Inequality and House Prices

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November 9, 2015

Job Market Paper

Abstract

This paper studies the interaction between inequality and house prices using an incomplete market heterogeneous agent model. The model links cross-sectional household portfolio saving decisions to housing market outcomes, contributing to the housing literature by illustrating a new price formation mechanism in which the investment motive among the wealthy plays a key role. A quantitative application of the theory rationalizes puzzling phenomena in China – notably, its recent substantial housing boom accompanied by rising savings rates. The theory in this paper shows that market frictions can have differential impacts cross-sectionally, increasing the risk of an explosion of inequality. This adds to the understanding of a broader topic: how inequality and macroeconomic forces can interact.

JEL Codes: E21, G11, O16, O18, P22, R21.
Keywords: housing, inequality, Chinese economy.

*University of Michigan, Department of Economics, fudongzh@umich.edu. I am indebted to my advisors Dmitriy Stolyarov, Rüdiger Bachmann, Miles Kimball, and Joshua Hausman for their continued guidance and support, and I am grateful to Dominik Menno, John Leahy, Andrei Levchenko, Gabriel Ehrlich, and Linda Tesar for their valuable feedback. All errors are my own. This is the link to the current version of the paper.
1 Introduction

Rising inequality in many countries over recent decades (exemplified by the top income shares series documented in Atkinson et al., 2011) has drawn increasing attention from policy makers and researchers. At the same time, global house prices (in real terms) have also been on an upward trajectory (see Knoll et al., 2014). This paper argues that the two simultaneous trends are not a mere coincidence.

Figure 1.1: Top income share growth and appreciation of house prices

Notes: For each observation in each panel, the average growth rates are obtained by averaging the yearly growth rates over the entire sample periods. In Panel (a), cross-country (real) house price indexes are taken from Federal Reserve Bank of Dallas’s International House Price Database as described in Mack and Martínez-García (2011); cross-country top 5% income shares are taken from Alvaredo et al. (The World Top Incomes Database) (income excludes capital gains). In Panel (b), US cross-state (nominal) house prices are from FHFA All-Transactions Indexes; US cross-state top 5% income shares are from Frank-Sommeiller-Price Series (income excludes capital gains) as described in Frank et al. (2015). Appendix A contains more details about data source, variable construction, robustness check, and other related empirical analysis.

As shown in Figure 1.1, growth in top 5 percent income share and house price appreciation are positively correlated both across countries and across states in the United States. Importantly, unlike house prices, rents do not co-move with top income shares (see Appendix A for details). This suggests that investment motives of the wealthy, rather

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1 The rise in house prices is the main contributor to the well-publicized “coming back” of aggregate capital-to-income ratios documented by Piketty (2013) and Piketty and Zucman (2014), as noted in Bonnet et al. (2013). On a similar note, Rognlie (2015) found that the increase in net capital share among developed economies during the last several decades also comes entirely from the housing sector.

2 The inequality measure adopted here is the top 5 percent income share, to approximate the financial affluence of upper class population (cross-sectional wealth data is very limited). The reason to focus on upper class households is because they are the empirically more relevant group who are affluent enough to buy houses only for investment purposes. Investment motive in housing is the focus of this paper.
than consumption needs of the poor, hold the key to understanding the dynamics of house prices. This observation is particularly relevant for economies with high shares of housing wealth, where housing market outcomes inevitably affect inequality. Thus, understanding the interacting forces between inequality and housing market outcomes can be essential for managing cycles in house prices and inequality.

To study the interaction between inequality and house prices, I set up an incomplete-market equilibrium model in which households face uninsurable idiosyncratic earnings risk. Markets for housing consumption and housing ownership are separately specified. Households in the model consume goods and rental services, and can save in either liquid bonds or lumpy housing. Housing investment is lumpy due to frictions in adjusting one’s housing stock. The lumpiness induces an “illiquidity premium” in the financial rate of return of housing over bonds.

In equilibrium, rents depend mostly on average resources, while house prices also respond to changes in inequality. All else equal, a rise in inequality shrinks the illiquidity premium and pushes up house prices. This result is due to differential responsiveness in housing demand across wealth groups, owing in turn to the differential impact of lumpy housing. In particular, housing demand from wealthy households is responsive since housing adjustment costs matter less as households get wealthier. Housing demand from the poor, however, is rigid because their housing investment is constrained (or close to constrained) by the lumpiness. As a result, the aggregate demand and aggregate price of housing respond positively to a rise in inequality (a shift of resources from the poor to the wealthy).

In a dynamic setting, the model implies an endogenous feedback loop between house prices and wealth inequality. This feedback loop causes house prices to overreact to changes in fundamentals. The mechanism is, again, based on the differential responsiveness in housing demand. Importantly, the above mechanism can rationalize persistent episodes where house price growth outstrips income growth in response to income growth shocks. Commentators and policy makers often view these episodes as a prelude to a market crash.

Consider a temporary income growth shock to the model. In a perfect foresight equilibrium, the house price-to-income ratio will increase at first and then subsequently adjust back to normal. The anticipated large swing of house prices is linked to optimal portfolio adjustments of the wealthy and the poor. The initial price run-up is driven by

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3 The word “inequality” is used here in a general sense and can be associated with several different distributions. The distribution for non-capital income is exogenously specified in the model, while capital income and wealth distributions are endogenously determined.

4 There are several frictions in the housing asset market, with fixed adjustment cost as the main one. Housing consumption, however, is flexible in the model: a homeowner is allowed to become a net renter or a landlord by trading housing services in a frictionless rental market. This modeling approach makes housing purchases a purely financial decision. More discussions about this setup will follow in the main text.

5 This result is loosely stated here for the purpose of this discussion. The exact result depends on details of the distribution of households and of changes in inequality. Section 2 discusses this result under a simplified model specification.
a portfolio shift among the wealthy, whose housing demand is the most responsive, from bonds to housing. High price growth sidelines households with modest resources; these households increase housing share in their portfolios after a delay. Wealth concentrates as the price run-up continues. Eventually the wealthy choose to consume their capital gains, and in doing so, they sell accumulated housing stock back to the low resource first-time buyers, who are forced to save for extended periods. Wealth becomes more dispersed as the house price adjusts back to trend. In the end, house prices experience an initial steep run-up followed by a soft landing, due to differential responsiveness across wealth groups.

To quantitatively evaluate how much the proposed mechanism can account for the dynamics of housing prices and wealth inequality, I apply the model to study the recent housing boom in China. I first parameterize the model to match important dimensions of aggregate and cross-sectional aspects of the Chinese economy in 2002. I then feed in changes in both aggregate income and income inequality from 2002 to 2012 taken from the data to the calibrated model and solve for the perfect foresight transition path equilibrium.

Not only can the model reproduce the observed housing price run-up, it can also match trajectories for wealth inequality and rising aggregate saving. Since none of the data moments after 2002 is used in the calibration of the model, this adds substantial validity to the proposed mechanism. The resulting housing price over-reaction driven by investment motives from the rich has broad policy implications for both developing and developed economies (housing booms and busts have been a common feature for both developed and developing countries in recent decades).

Finally, the theory in this paper illustrates a new channel through which inequality and macroeconomic forces can interact. In particular, market frictions can have differential impact cross-sectionally, increasing the risk of an explosion of inequality.

**Related Literature**

The model in this paper builds on a growing strand of literature that studies housing with incomplete market models and heterogeneous households. See, for example, Kiyotaki et al. (2011), Sommer et al. (2013), Iacoviello and Pavan (2013), and Favilukis et al. (forthcoming). The model in this paper differs from the rest of the literature in the sense that it stresses the interaction between inequality and housing market outcomes. In the model, housing purchases are purely financial decisions, and the responsiveness in housing demand differs across wealth groups due to the differential impact of lumpy housing. Thus,

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6 Although the main application of the model is to the Chinese housing boom, the theory in the paper generally applies as long as sufficient frictions exist in the housing market. Factors such as institutional details that can affect the lumpiness of housing, financial market developments, and the inequality level will determine to what extent the mechanism will be effective.

7 Favilukis et al. (forthcoming) also point out the importance of realistic wealth distribution in determining housing price dynamics. However, their mechanism is very different from that of this paper. In their paper, since housing asset cannot be a vehicle for investment (because housing asset is non-tradable), wealth heterogeneity only serves the purpose of generating a large fraction of housing demand from financially constrained households. This aspect of their mechanism makes equilibrium housing price sensitive to changes in financing constraints and interest rates.
housing market outcomes are closely linked to cross-sectional household portfolio saving decisions. The model highlights the investment motives of wealthy households in driving house price dynamics.

In addition, the modeling approach of this paper also takes elements from the literature initiated by Grossman and Laroque (1990) that studies optimal consumption problems under fixed adjustment costs. Of particular interest here are those papers that focus on portfolio choice in the presence of housing. (See, for example, Flavin and Yamashita, 2002, Fischer and Stamos, 2013, and Corradin et al., 2014.) In general, this strand of literature adopts a representative agent framework and studies partial equilibrium implications of lumpy housing for consumption and investment decisions with exogenously specified processes for house price. However, this paper explores the differential impact of the fixed adjustment costs on portfolio choice across wealth groups and investigates its implications for endogenous house price formation.

There are several papers close to this paper in subject matter in the sense that they directly associate house prices with inequality. See, for example, Nakajima (2005), Nieuwerburgh and Weill (2010), Määttänen and Terviö (2014), and Landvoigt et al. (2015). However, both their modeling approaches and the questions they address differ from those of this paper. Nakajima (2005) interprets rising cross-section earnings inequality as increasing individual earning uncertainty and studies the effects on portfolio allocation and housing prices. Nieuwerburgh and Weill (2010) adopt a spatial model with changes in cross-sectional productivity differences to explain the increase in house price dispersion across US metropolitan areas. Määttänen and Terviö (2014) and Landvoigt et al. (2015) both use assignment models to study the distributions of house prices within a metro area. The focus of this paper is to study how aggregate housing price and inequality interact with each other when houses are lumpy investment goods.

A few papers other than this one illustrate mechanisms whereby house price may overreact to changes in income. Ortalo-Magn and Rady (2006) show that house price can overshoot when changes in income affect the number of credit-constrained owners moving up the property ladder, while in this paper the overreaction in house price is caused by the investment motives of the unconstrained wealthy housing investors. Kahn (2008) finds that regime-switching productivity growth trends in output can generate house prices that are substantially more volatile than output. But the model in Kahn (2008) abstracts from heterogeneity and financial frictions, both of which are central for the mechanisms in this paper.

Finally, the quantitative exercise in this paper links the two strands of literature that study house prices and saving rates in China. Unlike this paper, which focuses on theoretically explaining the overreaction of house price to changes in income, most existing works in the emerging literature that studies China’s house price try to find the reasons why the level of house price is high in China. (See Wei et al., 2012 and Wang and Zhang, 2013 for example.) Garriga et al. (2013) and Chen and Wen (2014) are two exceptions that study the growth rate of housing price explicitly. Garriga et al. (2013) use a spatial model
China’s high and rising household saving rates attract a growing literature to investigate the mechanisms behind them. Many factors have been proposed as the possible drivers of these rising saving rates, but no answer is conclusive (see, for example, Modigliani and Cao, 2004, Chamon and Prasad, 2011, Wang and Wen, 2012, and Chamon et al., 2013, among many others). Among those factors, rising house prices is one of the usual suspects. Under a partial equilibrium setting (where house prices are exogenously given), Wang and Wen (2012) and Bussière et al. (2013) both show that the saving rates of certain household groups can increase with house prices, provided that certain conditions hold. However, there is no existing theory that can endogenously generate fast-growing house prices and rising saving rates at the same time, as this paper does.

Sectioning

The rest of the paper is organized as follows. Section 2 sets up a simplified version of the model that can be solved analytically to illustrate key mechanisms. Section 3 presents the main model and its calibration. Section 4 employs the calibrated model to study the Chinese housing boom between 2002 and 2012, and Section 5 concludes. An Appendix that contains additional empirical analysis, proofs, and numerical algorithms follows at the end.

2 A Basic Model

This section sets up a simplified version of the model. Basic elements of the theory are introduced and analytical solutions are derived. The results produced in the model indicate that house prices positively depend on inequality measures. Mechanisms behind the results are illustrated and followed by a discussion of general intuition. The basic model will be extended with more realistic features in the next section for quantitative applications.

2.1 Environment

The endowment economy considered here has two types of earners and two different assets: housing and bonds. Housing pays rent as a dividend but can only be held at a size no
smaller than a minimum threshold. Both price and return for housing are determined in equilibrium, and housing demand responsiveness differs depending on income. This simple setup allows for interactions between inequality and housing market outcomes.

**Endowments** In this economy, time is discrete and extends from \( t = 0, \ldots, \infty \). There are no aggregate or household-specific uncertainties.

Aggregate output \( Y \) is constant every period in this economy. Two types of young households are born at the beginning of each period, referred to respectively as top earners (with constant population share \( \mu \) in each generation) and bottom earners (\( 1 - \mu \)). Households live 2 periods and are productive only when they are young. The share of output received by top earners is \( \pi \). Each generation has a constant population mass of one half. Note that the relative endowment between the top earner and bottom earner is \( \pi \frac{1 - \mu}{\mu (1 - \pi)} \), which increases with both top income share \( \pi \) and bottom population share \( 1 - \mu \). \( \pi \) and \( 1 - \mu \) will be referred to as the two inequality measures of this economy.

