Inequality and House Prices

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Abstract

This paper studies the interaction between inequality and house prices using an incomplete market model with heterogeneous households. The model links cross-sectional household portfolio saving decisions to housing market outcomes. It illustrates a new price formation mechanism in which an investment motive among the wealthy plays a key role. A quantitative application of the theory rationalizes the recent substantial housing boom accompanied by rising saving rates in China. The theory in this paper shows that market frictions can have a differential impact cross-sectionally, increasing inequality. Inequality can in turn amplify frictions in the market.

JEL Codes: E21, G11, P22, R20.

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1 Introduction

Rising inequality in many countries over recent decades (exemplified by the top income share series documented in Atkinson et al., 2011) has drawn increasing attention from policy makers and researchers. At the same time, global house prices (in real terms) have also been on an upward trajectory (see Knoll et al., 2014). This paper argues that these two trends are not a coincidence.

Figure 1.1: Top income share growth and appreciation of house prices

![Graphs showing cross-country and cross-state income share growth vs. house price appreciation]

Notes: For each observation in each panel, the average growth rates are obtained by averaging the yearly growth rates over the entire sample periods. In Panel (a), cross-country (real) house price indexes are taken from Federal Reserve Bank of Dallas's International House Price Database as described in Mack and Martínez-García (2011); cross-country top 5% income shares are taken from Alvaredo et al. (The World Top Incomes Database) (income excludes capital gains). In Panel (b), US cross-state (nominal) house prices are from FHFA All-Transactions Indexes; US cross-state top 5% income shares are from Frank-Sommeiller-Price Series (income excludes capital gains) as described in Frank et al. (2015). Appendix A contains more details about data source, variable construction, robustness check, and other related empirical analysis.

As shown in Figure 1.1, growth in top 5 percent income share and house price appreciation are positively correlated both across countries and across states in the United States. Importantly, unlike house prices, rents do not co-move with top income shares.

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1 Inequality has many dimensions. This paper mostly looks at the dimensions over income and wealth. Moreover, because of the lack of administrative data on wealth, cross-sectionally comparable wealth data are limited. Therefore, inequality measures in most of the empirical analyses of this paper are along the income dimension. However, wealth inequality and income inequality are closely related, at least to certain degrees of aggregation (see Saez and Zucman, 2014).

2 The rise in house prices is the main contributor to the well-publicized “coming back” of aggregate capital-to-income ratios documented by Piketty (2014) and Piketty and Zucman (2014), as noted in Bonnet et al. (2014). On a similar note, Rognlie (2015) found that the increase in net capital share among developed economies during the last several decades also comes entirely from the housing sector.

3 There is no natural divide between high-income households and low-income households in an economy. However, the top 5 percent are often considered as high-income households in the literature that studies...
(see Appendix A for details). This suggests that investment motives of the wealthy might play an important role in understanding the dynamics of house prices. Given that wealthy households tend to own investment houses, housing market performance can in turn affect inequality. Thus, understanding the interacting forces between inequality and housing market outcomes can be important for studying cycles in house prices and inequality.

To study the interaction between inequality and house prices, I set up an incomplete-market equilibrium model in which households face uninsurable idiosyncratic earnings risk. Markets for housing consumption and housing ownership are separately specified. Households in the model consume goods and rental services, and can save in either liquid bonds or housing. The adjustment in housing asset is both large and infrequent, which I refer to as lumpy, due to market frictions. The lumpiness induces an “illiquidity premium” in the financial rate of return of housing over bonds.

In equilibrium, rents depend mostly on average resources, while house prices also respond to changes in inequality. All else equal, a rise in inequality tend to shrink the illiquidity premium and push up house prices. This result is due to “differential responsiveness” in housing demand across wealth groups, owing in turn to the differential impact of lumpy housing on household portfolio saving decision. In particular, housing demand from wealthy households is responsive to changes in individual and market conditions since housing adjustment costs matter less as households get wealthier. Housing demand from the poor, however, is rigid because their housing investment is constrained (or close to constrained) by the lumpiness. As a result, the aggregate demand and aggregate price of housing respond positively to a rise in inequality (a shift of resources from the poor to the wealthy).

Dynamically, the model implies an endogenous feedback loop between house prices and wealth inequality. This feedback loop causes house prices to respond strongly to changes in fundamentals. The mechanism is, again, based on the differential responsiveness in cross-sectional housing demand. Importantly, the above mechanism can rationalize persistent episodes where house price growth outstrips income growth in response to income growth shocks. Commentators and policy makers often view these episodes as a prelude to a market crash.

Consider a temporary aggregate income growth shock to the model. In a perfect foresight equilibrium, the house price-to-income ratio in the model will increase at first and then subsequently adjust back to normal. The anticipated swing of house price-to-income ratio reflects the cross-sectional heterogeneity in household consumption and saving behavior, see Kumhof et al. (2015) for example. This decision also fits the purpose of this study since the top 5 percent households are likely to be wealthy enough to buy houses for investment purposes. Analyzing the investment motive in housing is the focus of this paper.

There are several frictions in adjusting the housing asset, with fixed adjustment cost as the main one. Housing consumption, however, is flexible in the model: a homeowner is allowed to become a net renter or a landlord by trading housing services in a frictionless rental market. This modeling approach makes housing purchase a purely financial decision.

One such example is the common reaction to the recent Chinese housing boom. China’s fast house price appreciation has been frequently labeled as a bubble by commentators (as summarized in a Wikipedia article at this link: https://en.wikipedia.org/wiki/Chinese_property_bubble_(2005%E2%80%9311)).
income ratio is linked to optimal portfolio adjustments of the wealthy and the poor. The initial price run-up is driven by a portfolio shift among the wealthy, whose housing demand is the most responsive, from bonds to housing. High price growth sidelines households with modest resources; these households can increase housing share in their portfolios only after a delay. Wealth concentrates as the price run-up continues. Eventually the wealthy choose to consume their capital gains, and in doing so, they sell the accumulated housing stock back to the low resource first-time buyers, who are forced to save for extended periods. Wealth becomes more dispersed as the house price adjusts back to trend. In the end, there is no house price crash even after big booms – market frictions cause households taking turns to be active in the housing market: wealthy households are the initial investors and the poor are forced to save longer and delay their house purchases which prevents market crash from happening.

