the one dimensional space and determines whether (in equilibrium) they cluster in the middle or disperse all over the (one-dimensional) space in order to maximize the number of votes, we keep the parties in their initial location and have the voters select the parties that are closer to them (as the different parameters of the model permit). The outcome of our model is that centrist parties get better results with multiple votes. So, our model demonstrates that the single dimension is not a necessary condition for convergence, but that the single peaked preferences of the voters is.

Beyond this mechanical effect, however, we believe that the adoption of this multiple vote system may imply several additional long-term. With respect to voters, this system presents an exponential increase in the number of voting alternatives. Indeed, if we permit voters to have number of votes equal to half the number of parties, the number of choices is:

$$\sum_{i=1}^{\frac{N}{2}-1} \binom{N}{i} + 1$$

where N equals the total number of parties in a country. This increase of choices is likely to reduce the number of abstentions (since it eliminates abstention from indifference (e.g., Plane and Gershtenson 2004). Indeed, a voter who does not know if she should vote for party A or B in a multiparty system may now vote for both. Moreover, she may do so without confronting the cognitively taxing task of ranking candidates: all votes in this setting are "worth" the same.

In addition to its potential for decreasing abstention, we believe that a multiple vote system may help to increase voter information. In order to evaluate different candidates under such a system, voters will have to pay attention to the positions of a larger number of candidates—understanding that they will ultimately be voting for more than a single candidate. Moreover, understanding that actually casting multiple votes increases their impact on the outcome, voters face incentives both to cast more votes and improve their information in the process. We are hopeful that particular feature of the multiple voting system will have a significant impact on the voting habits of the public. With respect to parties, our results—particularly in the perturbation exercise—suggest that the total number of parties will multiply, since there is no reason for any political

entrepreneur not to create their own party and try their chances. This is particularly true given that they can reasonably expect many second or third votes from major parties around them, if the party is situated appropriately. In order to reduce this tendency, countries may consider strict rules of which parties are allowed to compete should be enforced (for example, parties have to exist 6 months before the election, and a large number of signatures is required for the creation of a new party). These restrictions will enable to voters to know the positions of the parties in competition, and choose them according to their preferences.

Third, with respect to the party positions, we showed that centrist parties are privileged in a multiple vote system, except for the cases where party clusters emerge in large distance from each other (like the case of Netherlands in our examples). Still, when the system is applied several times, the *emergence* of a centrist party--or the convergence of exiting parties close to the multidimensional median--is likely, because political entrepreneurs will understand the potential for success of such a party.

Finally, perhaps the most important consequence of such a voting system (although not directly demonstrated in this paper) is the potential promotion of a critical attitude of voters vis a vis parties, as opposed to an identification attitude. That is, instead of voters trying to find a party to identify with, they can be more critical and express their preferences more fully (if they so wish). This result carries with it both pros and cons. On one hand, party identification fulfills a variety of positive societal functions, such as increasing voter turnout, serving as a policy evaluation heuristic, and encouraging other types of political participation (see Dalton 2016 for a review). However, as Lavine, Johnston, and Steenbergen (2012) and others have underscored, intense partisan identification can lead to narrow-mindedness on the part of partisans. Indeed, such identifications may lead partisans to disregard important and informative information that does not confirm their partisan biases. Doing so could empower demagogic leaders or create partisan informational asymmetries and fracture a society according to partisan identifications. Regardless of whether a weakening of identification is a positive or negative externality, such a possibility is an important possible implication of the prosed electoral system, beyond the potential moderating effects examined here.

Works Cited

Bowler, S. & Grofman, B. (2000) *Elections in Australia, Ireland and Malta under the Single Transferable Vote: Reflections on an Embeddend Institution* (Ann Arbor: Michigan University Press).

Brams, and Fishburn P.C., 2010. Going from Theory to Practice: The Mixed Success of Approval Voting, in Laslier, J.F. and Sanver, M.R. eds., 2010. *Handbook on approval voting*. Springer Science & Business Media.