There is also a fixed amount of housing stock \( H \) in the economy, which produces flow services each period. Newborn households are not endowed with any housing assets (i.e., the older generation owns all the housing stock).

**Household preferences** Households share the same preference in this economy. In particular, they all receive utility from consuming output \( c \) and housing service \( s \). For the sake of simplicity, we assume the following separable flow utility function:

\[
U(c, s) = \varphi \ln c + (1 - \varphi) \ln s,
\]

where \( \varphi \in (0, 1) \) reflects the utility weight of housing services in total consumption.\(^8\) The period-by-period discount factor is constant at \( \beta \) across all the households.

**Markets and timing** Households can trade the ownership of housing assets. Other than housing, households can also invest in a one-period fully enforceable bond \( b \), which pays a fixed gross interest rate of \( 1 + r \). We assume a small open economy so that bonds are supplied by an outside intermediary with infinite supply elasticity. Households enter a period with last period’s bond holdings \( b \) and housing assets \( h \) (newborn households have no assets). At the beginning of the period, a household receives its income, which will be the output share for the young, and returns from financial assets for the old (the return from housing assets is the rental income earned).

Asset markets open first in order to allow the older generation to sell off assets for consumption. Households can buy or borrow bonds from the outside intermediary at the fixed price \( 1/(1 + r) \), with a collateralized constraint when borrowing. In particular,

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\(^8\)The log specification over consumption and housing services provides tractability for the model. However, this specification is not uncommon in the housing literature since it is consistent with the empirical finding that, over time and across cities, the expenditure share on housing is constant (see Iacoviello and Pavan, 2013). A more general utility function will be considered in the quantitative model in section 3.
households cannot borrow more than a fraction \( \lambda_h \) of their housing assets. Households can also trade housing assets, but there is a minimum holding size requirement in the housing asset market.\(^9\) To be specific, a household can only hold housing stock at a quantity larger than or equal to \( h \) with unit price \( P \).\(^{10}\)

After asset markets close, households participate in the goods market and housing rental market. Unlike housing asset markets, the housing rental market is frictionless in that households can buy any amount of services \( s \) at the market unit price \( R \). The model’s frictionless rental market may be a bit extreme, but it captures the idea that a person can rent a very small living space that is not, however, available for sale; for example, it is not uncommon to have several renters share one bedroom (depending on the culture).\(^{11}\)

**The nature of housing** The market arrangement in this economy has interesting implications about the nature of the housing asset. Housing in this economy is essentially a financial asset that delivers a dividend at the rate of \( \frac{R}{P} \), due to our assumption of two separate markets for housing services and housing stock.

Considering a household that owns \( h \) units of housing stock, the housing consumption of this household is not necessarily \( h \), since household members are allowed to either rent out part of their housing stock to become landlords \( (s < h) \) or to purchase more rental services to be net renters \( (s > h) \). This means that although housing services and housing stock are necessarily the same in aggregate \( (S = H) \), there is no cost for \( s \neq h \) at the individual level.\(^{12}\) This structure conveniently separates individual decisions about housing consumption and housing investment. It stresses the financial aspects of housing, allowing housing purchases to be a purely investment decision for households to make.\(^{13}\)

More discussions of this modeling approach will follow in the rest of the paper.

**Young household’s problem** Before proceeding with the solution, we note that our assumption on preferences leads to constant optimal budget shares of housing and non-housing consumption. Define total expenditure \( x = c + Rs \), the household’s univariate

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\(^9\)This minimum size friction here serves the purpose of generating lumpiness in the housing market in a tractable way. Although the minimum size friction is realistic and frequently adopted in the quantitative housing literature (see, for example, Iacoviello and Pavan, \( \text{2013} \)), it is not essential for the mechanisms in this paper. See section \( \text{2.3} \) for more discussion.

\(^{10}\)The specification of a (quality-adjusted) per-unit price of housing is natural here since the total stock is fixed and individual housing choice is continuous when it is above the minimum threshold size \( h \). The general focus of this paper is the aggregate housing price, instead of the cross-section price dispersion.

\(^{11}\)In the quantitative housing literature, when rental decision is explicitly modeled, it is usual to assume that rental units come in smaller sizes than houses, see Iacoviello and Pavan \( \text{2013} \) and Sommer et al. \( \text{2013} \) for example.

\(^{12}\)Although in reality becoming a landlord bears with it various costs, such as management costs and differential tax treatments, this paper abstracts from those costs, in order to make the exposition clean. Taking them into account will not affect the main message of this paper.

\(^{13}\)Sommer et al. \( \text{2013} \) allow households to invest in rental properties. However, housing investment decision in their model cannot be easily separated from housing consumption decision, since becoming a landlord is assumed to shift one’s utility function.
indirect flow utility function is the following:

\[ u(x) = \ln x + (1 - \varphi) \ln(1 - \varphi) + \varphi \ln \varphi - \varphi \ln R. \]

And the young household’s decision is reduced to a standard consumption-saving problem, with a portfolio choice between bonds and housing:

\[
\begin{align*}
\max_{x,h,b} & \quad \ln x + \beta \ln x' \\
\text{s.t.} & \quad x + b + Ph = y \\
& \quad x' = (1 + r)b + (P + R)h \\
& \quad b \geq -\lambda h Ph \\
& \quad h \in [0, [h, +\infty]).
\end{align*}
\]  

**Equilibrium** A stationary equilibrium is a set of prices \( R \) and \( P \), and a collection of decision rules \( b(y, R, P), h(y, R, P) \), and \( x(y, R, P) \), such that decision rules are optimal and markets clear.\(^{14}\)

Note that in the model the distribution of \( y \) is exogenously given, and both rent \( R \) and house price \( P \) are endogenously determined by market clearing conditions. Although there are separate market clearing conditions for housing rental and ownership, \( R \) and \( P \) are jointly determined in the equilibrium since housing demand is a function of its financial rate of return \( \frac{R}{P} \). Moreover, since housing asset is risk-free in a stationary environment, due to the minimum holding size friction, the equilibrium return from housing has to be at least as high as the return from the risk-free bond (i.e., \( \frac{R}{P} \geq r \)).

### 2.2 Analytical result

Although the setup of the model is simple, multiple equilibria still arise when there is no restriction on parameter values. In this section, I restrict attention to one particular type of equilibrium that is most relevant for realistic settings. Appendix B contains the full characterization of model equilibria.

**Theorem 1.** When the parameters of the model satisfy the condition that

\[
\frac{(1 + \beta)(2H - (1 - \mu)h)}{\beta \pi h} \leq \frac{(1 - \pi)}{(1 - \mu)} \leq \frac{\pi h}{2H - (1 - \mu)h},
\]

the equilibrium housing price

\[
P = \frac{\pi}{-(1 - \mu)h + 2H} \times \frac{\beta Y}{(1 + \beta)(1 - \lambda h)}
\]

\(^{14}\)We briefly describe the equilibrium condition here to save space. The equilibrium definition will be stated more precisely in Appendix B and also in the quantitative model in Section B.
increases with both inequality measures $\pi$ and $1 - \mu$. Moreover, the equilibrium return in housing is higher than bonds (i.e., $\frac{R}{P} > r$), and it decreases with both inequality measures.

We leave the proof of Theorem 1 to Appendix B rather than discuss the intuition in the main text. To understand the intuition for the theorem, we first explain why housing can have a higher return than bonds in equilibrium. In a stationary equilibrium, the return for housing is simply the rent-over-price ratio. Since households spend a constant share on rental consumption, rental price is roughly proportional to aggregate output. House price depends on aggregate saving, which is also roughly a constant share of aggregate output due to the log specification for utility function. Thus, $\frac{R}{P}$ depends on the trade-off between the importance of housing consumption and saving motive. When the utility weight on housing is high relative to a household’s patience level, $\frac{R}{P}$ can be higher than $r$.

Figure 2.1: Policy functions for housing and bonds

Notes: This figure illustrates the equilibrium policy functions for both housing asset and bonds. The policy functions correspond to one set of parameter values that satisfies the conditions for equilibria in which $\frac{R}{P} > r$. Moreover, for the ease of exposition, borrowing is not allowed ($\lambda_h = 0$).

Now assuming $\frac{R}{P} > r$ is the equilibrium outcome, Figure 2.1 plots optimal rules for saving in housing and bonds as functions of lifetime resources. If there is either no friction in the housing market ($\lambda_h = 0$) or no limit in borrowing ($\lambda_h = 1$), denoted as the no-friction benchmark, households will simply save a constant share of total resources. And all of the savings will go to housing (the dotted line). However, with housing market

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15 There are many other realistic reasons for housing to have a higher return in equilibrium. For example, in the quantitative model of this paper, adjustment costs justify the higher return in housing assets.
frictions and borrowing limits, certain households’ saving decisions are distorted. In
particular, households with low levels of resources cannot enter the housing market and
save only in bonds (shown as “outsiders” in Figure 2.1); although households with modest
resources participate in the housing market, they are constrained by the frictions and
have to over-save relative to the no-friction benchmark (as “constrained participants”).
Only those households with high enough resources are not affected by the frictions (as
“unconstrained” participants). As a result, households with low and modest resources are
in large zones of inaction: their housing demand is “unresponsive.” In contrast, affluent
households have “responsive” housing demand.

Theorem 1 corresponds to a situation where the underlying parameter values of the
model imply a modest income inequality relative to frictions in the housing market. The
equilibrium outcome is that the bottom earners participate in the housing market but
are constrained while top earners are unconstrained. So it is easy to understand why an
increase in inequality will cause house prices to go up: an increase in either π or 1 − μ
generates more housing demand from the top earners (who are “responsive”) but does
not suppress demand from the bottom earners (who are “unresponsive”).

Although this result is based on an extreme assumption about income distribution,
it is straightforward to generalize the basic model to include a continuous distribution
of household types, and one can get qualitatively similar results with realistic income
distributions. More importantly, the intuition in this model holds more generally, as we
will discuss in the next section.

2.3 Discussion

The model presented in this section is relatively simple; strong assumptions are made for
tractability reasons. However, this simple model does allow two distinct relative prices –
rents and house prices – to be determined in equilibrium via market clearing. Moreover,
housing market outcomes are linked with cross-section saving decisions and thus interact
with inequality. Most results and intuitions from the model are generalizable to realistic
settings.

First, housing is a preferred asset. This is for purely financial reasons in the model:
it provides a higher rate of return than bonds do. In reality, there are other reasons for
a household to favor housing. For example, housing ownership can signal social status,
as Wei et al. (2012) have emphasized, and thereby strengthen one’s competitiveness in
the marriage market. Higher returns on housing are especially sustainable in an economy
where households place a high value on housing but are not wealthy enough to make
sufficient housing investment.

**Note:** Housing demand is very responsive (it jumps) at one critical point where switching occurs
between “outsiders” and “constrained participants”.

**Note:** This relates to efficient housing provision, which will not be the current focus of this paper. We will
come back to this point in the concluding section.
Second, housing purchases are lumpy. In the simple model, the lumpiness is built in by the minimum purchase size friction. In reality, there are many other reasons for housing purchases to be lumpy (although the minimum size friction is still relevant). Non-convex adjustment cost, considered later in the quantitative model, is one obvious reason.

When housing is both desirable and lumpy, as shown in the model, the responsiveness in housing demand will differ across wealth groups. This is especially the case if credit conditions are tight. The differential responsiveness in turn causes inequality and housing market outcomes to interact. This mechanism is very relevant in the data. Due to the concentrated wealth distribution, wealthy households can comfortably invest in multiple properties while regular households have to borrow for just one residence since a house can cost multiple years of income for a regular household in most countries.

The quantitative model in the next section will incorporate realistic features but lead to intuitions similar to the ones discussed above.

3 A Quantitative Model

The model in Section 2 serves the purpose of setting up the basic framework and illustrating some qualitative implications of the theory. However, simplified assumptions prevent the model from delivering quantitative implications. This section extends the basic model to make it suitable for quantitative study of the recent housing market boom in China. To illustrate why China is a good test ground for the theory, Section 3.1 presents some institutional backgrounds and stylized facts about the Chinese economy and its housing market. Some of the facts will inform the specification and calibration of the model in Section 3.2 and Section 3.4, and others will be used to evaluate the model and discipline the transitional exercise in Section 4.

3.1 Backgrounds

As the economic power of China emerges, there is growing interest in understanding “puzzling” economic phenomena in China. In order to understand China, research must take into account special institutional ingredients as well as characteristics of the Chinese data. Several recent studies, such as Song et al. (2011) and Chang et al. (2015), make this point clear. This section thus briefly summarizes relevant backgrounds about China’s housing market. More details will be supplemented in Appendix C.

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18 “Lumpy housing” in this paper means that housing adjustment at the household level is large and infrequent compared to that of the liquid bond.

19 See Iacoviello and Pavan (2007) for a discussion of various realistic frictions that make housing investment lumpy within the context of the US. The lumpiness is even more relevant for economies with underdeveloped financial markets. Section 3.4 contains more details about the formulation, calibration, and evaluation of the frictions that make housing lumpy in the quantitative model.
3.1.1 China’s economic growth, inequality, and housing market after 2002

We first show a series of top wage share and residential land prices in urban China in Figure 3.1, together with private sector employment share and aggregate income. According to Figure 3.1, it is clear that the recent Chinese experience fits well with the pattern shown in Panel (a) in Figure 1.1, only it is much more dramatic. Wage share of the top 10 percent grew from 26% to 44% between 2002 and 2012: in other words, it almost doubled. During the same time, residential land prices grew significantly faster than the already fast income growth.