This above mechanism is particularly relevant for economies in which there are limited channels through which households can investment, such as China. Therefore, to quantitatively evaluate how much the proposed mechanism can account for the dynamics of house prices and wealth inequality, I apply the model to study the recent housing boom in China. I first parameterize the model to match important dimensions of aggregate and cross-sectional data of the Chinese economy in 2002. I then feed in changes in both aggregate income and income inequality from 2002 to 2012 taken from the data to the calibrated model and solve for the perfect foresight transition path equilibrium.

Not only can the model reproduce the observed housing price run-up, it can also match trajectories for wealth inequality and aggregate household saving rates. Since none of the data moments after 2002 is used in the calibration of the model, this adds validity to the proposed mechanism. Broadly speaking, the mechanism in this paper illustrates a new channel through which inequality and macroeconomic forces can interact. In particular, market frictions can have a differential impact cross-sectionally, increasing inequality. An increase in inequality can in turn amplify frictions in the market.

Related Literature The model in this paper builds on a growing strand of literature that studies housing with incomplete market models and heterogeneous households. See, for example, Kiyotaki et al. (2011), Sommer et al. (2013), Iacoviello and Pavan (2013), and Favilukis et al. (forthcoming). The model in this paper differs from the rest of the literature in the sense that it stresses the interaction between inequality and housing market outcomes. In the model, housing purchases are purely financial decisions, and the lumpiness in housing affects housing choices differently across wealth groups. Thus, housing market outcomes are closely linked to cross-sectional household portfolio saving.

Favilukis et al. (forthcoming) also point out the importance of a realistic wealth distribution in determining housing price dynamics. However, their mechanism is very different from that of this paper. In their paper, since housing assets cannot be a vehicle for investment (because housing assets are non-tradable), wealth heterogeneity only serves the purpose of generating a large fraction of housing demand from financially constrained households. This aspect of their mechanism makes equilibrium house price sensitive to changes in financing constraints and interest rates.
decisions. The model highlights the investment motives of wealthy households in driving house price dynamics.

In addition, the modeling approach of this paper also takes elements from the literature initiated by Grossman and Laroque (1990) that studies optimal consumption problems under fixed adjustment costs. Of particular interest here are those papers that analyze portfolio choices in the presence of housing, see, for example, Flavin and Yamashita (2002), Fischer and Stamos (2013), and Corradin et al. (2014). In general, this strand of literature adopts a representative agent framework and studies partial equilibrium implications of lumpy housing for consumption and investment decisions with exogenously specified processes for house prices. In contrast, this paper explores the differential impact of the fixed adjustment costs on portfolio choices across wealth groups and investigates its implications for endogenous house price formation.

There are several other papers that directly associate house prices with inequality, see, for example, Nakajima (2005), Nieuwerburgh and Weill (2010), Määttänen and Terviö (2014), and Landvoigt et al. (2015). Nakajima (2005) interprets rising cross-section earnings inequality as increasing individual earning uncertainty and studies the effects on portfolio allocation and housing prices. Nieuwerburgh and Weill (2010) adopt a spatial model with changes in cross-sectional productivity differences to explain the increase in house price dispersion across US metropolitan areas. Määttänen and Terviö (2014) and Landvoigt et al. (2015) both use assignment models to study the distributions of house prices within a metro area. By contrast, this paper uses a different modeling approach to investigate how aggregate house prices and inequality interact with each other when houses are lumpy investment goods.

A few papers study mechanisms whereby house prices may “overreact” to changes in fundamental factors. Ortalo-Magn and Rady (2006) show that house prices can overshoot when changes in income affect the number of credit-constrained owners moving up the property ladder, while in this paper the overreaction in house prices is caused by the investment motives of the unconstrained wealthy housing investors. Kahn (2008) finds that regime-switching productivity growth trends in output can generate house prices that are substantially more volatile than output. But the model in Kahn (2008) abstracts from heterogeneity and financial frictions, both of which are central for the mechanisms in this paper. Piazzesi and Schneider (2009) provide a search model to illustrate that a small number of optimistic traders can have a big impact on house prices. This paper shows that wealthy housing investors can drive the house price dynamics.

Finally, the quantitative exercise in this paper links the two strands of literature that study house prices and household saving rates in China. Unlike this paper, which tries to theoretically rationalize the fast appreciation of house prices, most existing papers in the emerging literature that studies China’s house prices focus on empirically explaining the high house price level in China. See Wei et al. (2012) and Wang and Zhang (2014) for example. Garriga et al. (2014) and Chen and Wen (2014) are two exceptions that study the growth rate of housing price theoretically. Garriga et al. (2014) use a spatial model to
explore the role of structural transformation and the resulting rural-urban migration in the house price dynamics in urban areas of China. However, their model forces housing demand to be determined only by migrants moving from rural areas to cities, which does not seem to be consistent with the excessively strong investment housing demand from existing homeowners found in the data (Chen and Wen, 2014). Chen and Wen (2014) propose a growing-bubble theory to explain the fast house price growth, but their basis is a general asset bubble framework with no particular relevance to the nature of housing assets (since housing is an intrinsically valueless asset in their baseline setup). In contrast, the theory provided by this paper stresses the crucial role played by the lumpy nature of housing as an investment vehicle.

China’s high and rising household saving rates have attracted a growing literature investigating the mechanisms behind them. Many factors have been proposed as the possible drivers of these rising saving rates, but no answer has been found conclusive (see, for example, Modigliani and Cao, 2004; Chamon and Prasad, 2010; Chamon et al., 2013; and Curtis et al., 2015, among many others). Among those factors, rising house prices is one of the usual suspects. Under a partial equilibrium setting (where house prices are exogenously given), Wang and Wen (2012) and Bussière et al. (2013) both show that the saving rates of certain household groups can increase with house prices, provided that certain conditions hold. However, to my best understanding there is no existing theory that can endogenously generate fast-growing house prices and rising saving rates at the same time, as this paper does.

The rest of the paper is organized as follows. Section 2 sets up a simplified version of the model that can be solved analytically to illustrate the key mechanisms. Section 3 presents the main model and its calibration. Section 4 employs the calibrated model to study the Chinese housing boom between 2002 and 2012, and Section 5 concludes. An Appendix that contains additional empirical analyses, proofs, and numerical algorithms follows.