Brams, S. and Fishburn, P.C., 2007. Approval voting. Springer Science & Business Media.

Brams, S.J. and Fishburn, P.C., 1978. Approval voting. American Political Science Review, 72(3), pp.831-847.

Colomer, J.M. and McLean, I., 1998. Electing popes: approval balloting and qualified-majority rule. The Journal of interdisciplinary history, 29(1), pp.1-22.

Cox, G.W., 1990. Centripetal and centrifugal incentives in electoral systems. *American Journal of Political Science*, pp.903-935.

Crosson, J., 2020. Extreme districts, moderate winners: Same-party challenges, and deterrence in top-two primaries. *Political Science Research and Methods*, pp.1-17.

Dalton, R. 2016. "Party Identification and Its Implications." *Oxford Research Encyclopedia of Politics*. Retrieved from http://politics.oxfordre.com/view/10.1093/acrefore/ 9780190228637.001.0001/acrefore-9780190228637-e-72.

Fromuth, Peter. 2019. Ranked choice voting is easier than it sounds. Maybe it would cure our awful politics. *USA Today*. Available at: https://www.usatoday.com/story/opinion/2018/09/06/ranked-choice-voting-elevate-moderates-reduce-polarization-column/1196693002/ [Accessed 5 Sep. 2019].

Girard, C., 2010. Acclamation voting in Sparta: an early use of approval voting. In Laslier, J.F. and Sanver, M.R. eds., 2010. *Handbook on Approval Voting*, pp.15-17.

Kellett, J. and Mott, K., 1977. Presidential primaries: Measuring popular choice. *Polity*, *9*(4), pp.528-537.

Laver, M. and Benoit, K., 2015. The basic arithmetic of legislative decisions. *American Journal of Political Science*, *59*(2), pp.275-291.

Lavine, H.G., Johnston, C.D. and Steenbergen, M.R., 2012. *The ambivalent partisan: How critical loyalty promotes democracy*. Oxford University Press.

Mitchell, P., 2014 The single transferable vote and ethnic conflict: The evidence from Northern Ireland. *Electoral Studies* 33: 246-257

Maskin, E., & Sen, A. (2016). How Majority Rule Might Have Stopped Donald Trump. *New York Times*. April 28. Retrieved from https://www.nytimes.com/2016/05/01/opinion/sunday/how-majority-rule-might-have-stopped-donald-trump.html

Plane, D.L. and Gershtenson, J., 2004. Candidates' ideological locations, abstention, and turnout in US midterm Senate elections. *Political Behavior*, *26*(1), pp.69-93.

Puri, Madan L.; Sen, Pranab K.; Nonparametric Methods in Multivariate Analysis, John Wiley & Sons, New York, NY, 1971.

Santucci, J., 2018. Maine ranked-choice voting as a case of electoral-system change. *Representation*, 54(3), pp.297-311.

Shea, I.E., 1984. *The grange was my life*. Ye Galleon Press.

Tsebelis, G. 2014. The Greek Constitution from a Political Science Point of View" in *Greek Political Science Review* (July; 42): 145-72

Tsebelis G. 2018 "How Can We Keep Direct Democracy and Avoid "Kolotoumba"" *Homo Oeconomicus* 35(1): 81-90