Figure 3.1: China in transition

(a) Top wage share and employment structure  (b) Land price growth and income growth

Notes: In Panel (a), private employment share (including various private employment types) is calculated from the China Labor Statistical Year Book 2002–2012. The top 10% wage share series is calculated by author from Chinese Household Income Project (CHIP) 2002, 2007 and China Family Panel Studies (CFPS) 2010, 2012. Both CHIP and CFPS are large-scale national household surveys, which are arguably the best available data sources on household income and financial assets in China. More statistics from the two data sets will be utilized in later parts of this paper. Appendix C has more descriptions of the two data sets. In Panel (b), the residential land price indexes are Wharton/NUS/Tsinghua Chinese Residential Land Price Indexes taken from Deng et al. (2014). Real per capita income is the Average Wage of Employed Persons in Urban Units taken from National Bureau of Statistics of China (deflated by the Urban Household Consumer Price Index).

Note that we only present here a land price index after 2004 because reliable data was unavailable for the period preceding that date. To simplify the discussion, we will refer

Note that the income concentration in China at 2002 is relatively low compared to most countries in Panel (a) in Figure 1.1, thus, we choose the top 10% share instead of the top 5% share. Section 3.5 discusses more about the inequality comparison between China and the United States.

According to China’s official house price indices, the average housing price (urban) appreciation is only mild compared to aggregate income growth. However, these indices are mistrusted and widely criticized due to the lack of quality adjustments in their construction. Constant quality price series developed by
to this land price index as an approximation of house prices in the rest of the paper.\footnote{Admittedly, land prices can be very different from house prices. However, many recent studies have identified land prices as the key factor that affects house prices across countries in recent decades (see Knoll et al., 2013 for example). In particular, according to Wu et al. (2017), house price growth in China is driven by rising land values, not by construction costs. We will discuss more of this in Section 4.3.}

This paper focuses on periods after the year 2002, mainly because it is not until 2002 that housing market in China roughly finished the transition from a state welfare provision system to private market provision. In China, housing became a commodity that individuals can purchase only after the early 1990s. (Before that local governments or work units allocated housing.) It took about one decade to complete the privatization process. In 2002, 78% of all households owned their homes with partial or full property rights. Moreover, more than 50% of the new housing is provided by private developers. Readers are referred to Walder and He (2014) for a more detailed discussion of the housing market privatization process in China.

The early 2000s is also the time when a mass retrenchment program in the state sector arrived at completion. The retrenchment program laid off roughly one fourth of workers from the state-owned sector in China. Starting from the early 2000s, China saw deepening privatization (especially the restructuring of firm ownership) and increasing openness (China joined the World Trade Organization in 2001). As a result, economic growth accelerated, as did to an even greater extent wage inequality. A later part of this paper treats changes in economic growth and wage inequality after 2002 as exogenous shocks.\footnote{Note that this paper focuses only on the macroeconomic implications of changes in economic growth and wage inequality; it does not take a stand on the fundamental reasons for those changes, although evidence suggests that the causes are structural. See Chang-Tai and Song (2011) for an analysis of how the transformation of China’s state sector affects productivity; see Han et al. (2012) for an investigation of the impact of globalization on wage inequality; see Appleton et al. (2013) for a discussion of increased wage inequality due to changes in the wage structure.}

### 3.1.2 Household finance in China

It’s not news that China as a nation has placed a great deal of emphasis on saving during recent decades: not only the government and firms, but also the households have prioritized savings (see Yang et al., 2013). Moreover, as documented in Chamon and Prasad (2010), the average urban household saving rate in China rose significantly during late 1990s and early 2000s, for all demographic groups.

Where do the enormous savings in the household sector go? Panel (a) in Figure 5.2 compares the components of total assets in 2012 between China and the United States. Households in China have an asset portfolio dominated by housing and fixed bank deposits. The share of housing (net of mortgages) in total asset in China is more than two times as big as that in the United States. Moreover, the share of financial asset in China...
is only about one fourth of the US share. What’s worse, half of the financial assets in Chinese households’ portfolios are in terms of fixed bank deposits, which gain robust low returns (sometimes even negative in real terms, as shown in Panel [b] in Figure 3.2). In contrast, the share of fixed deposits is negligible for US households.

Figure 3.2: Household asset position and interest rate

<table>
<thead>
<tr>
<th></th>
<th>China</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing net</td>
<td>79.4%</td>
<td>34.5%</td>
</tr>
<tr>
<td>Housing gross</td>
<td>81.8%</td>
<td>44.6%</td>
</tr>
<tr>
<td>Housing debt</td>
<td>-2.4%</td>
<td>-10.1%</td>
</tr>
<tr>
<td>Fixed deposits</td>
<td>5.3%</td>
<td>0.9%</td>
</tr>
<tr>
<td>Other financial</td>
<td>5.5%</td>
<td>40.1%</td>
</tr>
<tr>
<td>Other</td>
<td>9.7%</td>
<td>24.6%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

(a) Asset composition

(b) Interest rate

Notes: The statistics in Panel (a) are calculated by the author using the CFPS 2012 and the Survey of Consumer Finance (SCF) 2013. See Appendix C for the detailed calculation procedure. In Panel (b), the 1-year deposit rate is from the People’s Bank of China and the inflation rate is from the National Bureau of Statistics of China.

Chinese households have good reason to save in housing and bank deposits: in fact, they have no good alternatives. First of all, the Chinese government implements strict capital controls so that saving in capital markets outside of China is mostly off the table for regular households. Second, domestic stock markets were underdeveloped and offered shabby returns in the last two decades. As shown in Table 3.1, at least during 2003–2013, stock market index returns are dominated by the housing price index according to a simple mean and variance comparison.

Table 3.1: Returns of Stock Market Index vs. Housing Price Index (2003–2013)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stock</td>
<td>Housing</td>
</tr>
<tr>
<td>2003-2013</td>
<td>0.073</td>
<td>0.157</td>
</tr>
<tr>
<td>2003-2008</td>
<td>0.090</td>
<td>0.204</td>
</tr>
<tr>
<td>2009-2013</td>
<td>0.053</td>
<td>0.109</td>
</tr>
</tbody>
</table>

Notes: The numbers in this table are directly extracted from Table 3 and Table 4 in Fang et al. (2015). The housing price index is for first-tier cities only. See Fang et al. (2015) for more details and discussion.

The facts about China summarized in this section make it clear that the basic setup of the model (i.e., households choosing portfolios of housing and fixed return bonds) is
3.2 Model environment

The setup of the quantitative model is still kept as clean as possible in order to allow for a clear understanding of the mechanisms at work. Compared to the basic model in Section 2, there are only two main extensions: households become infinite-lived and they face uninsurable idiosyncratic earnings risks. As a result the setup is similar to standard incomplete-market settings such as in Huggett (1993) and Aiyagari (1994). In discussing the model environment, we focus on the new extensions and give only brief consideration to the parts that are similar to those of the basic model. Moreover, variable definition and notation are kept consistent with those in Section 2 whenever possible.

3.2.1 Household preferences and endowments

The basic setup is still an endowment economy, with no aggregate uncertainty and populated by two types of earners differing in income processes. Superscript \( i \in \{B, T\} \) will be used to denote bottom and top earners, respectively. In addition to the basic model, there are heterogeneities within each type of earner, both in terms of patience level and endowment condition, as described in the following.

Each type \( i \) earners are ex-ante identical and infinitely lived, with lifetime utility given by

\[
E_0 \sum_{t=0}^{\infty} \beta_t^i u(c_t, s_t),
\]

where \( \beta_t^i \) describes the cumulative discount factor between period 0 and period \( t \), and it differs between bottom and top earners. In particular, \( \beta_t^B = \tilde{\beta}^B \beta_t^{B-1} \), where \( \tilde{\beta}^B \) is an idiosyncratic shock following a three-state, first-order Markov process.\(^{24}\) But \( \beta_t^T = \tilde{\beta}^T \beta_t^{T-1} \), where \( \tilde{\beta}^T \) is a standard constant one period discount factor. Unlike in the basic model, the instantaneous utility function \( u(c_t, s_t) \) is modeled as a non-separable form, following Kiyotaki et al. (2011) and others:

\[
u(c_t, s_t) = \left( \frac{c_t}{1-\varphi} \right)^{1-\varphi} \left( \frac{s_t}{\varphi} \right)^{1-\frac{1}{\sigma}} - 1,
\]

where \( \sigma \) is the risk aversion parameter and \( \varphi \in (0, 1) \) again reflects the utility weight of housing services in the consumption aggregate of \( c \) and \( s \). Note that although the Cobb-Douglas preference has implications for the degree of substitutability between housing and non-housing consumption, it still produces the convenient feature that the housing expenditure share is constant. This specification will be discussed in more detail in Section 3.3.

---

\(^{24}\)The preference heterogeneity is introduced in a similar manner as that of Krusell and Smith (1998) and for similar reasons: to generate a more realistic wealth distribution.
Households in the economy also face idiosyncratic endowment shocks denoted as \( \varepsilon \) (the same between bottom and top earners), which are treated as being similar in spirit to unemployment shocks and will therefore be referred to as unemployment shocks hereafter. In particular, \( \varepsilon \) follows a first-order Markov process with two states \{0, 1\}: \( \varepsilon = 1 \) means that the corresponding household is employed, receives a normal endowment, according to her type (\( \tilde{y}^B \) for a bottom earner and \( \tilde{y}^T \) for a top earner). If \( \varepsilon = 0 \), the household is unemployed, receiving a safety-net endowment that equals a fraction \( \omega \) of the endowment at normal times. Note that the sum of the bottom earners’ income always equals \((1 - \mu)Y\) and the sum of the top earners’ income always equals \( \mu Y \).  

\[ 3.2.2 \text{ Asset market} \]

The basic structure of the asset market is similar to that in Section \[ 2.1 \], with the following two extensions about borrowing constraints and housing adjustment costs:

**Borrowing constraints** In addition to the loan to value ratio criterion in the basic model, there is an additional loan to income criterion. In particular, not only are households unable to borrow more than a fraction \( \lambda_h \) of their housing stock, they cannot borrow more than a fraction \( \lambda_y \) of their expected earnings either; i.e.,

\[ b' \geq \max \{-\lambda_h P h', -\lambda_y \mathcal{Y}(\varepsilon, \tilde{y}^i)\}, \]

where \( \mathcal{Y}(\varepsilon, \tilde{y}^i) = E[\sum_{k=0}^{N} \frac{y_k}{(1+r)^k} | \varepsilon, \tilde{y}^i] \) approximates the expected present discounted value of lifetime endowment. The endowment at period \( k \) is denoted as \( y_k \), and \( N \) is the approximate length of a life-cycle. This constraint is consistent with the usual lending criteria in the mortgage market that take into account minimum down payments, ratios of debt payments to income, and current and expected future employment conditions. Section \[ 3.4 \] discusses the implication of this mortgage lending structure in more detail.

**Housing adjustment costs** Unlike in the basic model, a homeowner incurs a cost \( \Phi(h', h) \) whenever she adjusts her housing stock:

\[ \Phi(h', h) = f_1 P|h' - h| + f_2 1_{\{h' \neq h\}} P h. \]  

(3.1)

Note that there are both linear and fixed components in the adjustment cost function. The linear component \( f_1 P|h' - h| \) captures common practices in the housing market that

---

\[ ^{25} \text{Note that the endowment process is trivialized in the model, which does not take certain cross-section and life-cycle aspects of income distribution into account. There are two main reasons for adopting this modeling approach. The first simply has to do with data availability. A panel data set with close track of individual income is needed to identify the persistence parameter in the income process. The second reason is that a more elaborate endowment process will affect not only inequality in earnings in the cross-section but also the individual earning uncertainty. However, for the transition exercise in Section 4, we need to isolate the effect of changes in cross-section inequality from changes in individual earning uncertainty. The current modeling approach for the endowment process better fits that purpose.} \]
require, for instance, commissions paid to realtors to be equal to a fraction of the value of
the house being sold. The fixed component $f_2 \mathbb{1}_{\{h' \neq h\}} P h$, which is in terms of a percentage
value of the minimum size house, captures other costs associated with housing transaction,
such as registration fees and search costs.

### 3.3 The household decision problem and competitive equilibrium

Households in this model accumulate wealth to insure against idiosyncratic endowment
shocks. In choosing vehicles to save in, they face a portfolio choice between liquid wealth
(bonds) and illiquid wealth (housing). In particular, the state variables relevant to a
type $i$ earner’s decision making include the individual state vector $(\varepsilon, \tilde{\beta}^i, b, h)$ and the
aggregate state variable $\Lambda$, which denotes the measure of households over $(\varepsilon, \tilde{\beta}^i, b, h, i)$.

Denote $V^i(\varepsilon, \tilde{\beta}^i, b, h; \Lambda)$ as the value function for a type $i$ earner, and the dynamic
programming problem is the following:

$$V^i(\varepsilon, \tilde{\beta}^i, b, h; \Lambda) = \max_{x, b', h'} u(x; R) + \tilde{\beta}^i E[V^i(\varepsilon', \tilde{\beta}^i', b', h'; \Lambda') | \varepsilon, \tilde{\beta}^i]$$

s.t.

$$x + b' + Ph' + \Phi(h', h) = \varepsilon y^i + (1 - \varepsilon) \omega y^i + (1 + r)b + Rh + (1 - \delta)Ph$$

$$b' \geq \max\{-\lambda_h Ph', -\lambda_y Y(\varepsilon, y^i)\}$$

$$h' \in \{0, [h, +\infty)\},$$

for $i \in \{B, T\}$. Similarly as in the basic model, $x$ denotes total expenditure and $u(x; R)$
denotes the indirect flow utility function.