2 A Basic Model

This section sets up a simplified version of the model. I introduce basic elements of the theory and derive analytical solutions. The results illustrate that house prices positively depend on exogenous inequality measures. I discuss mechanisms behind the results and the general intuition.

2.1 Environment

The endowment economy considered here has two types of earners: top and bottom; and two different assets: housing and bonds. Housing pays rent as a dividend but can only be

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7The distribution for non-capital income is exogenously specified in the model, while capital income and wealth distributions are endogenously determined.
held at sizes bigger than or equal to a minimum threshold. Both house prices and rents
are determined in equilibrium.

**Endowments and generational structure** In this economy, time is discrete and
extends from \( t = 0, \ldots, \infty \). There are no aggregate or household-specific uncertainty.

Aggregate output \( Y \) is constant every period. Two types of young households are born
at the beginning of each period, referred to, respectively, as top earners (with constant
population share \( \mu \) in each generation) and bottom earners \((1 - \mu)\). Households live 2
periods and are productive only when they are young. The share of output received by
top earners is \( \pi \). Each generation has a constant population mass of one half. Note
that the relative endowment between the top earner and bottom earner is \( \frac{\pi(1-\mu)}{\mu(1-\pi)} \), which
increases with both top income share \( \pi \) and bottom population share \( 1 - \mu \). Note that
\( \pi \) and \( 1 - \mu \) are the two parameters that govern the exogenous endowment inequality of
this economy.

There is also a fixed amount of housing stock \( H \) in the economy, which produces flow
services each period. Newborn households are not endowed with any housing assets (i.e.,
the older generation owns all the housing stock).

**Household preferences** Households have the same individual preferences. In partic-
ular, they all receive utility from consuming output \( c \) and housing services \( s \). I assume
the following separable flow utility function:

\[
U(c, s) = (1 - \varphi) \ln c + \varphi \ln s,
\]

where \( \varphi \in (0,1) \) reflects the utility weight of housing services in total consumption.\(^6\) The
discount factor is constant at \( \beta \) across all the households.

**Markets and timing** Households can trade the ownership of housing assets. Other
than housing, households can also invest in a one-period fully enforceable bond \( b \), which
pays a fixed gross interest rate of \( 1 + r \). I assume a small open economy so that bonds
are supplied by an outside intermediary with infinite supply elasticity. Households enter
a period with last period’s bond holdings \( b \) and housing assets \( h \) (newborn households
have no assets). At the beginning of the period, a household receives its income, i.e., the
output share for the young, and the returns from financial assets for the old (the return
from housing assets is the rental income earned).

Asset markets open first in order to allow the older generation to sell off assets for
consumption. Young households can buy or borrow bonds from the outside intermediary
at the fixed price \( 1/(1+r) \). Since households become unproductive when old, to guarantee
debt payment (default is not allowed in the model), there is a collateralized constraint

\(^6\)The log-specification over consumption and housing services provides tractability. A more general utility
function will be considered in the quantitative model in section \([5]\)
for borrowing. In particular, households cannot borrow more than a fraction \( \lambda_h \) of their housing assets. Households can also trade housing assets, but there is a minimum holding size requirement in the housing asset market. To be specific, a household can only hold housing stock at a quantity larger than or equal to \( h \) with unit price \( P \).

After asset markets close, households participate in the goods market and housing rental market. Unlike housing asset markets, the housing rental market is frictionless in that households can buy any amount of services \( s \) at the market unit price \( R \). The model’s frictionless rental market may be a bit extreme, but it captures the idea that a person can rent a very small shared living space that is not, however, available for sale.

The nature of housing The market arrangement in this economy has interesting implications about the nature of the housing asset. Housing in this economy is essentially a financial asset that delivers a dividend at the rate of \( \frac{R}{P} \), due to our assumption of two separate markets for housing services and housing stock. Considering a household that owns \( h \) units of housing stock, the housing consumption of this household is not necessarily \( h \), since household members are allowed to either rent out part of their housing stock to become landlords \((s < h)\) or to purchase more rental services and be net renters \((s > h)\). This means that although housing services and housing stock are necessarily the same in aggregate \((S = H)\), individual housing ownership and consumption can be different \((s \neq h)\) without any friction. This structure separates individual decisions about housing consumption and housing investment. It stresses the financial aspects of housing, allowing housing purchase to be an investment decision only.

Household’s problem Old households only make consumption choices. The assumption on preferences leads to constant optimal budget shares of housing and non-housing consumption. Define total expenditure \( x = c + Rs \), the household’s one-period univariate indirect flow utility function is the following:

\[
    u(x) = \ln x + (1 - \varphi) \ln(1 - \varphi) + \varphi \ln \varphi - \varphi \ln R.
\]

This minimum size friction here serves the purpose of generating lumpiness in the housing market in a tractable way. Although the minimum size friction is frequently adopted in the quantitative housing literature (see, for example, Iacoviello and Pavan, 2013), it is not essential for the mechanisms in this paper as long as there are other frictions in place that make housing lumpy. See section 2.3 for more discussion.

The specification of a (quality-adjusted) per-unit price of housing is natural here since the total stock is fixed and the individual housing choice is continuous when it is above the minimum threshold size \( h \). The general focus of this paper is the aggregate housing price, not the cross-sectional price dispersion.

In the quantitative housing literature, when the rental decision is explicitly modeled, it is common to assume that rental units come in smaller sizes than houses, see Iacoviello and Pavan (2013) and Sommer et al. (2013) for example.

Although in reality becoming a landlord bears with it various costs, such as management costs and differential tax treatments, this paper abstracts from those costs, in order to keep the exposition clean.

Sommer et al. (2013) allow households to invest in rental properties. However, housing investment decisions in their model can not be easily separated from housing consumption decisions, since becoming a landlord is assumed to shift one’s utility function.
The young household’s decision is thus reduced to a standard consumption-saving problem, with a portfolio choice between bonds and housing:

$$\max_{\{x,h,b\}} \ln x + \beta \ln x'$$

s.t. $x + b + Ph = y$

$$x' = (1 + r)b + (P + R)h$$

$$b \geq -\lambda h Ph$$

$$h \in \{0, [h, +\infty]\}.$$  \hfill (2.1)

**Equilibrium** A stationary equilibrium is a set of prices $R$ and $P$, and a collection of decision rules $b(y, R, P)$, $h(y, R, P)$, and $x(y, R, P)$, such that decision rules are optimal and markets clear.\[14\]

Note that, in the model, the distribution of $y$ is exogenously given, and both rent $R$ and house price $P$ are endogenously determined by market clearing conditions. Although there are separate market clearing conditions for housing rental and ownership, $R$ and $P$ are jointly determined in the equilibrium since housing demand is a function of its financial rate of return $\frac{R}{P}$. Moreover, since housing assets are risk-free in a stationary environment, due to the minimum holding size friction, the equilibrium return from housing has to be at least as high as the return from the risk-free bond (i.e., $\frac{R}{P} \geq r$).