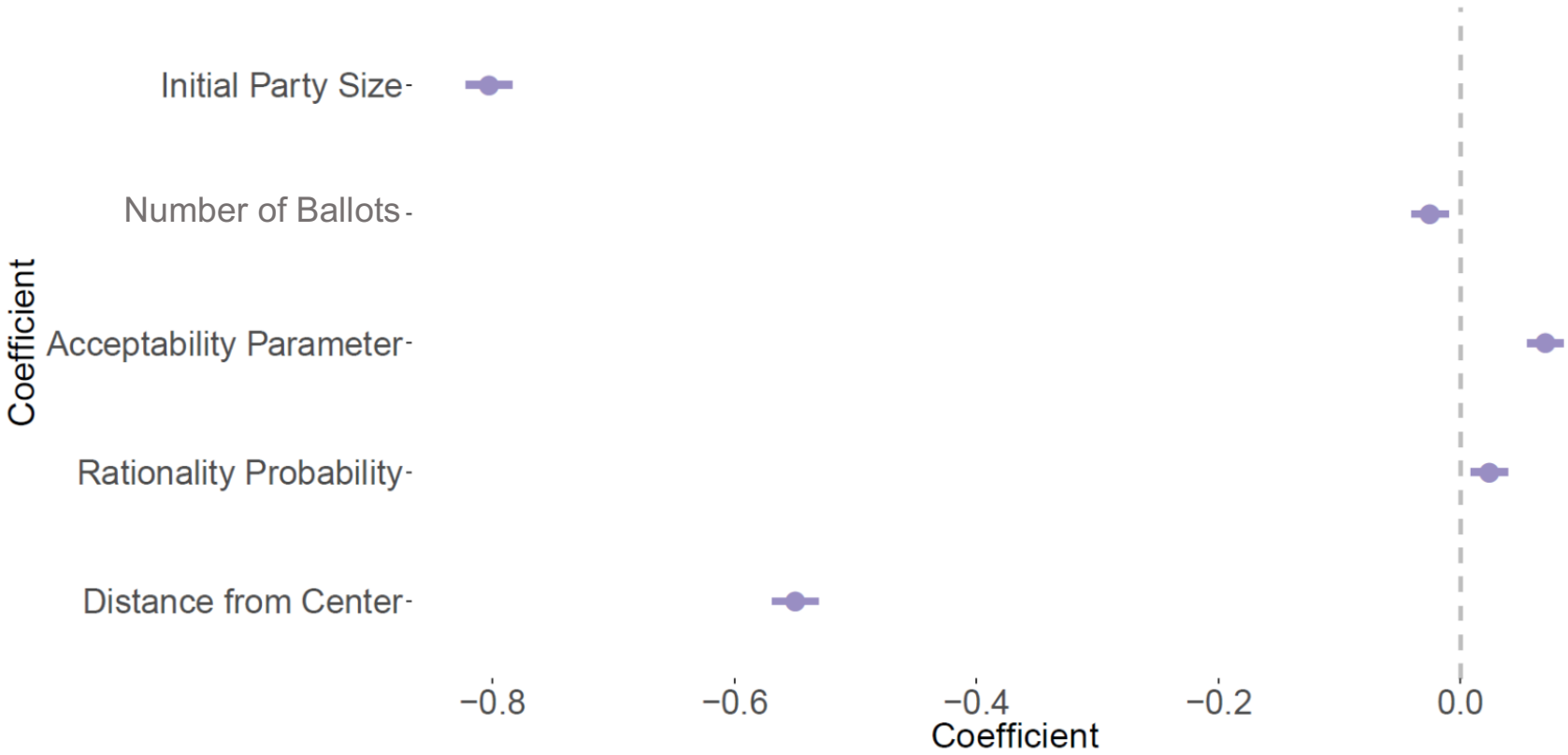

Figure 1a. Ideological Centrism and Gains from *n*-vote System; Coefficient Plot
(pooled fixed-effects regression; Model 1 in Table 1)

Table 1. Country-Specific *n*-vote Regression Results
(Center of Gravity)