**Recursive Competitive Equilibrium** A recursive competitive equilibrium is then de-
defined as: individual value and policy functions $\{V_i, b_i, h_i\}_{i \in \{B, T\}}$, pricing functions $\{P, Q\}$,
such that,

1. $\{V_i, b_i, h_i\}_{i \in \{B, T\}}$ solve household’s problem.


   (i) Housing market: $\int_{\Lambda} \sum_i h_i(\varepsilon, \tilde{\beta}, b, h; \Lambda) = H$

   (ii) Rental market: $\int_{\Lambda} \sum_i s_i(\varepsilon, \tilde{\beta}, b, h; \Lambda) = H$

   (iii) Goods market: $\int_{\Lambda} \sum_i \Phi(h_i, h_{i-1}) + \delta PH + C = Y + rA$

3. $\Lambda$ is generated by $\{b_i, h_i\}$ and exogenous processes $\varepsilon$ and $\tilde{\beta}^i$.

26Note that the housing adjustment cost is really modeled as a transaction cost in the model. By
paying the cost, housing stock can be adjusted to any future level $h' \in \{0, [h, +\infty)\}$. The assumption
that the fixed cost component applies to adjustments that are even sufficiently minor relative to the
existing housing stock essentially prevents homeowners making small improvements to their houses. More
discussion of this adjustment cost setup will follow in Section 3.4 when we calibrate the model.
Here $C$ denotes the aggregate non-housing consumption and $A$ denotes the aggregate bonds savings. The formula $\int_A \sum_i \Phi(h_i, h_{t-1})$ summarizes the total transaction costs incurred by homeowners for adjusting housing stock, which depends on both the total transaction volume and the transaction frequency due to our specification of $\Phi$ in Equation (3.1). Therefore, according to the goods market clearing condition, prices $P$ and $R$ will depend on both the transaction volume and the transaction frequency of housing stock. Note that this means that households need to know the transaction volume and the transaction frequency for their decision making. This observation has implication on the computation of the model. Although households do not need to know the exact $A$ to solve their individual problems, they need to know how prices $P$ and $R$ depend on $A$. We leave the details on our computational strategy to Appendix D.1.

### 3.4 Calibration

The frequency of the model is yearly. The model is parameterized to match important dimensions of aggregate and distributional statistics of the Chinese economy in 2002. Table 3.2 summarizes the parameters used in the baseline model. Detailed descriptions of the calibration procedure follow afterward.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional values</td>
<td>$\sigma$</td>
<td>1.00</td>
<td>Standard value</td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\gamma$</td>
<td>1.00</td>
<td>Normalization</td>
</tr>
<tr>
<td>Total endowment</td>
<td>$Y$</td>
<td>1.00</td>
<td>Normalization</td>
</tr>
<tr>
<td>Total housing stock</td>
<td>$H$</td>
<td>1.00</td>
<td>Normalization</td>
</tr>
<tr>
<td>Transition matrix of discount factor</td>
<td>$\Pi_{\beta, \beta'}$</td>
<td>-</td>
<td>Krusell and Smith (1998)</td>
</tr>
<tr>
<td>Estimated from the data</td>
<td>$r$</td>
<td>0.02</td>
<td>People’s Bank of China</td>
</tr>
<tr>
<td>Risk free interest rate</td>
<td>$\pi$</td>
<td>0.26</td>
<td>Top 10% wage share$^a$</td>
</tr>
<tr>
<td>Top 10% wage share</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transition matrix of employment process</td>
<td>$\Pi_{\beta, \beta'}$</td>
<td>-</td>
<td>Giles et al. (2005)</td>
</tr>
<tr>
<td>Mean endowment for bottom earner</td>
<td>$\mu$</td>
<td>0.90</td>
<td>Derived from $\Pi_{\beta, \beta'}$</td>
</tr>
<tr>
<td>Mean endowment for top earner</td>
<td>$\mu^T$</td>
<td>2.86</td>
<td>Derived from $\Pi_{\beta, \beta'}$</td>
</tr>
<tr>
<td>Unemployment replacement rate</td>
<td>$\omega$</td>
<td>0.07</td>
<td>China Labor Statistical Yearbook</td>
</tr>
<tr>
<td>Minimum size housing</td>
<td>$h$</td>
<td>0.57</td>
<td>First quartile value$^b$</td>
</tr>
<tr>
<td>Borrowing constraint against housing</td>
<td>$\lambda_h$</td>
<td>0.60</td>
<td>Down payment requirement</td>
</tr>
<tr>
<td>Linear transaction cost</td>
<td>$f_1$</td>
<td>0.03</td>
<td>Association of Realtors</td>
</tr>
<tr>
<td>Calibrated in the model</td>
<td>$\delta$</td>
<td>0.03</td>
<td>Wu et al. (2012)</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utility share of housing service</td>
<td>$\varphi$</td>
<td>0.24</td>
<td>Bond share in total: 0.23$^a$</td>
</tr>
<tr>
<td>Discount factor for top earners</td>
<td>$\beta^B_{\mu}$</td>
<td>0.97</td>
<td>Top 10% asset-over-income: 10.5$^a$</td>
</tr>
<tr>
<td>Discount factor for medium-discerning earners</td>
<td>$\beta^B_{\mu}$</td>
<td>0.96</td>
<td>Aggregate asset-over-income: 5.4$^a$</td>
</tr>
<tr>
<td>Discount factor variation for top earners$^b$</td>
<td>$\Delta \beta^B$</td>
<td>0.01</td>
<td>Wealth gini: 0.48$^b$</td>
</tr>
<tr>
<td>Borrowing constraint against income</td>
<td>$\lambda_y$</td>
<td>0.10</td>
<td>Aggregate loan-to-value: 0.05$^a$</td>
</tr>
<tr>
<td>Fixed transaction cost</td>
<td>$f_2$</td>
<td>0.12</td>
<td>Housing-transaction-over-GDP: 0.06</td>
</tr>
</tbody>
</table>

$^a$: Calculated using CHIP 2002. See Appendix C for detailed procedure.

$^b$: The discount factor grids are symmetric; i.e., $\Delta \beta^B = \beta^B_h - \beta^B_m = \beta^B_m - \beta^B_l$. 

18
Endowments  Both output and housing stock are normalized to be 1. We set \( \pi \) at 26%, according to the top 10% wage income share in CHIP 2002. The normal endowment levels for bottom earners and top earners, \( \bar{y}^B \) and \( \bar{y}^T \) respectively, are then set to meet the conditions that \( (1 - u)\bar{y}^B + uy^B = (1 - \mu)Y \) and \( (1 - u)\bar{y}^T + uy^T = \mu Y \), where \( u \) denotes the aggregate unemployment rate and \( \mu = 0.1 \).

The employment process \( \Pi_{e,e'} \) is chosen so that the average duration of an unemployment spell is two years and the unemployment rate \( u \) is 10%, which is roughly in line with the findings in Giles et al. (2005). The transition matrix of the employment status is as follows (rows indicate the current state, and columns indicate next period’s state; both the first row and the first column correspond to \( e = 1 \) – i.e., employment):

\[
\begin{bmatrix}
17/18 & 1/18 \\
1/2 & 1/2
\end{bmatrix}
\]

The unemployment insurance replacement rate \( \omega \) is set to be 7%. This level of unemployment insurance is in line with the data. According to the China Labor Statistical Yearbook 1999 to 2005, the overall replacement rate from unemployment insurance is about 14% of the worker’s wage, and the maximum duration of benefits for unemployment insurance recipients is 2 years. Moreover, the China Labor Statistical Yearbook reports that about 50% of all unemployed workers were eligible for unemployment benefits from the unemployment insurance system from 1999 to 2005. Thus, we set \( \omega \) to be 7%, the same for both the top and the bottom earners.

Preferences  The baseline risk aversion parameter is set at \( \gamma = 1 \). The utility share of housing service \( \varphi \) is set at 0.24 to match the housing wealth and bonds wealth split in the data. Note that we could alternatively calibrate \( \varphi \) according to the housing consumption share in total consumption. However, a good measurement of aggregate housing consumption, especially imputed rents, is not available for 2002. Data availability is also the reason for choosing the Cobb-Douglas preference specification, which fixes the degree of substitutability between housing and non-housing consumption. The intra-temporal substitutability between housing and non-durable consumption can be inferred provided appropriate data.

To generate a net asset over income ratio of 10.5 for the population’s top 10% wealthiest, we calibrate \( \tilde{\beta}^T \) to be 0.9657. The shock to the patience level of bottom earners, \( \tilde{\beta}^B \), takes on values from a symmetric grid, \( \{\tilde{\beta}^B_l = 0.9427, \tilde{\beta}^B_m = 0.9527, \tilde{\beta}^B_h = 0.9627\} \), with 80% of the bottom earners adopting the middle value and 10% each adopting the extreme points in the invariant distribution. The expected duration of the extreme discount factors is 50 years. As in Krusell and Smith (1998), this is meant to capture, albeit in a somewhat crude way, a dynastic element in the evolution of preferences. Transitions can only occur to adjacent values, where the transition probability from either extreme value to the middle grid is 1/50 and from the middle grid to either extreme value is 1/400. This Markov chain for \( \tilde{\beta}^B \) has been chosen to match the aggregate wealth-over-income ratio.
and the Gini coefficient (0.48) of the wealth distribution. The transition matrix of \( \hat{\beta} \) is as follows (rows indicate the current state, and columns indicate next period’s state; both the first row and the first column correspond to \( \hat{\beta} = \hat{\beta}_l \); both the second row and the second column correspond to \( \hat{\beta} = \hat{\beta}_m \)):

\[
\begin{bmatrix}
49/50 & 1/50 & 0 \\
1/400 & 199/200 & 1/400 \\
0 & 1/50 & 49/50
\end{bmatrix}
\]

**Market arrangements**  Risk free interest rate \( r \) is set as the one year bank deposit rate in 2002 taken from the People’s Bank of China. Alternatively, \( r \) can be chosen according to the long-term bank deposit rate. Since interest rate in this model is not used to target any specific data moments, a different \( r \) will simply change the calibrated patience levels. The model does not consider the markup of mortgage borrowing either. Although the cost of mortgage credit clearly affects the borrowing decision, it is less relevant in the model since the loan-to-value ratio is targeted by the loan to income criterion.

The minimum size house that can be owned (\( h \)) is set according to the first quartile housing asset value among all the homeowners in CHIP 2002. This value is roughly 2 times the average annual wage income among all the homeowners in CHIP 2002. The maximum loan-to-value ratio restriction is set at \( \lambda_h = 0.60 \), corresponding to a constant down payment requirement of \( 40\% \). The maximum loan-to-future-income ratio restriction \( \lambda_y \) is chosen to be 0.10. The two borrowing requirements in the model together determine the overall tightness of credit conditions in the model, generating an aggregate loan-to-value ratio of 5%.

Note that the mortgage option setup in the model allows households to draw on a home equity line of credit (subject to a loan to income criterion). Thus, loans are essentially payment-option mortgages with a required interest rate payment and a pre-approved home equity line of credit. In principle, this means that a borrower can choose to only cover mortgage interest payments but not to pay down principal every period. However, mortgage loans in China are all installment loans: equal installments of the loan must be paid each period before maturity, although early retirement is allowed. It turns out that this is not a problem for our model. In the model, households usually do borrow when they make new housing purchases, but they pay back their debts gradually afterward. This is because unemployment shocks and sizable adjustment costs in housing asset induce households to desire a certain amount of liquid wealth buffer. Moreover, the

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27 The minimum down payment requirement for first home purchases in China is strictly regulated by the People’s Bank of China. It has changed over time but within a narrow band between two levels: 30% or 40%. Mortgages to finance purchases of second homes can be subject to even higher down payment requirements.

28 In China, the most important factor considered by the bank for mortgage approval is usually the applicant’s income (no official credit record system exists until 2006). However, there is no clear-cut loan to income criterion universally set for all the banks. Guidelines from bank regulators usually require that a borrower’s ratio of monthly mortgage payment to income should be lower than 50%. For a comprehensive discussion of residential mortgages in China, see Fang et al. (2015).
tight loan to income criterion further dampens the effect of a home equity line of credit since the borrowing constraints become much tighter when households are unemployed. Overall, the convenient setup for borrowing constraints does not deviate much from the reality in China.

The depreciation rate of housing asset \( \delta \) is set at 0.03, to approximate both the maintenance and depreciation cost of holding housing wealth.\(^{29}\) We choose \( f_1 = 0.03 \) roughly according to the amount of commissions for realtors when transacting houses, and we calibrated \( f_2 = 0.12 \) to target the housing transaction volume-over-GDP ratio. Overall, the adjustment cost function is used to approximate various realistic costs associated with transacting houses.\(^{30}\) Admittedly, the setup of the adjustment cost is crude in many ways. For example, it does not allow homeowners to make small adjustments avoiding paying fixed costs and it does not capture the reality that costs are asymmetric between buying and selling houses. But the setup does serve the purpose of generating lumpiness in housing that has differential impact across wealth groups, and that feature is essential for the mechanism in the paper.

### 3.5 Steady-state properties

In this section, we illustrate the steady-state properties of the model. We will pay special attention to the distribution of wealth (including its composition) and cross-sectional housing adjustment behaviors, since they are central to the main mechanism of the paper.