### 2.2 The analytical result

Although the setup of the model is simple, multiple equilibria still arise when there is no restriction on parameter values. In this section, I restrict attention to one particular type of equilibrium in which all households own some housing assets in equilibrium. This is more relevant for realistic settings. Appendix B contains the full characterization of the model’s equilibria.

**Theorem 1.** When the parameters of the model satisfy the condition that

$$\frac{\beta \pi h}{(1 + \beta)(2H - (1 - \mu)h)} \leq \frac{1 - \pi}{1 - \mu} \leq \frac{\pi h}{2H - (1 - \mu)h'},$$

the equilibrium housing price

$$P = \frac{\pi}{-(1 - \mu)h + 2H} \times \frac{\beta Y}{(1 + \beta)(1 - \lambda h)}$$

increases with both inequality measures $\pi$ and $1 - \mu$. Moreover, the equilibrium return in housing is higher than bonds (i.e., $\frac{R}{P} > r$), and it decreases with both inequality measures.

\[14\] I briefly describe the equilibrium conditions here to save space. The equilibrium definition will be stated more precisely in Appendix B and also in the quantitative model in Section 3.3.
I leave the proof of Theorem 1 to Appendix B and rather discuss the intuition behind in the main text. I first explain why housing can have a higher return than bonds in equilibrium. In a stationary equilibrium, the return for housing is simply the rent-over-price ratio. Since households spend a constant share on rental consumption, rental price is roughly proportional to aggregate output. House prices depend on aggregate saving, which is also roughly a constant share of aggregate output due to the log-specification for the utility function. Thus, when the utility weight on housing is relatively high and household’s patience level is relatively low, \( \frac{R}{P} \) will be higher than \( r \).

Figure 2.1: Policy functions for housing and bonds

Notes: This figure illustrates the equilibrium policy functions for both housing asset and bonds. The policy functions correspond to one set of parameter values that satisfies the conditions for equilibria in which \( \frac{R}{P} > r \). Moreover, for the ease of exposition, borrowing is not allowed (\( \lambda_h = 0 \)).

Now assuming \( \frac{R}{P} > r \) is the equilibrium outcome, Figure 2.1 plots optimal rules for saving in housing and bonds as functions of lifetime resources \( (y) \). If there is either no friction in the housing market (\( h = 0 \)) or no limit in borrowing (\( \lambda_h = 1 \)), denoted as the no-friction benchmark, households will simply save a constant share of total resources. And all of the savings will go to housing (the dotted line). However, with housing market frictions and borrowing limits, certain households’ saving decisions are distorted. In particular, households with low levels of resources cannot enter the housing market and save only in bonds (shown as “outsiders” in Figure 2.1); although households with modest resources participate in the housing market, they are constrained by the frictions and have to save more relative to the no-friction benchmark (as “constrained participants”). Only those households with high enough resources are not affected by the frictions (as “unconstrained” participants). As a result, households with low and modest resources are
in large zones of inaction: their housing demand is “unresponsive” to changes in income\(^{15}\). In contrast, affluent households have “responsive” housing demand.

Theorem 1 corresponds to a situation where the underlying parameter values of the model imply modest income inequality and market frictions. The equilibrium outcome is that the bottom earners participate in the housing market but are constrained while top earners are unconstrained\(^{15}\). So it is easy to understand why an increase in inequality will cause house prices to go up: an increase in either \(\pi\) or \(1 - \mu\) generates more housing demand from the top earners (who are “responsive”) but does not suppress demand from the bottom earners (who are “unresponsive”).

2.3 Discussion

The model presented in this section is relatively simple; I made strong assumptions for tractability reasons. However, this simple model does allow two distinct relative prices – rents and house prices – to be determined in equilibrium via market clearing. Moreover, housing market outcomes are linked with cross-sectional saving decisions and thus interact with inequality. Results from the model are generalizable to more realistic settings.

First, housing is a preferred asset. This is for purely financial reasons in the model: because of lumpiness, it provides a higher rate of return than bonds. In reality, there are other reasons for a household to favor housing. For example, housing ownership can signal social status, as Wei et al. (2012) have emphasized, and thereby strengthen one’s competitiveness in the marriage market. Higher returns on housing are especially sustainable in an economy where households place a high value on housing but are not wealthy enough to make sufficient housing investment.

Second, housing purchases are lumpy. In the simple model, the lumpiness is built in by the minimum purchase size friction. In reality, there are many other reasons for housing purchases to be lumpy (although the minimum size friction is still relevant). Non-convex adjustment cost, considered later in the quantitative model, is one obvious reason\(^{17}\).

When housing is both desirable and lumpy, as shown in the model, the responsiveness in housing demand will differ across wealth groups. This is especially the case if credit conditions are tight. The differential responsiveness in turn causes inequality and housing market outcomes to interact. Due to the concentrated wealth distribution, wealthy households can comfortably invest in multiple properties while less wealthy households have to borrow for just one residence since a house can cost multiple years of income for most households.

\(^{15}\)Note that housing demand is very responsive (it jumps) at one critical point where switching occurs between “outsiders” and “constrained participants”.

\(^{16}\)There are only two income levels among all households in the basic model. When allowing for more household types, the constrained participants would be households in the middle income range.

\(^{17}\)See Iacoviello and Pavan (2007) for a discussion of various realistic frictions that make housing investment lumpy within the context of the US. The lumpiness is even more relevant for economies with underdeveloped financial markets. Section 3.4 contains more details about the formulation, calibration, and evaluation of the frictions that make housing lumpy in the quantitative model.
3 A Quantitative Model

The model in Section 2 serves the purpose of setting up the basic framework and illustrating some qualitative implications of the theory. This section extends the basic model to make it suitable for quantitative study of the recent housing market boom in China. To illustrate why China is a good test ground for the theory, Section 3.1 presents some institutional background and stylized facts about the Chinese economy and its housing market. Some of the facts will inform the specification and calibration of the model in Section 3.2 and Section 3.4 and others will be used to evaluate the model and discipline the transitional exercise in Section 4.