	Dependent variable:				
	Gains from *n*-vote System				
	(1)	(2)	(3)	(4)	(5)
Distance from Center	−0.258***	−0.632***	−0.627***	−0.099***	−0.120***
	(0.008)	(0.016)	(0.031)	(0.007)	(0.009)
Initial Party Size	−7.998***	−14.680***	−37.423***	−6.522***	−1.512***
	(0.100)	(0.215)	(0.739)	(0.077)	(0.113)
Acceptability Parameter	0.010	0.055***	−0.002	−0.007	−0.001
	(0.007)	(0.012)	(0.018)	(0.007)	(0.002)
Number of Votes	0.209***	0.086***	0.356***	0.109***	0.123***
	(0.012)	(0.017)	(0.027)	(0.024)	(0.007)
Proximity Voting	−0.100	0.276*	−0.029	−0.931***	−0.010
	(0.093)	(0.164)	(0.258)	(0.106)	(0.030)
Constant	2.485***	5.219***	7.106***	3.259***	0.581***
	(0.122)	(0.212)	(0.383)	(0.135)	(0.075)
Observations	24,000	6,000	6,000	6,000	6,000
R^2	0.235	0.476	0.309	0.499	0.076
Adjusted R^2	0.235	0.476	0.309	0.498	0.076
Note:					*p<0.1; **p<0.05; ***p<0.01