#### Wealth distribution

Table 3.3 examines the steady-state wealth distribution produced by our model. Compared to the data, the model matches almost all the moments.

<table>
<thead>
<tr>
<th>% of wealth held by top</th>
<th>Fraction with wealth &lt; 0</th>
<th>Gini coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% 5% 10% 20% 30%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model (baseline)</td>
<td>10% 24% 36% 55% 66%</td>
<td>1% 0.48</td>
</tr>
<tr>
<td>Data (CHIP 2002)</td>
<td>8% 23% 35% 53% 64%</td>
<td>2% 0.48</td>
</tr>
</tbody>
</table>

Notes: The wealth distribution in the data is calculated from CHIP 2002. Appendix C contains more details about the calculation procedure.

Note that, when compared to the US data, wealth concentration is at a much lower level in China.\(^{31}\) Although this might be a reflection of the true inequality level difference

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\(^{29}\) There is no good direct source to calculate the depreciation rate of housing in China. Previous literature routinely assumes a depreciation rate of 2.5%–3%. See Wu et al. (2012) and Chivakul et al. (2015), for example.

\(^{30}\) There are more than 20 different fees and taxes associated with housing transactions in China. Moreover, fees and taxes differ substantially according to the type of transaction and ownership rights (houses in China have different types of ownership rights due to institutional reasons). Details of those fees and taxes can be found at [http://esf.fang.com/zt/201403/estjyefmx.html](http://esf.fang.com/zt/201403/estjyefmx.html) (in Chinese, provided by a NYSE listed private realtor company SouFunHoldings).

\(^{31}\) According to the SCF 2001, the net worth shares of the top 1 and 5 percent wealthy households in the US are 32% and 57%, respectively.
between the two economies, it might also due, at least in part, to the underrepresentation of the top richest households in CHIP 2002. Even if it were possible to find ways to adjust CHIP 2002 to account for the missing richest households, it would be extremely hard to get an accurate account of the asset composition of those at the top. Thus, this paper will not attempt to address the potential underrepresentation of the wealthiest households in the Chinese data. More importantly, those at the top who are missing from the survey data are unlikely to hold mainly housing in their portfolios since they tend to have access to assets with better returns. Thus, overlooking the richest households here is unlikely to affect the results in this paper.

Cross-sectional asset composition Since the difference in cross-sectional portfolio choice is central to the mechanisms in the paper, it is important to examine how well the model matches with the real portfolio composition in the data. Figure 3.3 compares the model produced portfolio composition with the data counterpart.

Figure 3.3: Comparison of cross-sectional asset composition

Notes: Panel (a) plots the distributions of net housing-wealth-over-income ratio both in the model and in the data. Each distribution is normalized by dividing its maximum value, so that the range is between 0 and 1. Panel (b) plots the distributions of bond wealth share in total wealth among homeowners. To be consistent with the model, bond wealth in the data is defined as fixed bank deposits, and total wealth is defined as bank deposits plus net housing wealth. The two data distributions in both panels are obtained from CHIP 2002.

32Xie and Jin (forthcoming) find that, after supplementing CFPS 2012 with data from the China Rich List reports, wealth concentration in China is comparable to that of the United States, at least according to measures of top wealth shares.

33The rate of return to capital has stayed comfortably over 20% throughout 1993 to 2005, according to Bai et al. (2006). Although these high corporate returns are not available to normal households, the wealthiest do have access to them.
Panel (a) in Figure 3.3 shows that the model does a good job of capturing the fact that most households own similar housing wealth, a manifestation of the minimum size friction. Panel (b) in Figure 3.3 shows that liquidity conditions vary a lot among homeowners, both in the data and in the model. Households in the model ideally desire to hold a balanced composition between liquid wealth and housing wealth. Although housing asset earns a higher return, liquid wealth is more suitable for the purpose of smoothing temporary income shocks. The model can produce a wide range of liquidity conditions because, due to the adjustment costs, households opt for depleting liquid wealth when making new housing purchases and gradually accumulating it back to the ideal level afterward.

**Housing adjustments**  As discussed in great detail under the context of the basic model in Section 2.3, the cross-sectional difference in housing demand responsiveness is the key to generating interactions between housing market outcomes and inequality. Thus, we examine to what extent the differential responsiveness in housing demand holds true in the quantitative model here.

We first discuss the housing decisions for households who do not own any housing assets. Similarly as in the basic model, housing has a higher financial rate of return than bonds. Therefore, housing decisions for non-homeowners are standard: there is a threshold amount (depending on the exact household type) of liquid assets such that, if assets exceed the threshold, non-homeowners become homeowners. Also, the larger the initial liquid assets are, the less likely a household is to borrow for the purpose of financing a housing purchase.

Figure 3.4 plots homeowners’ housing adjustment choices as a function of initial housing and liquid wealth, across 4 different types, employed or unemployed, top earners or bottom earners (medium patience level). A homeowner can stay put, upgrade, downsize, or completely sell off her housing stock (thus exiting the housing market).

Across all types of households, larger liquid assets increase the chance of upgrading one’s housing stock, and transaction costs create a region of inaction where the housing stock is constant. If liquid wealth falls below a certain level, the household either downsizes or exits the housing market. Moreover, when a household does adjust her housing stock, the size of the adjustment also depends a lot on her liquid wealth level (although this feature cannot be illustrated through Figure 3.4).

One important feature of the model is that bottom earners have much larger inaction zones – i.e., unresponsive housing demand; while top earners adjust their housing stock much more frequently – i.e., responsive housing demand. The unresponsive housing demand from the bottom earners is also reflected in the feature that when the amount of liquid assets is small, the housing tenure decision depends on the initial level of housing wealth non-monotonically. Consider, for instance, an unemployed homeowner with liquid assets equal to about thirty percent of annual income (i.e., a bottom earner). If the initial housing stock size is really small, the homeowner pays the adjustment cost and, because of his low liquid assets, completely quits the housing market. If the initial housing stock
size is medium, the homeowner does not change house size since, given the modest size, quitting the housing market is too big an adjustment, while downsizing a bit is not economical given the sizable fixed adjustment costs. If the initial house size is large, there is enough room to optimally downsize the housing stock instead of completely selling it off.

Figure 3.4: Housing adjustment functions

Notes: The figure illustrates the equilibrium optimal housing adjustment policy as functions of initial housing wealth and bond wealth. The plots in Panel (a) and Panel (c) correspond to a medium patient bottom earner ($\tilde{\beta} = \tilde{\beta}_m$).

It’s clear that the calibrated model not only captures the wealth distribution of Chinese households in 2002, it also matches well the distribution of household portfolio composition. More importantly, the differential responsiveness in housing demand channel is
at work. Thus, the calibrated model is well suited to study the impact of changes in macroeconomic factors on equilibrium house prices in China after 2002.

4 Transition experiment

Recently, a slowdown of the Chinese economy has incited rising concerns of another setback in the fragile global economy. A possible housing market crash is at the forefront of these worries: housing prices in China have been growing too fast for too long, and a crash may be inevitable. However, as noted by Fang et al. (2015) and Wu et al. (2015), it is an ambitious task to reliably identify whether the enormous house price appreciation in China is a bubble since very special features of the Chinese economy, such as fast economic growth, large-scale urbanization, and severe financial frictions, must be taken into account in a systematic framework.

This section takes this challenge seriously. In particular, we conduct the following transition experiment: starting from the calibrated model in Section 3.4, and introducing income growth of the same magnitude as in the data, we analyze the response of house prices. Moreover, the model will also take into account another factor, changes in income inequality, which has not been discussed much in the literature focusing on house prices in China. The main goal of this transition exercise is to quantitatively evaluate how much the proposed mechanism in this paper can account for the dynamics of house prices in China. The model will also speak to the dynamics of aggregate saving as well as wealth inequality.

4.1 Transition problem

In this section, we formulate the transition exercise mathematically. We will carefully specify the shocks according to the data, define the transition problem, and briefly discuss the solution methodology.

In the main exercise below, we engineer an increase in both income inequality and income growth of the same magnitude as in the data. The discipline in the transition experiment comes from assuming that the entire wage inequality increase is generated by an increase in the top income share over time. The individual income process and all other parameters stay constant during the transition. This way, we control for the earnings uncertainty faced by households during the whole transition exercise so that there is no change in precautionary saving motives.

To keep things simple, we assume a linear increase in both the top income share and total income between the initial steady state (2002) and period 10 of the transition (2012). From period 11 (2013) onward, we assume that both the top income share and total income stay constant at their final steady-state values (the same as the 2012 value). Note that we set top income share and total income to be constant from 2013 onward, in order to cleanly illustrate the strength of the mechanism in generating house price run-up
with only a temporary growth shock. In the following, we formally define the transition problem. Denote $t \leq T$ as a transition period, the period $t$ value function $V^t_i(\varepsilon, \tilde{\beta}, b, h; \Lambda_t, Y_t, \pi_t)$ solves

$$V^t_i(\varepsilon, \tilde{\beta}, b, h; \Lambda_t, Y_t, \pi_t) = \max_{x, b', h'} u(x, R_t) + \tilde{\beta}E[V^{t+1}_i(\varepsilon', \tilde{\beta}', b', h'; \Lambda_{t+1}, g_{t+1}, \pi_{t+1})|\varepsilon, \tilde{\beta}]$$

s.t.

$$x + b' + P_t h' + \Phi(h', h) = \varepsilon y_t^i + (1 - \varepsilon)\omega y_t^i + (1 + r)b + R_t h + (1 - \delta)P_t h$$

$$b' \geq B(\lambda h, h', \lambda y, \varepsilon)$$

$$h' \in \{0, [h, +\infty)\},$$

for $i \in \{B, T\}$. Here $\Lambda_0$ corresponds to the calibrated initial steady state distribution, and $T$ is the ending period of the transition when $\Lambda_T$ will stay constant at the new steady state level after $t \geq T$. Note that $T$ is an endogenous equilibrium object.

We consider a perfect foresight transition path to the new steady state (no aggregate uncertainty); i.e., the whole sequences of $\{Y_t, \pi_t\}_{t=1}^\infty$ are shocks known at $t = 1$. In particular,

$$\pi_t = \begin{cases} 
\pi_{t-1} + 0.01 & \text{if } t < 10 \\
\pi_{t-1} & \text{otherwise} \end{cases}, \quad Y_t = \begin{cases} 
1.08 \times Y_{t-1} & \text{if } t < 10 \\
Y_{t-1} & \text{otherwise} \end{cases}.$$

Markets are required to clear at all times during the transition, as in the steady-state equilibrium. Note that solving the household optimization problem along the transition path requires adding time to the state variables listed in the steady-state problem described earlier in the paper because both current-period states and future states affect households’ optimal decisions. Not surprisingly, it turns out that households need not know the full information of $\{\Lambda_t\}_{t=1,\ldots}$; their decisions depend on $\Lambda_t$ only through the following aggregate state variables: house price $P_t$, aggregate saving in bonds $A_t$, and housing transaction volume and housing transaction frequency at each period $t$. This is because the dynamics of those aggregate variables contains enough information for households to pin down the whole return sequence of housing asset.

Therefore, solving the transition path problem is a fixed point iteration process of the sequences of prices $P_t$, bond savings $A_t$, transaction volumes at $t$, and transaction frequencies at $t$ along which the optimal decisions of households clear all the three markets (good market, rental market, and housing market). Given a sequence of those aggregate variables, the dynamic programming problem can be solved recursively, moving backward in time from time period $T$. Note that we also need to solve for the new steady-state equilibrium with different top income shares $\pi_T$ and output level $Y_T$. Details of the

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\[\text{If we assume households in the model expect the income growth will continue after 2013, the resulting house price will respond more. We will leave the investigation of how different expectations of economic growth can affect the dynamics of house prices to future sensitivity analysis.}\]
computation procedure can be found in Appendix D.2.

4.2 Result

In this section, we investigate the perfect foresight equilibrium dynamics. We study the economy’s transition from 2002 until 2012, and ultimately towards the new steady state. We particularly focus on the outcomes of three aggregate variables: house price, wealth inequality and aggregate savings rate.

In the three panels we present below in Figure 4.2, the starting point is the initial 2002 steady state; the solid line denotes the transition path outcomes of the model, and the dotted line with circles denotes the data. Note that we only present the transitional dynamics until 2020, since the economy is pretty much stabilized by then and the transition to new steady state afterward is very gradual.

Our main object of interest is the evolution of house prices. In particular, the central question of the paper is whether the large-scale housing price appreciation (faster than income) can be accounted for by the relatively parsimonious model setup within a rational expectation regime. Panel (a) in Figure 4.2 shows the model’s prediction for house price dynamics along the transition path (solid line), presented as the aggregate price-over-income ratio relative to the initial steady state. Note that aggregate income in the model is assumed to grow at 8% annually between 2002 and 2012, and at 0% afterward.

The results show that the amplification mechanism of our model is sufficient for rationalizing a large run-up of house prices. In particular, the growth rate of house prices in the model is significantly higher than the growth rate of income during early periods of the transition and then gradually drops below income growth and even becomes negative toward the end of the time period examined. The existing data show a similar price-over-income trend in the first phase of the model outcome although the data trend begins less abruptly and is sustained over a longer period than that in the model. As will be discussed in the next section, this difference might be reconcilable by incorporating more realistic features such as continued income growth or urbanization.