3.1 Background

In order to understand economic phenomena in China, special institutional ingredients of the Chinese economy need to be taken into account. Several recent studies, such as Song et al. (2011) and Chang et al. (2015), make this point clear. This section thus briefly summarizes the relevant background about housing market and household finance in urban China. More details can be found in Appendix C.

3.1.1 China’s economic growth, inequality, and housing market after 2002

Panel (a) in Figure 3.1 show four series of income inequality measures in urban China: income and wage shares of the top 5 and top 10 percent. Panel (b) in Figure 3.1 presents the residential land prices series in urban China, together with a series of per capita income. Note that I only present here a land price index because reliable house prices data is publicly unavailable for China. To simplify the discussion, I will refer to this land price index as an approximation of house prices in the rest of the paper. All the income inequality measures grew significantly during the considered time periods. In

18There is a big urban-rural divide in China. This paper does not consider rural China.
19Note that, unlike the income-tax based cross-sectional income data used in Figure 1.1, administrative income statistics for China are not available. Existing data on income distribution in China are largely based on household surveys (see Piketty and Qian, 2009). Income inequality measures based on household surveys are often less comparable to those based on administrative sources. Moreover, the income concentration in China at 2002 (calculated by author) is much lower than most countries in Panel (a) in Figure 1.1 (the top 5 percent income share is 14% in China at 2002, while it is 30% at the same time in the United States); thus, I show four different income inequality series for China. Section 3.5 discusses the wealth inequality comparison between China and the United States.
20According to China’s official house price indices, the average housing price (urban) appreciation is only mild compared to aggregate income growth. However, these indices are mistrusted and widely criticized due to the lack of quality adjustments in their construction. Constant quality price series developed by academic researchers mostly appreciate at a faster pace than aggregate income, although with regional variations. See Wu et al. (2014), Deng et al. (2014), Wu et al. (2015) and Fang et al. (2015) for more details. Still, a well trusted aggregate house price index is not publicly available for China. The residential land price index presented here is relatively more developed and taken from Deng et al. (2014).
21In general, land prices can be very different from house prices. However, house prices and land prices go hand in hand in China during the time periods considered in this paper. According to Fang et al. (2015) and Wu et al. (2015), house prices growth roughly matches with the growth in land prices in China over the past decade. Moreover, Wu et al. (2015) show that house prices growth in China is driven by rising land values, not by construction costs.
particular, the income share of the top 5 percent grew from 14% to 26% between 2002 and 2012: in other words, it has almost doubled. During the same time, house prices grew significantly faster than the already fast income growth. It is clear that the recent Chinese experience fits well with the pattern shown in Panel (a) in Figure 1.1, only it is much more dramatic.

Figure 3.1: China in transition

Notes: All the income inequality measures in Panel (a) are calculated by author from Chinese Household Income Project (CHIP) 2002, 2007 and China Family Panel Studies (CFPS) 2010, 2012. Both CHIP and CFPS are large-scale national household surveys, which are arguably the best available data sources on household income and financial assets in China. More statistics from the two data sets will be utilized in later parts of this paper. Appendix C has more descriptions of the two data sets. In Panel (b), the residential land price indexes are Wharton/NUS/Tsinghua Chinese Residential Land Price Indexes taken from Deng et al. (2014). Note that I only present here a land price index because reliable house prices data is publicly unavailable for China. See footnote 20 for more details. Real per capita income is the Average Wage of Employed Persons in Urban Units taken from National Bureau of Statistics (NBS) of China (deflated by the Urban Household Consumer Price Index).

This paper focuses on periods after the year 2002, mainly because it is not until 2002 that housing markets in China finished the transition from a state welfare provision system to private market provision in large part. In China, housing became a commodity that individuals can purchase only after the early 1990s (before that local governments or work units allocated housing). It took about one decade to complete the privatization process. In 2002, 78% of all households owned their homes with partial or full property rights (Walder and He, 2014). Moreover, more than 50% of the new housing is provided by private developers in 2002 (Wu et al., 2014). A more detailed discussion of the housing market privatization process in China can be found in Appendix C.1.

The early 2000s is also the time when a mass retrenchment program in China’s state sector arrived at completion. The retrenchment program laid off roughly one fourth of
workers from the state-owned sector in China. Starting from the early 2000s, China saw deepening privatization (especially the restructuring of firm ownership) and increasing openness (China joined the World Trade Organization in 2001). As a result, economic growth accelerated, as did to an even greater extent income inequality.

3.1.2 Household finance in China

It is well documented that Chinese households save a lot. Moreover, the average urban household saving rate in China rose significantly during most of the first decade of the 21st century (see Chamon and Prasad, 2010; Yang et al., 2013; and Curtis et al., 2015). Where does the saving in China’s household sector go?

![Figure 3.2: Household asset position and interest rate](image)

(a) Asset composition

<table>
<thead>
<tr>
<th>Component</th>
<th>China</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing net</td>
<td>79.4%</td>
<td>34.5%</td>
</tr>
<tr>
<td>Housing gross</td>
<td>81.8%</td>
<td>44.6%</td>
</tr>
<tr>
<td>Housing debt</td>
<td>-2.4%</td>
<td>-10.1%</td>
</tr>
<tr>
<td>Fixed deposits</td>
<td>5.3%</td>
<td>0.9%</td>
</tr>
<tr>
<td>Other financial</td>
<td>5.5%</td>
<td>40.1%</td>
</tr>
<tr>
<td>Other</td>
<td>9.7%</td>
<td>24.6%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

(b) Interest rate

Panel (a) in Figure 3.2 compares the components of total assets in 2012 between China and the United States. Households in China have an asset portfolio dominated by housing and fixed bank deposits. The share of housing (net of mortgages) in total assets in China is more than two times as big as that in the United States. Moreover, the share of financial assets in China is only about one fourth of the US share. Half of the financial assets in Chinese household portfolios are in terms of fixed bank deposits, which achieve low returns (sometimes even negative in real terms, as shown in Panel (b) in Figure 3.2). In contrast, the share of fixed deposits is negligible for US households.

