WARMUP: ARITHMETIC WITH LETTERS

Here are a few problems taken from the book "Sideways Arithmetic from Wayside School" by Louis Sachar. In each problem, the numbers in an arithmetic problem have been replaced by letters, and you have to figure out what number each letter represents. The rules are that every letter stands for a number between 0 and 9, each letter in a problem always stands for the same number, and two different letters can't represent the same number.

Problem 1:

$$
\begin{array}{r}
E\ G\ G \\
+\ \ E\ G\ G \\
\hline
P\ A\ G\ E
\end{array}
$$

Problem 2:

$$
\begin{array}{r}
S\ H\ E \\
+\ \ E\ E\ L \\
\hline
E\ L\ S\ E
\end{array}
$$

Problem 3:

$$
\begin{array}{rcccc}
 & & D & O & G \\
\times & & & A & D \\
\hline
 & & D & O & G \\
 & A & D & O & \\
\hline
 & A & G & O & G \\
\end{array}
$$

Problem 4:

$$
\begin{array}{rcccc}
 & & R & Y & E \\
\times & & & B & E \\
\hline
 & & A & R & E \\
 & E & A & T & \\
\hline
 & W & E & R & E \\
\end{array}
$$