Panel (b) and Panel (c) in Figure 4.2 show that the hump-shaped house price-over-income dynamics is accompanied by similar patterns in the evolutions of wealth inequality and the aggregate savings rate. Again, the model outcomes are a bit more dramatic than the data counterpart, for similar reasons as argued above. Overall, the results in Figure 4.2 show that the model can roughly account for both the trend dynamics and the changes in magnitude for all the three aggregate variables in consideration.

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35 The steady-state aggregate variables are close to their levels in the year 2020 although the full transition will take about 20 more periods forward.
36 We focus on the data trend here since the raw data is volatile, especially due to the interruption of the 2008 economic crisis, while the model outcomes are smooth since we only consider a one-time period of anticipated temporary shocks.
37 Wealth inequality only falls a bit after the early surge and remains at a much higher level than the initial steady state afterwards. This is not the case for savings rate and price-over-income ratio. The reason for this difference is the model’s assumption that the increase in top income share is permanent.
Figure 4.2: Transitional dynamics

Notes: The raw data for Panel (a) is constructed from the same sources as those in Panel (b) of Figure 3.1. The trend is constructed by HP filtering the raw data with frequency parameter $\lambda = 6.25$. Note that since the price index is available only after 2004, the comparison in Panel (a) is done by “pretending” the starting year in the model is 2004. The data in Panel (b) is calculated by author from the same sources as those in Panel (a) of Figure 3.1. See Appendix C for more details. The data for Panel (c) is taken from Yang et al. (2013), where the savings rates are calculated from Urban Household Surveys conducted by National Bureau of Statistics of China. Note that the level of savings rate in the model is not directly comparable to that of the data; we only compare the relative changes here.
In the following, we investigate the mechanism behind the model outcome. As discussed in earlier parts of the paper, one key feature of the model is the differential responsiveness in housing demand across wealth groups, which plays important roles in the stationary results. The differential responsiveness turns out to be crucial for the transitional analysis, too. In particular, it implies an endogenous feedback loop between house price and wealth inequality. It is exactly this feedback loop that causes house price to overreact to temporary shocks. The general intuition about the mechanism is the following.

The feedback mechanism is initiated by expected income growth, which will ultimately lead to house price appreciation. Due to potential capital gains, a period of high premium in housing is thus anticipated among households. In response, the most responsive wealthy households stock up on housing wealth initially, causing a surge in house prices. Due to the skewed housing wealth distribution, the price surge causes a higher dispersion in capital income and thus higher wealth inequality follows. Higher wealth inequality, in turn, helps sustain the growth in house prices. This is because, while housing demand from wealthy households saturates, demand from households with modest wealth starts to pick up after a short delay. Even with high premium, transaction frictions and liquidity constraints still limit regular households’ response in the housing market. Accordingly, the inequality gap between wealthy housing investors and the rest continues to widen as house prices keep creeping up. This creates a self-reinforcing feedback loop between house price and wealth inequality.

The loop ends eventually, when most of the capital gains (due to income growth) in the housing market are realized. The return premium sinks to a normal level, wealthy households begin to draw down their over-sized housing stock, and at about the same time, households that initially had meager resources are able to enter the housing market after continued saving. Housing wealth becomes more dispersed as house prices adjust back to trend.

Figure 4.3 plots the transitional dynamics for housing premium and buyer distribution, to help confirm the above intuition. According to the top graph in Figure 4.3, the housing premium stays high only for early periods when capital gains in housing present. Although the high premium dwarfs the transaction costs in housing, early buyers who are able to enjoy the whole high return periods are still mostly households from the top wealth tertile. This is because, tight borrowing limits and a high entry barrier prevents households with low wealth from responding quickly. Therefore, most of the initial housing demand comes from a portfolio shift from bonds to housing among the responsive wealthy. Households with relatively plentiful resources can tap into some of the high returns in the housing market during later periods of the high return phase, with modest saving efforts. Households with initial low resources (mostly non-homeowners) are forced to save more and wait longer to enter the housing market.
Figure 4.3: Housing premium and buyer distribution during transition

Notes: The housing premium in period $t$ is calculated as $\frac{R_t}{y_t} + \frac{P_t}{y_{t-1}} - \delta - r$. Each tertile buyer share in period $t$ is obtained by accruing all the households who purchased housing assets (either by upgrading or making first-time purchases) in period $t$ according to wealth tertile in period $t$.

The transition dynamics of house price and buyer distribution makes it clear that aggregate saving can increase due to strong saving motives for housing purchases from households with few resources. However, the initial sharp rise in aggregate saving is also partly due to the portfolio re-balancing behavior of the wealthy. As explained earlier, households in the model desire a certain level of liquid wealth. After the initial portfolio shift toward housing wealth, while waiting for the capital gain in housing asset to fully realize, the wealthy need to save up more liquid wealth since it is still the better form of buffer against idiosyncratic income shocks.
In sum, the model rationalizes a house price run-up relative to income growth and its adjustment back to trend in a perfect foresight equilibrium with one-time shocks. It also explains increased wealth concentration and a rising aggregate savings rate. The mechanism links portfolio choices of the wealthy and the poor with price dynamics and expectations, stressing the role of responsive investment behavior among wealthy households. There is no crash even after a big house price boom: the barrier to entry forces the poor to save and delay their house purchases, and this in turn creates conditions for the soft landing of house prices.

A final note is that the strength of the mechanism described above crucially depends on the exogenous inequality level in the economy, which is largely controlled by the top income share in the model. Thus, the inequality shocks fed into the model are not only empirically relevant but also important for amplifying the extent of the housing market responses and matching them with the observed magnitude of house price run-ups. We will investigate more of the role played by inequality shocks in future sensitivity analyses.

4.3 Discussion

In this section, we discuss missing elements in the baseline transition exercise, additional sensitivity analysis, and potential extensions.

Other demand factors  The baseline transition exercise only considers one-time anticipated income shocks (both growth and inequality), abstracting from changes in many other demand factors. Although this choice makes our results clean for interpretation, other demand factors might play important roles in the housing boom. We briefly discuss the implications of other demand factors here.

In particular, this paper does not consider changes in credit conditions. This choice is consistent with the empirical evidence. Although the amount of mortgage borrowing did increase over the years in China, the regulation policy changes are limited, in the sense that both the mortgage interest rates and down payment requirements vary within a relatively small range (see Fang et al., 2015 for more details). One might worry that borrowing from the shadow banking sector might mean that some lenders actions may work independently from official regulations. However, according to the two sources of household survey data analyzed in this paper (CHIP 2002 and CFPS 2012), the loan-to-value ratio at the household level remains low in the two survey waves (about 5% in both 2002 and 2012), this figure suggests that shadow financing is not a significant concern. Theoretically speaking, if we allow a controlled relaxation in credit conditions during the

\[38\] Recent literature explaining the early 2000s US housing booms focuses on credit expansions (see, for example, Kiyotaki et al., 2011, Sommer et al., 2013, and Favilukis et al., forthcoming). Housing market outcomes are sensitive to credit conditions in those models because house prices are mainly driven by consumption demand among the majority poor agents. Due to the very little cross-sectional variation in wealth across low income agents, small changes in credit availability can cause large groups of identical, low income, low wealth households to move between renting and homeownership. However, the large swings in homeownership often produced by those models contradict the data.
transition exercise, the model is able to produce larger housing booms.

The large-scale urbanization process, rural-urban migration in particular, is a more relevant exogenous demand factor. According to the two latest National Population Censuses, the urban population in China increased at an average annual rate of about 4%. Chen and Song (2014) find that urbanization accounts for 80.4% of the total urban population growth. Moreover, among the urbanized population, rural-urban migration accounts for more than half. This means that including urbanization factors will introduce a large number of poor households who need to save up a long time for housing purchases to the model. Accordingly, the mechanism in the paper will get strengthened. In particular, this will likely cause the housing boom to last longer.

This paper does not consider demographic factors either. In particular, life-cycle components are not explicitly modeled. For one thing, they are less relevant in the short and medium run than factors such as large-scale urbanization. Moreover, the intergenerational link is strong in China in the sense that parents are usually able and willing to provide financial support to their children’s home purchases. This means that the “actual” housing demand in China might present less life-cycle pattern than a standard life-cycle housing demand model would imply. This is partly supported by the empirical observation that the saving behavior does not vary significantly along the life-cycle dimension. Thus life-cycle does not seem to be a factor of the first order here.

Aggregate uncertainty and expectations  Aggregate uncertainties are not currently under consideration in this paper. Due to the modeling approach of this paper (which treats housing as a lumpy financial asset), adding aggregate uncertainty is expected to strength the mechanism in the model. Since housing becomes a risky asset in the presence of aggregate uncertainty, it is expected that housing will be compensated with a risky premium in addition to the illiquidity premium. The lumpiness of housing will thus increase. This might amplify the differential responsiveness in housing demand.

Taking aggregate uncertainties into account will also help the model in matching the volatility in house prices in the data. However, since the effects from aggregate uncertainties tend to be mostly short term, the qualitative result of the large-scale transition we considered here is thus less likely to be changed by incorporating aggregate uncertainty. Similar reasoning applies to different ways of modeling expectations other than the current simple approach in the model. In general, incorporating aggregate uncertainties and changes in expectations is less likely to affect our main qualitative results, although it might add interesting dimensions to an extended model.

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39 This is partly because of the Chinese tradition that children support parents in their old age (and sometimes even live in the same house).

40 According to Chamon and Prasad (2010), the age profile of savings among Chinese households has an unusual pattern in recent years, with younger and older households having relatively high savings rates.
Supply side factors Finally, housing supply is fixed in the model.\footnote{Combining with the fixed population mass, the model essentially assumes roughly matching exogenous changes in housing demand and supply. This turns out not to be a bad assumption. According to Wu et al. (2015), in aggregate, the growth in households demanding housing units driven by fundamentals roughly matches with new housing construction in urban China, although with considerable regional variation.} Allowing for housing production is unlikely to affect the qualitative result of this paper either, especially since land is an important factor in housing construction. As discussed earlier, according to Wu et al. (2015), house price growth in China is driven by rising land values, not by construction costs. However, incorporating housing production can be an important future avenue to explore. The cross-section variation in housing supply elasticity can be an important factor in accounting for the regional variations in house price growth.

In sum, the qualitative results from the baseline transition experiment are expected to hold even considering various additional elements. At the same time, there are relevant extensions to the model that might deliver interesting implications.

5 Conclusion

This paper developed a heterogeneous agent equilibrium model to study the interactions of inequality, income growth, and house price dynamics. Important features of the housing market environment are explicitly captured: housing delivers returns both in terms of rental dividends and capital gains; market frictions make housing adjustment lumpy. House prices and rents (and therefore the financial rate of return on housing) are jointly determined in equilibrium. The unique modeling approach of the paper allows housing purchases to be purely financial decisions. Thus, housing market outcomes are closely linked to cross-sectional household portfolio saving decisions: prices and inequality endogenously interact.

The model can account for the positive empirical relationship between inequality and house prices. More importantly, a transitional equilibrium analysis of the Chinese economy can rationalize its observed house price growth accompanied by a rise in the private savings rate. Key mechanisms behind the results are based on the differential responsiveness in housing demand across wealth groups, which is due to the differential impact of the lumpy housing.

This work contributes to the housing literature by illustrating a new price formation mechanism, which highlights the role played by the investment motive among the wealthy.\footnote{This mechanism might be an important force behind the recent housing boom in the United States: evidence from the SCF shows that the housing stock owned by wealthy households increased much more dramatically than the rest of the housing stock during the housing price run-up in the early 2000s.} Another important contribution is that the paper provides a tractable quantitative model that rationalizes puzzling phenomena in China – notably, the recent big housing boom and rising saving rate. Moreover, the theory in this paper shows that market frictions can have differential impact cross-sectionally, increasing the risk of an explosion of inequality. This adds to the understanding of a broader topic: how inequality...
and macroeconomic forces can interact.

At its current stage, this research only focuses on explaining the mechanisms behind certain housing market outcomes. Important welfare and policy implications could easily be the next step in this analysis. For example, the interaction between inequality and return premium in the housing market directly speaks to the efficient provision of housing services in a steady state. If housing supply is explicitly modeled, increased inequality might boost housing production and ultimately improve welfare. Moreover, in the short run, low-income households can be priced out of housing market due to the feedback loop between house prices and wealth inequality. This highlights the potentially large welfare cost to low-income households during fast economic growth, a situation that clearly calls for appropriate policy interventions.

Possible extensions of this work may have applications beyond explaining housing market outcomes. For example, one can develop a related model to explain the cross-sectional debt-to-income ratios and aggregate default risk. In an environment in which heterogeneous households face investment opportunities with different risk-return perspectives and different investment costs, wealthy households can afford to load on riskier and costlier assets with a low debt-to-income ratio, earning high returns. While the wealthy households bid up the prices for high-return assets, the poor households are either “trapped” in risk-free assets with meager returns or forced to borrow a lot (relative to income) and bear higher default risks. The mechanism can potentially explain the cross-sectional debt-to-income ratio patterns in the United States and shed light on the development of financial crises.
References


A Stylized facts about inequality and house prices

Figure 1.1 in Section 1 presents the raw correlation between top income share growth and appreciation of house prices both across countries and across states in the United States. In this section, I explain more details of the data source and variable construction. I also conduct some robustness checks on these correlations by controlling for some factors such as income and population growth and housing supply elasticity. Moreover, I show that rents do not co-move with top income shares. But due to data availability, the analysis with rental data is limited only to the US cross-state level. In the end, I discuss potential directions for future empirical work.