Chinese households have good reason to save in housing and bank deposits: in fact, they have no good alternatives. First of all, the Chinese government implements strict

Note that this paper focuses only on the macroeconomic implications of changes in economic growth and wage inequality; it does not take a stand on the fundamental reasons for those changes, although evidence suggests that the causes are structural. See Chang-Tai and Song (2015) for an analysis of how the transformation of China’s state sector affects productivity; see Han et al. (2012) for an investigation of the impact of globalization on wage inequality; see Appleton et al. (2014) for a discussion of increased wage inequality due to changes in the wage structure.
capital controls so that saving in capital markets outside of China is off the table for most households. Second, domestic stock markets were underdeveloped and offered meager returns in the last two decades. As shown in Table 3.1, at least during the 2003–2013 period, stock market index returns are dominated by the housing price index according to a simple mean and variance comparison.

Table 3.1: Returns of Stock Market Index vs. Housing Price Index (2003–2013)

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean Stock</th>
<th>Mean Housing</th>
<th>Std. Dev. Stock</th>
<th>Std. Dev. Housing</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003-2013</td>
<td>0.073</td>
<td>0.157</td>
<td>0.515</td>
<td>0.154</td>
</tr>
<tr>
<td>2003-2008</td>
<td>0.090</td>
<td>0.204</td>
<td>0.662</td>
<td>0.105</td>
</tr>
<tr>
<td>2009-2013</td>
<td>0.053</td>
<td>0.109</td>
<td>0.339</td>
<td>0.191</td>
</tr>
</tbody>
</table>

Notes: The numbers in this table are directly extracted from Table 3 and Table 4 in Fang et al. (2015). The housing price index is for first-tier cities only. See Fang et al. (2015) for more details and discussion.

To sum up, the basic setup of the model (i.e., households choosing portfolios of housing and fixed return bonds) is well suited for a case study of China.

3.2 Model environment

Compared to the basic model in Section 2, there are two main extensions in the quantitative model: households become infinite-lived and they face uninsurable idiosyncratic earnings risks. As a result, the setup is similar to standard incomplete-market settings such as in Huggett (1993) and Aiyagari (1994). In discussing the model environment, I focus on the extensions and give only brief consideration to the parts that are similar to those of the basic model. Moreover, I try to keep variable definitions and notations consistent with those in Section 2 whenever possible.

3.2.1 Household preferences and endowments

The basic setup is still an endowment economy, with no aggregate uncertainty and populated by two types of earners differing in income processes. Superscript $i \in \{B, T\}$ will be used to denote bottom and top earners, respectively. In addition to the basic model, there are heterogeneities within each type of earner, both in terms of patience level and endowment condition.

Each type $i$ earner is ex-ante identical and infinitely lived, with lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta_t^i u(c_t, s_t),$$

where $\beta_t^i$ describes the cumulative discount factor between period 0 and period $t$, and it differs between bottom and top earners. In particular, $\beta_t^B = \tilde{\beta} B \beta_{t-1}^B$, where $\tilde{\beta}^B$ is
an idiosyncratic shock following a three-state, first-order Markov process.\footnote{The preference heterogeneity is introduced in a similar manner as that of Krusell and Smith (1998) and for similar reasons: to generate a more realistic wealth distribution.} But $\beta_t = \beta T \beta_{t-1}$, where $\beta T$ is a standard constant one period discount factor. Unlike in the basic model, the instantaneous utility function $u(c_t, s_t)$ is modeled as a non-separable form, following Kiyotaki et al. (2011) and others:

$$u(c_t, s_t) = \left( \frac{(c_{1-t}^{1-\varphi})^{\varphi} (s_{1-t}^{\varphi})^{\varphi}}{1 - \frac{1}{\sigma}} - 1 \right),$$

where $\sigma$ is the risk aversion parameter and $\varphi \in (0, 1)$ again reflects the utility weight of housing services in the consumption aggregate of $c$ and $s$. Note that although the Cobb-Douglas preference has implications for the degree of substitutability between housing and non-housing consumption, it still produces the feature that the housing expenditure share is constant.\footnote{This feature is consistent with the empirical finding that, over time and across cities, the expenditure share on housing is constant (see Iacoviello and Pavan, 2013).}

Households in the economy also face idiosyncratic endowment shocks denoted as $\varepsilon$ (the same for bottom and top earners), which are the same as unemployment shocks. In particular, $\varepsilon$ follows a first-order Markov process with two states $\{0, 1\}$; $\varepsilon = 1$ means that the corresponding household is employed, receives a normal endowment, according to her type ($y^B$ for a bottom earner and $y^T$ for a top earner). If $\varepsilon = 0$, the household is unemployed, receiving a safety-net endowment that equals a fraction $\omega$ of the endowment at normal times. Note that the sum of the bottom earners’ income always equals $(1 - \mu)Y$ and the sum of the top earners’ income always equals $\mu Y$.\footnote{Note that the endowment process is kept simple in the model, which does not take certain cross-section and life-cycle aspects of the income distribution into account. There are two main reasons for adopting this modeling approach. The first simply has to do with data availability. A panel data set with a close tracking of individual income would be needed to identify the persistence parameter in the income process. The second reason is that a more elaborate endowment process will affect not only inequality in earnings in the cross-section but also the individual earnings uncertainty. However, for the transition exercise in Section 4 I want to isolate the effect of changes in cross-section inequality from changes in individual earnings uncertainty. The current modeling approach for the endowment process better fits that purpose.}

### 3.2.2 Asset market

The basic structure of the asset market is similar to that in Section 2.1, with the following two extensions about borrowing constraints and housing adjustment costs:

**Borrowing constraints** In addition to the loan to value ratio criterion in the basic model, there is an additional loan to income criterion. In particular, not only are households unable to borrow more than a fraction $\lambda_h$ of their housing stock, they cannot borrow more than a fraction $\lambda_y$ of their expected earnings either; i.e.,

$$b' \geq \max\{-\lambda_h Ph', -\lambda_y \bar{Y}(\varepsilon, \bar{y}^i)\},$$

\footnote{Note that the endowment process is kept simple in the model, which does not take certain cross-section and life-cycle aspects of the income distribution into account. There are two main reasons for adopting this modeling approach. The first simply has to do with data availability. A panel data set with a close tracking of individual income would be needed to identify the persistence parameter in the income process. The second reason is that a more elaborate endowment process will affect not only inequality in earnings in the cross-section but also the individual earnings uncertainty. However, for the transition exercise in Section 4 I want to isolate the effect of changes in cross-section inequality from changes in individual earnings uncertainty. The current modeling approach for the endowment process better fits that purpose.}
where \( Y(\varepsilon, \bar{y}^i) = E[\sum_{k=0}^{N} \frac{y_k}{(1+r)^k} | \varepsilon, \bar{y}^i] \) approximates the expected present discounted value of lifetime endowment. The endowment at period \( k \) is denoted as \( y_k \), and \( N \) is the approximate length of a life-cycle. This constraint is consistent with the usual lending criteria in the mortgage market that take into account minimum down payments, ratios of debt payments to income, and current and expected future employment conditions.