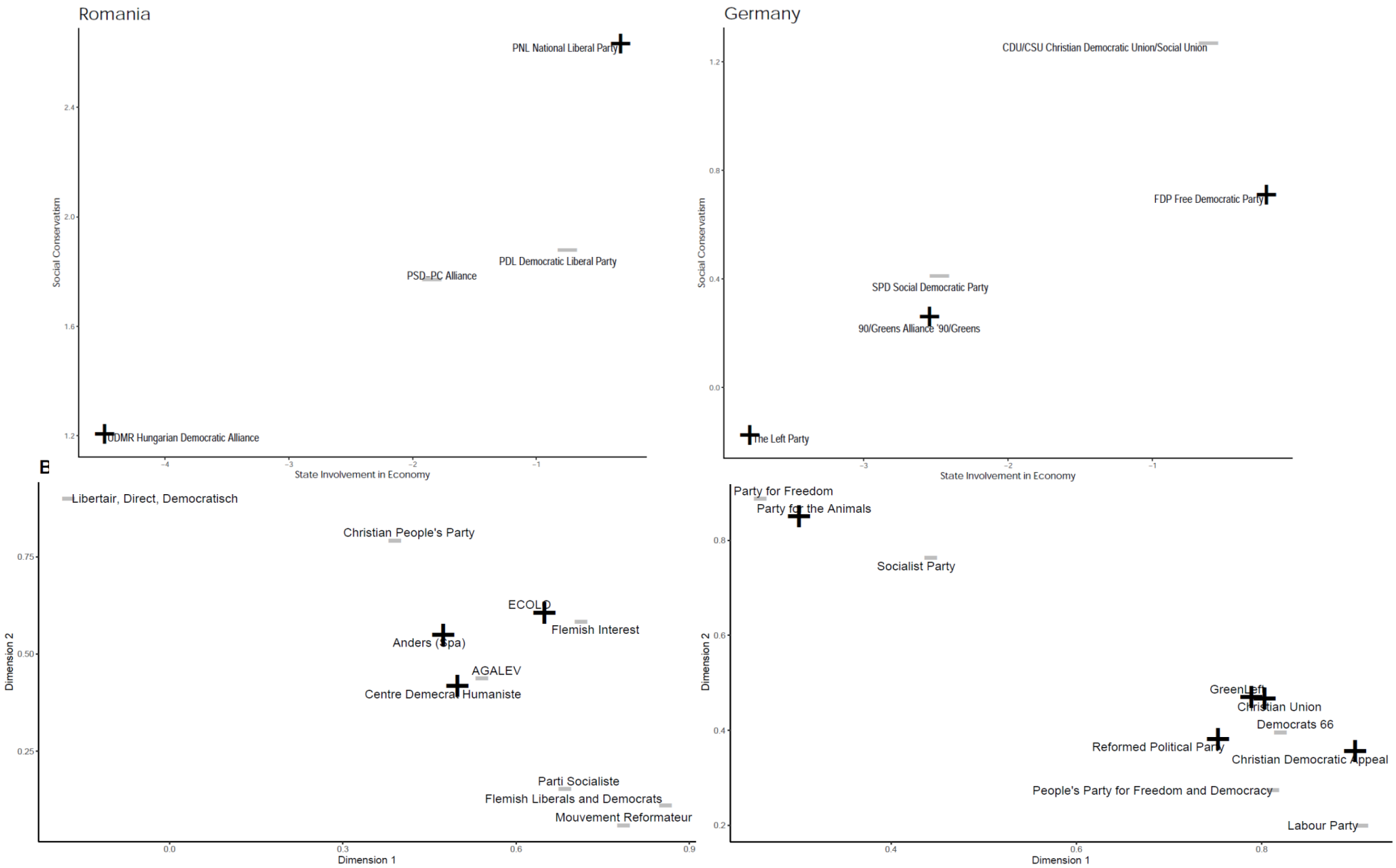

Figure 2. Two-Dimensional Depictions of Parties, Gains and Losses

Two-dimensional projections of parties' Manifesto scores, depicted with gains and losses in an n-vote (n = 3; a = 12) setting. Here, the larger the plus sign, the larger the gains for a party, whereas the larger the minus sign the larger the losses. As the figures depict, centrally located parties generally benefit in the n-vote setting, relative to peripheral parties.

	Dependent variable:		
	Gains from n-vote System		
	(1)	(2)	(3)
	Small Perturb.	Larger Perturb.	Largest Perturb.
Distance from Center	−2.879***	−2.310***	−1.162***
	(0.018)	(0.023)	(0.020)
Initial Party Size	−23.057***	−19.909***	−19.182***
	(0.416)	(0.513)	(0.361)
Acceptability Parameter	−0.002	0.137***	0.292***
	(0.018)	(0.021)	(0.015)
Number of Votes	−0.108***	−0.072**	0.001
	(0.026)	(0.031)	(0.022)
Proximity Voting	0.266	0.334	0.018
	(0.242)	(0.307)	(0.214)
Constant	22.212***	16.533***	7.564***
	(0.299)	(0.375)	(0.271)
Observations	6,600	6,600	6,600
R^2	0.833	0.693	0.574
Adjusted R^2	0.833	0.693	0.573
Residual Std. Error (df = 6594)	2.332	2.838	1.989
F Statistic (df = 5; 6594)	6,577.605***	2,973.321***	1,774.153***
Note:			*p<0.1; **p<0.05; ***p<0.01