The choice of house price indexes Every house price index is representative in its own way and has different strengths and weaknesses. Even within the same geographic area, house price indexes from different sources can differ significantly, depending on construction methodology, sample composition and data aggregation. Therefore, sometimes it is crucial to choose the appropriate index according to specific purposes.

The focus of this paper is about the relationship between house prices and inequality. One central argument, both empirically and theoretically, in the paper is that investment motives of the wealthy households play important roles in driving house price dynamics. Since wealthy households tend to purchase more expensive houses than the rest of the population, using a value-weighted index, in which expensive houses have a greater influence on estimated price changes, will naturally bias the result of the analysis. This is especially true if more expensive houses have a different price dynamics than less expensive ones. Therefore, for this study, it is important to choose the house price indexes that weights price trends equally for all residential properties. To this end, I choose the all-transactions index produced by the Federal Housing Finance Agency (FHFA) for the US cross-state analysis.

The FHFA indexes are available from 1976-present for all states in the US, which is great for cross-section analysis. Each state-level index is constructed using a weighted, repeat sales method which compares transaction prices of the same property over time. This method is preferred because it avoids composition biases from quality changes in the stock of houses in transaction.

For cross-country analysis, I try to use house price indexes that are consistent with the US FHFA index methodology, which are readily available for many countries from The Federal Reserve Bank of Dallas’s International House Price Database as described in Mack and Martínez-García (2011). For the purpose of this study, I use the house price indexes expressed in real terms to control for potential correlation between changes in inequality and the overall price level in a country.

43In a related project, from various data sources, we document that the price dynamics of houses from different price ranges does differ. In particular, more expensive houses tend to appreciate faster than the rest of the houses.
Robustness checks with other data sources  The weakness of the FHFA index is that its underlining sample composites only houses purchased with conforming mortgages, which might understate the sensitivity of house prices to alternative credit linking to investment purposes, especially those cash transactions. Moreover, in specific regions and during certain times, housing purchases with sub-prime, jumbo, and other non-conforming loans can contribute significantly to changes in house prices as shown by some researchers. To address these issues, I repeat the same analysis using an alternative index: the Zillow Home Value Index (ZHVI), which is also available across all states in the US, although extending only back to 1996. ZHVI uses conceptually similar repeat-sales methods to address composition biases like the FHFA index, with the additional advantage of a broader sample coverage.\(^4\) The results of the analyses are similar when using either the FHFA index or the ZHVI, for the same time periods. Similarly, for the cross-country analysis, the robustness of the result is verified by using an alternative set of comparable house price indexes from the BIS Residential Property Price database and National sources.

The choice of inequality measures  The inequality measure in this paper focuses on households in the upper end. This is due to the theme of this paper: investment motive among the wealthy in driving housing price dynamics. The theory in the paper have several empirical implications regarding the relationship between income inequality, housing prices and wealth inequality. However most of them are hard to test due to data availability and cross-section comparability. In particular, cross-sectional data on wealth distribution is very limited (see Saez and Zucman, 2013 for an account of the series of wealth distribution in the United States using capitalized income tax data). Cross-sectional data on income distribution has been better documented, thanks to the recent effort by Alvaredo et al. (The World Top Incomes Database) and Frank et al. (2015). Thus, the measurement for inequality in this paper is consistently the top income (excluding capital incomes) shares.

Control for average income growth and housing supply elasticity  Some important factors that affect the dynamics of house prices might also affect changes in inequality. Two obvious ones are average income and housing supply elasticity. Figure 1.3 conduct the same correlation analysis as in Figure 1.1 in Section 1 with additional elements to control for those factors. In particular, I add cross-sectional series on personal disposable income per capita, to control for changes in average income and population growth. Consistent with the cross-country house prices data, cross-country disposable income per capita series are expressed in real terms. Moreover, I also try to control for cross-sectional differences in housing supply elasticity, although in very crude ways.

\(^4\)Details of the ZHVI construction methodology can be found at https://www.zillow.com/research/zhvi-methodology-6032/. In general, ZHVI is in nature similar to the S&P/Case-Shiller indexes, but with subtle differences in sample collection (see http://cdn1.blog-media.zillowstatic.com/9/ZHVI-infoSheet-U4ed2b.pdf for a detailed comparison of the two indexes). I choose ZHVI because it has better geographic coverages.
For both cross-country and cross-state analysis, I only have supply elasticity measures at cross-section but not overtime. This is only going to be problematic if changes in the supply elasticity correlates with changes in inequality. Potential corrections of this issue will be addressed in future work.

Figure A.1: Top income share growth and appreciation of house prices, with controls

Notes: House prices and inequality measures are exactly the same as those in Figure 1.1 in Section 1. Income is measured by personal disposable income per capita, with the same panel structure as the price and inequality data. The average growth rates of income are obtained by averaging the yearly growth rates over the entire sample periods, as close substitutes. Supply elasticity measures are available only as a cross-section. In Panel (a), cross-country personal disposable income per capita (real) series are taken from Federal Reserve Bank of Dallas’s International House Price Database as described in Mack and Martínez-García (2011); cross-country housing supply elasticities are cross-sectional estimates from Caldera and Johansson (2013). In Panel (b), US cross-state (nominal) personal disposable income per capita series are from BEA Regional Economic Accounts; US cross-state housing supply elasticities are approximated by the cross-sectional Wharton Land-Use Regulatory Index (WRLURI) from Gyourko et al. (2008).

As can be seen in Figure A.1, the positive association between top income share growth and appreciation of house prices remain significant after those controls. It even gets stronger at the cross-country level. These evidences provide us with more confidence in the causal relationship between inequality and house prices.

US cross-state rents and inequality The theory in this paper suggests that rents depend mostly on average resources instead of the inequality level. It is important to verify this result to justify the model setup in this paper. However, long enough series of rents data are hardly available at the cross-country level, especially if we want them to be comparable. Thus, the analysis with rental data is limited only to the US cross-state level. Figure A.2 plots the correlation between growth in top 5 percent income share and rental prices across states in the United States, both with and without controls for
average income and housing supply elasticity.

Figure A.2: Top income shares growth and rents appreciation

(a) Cross-country

(b) Cross-state, US

Notes: In both panels, inequality measures, income measures, and supply elasticity measures are exactly the same as those in Panel (b) of Figure A.1. In Panel (b), US cross-state (nominal) rents are taken from Historical Census of Housing Tables (decennial), provided by U.S. Census Bureau, Housing and Household Economic Statistics Division. Gross rent is the monthly amount of rent plus the estimated average monthly cost of utilities and fuels. Monthly rents were computed for specified renter-occupied units paying cash rent, which exclude one-family houses on ten or more acres.

As can be seen in Figure A.2, rents do not co-move with top income shares (there is a hint of a positive correlation between those two in the raw data in Panel [a], which is gone after we add the controls in Panel [b]). This adds validity to the mechanisms proposed in this paper.

Future work Although the focus of this paper is about understanding the theoretical interaction between housing market outcomes and inequality dynamics, related empirical analysis is relevant both in the sense of providing validation to the theory in the paper and of its own importance. To empirically establish the causality behind the co-movements between house prices and inequality, the analysis in this paper is clearly inadequate. My empirical analysis of disaggregated data, especially across US states, Metro areas, and even counties, is of further interest. Some preliminary analysis suggests that the correlation between growth in inequality and house prices is robust. A more detailed analysis will not only help confirming the relationship, but also help identifying contributing factors and mechanisms behind this correlation.
B Equilibrium characterization of the basic model

Before the equilibrium characterization, we first recast the definition of equilibrium in more details.

Definition of equilibrium The steady state stationary equilibrium associated with \( \mu, \pi, Y, r, l, H \) and \( h \) is defined as: a set of prices \( R \) and \( P \), a set of policy functions \( b(y, R, P) \), \( h(y, R, P) \) and \( x(y, R, P) \) for the young households, as well as a stationary distribution \( \lambda(y, b, h) \), such that

1. Given \( R \) and \( P \), \( b(y, R, P) \), \( h(y, R, P) \) and \( x(y, R, P) \) solve the young household’s problem defined in (2.1).

2. The stationary distribution \( \lambda(y, b, h) \) is induced by \( \mu, \varphi, Y, r, H \) and \( b(y, R, P) \), \( h(y, R, P) \), \( x(y, R, P) \).


\[(i) \int \int \lambda(y, b, h) h(y, b, h) = H \]
\[(ii) \int \int \lambda(y, b, h) s(y, b, h) = H \]

B.1 Equilibrium with \( \frac{R}{P} = r \)

For equilibrium with \( \frac{R}{P} = r \), there is no strict portfolio choice problem. Regardless of the level of \( y \), the solution to problem (2.1) is simply

\[ x = \frac{1}{1 + \beta} y, \quad b + Ph = \frac{\beta}{1 + \beta} y \]

subject to constraints (2.2) and (2.3).

Now we can recover \( R \) from rental market clearing condition (31),

\[ \frac{1}{2} \frac{1}{1 + \beta} \varphi Y + \frac{1}{2} \frac{(1 + \beta) r}{1 + \beta} \varphi Y = RH \]

\[ \Rightarrow R = (1 + \frac{\beta r}{1 + \beta}) \frac{\varphi Y}{2H} \]

Note that we used the condition that \( Rs = \varphi x \). \( R \) increases with \( r \), due to a pure wealth effect from the old generation. But the increase is less than one for one, since there is still a substitution effect from the young generation.

Price simply follows from the no arbitrage condition,

\[ P = (\frac{1}{r} + \frac{\beta}{1 + \beta}) \frac{\varphi Y}{2H} \]

\( P \) decreases with \( r \), due to the fact that \( R \) falls behind the growth of \( r \), which makes housing less attractive than bond.
Note that here income inequality does not matter for neither $P$ nor $R$, but this is an artifact of the assumption that the marginal propensity to consume does not change along wealth dimension.

In the end, housing asset market clearing condition \((3i)\), together with constraints \((2.2)\) and \((2.3)\) imply,

$$lP_{h} \leq \frac{\beta \pi Y_{h}}{1 + \beta \frac{\pi}{\mu}}$$

$$lP_{H} \leq \begin{cases} \frac{\beta \pi Y_{H}}{1 + \beta \frac{\pi}{\mu}} & \text{if } lP_{h} > \frac{\beta \pi Y_{H}}{1 + \beta \frac{\pi}{\mu}} \\ \frac{\beta Y_{H}}{1 + \beta \frac{\pi}{\mu}} & \text{otherwise} \end{cases}$$

Plugging in the pricing function, we have the following two cases.

1. Everyone can afford a starter home.

$$2 \frac{(1 - \pi)H}{(1 - \mu)\Pi} > \varphi l$$

$$r \geq \max\{\frac{1 + \beta}{\beta} \frac{\varphi l}{2 \frac{(1 - \pi)H}{(1 - \mu)\Pi} - \varphi l}, \frac{1 + \beta}{\beta} \frac{\varphi l}{1 - \varphi l}\}$$

2. Only top earners can afford a starter home.

$$r \geq \frac{1 + \beta}{\beta} \frac{\varphi l}{\pi - \varphi l}, \text{ if } 2 \frac{(1 - \pi)H}{(1 - \mu)\Pi} < \varphi l$$

$$\frac{1 + \beta}{\beta} \frac{\varphi l}{2 \frac{(1 - \pi)H}{(1 - \mu)\Pi} - \varphi l} \geq r \geq \frac{1 + \beta}{\beta} \frac{\varphi l}{\pi - \varphi l}, \text{ if } 2 \frac{(1 - \pi)H}{(1 - \mu)\Pi} > \varphi l$$

### B.2 Equilibrium with $\frac{R}{P} > r$

For equilibrium with $\frac{R}{P} > r$, due to the minimum size friction in housing asset transaction and the borrowing constraint, the optimal portfolio choice will depend on the level of $y$. Moreover, whenever the optimal portfolio consists of non-zero housing, constraint \((2.2)\) will bind, i.e., it’s of the household’s interest to fully utilize the collateralized borrowing opportunity. In the following, we first identify the policy functions.

**Policy functions** First, note that constraint \((2.3)\) necessarily binds for households with sufficiently low income. In particular, let’s denote

$$\Upsilon \equiv lP_{h}.$$  

---

6 Top young earners have to be able to afford a starter home.

7 Bottom young earners could not afford a starter home.
which is the income level at which a starter house is affordable only with zero consumption. Then for households with an income level lower than $y$, they are forced to save in terms of bond and the solution to problem (2.1) for $0 < y \leq \bar{y}$ is simply

$$x = \frac{1}{1 + \beta} y, \quad h = 0, \quad b = \frac{\beta}{1 + \beta} y.$$ 

Second, for households with high enough income, constraint (2.3) will not bind. Due to the assumed log preference, the cutoff income level can be conveniently identified as

$$\bar{y} \equiv \frac{1 + \beta}{\beta} \bar{x}.$$ 

Thus, the solution to problem (2.1) for $y \geq \bar{y}$ is simply

$$x = \frac{1}{1 + \beta} y, \quad h = \frac{\beta}{1 + \beta} y \times \frac{1}{lP}, \quad b = (1 - l)P h.$$ 

For households with an income level $\underline{y} \leq y < \bar{y}$, they are liquidity-constrained, and are unable to optimally choose more than $h$ units of housing assets. Moreover, due to the log preference, their optimal decision must be either choosing exactly $h$ units of housing, or no housing at all.