**Housing adjustment costs** Unlike in the basic model, a homeowner incurs a cost \( \Phi(h', h) \) whenever she adjusts her housing stock:

\[
\Phi(h', h) = f_1 P|h' - h| + f_2 \mathbb{1}_{\{h' \neq h\}} Ph.
\]  

(3.1)

Note that there are both linear and fixed components in the adjustment cost function. The linear component \( f_1 P|h' - h| \) captures common practices in the housing market that require, for instance, commissions paid to realtors to be equal to a fraction of the value of the house being sold. The fixed component \( f_2 \mathbb{1}_{\{h' \neq h\}} Ph \), which is in terms of a percentage value of the minimum size house, captures other costs associated with housing transactions, such as registration fees and search costs.26

### 3.3 The household decision problem and competitive equilibrium

Households in this model accumulate wealth to insure against idiosyncratic endowment shocks. When choosing vehicles to save in, they face a portfolio choice between liquid wealth (bonds) and illiquid wealth (housing). In particular, the state variables relevant to a type \( i \) earner’s decision making include the individual state vector \((\varepsilon, \tilde{\beta}^i, b, h)\) and the aggregate state variable \( \Lambda \), which denotes the measure of households over \((\varepsilon, \tilde{\beta}^i, b, h, i)\).

Denote \( V^i(\varepsilon, \tilde{\beta}^i, b, h; \Lambda) \) as the value function for a type \( i \) earner, and the dynamic programming problem is the following:

\[
V^i(\varepsilon, \tilde{\beta}^i, b, h; \Lambda) = \max_{x, b', h'} u(x; R) + \tilde{\beta}^i E[V^i(\varepsilon', \tilde{\beta}'^i, b', h'; \Lambda') | \varepsilon, \tilde{\beta}^i] \\
\text{s.t.} \\
x + b' + Ph' + \Phi(h', h) = \varepsilon \bar{y}^i + (1 - \varepsilon) \omega \bar{y}^i + (1 + r)b + Rh + (1 - \delta)Ph \\
b' \geq \max\{-\lambda b Ph', -\lambda_y Y(\varepsilon, \bar{y}^i)\} \\
h' \in \{0, [h, +\infty)\},
\]

for \( i \in \{B, T\} \). Similarly as in the basic model, \( x \) denotes total expenditure and \( u(x; R) \) denotes the indirect flow utility function.  

26Note that the housing adjustment cost is really modeled as a transaction cost in the model. By paying the cost, housing stock can be adjusted to any future level \( h' \in \{0, [h, +\infty)\} \). The assumption that the fixed cost component applies to adjustments that are minor relative to the existing housing stock essentially prevents homeowners from making small improvements to their houses.
Recursive Competitive Equilibrium  A recursive competitive equilibrium is then defined as: individual value and policy functions \(\{V_i, b_i, h_i\}_{i\in\{B,T\}}\), pricing functions \(\{P, R\}\), such that,

1. \(\{V_i, b_i, h_i\}_{i\in\{B,T\}}\) solve household’s problem.

   
   (i) Housing market: \(\int_A \sum_i h_i(\varepsilon, \tilde{\beta}, b, h; \Lambda) = H\)
   
   (ii) Rental market: \(\int_A \sum_i s_i(\varepsilon, \tilde{\beta}, b, h; \Lambda) = H\)
   
   (iii) Goods market: \(\int_A \sum_i \Phi(h_i, h_{t-1}) + \delta PH + C = Y + rA\)

3. \(\Lambda\) is generated by \(\{b_i, h_i\}\) and exogenous processes \(\varepsilon\) and \(\tilde{\beta}'\).

Here \(C\) denotes the aggregate non-housing consumption and \(A\) denotes the aggregate bonds savings. The formula \(\int_A \sum_i \Phi(h_i, h_{t-1})\) summarizes the total transaction costs incurred by homeowners for adjusting the housing stock, which depends on both the total transaction volume and the transaction frequency (see Equation [3.1]). Therefore, according to the goods market clearing condition (2iii), prices \(P\) and \(R\) depend on both the transaction volume and the transaction frequency of housing stock. This observation has implications for the computation of the model. I leave the details on the computational strategy to Appendix D.1.

3.4 Calibration

The frequency of the model is yearly. The model is parameterized to match important dimensions of aggregate and distributional statistics of the Chinese economy in 2002. Table 3.2 summarizes the parameters used in the baseline model. Detailed descriptions of the calibration procedure follow in the main text.

Endowments  Both output and the housing stock are normalized to be 1. I set \(\pi\) to 26%, according to the top 10% wage income share in CHIP 2002. Wage income is the data counterpart for the exogenous endowment in the model. The reason to choose the top 10% as the top earners is because the income share of the top 5% in China is too low compared to the other countries in Panel (a) in Figure 1.1 (the calculated top 5 percent income share in China at 2002, while it is 30% at the same time in the United States). The normal endowment levels for bottom earners and top earners, \(\bar{y}_B\) and \(\bar{y}_T\) respectively, are then set to meet the conditions that \((1 - u)\bar{y}_B + u\bar{y}_B = (1 - \mu)Y\) and \((1 - u)\bar{y}_T + u\bar{y}_T = \mu Y\), where \(u\) denotes the aggregate unemployment rate and \(\mu = 0.1\).

