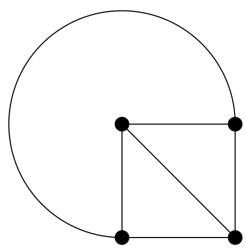
PLANAR GRAPHS, SOCCER BALLS, AND EULER'S FORMULA

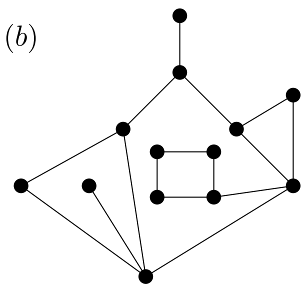
Definitions
A *graph* is a collection of vertices (dots or nodes) and edges (lines connecting the vertices). A *planar graph* is a graph that can be drawn[1] on a piece of paper so that no two edges intersect (except at vertices). The edges do not have to be straight.

For example, here are two planar graphs:

(*a*)

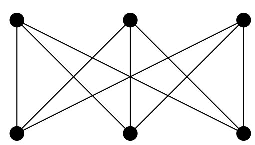


(*b*)



Here is a graph that is <u>not</u> planar:

(*c*)



A *face* of a planar graph is a region of "empty space" in the graph such that one can move between any two points in the region without crossing a vertex or an edge. The "outside" of the graph is also considered a face (called the *infinite* or *unbounded* face). For example, graph (a) has 4 faces and graph (b) has 5 faces.

Problem 1: For each of the planar graphs on the back of this page, write down the number of vertices, edges, and faces. Do you notice a rule that these three numbers always seem to obey? [Hint: if you know the number of vertices and faces, can you predict the number of edges?]

Problem 2: How many vertices, edges, and faces does a cube have? A pyramid? A tetrahedron? An octahedron? Do these numbers follow a pattern? If so, can you explain why?

Problem 3: Can you draw a planar graph with 12 faces, such that every vertex is connected to three edges, and every face is surrounded by five edges? Remember that the infinite face counts as a face! [Hint: start with a big pentagon, and then draw more pentagons inside it.]

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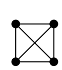
[1]This is a bit subtle. For instance, the following graph does not *look* planar, since the two diagonal edges cross:



However, if we move one of the diagonal edges outside the square, then we see that this is the same graph as (a) above, so this is in fact a planar graph. Graph (c) is not planar because *no matter how we draw it*, there will always be at least two edges that cross!