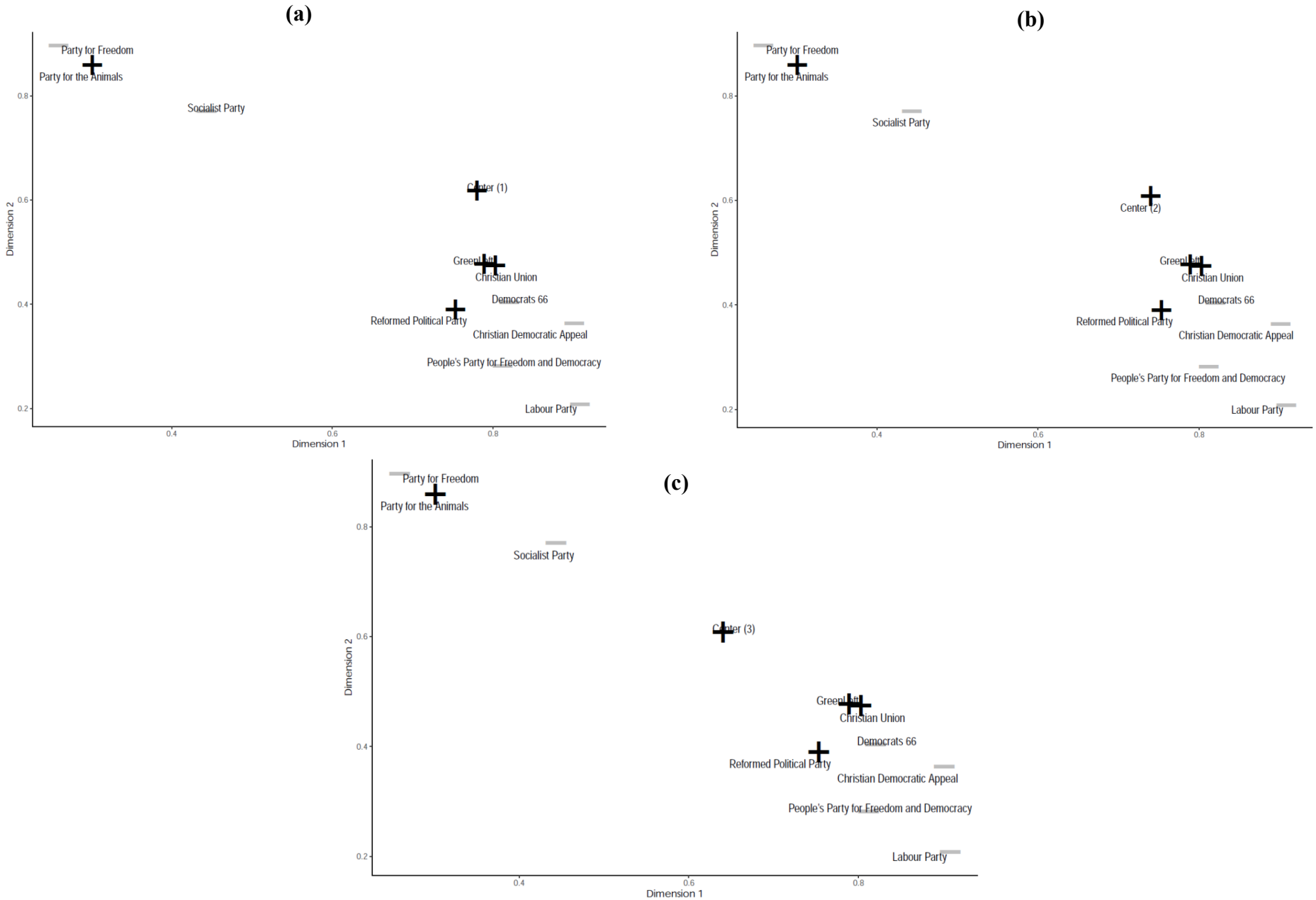

Figure 3. Centripetal Results in Netherland, Following Introduction of Centrist Party

Gains and losses in an n-vote (n = 3; a = 12) setting in the Netherlands, after the introduction of a small centrist party. Here again, the larger the plus sign, the larger the gains for a party, whereas the larger the minus sign the larger the losses. Centrist party, labeled "Center," makes clear, positive gains in the n-vote setting, as predicted.