If a household decide to choose $h$ units of housing, her current consumption will suffer, while she can enjoy a higher return from her saving. In the end, her lifetime utility level will be

$$\ln(y - \underline{y}) + \beta \ln((1 + \rho)),$$

where

$$\rho \equiv r + \frac{1}{l} \left( \frac{R}{P} - r \right),$$

which is the net financial return from investing in housing.

If a household were to choose zero housing, the optimal consumption-saving decision would be the same as the households with a lower income than $\underline{y}$, and the associate utility level is

$$\ln\left(\frac{1}{1 + \beta} y \right) + \beta \ln\left( \frac{\beta(1 + r)}{1 + \beta} y \right).$$

Denote $d(y)$ as the difference between the two utility levels, which can be simplified to the following

$$d(y) = \ln \left( (1 + \beta) \frac{y - \underline{y}}{y} \right) - \beta \ln \left( \frac{\beta(1 + r)y}{(1 + \beta)\underline{y}(1 + \rho)} \right).$$

It’s straightforward to show that $d(y)$ is monotonically increasing when $y \in [\underline{y}, \bar{y}]$. Moreover, $d(\underline{y}) = -\infty$ and $d(\bar{y}) > 0$. Then we know there is a unique solution to the equation $d(y^*) = 0$, and $d(y) < 0$ if and only if $y < y^*$. Thus, it’s optimal to purchase $h$ units of housing for households with income levels between $y^*$ and $\bar{y}$. While households will only save in bonds when they have an income...
level below $y^*$.

In the end, we arrive at the following policy functions.

$$h(y, R, P) = \begin{cases} \frac{\beta}{1+\beta} \frac{y}{P} & \text{if } y \geq \bar{y} \\ h & \text{if } y^* < y < \bar{y} \\ 0 & \text{otherwise} \end{cases}$$

$$b(y, R, P) = \begin{cases} -\frac{\beta}{1+\beta} \frac{(1-l)y}{P} & \text{if } y \geq \bar{y} \\ -(1-l)Ph & \text{if } y^* < y < \bar{y} \\ \frac{\beta}{1+\beta} y & \text{otherwise} \end{cases}$$

With the policy functions we identified above, equilibrium prices can be solved out using market clearing conditions (3i) and (3ii). In the following, we discuss there different equilibrium outcomes that arise case-by-case.

**Case 1:** \( \frac{(1-\pi)Y}{(1-\mu)} \geq \bar{y} \)  

Asset market clearing condition (3i) gives

$$\frac{\beta Y}{2(1+\beta)l} = PH$$

$$\Rightarrow P = \frac{\beta Y}{2(1+\beta)lH}.$$ 

Rental market clearing condition (3ii) gives

$$\varphi \frac{Y}{2(1+\beta)} + \frac{\beta \varphi (1+\rho)}{2(1+\beta)} = RH$$

$$\Rightarrow \frac{R}{P} = \frac{\varphi}{1-\varphi} \left( \frac{1}{\beta} + 1 + (l-1)r \right), \quad R = \frac{\varphi \beta Y}{2(1-\varphi)(1+\beta)H} \left( \frac{1}{\beta} + 1 + \frac{(l-1)r}{l} \right)$$

Note that all the prices and returns do not depend on any inequality measure (it will if the propensity to consume differs across agents). However, for this type of equilibrium to exist, we do need a relatively equal distribution and small $h$. In particular, \( \frac{(1-\pi)Y}{(1-\mu)} \geq \bar{y} \) translates into

$$\frac{(1-\pi)}{(1-\mu)} \geq \frac{h}{2H}.$$ 

Moreover, we have restriction on $r$, such that $\frac{R}{P} > r$, which is

$$r < \frac{\varphi (1+\beta)}{\beta (1-l\varphi)}.$$ 

**Case 2:** \( \frac{\pi Y}{\mu} > \bar{y} > \frac{(1-\pi)Y}{(1-\mu)} \geq y^* \)  

Asset market clearing condition (3i) gives
\[ \frac{\beta \pi Y}{2(1 + \beta)l} + \frac{(1 - \mu)}{2} P_h = PH \]

\[ \Rightarrow P = \frac{\beta \pi Y}{(1 + \beta)(2H - (1 - \mu)l)} \]

Rental market clearing condition (3ii) gives

\[ \frac{\varphi}{2} \left( \pi Y + \frac{\beta \pi Y}{(1 + \beta)} \rho + (1 - \pi)Y + (1 - \mu)P_h \rho \right) = RH \]

\[ \Rightarrow \frac{R}{P} = \frac{\varphi}{1 - m\varphi} \left( l\left( \frac{1}{\beta} + 1 \right)n + (l - 1)rm \right) \]

\[ R = \frac{\varphi \beta Y}{(1 - m\varphi)(1 + \beta)(2H - (1 - \mu)l)} \left( \left( \frac{1}{\beta} + 1 \right)n + (l - 1)rm \right) \]

where \( m \equiv \frac{1}{1 + (1 - \mu)\frac{1 + h}{2H}} \) and \( n = (1 - (1 - \mu)\frac{h}{2H}) \frac{l}{\varphi} \)

Note that all the prices and returns do not depend on any inequality measure (it will if the propensity to consume differs across agents). However, for this type of equilibrium to exist, we do need a relatively equal distribution and small \( h \). In particular, \( \bar{y} > \frac{(1 - \pi)Y}{(1 - \mu)} \geq y^* \) translates into

\[ \frac{\beta \pi h}{(1 + \beta)(2H - (1 - \mu)l)} \leq \frac{(1 - \pi)}{(1 - \mu)l} \leq \frac{\pi h}{2H - (1 - \mu)l} \].

Moreover, we have restriction on \( r \), such that \( \frac{R}{P} > r \), which is

\[ r < \frac{\varphi l(1 + \beta)n}{\beta(1 - l\varphi m)} \], \( \varphi m < 1 \).

**Case 3:** \( \frac{\pi Y}{\mu} > \bar{y} > y^* > \frac{(1 - \pi)Y}{(1 - \mu)} \)

Asset market clearing condition (3i) gives

\[ \frac{\beta \pi Y}{2(1 + \beta)l} = PH \]

\[ \Rightarrow P = \frac{\beta \pi Y}{2(1 + \beta)lH} \]

Rental market clearing condition (3ii) gives

\[ \frac{\varphi Y}{2(1 + \beta)} + \frac{\beta \varphi}{2(1 + \beta)} \left( \pi Y(1 + \rho) + (1 - \pi)Y(1 + r) \right) = RH \]

\[ \Rightarrow \frac{R}{P} = \frac{\varphi}{1 - \varphi} \left( l\left( \frac{1}{\beta} + 1 \right)\pi - r \right), \quad R = \frac{\varphi \beta \pi Y}{2(1 - \varphi)(1 + \beta)H} \left( \left( \frac{1}{\beta} + 1 \right)\pi - r \right) \]

Note that all the prices and returns only depend on the inequality measure \( \pi \). However, for this type of equilibrium to exist, we need a relatively unequal distribution and big \( h \).
In particular, \( y^* > \frac{(1-\tau)}{(1-\mu)} \) translates into

\[
\frac{(1 - \pi)}{(1 - \mu)} \leq \frac{m\beta h}{2(1 + \beta)H}.
\]

where \( m \equiv \frac{y^*}{\sum} \).

Moreover, we have restriction on \( r \), such that \( \frac{R}{\phi} > r \), which is

\[
r < \frac{\phi l (1 + \beta)}{\beta (\pi - \ell \phi)}.
\]

C Institutional backgrounds and data sources for China

C.1 Backgrounds

Readers are referred to Walder and He (2014) for more backgrounds about the housing market privatization process in China before the early 2000s, and to Fang et al. (2015) for more discussion about the housing market in China between 2002 and 2013. For

C.2 Household income and wealth in China

The availability of household-level data from China is limited. One common data source that many papers used to study problems related to cross-sectional income in China is the annual Urban Household Survey (UHS) conducted by the National Bureau of Statistics (see Piketty and Qian, 2009, Yang et al., 2013, and Chen and Song, 2014 for example). However, UHS does not contain a good account for household wealth. For data source that can account for both household income and household wealth, The Chinese Household Income Project (CHIP) is the first option for periods earlier than late 2000s. Due to increasing interest in the wealth distribution in China, more resources start to be available after late 2000s. The China Family Panel Studies (CFPS) used in this study especially fit the purpose of this study.

In the following, I describe more details of the two data sources. I will also provide detailed calculation procedures of all those distributional statistics used in the main text of this paper.


The China Family Panel Studies (CFPS) The China Family Panel Studies (CFPS) is a nationally representative, annual longitudinal survey of Chinese communities, families, and individuals launched in 2010 by the Institute of Social Science Survey of Peking
University, China. In particular, CFPS contains detailed information about household income and wealth. A detailed account of the advantage of CFPS and its application can be found in Xie and Jin (forthcoming).

D Computation

D.1 Solving for the stationary equilibrium

The equilibrium definition is formally defined by the conditions presented in Section 3.3. Those conditions require an economy with a stationary distribution of $\Lambda^*$ over $(\varepsilon, \hat{\beta}, b, h, i)$ in which households behave optimally and markets always clear with prices $\{P^*, Q^*\}$. We approximate the type distribution of households $\Lambda^*$ by a distribution of finite number of households. An agent is characterized by its individual type $(\varepsilon, \hat{\beta}, b, h, i)$, and we use the type distribution of large number of households to approximate the type distribution.

Note households in this economy do not need to know the exact to solve their individual problems, knowing $P^*$ and $R^*$ are enough. However, directly searching for a pair of $P^*$ and $R^*$ that can clear all the markets is very time consuming, since it’s hard to find a updating rule that can make $P$ and $R$ monotonically converge. A more efficient approach is to use the goods market clearing condition to identify the relationship between $P, R, A$, transaction volume ($\Omega$) and the transaction frequency ($F$) of housing stock.

By treating $R$ as a residue from condition and searching for a stationary combination of the following aggregate variables: $P, A, \Omega,$ and $F$, it is easy to find a monotonically converging update rule. The algorithm outlined in the following exploits this property of the model and solve the equilibrium efficiently.

Let $P^k, A^k, \Omega^k,$ and $F^k$ represent the $k$th guess of the set of aggregate state variables.

1. Guess a set of aggregate variables $\{P^k, A^k, \Omega^k, F^k\}$.

2. Given the set of aggregate variables, solve for solve the households’ optimal decision rules $\{b^k_i, h^k_i\}_{i\in\{B,T\}}$. This step of the algorithm requires solving the households’ value functions $V^k_i$. To find $V^k_i$, I first approximate the household’s continuation value function with a set the interpolation grids for $(b, h)$. I then use a value function iteration to solve the household’s parametric dynamic programming problem as defined in Section 3.3. Note that, by imposing $\{P^k, A^k, \Omega^k, F^k\}$ in the household optimization problem, we can find $R^k$.

3. Using the above obtained optimal decisions, we simulate the economy using $M$ households and $Z$ periods. $M$ needs to be large enough and $Z$ needs to be long enough such that the simulated economy becomes stationary within $Z$ periods. In practice, $M$ is chosen to be 100,000 and $Z = 1,000$.

4. Calculate aggregate statistics in the ending period where the economy is stable. In particular, total demand for housing assets and total demand for housing rentals
are needed to check equilibrium conditions. Note that the good market condition are automatically satisfied by the derivation of $R^k$.

5. Check whether housing asset market clearing condition and housing rental market clearing condition are satisfied. If not, update the guess for the set of aggregate variables $\{P^k, A^k, \Omega^k, F^k\}$ and go back to step 1. Note that the updating rule can be straightforward here since we only have one price in the set of aggregate variables.

### D.2 Solving for the transition path equilibrium

This appendix describes the solution of the model along the perfect foresight transition path between two steady states described in Section 4.1.

Similarly as solving for the steady-state equilibrium, the computation is done by a fixed point iteration process. The only difference is that, other than iterating over a set of aggregate variables, now we need to iterate over a set of sequences of aggregate state variables. Moreover, there is a setup stage when we need to get the following objects ready.

- Guess a length for the transition period $T$.
- Value functions from the new steady state equilibrium.
- $\Lambda_0$, the distribution over $\beta_i, b, h, i$ from the initial steady state.

Let $\{P^k_t, A^k_t, \Omega^k_t, F^k_t\}_{t=1}^{t=T}$ represent the $k$th guess of the set of sequences of aggregate state variables.

1. Guess a set of aggregate variables $\{P^k_t, A^k_t, \Omega^k_t, F^k_t\}_{t=1}^{t=T}$.

2. Given the set of aggregate variables, solve for solve the households’ optimal decision rules at each time period $t \leq T$. Note that the decision rules can be solved backward, starting from the $T$, the ending period of the transition, taking the sequence of aggregate variables $\{P^k_t, A^k_t, \Omega^k_t, F^k_t\}_{t=1}^{t=T}$.

3. Using the above obtained optimal decisions, we simulate the initial distribution $\Lambda_0$ for each period along the transition path.

4. Calculate aggregate statistics in each period, based on the simulation results.

5. Check whether market clearing conditions in each period are satisfied. If not, update the guess for the set of aggregate variables $\{P^k_t, A^k_t, \Omega^k_t, F^k_t\}_{t=1}^{t=T}$ and go back to step 1.

Finally, we update the length of the transition period $T$ if necessary. In practice, depending on the level of the convergence precision, any $T \geq 50$ will result in the same computed equilibrium.