APPENDIX

A. *R Function Used to Calculate New Vote Share*

```
require("plyr")
require("Gmedian")
require(SDMTools)
require("ggrepel")
require(ggthemes)
require("jtools")

COG_ndim <- function(ideology, parties){
  M <- sum(parties[,2])
  output_mean <- NA
  output_sd <- NA
  for(i in 1:ncol(ideology)){
    assign(paste("COG", i, sep=""), wt.mean(ideology[,i], parties[,2]))
    assign(paste("COG.sd", i, sep=""), wt.sd(ideology[,i], parties[,2]))
    output_mean[i] <- get(paste("COG",i,sep=""))
    output_sd[i] <- get(paste("COG.sd", i, sep=""))
  }
  data.frame(cbind(output_mean, output_sd))
}


result <- function(population, parties, ideology, acceptability, ballots, rational){
  voters <- c()
  lengths <- round_preserve_sum(parties$percentages, 3)
  for(i in parties$names){ # creates the party-voter vector
    subvector <- c()
    length = population*lengths[i]
    length[is.na(length)] <- 0
    subvector <- c(rep(i, times=length))
```

```
    voters <- append(voters, subvector)
  }
  vote.mat <- matrix(ncol = ncol(ideology)) #dimensionality
  for(i in voters){
    vote.mat <- rbind(vote.mat, ideology[i,])
  }
  ### VOTES ###
  # First Vote
  votes <- c()
  vote1 <- c()
  for(i in 2:nrow(vote.mat)){
    distances <- c()
    for(j in 1:nrow(ideology)){
      distances <- append(distances, dist(rbind(vote.mat[i,], ideology[j,])))
    }
    vote1 <- append(vote1, which.min(distances))
  }
  votes <- vote1
  # Second vote
  vote2 <- c()
  for(i in 2:nrow(vote.mat)){
    if(runif(1) <= rational){
      distances <- c()
      for(j in 1:nrow(ideology)){
        distances <- append(distances, dist(rbind(vote.mat[i,], ideology[j,])))
      }
      ifelse(sort(unique(distances))[2] < acceptability, vote2 <-
append(vote2,which(distances == sort(unique(distances))[2])), vote2 <- append(vote2,
NA))
    } else{vote2 <- append(vote2,sample(parties$names,1))}
  }
  votes <- append(vote1, vote2)
  recorded <- cbind(vote1, vote2)
  # 3rd vote and beyond
```

```
  if(ballots>2){
    for(b in 3:ballots){
      voters <- NA
      vote.vec.init <- votes
      vote.total <- data.frame(table(vote.vec.init, useNA = "always"))
      vote.vec.perc <- round_preserve_sum(vote.total$Freq/sum(vote.total$Freq),3)
      vote.total <- cbind(vote.total, vote.vec.perc)
      lengths <- round_preserve_sum(vote.total$vote.vec.perc, 3)
      for(i in 1:length(as.character(vote.total$vote.vec.init))){ # creates the party-
voter vector
        subvector <- c()
        length = round_preserve_sum(sum(vote.total$Freq)/(b-1)*lengths[i], 3)
        sum(vote.total$Freq)
        length[is.na(length)] <- 0
        subvector <- c(rep(as.character(vote.total$vote.vec.init[i]), times=length))
        voters <- append(voters, subvector)
      }
      voters <- as.numeric(voters[2:length(voters)])
      vote.mat.init <- matrix(ncol = ncol(ideology))
      for(i in voters){
        vote.mat.init <- rbind(vote.mat.init, ideology[i,])
      }
      vote.mat.init <- vote.mat.init[2:nrow(vote.mat.init),]
      assign(paste("vote", b, sep=""), c())
      for(i in 1:nrow(vote.mat.init)){
        distances <- c()
        loopnums <- 1:nrow(ideology)
        loopideo <- subset(loopnums, loopnums%in%recorded[i,]==F)
        if(runif(1) <= rational){
          for(j in loopideo){
            distances <- append(distances, dist(rbind(vote.mat.init[i,],
ideology[j,])))
          }
```

```
          ifelse(is.na(sort(unique(distances))[1])==F & sort(unique(distances))[1] <
acceptability, assign(paste("vote", b, sep=""),

append(get(paste("vote", b, sep="")),loopideo[which.min(distances)])),
                 assign(paste("vote", b, sep=""),append(get(paste("vote", b, sep="")),
NA)))
        } else{assign(paste("vote", b, sep=""), append(get(paste("vote", b,
sep="")),sample(c(1:length(loopideo)),1)))}
      }
      votes <- append(votes, get(paste("vote", b, sep="")))
      recorded <- cbind(recorded,get(paste("vote", b, sep="")))
    }
  }else{NA}
  counts <- data.frame(table(votes))
  percentage <- (counts$Freq)/sum(counts$Freq)
  vote1count <- data.frame(table(vote1))
  vote1count <- (vote1count$Freq)/sum(vote1count$Freq)
  distance_from_center <- c()
  for(i in 1:nrow(ideology)){
    distance_from_center <- append(distance_from_center,
dist(rbind(ideology[i,],COG_ndim(ideology, parties)$output_mean)))
  }
  results <- data.frame(cbind(counts$votes, percentage, vote1count,
distance_from_center))
  names(results) <- c("Party", "Votes", "Initial Vote", "Distance from Center")
  results
}
```