The employment process \(\Pi_{x,x'}\) is chosen so that the average duration of an unemployment spell is two years and the unemployment rate \(u\) is 10%, which is roughly in line with the findings about China’s unemployment rate in Giles et al. (2005). The transition matrix of the employment status is as follows (rows indicate the current state, and columns
Table 3.2: Summary of Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conventional values</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\sigma$</td>
<td>1.00</td>
<td>Standard value</td>
</tr>
<tr>
<td>Total endowment</td>
<td>$Y$</td>
<td>1.00</td>
<td>Normalization</td>
</tr>
<tr>
<td>Total housing stock</td>
<td>$H$</td>
<td>1.00</td>
<td>Normalization</td>
</tr>
<tr>
<td><strong>Estimated from the data</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk free interest rate</td>
<td>$r$</td>
<td>0.02</td>
<td>People’s Bank of China</td>
</tr>
<tr>
<td>Top 10% wage share</td>
<td>$\pi$</td>
<td>0.26</td>
<td>Top 10% wage share(^a)</td>
</tr>
<tr>
<td>Transition matrix of employment process $\Pi_{\varepsilon,\varepsilon'}$</td>
<td></td>
<td></td>
<td>Giles et al. (2005)</td>
</tr>
<tr>
<td>Mean endowment for bottom earner $y^B$</td>
<td></td>
<td>0.90</td>
<td>Derived from $\Pi_{\varepsilon,\varepsilon'}$</td>
</tr>
<tr>
<td>Mean endowment for top earner $y^T$</td>
<td></td>
<td>2.86</td>
<td>Derived from $\Pi_{\varepsilon,\varepsilon'}$</td>
</tr>
<tr>
<td>Unemployment replacement rate $\omega$</td>
<td></td>
<td>0.07</td>
<td>China Labor Statistical Yearbook</td>
</tr>
<tr>
<td>Minimum size housing</td>
<td>$h$</td>
<td>0.57</td>
<td>First quartile value(^b)</td>
</tr>
<tr>
<td>Borrowing constraint against housing $\lambda_h$</td>
<td></td>
<td>0.60</td>
<td>Down payment requirement</td>
</tr>
<tr>
<td>Linear transaction cost</td>
<td>$f_1$</td>
<td>0.03</td>
<td>Association of Realtors</td>
</tr>
<tr>
<td><strong>Calibrated in the model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.03</td>
<td>Wu et al. (2012)</td>
</tr>
<tr>
<td>Utility share of housing service $\varphi$</td>
<td></td>
<td>0.24</td>
<td>Bond share in total: 0.23(^a)</td>
</tr>
<tr>
<td>Discount factor for top earners $\beta^T$</td>
<td></td>
<td>0.97</td>
<td>Top 10% asset-over-income: 10.5(^a)</td>
</tr>
<tr>
<td>Medium discount factor for bot earners $\beta^B_m$</td>
<td></td>
<td>0.96</td>
<td>Aggregate asset-over-income: 5.4(^a)</td>
</tr>
<tr>
<td>Discount factor variation for bot earners $\Delta^B$</td>
<td></td>
<td>0.01</td>
<td>Wealth gini: 0.48(^a)</td>
</tr>
<tr>
<td>Transition matrix of discount factor $\Pi_{\beta,\beta'}$</td>
<td></td>
<td></td>
<td>See main text</td>
</tr>
<tr>
<td>Borrowing constraint against income $\lambda_y$</td>
<td></td>
<td>0.10</td>
<td>Aggregate loan-to-value: 0.05(^a)</td>
</tr>
<tr>
<td>Fixed transaction cost</td>
<td>$f_2$</td>
<td>0.12</td>
<td>Housing-transaction-over-GDP: 0.06</td>
</tr>
</tbody>
</table>

\(^a\): Calculated using CHIP 2002. See Appendix C for detailed procedure.

\(^b\): The discount factor grids are symmetric; i.e., $\Delta^B = \beta^B - \beta_m = \beta^B - \beta_1$.

indicate next period’s state; both the first row and the first column correspond to $\varepsilon = 1$ – i.e., employment):

$$
\begin{bmatrix}
17/18 & 1/18 \\ 1/2 & 1/2
\end{bmatrix}
$$

The unemployment insurance replacement rate $\omega$ is set to be 7%. This level of unemployment insurance is in line with the data. According to the China Labor Statistical Yearbook 1999 to 2005, the overall replacement rate from unemployment insurance is about 14% of the worker’s wage, and the maximum duration of benefits for unemployment insurance recipients is 2 years (the same as the unemployment spell in the model). However, the China Labor Statistical Yearbook also reports that about half of all unemployed workers were eligible for unemployment benefits from the unemployment insurance system from 1999 to 2005. Thus, I set $\omega$ to be 7% (half of the 14%), the same for both the top and the bottom earners.

**Preferences** The baseline risk aversion parameter is set at $\gamma = 1$. The utility share of housing service $\varphi$ is set at 0.24 to match the housing wealth and bonds wealth split
among all households in CHIP 2002. I calibrate $\tilde{\beta}_T$ to be 0.9657 according to the net asset-over-income ratio (10.5) of the population’s top 10% wealthiest. The shock to the patience level of bottom earners, $\tilde{\beta}_B$, takes on values from a symmetric grid, $(\tilde{\beta}_h = 0.9427, \tilde{\beta}_m = 0.9527, \tilde{\beta}_l = 0.9627)$, with 80% of the bottom earners adopting the middle value and 10% each adopting the extreme points in the invariant distribution. The expected duration of the extreme discount factors is 50 years. As in Krusell and Smith (1998), this is meant to capture, albeit in a somewhat crude way, a dynastic element in the evolution of preferences. Transitions can only occur to adjacent values, where the transition probability from either extreme value to the middle grid is 1/50 and from the middle grid to either extreme value is 1/400. This Markov chain for $\tilde{\beta}_B$ has been chosen to match the aggregate wealth-over-income ratio (5.4) and the Gini coefficient (0.48) of the wealth distribution. The transition matrix of $\tilde{\beta}$ is as follows (rows indicate the current state, and columns indicate next period’s state; both the first row and the first column correspond to $\tilde{\beta} = \tilde{\beta}_l$; both the second row and the second column correspond to $\tilde{\beta} = \tilde{\beta}_m$):

$$\begin{bmatrix}
49/50 & 1/50 & 0 \\
1/400 & 199/200 & 1/400 \\
0 & 1/50 & 49/50
\end{bmatrix}.$$  

**Market arrangements** The risk-free interest rate $r$ is set as the one year bank deposit rate in 2002 taken from the People’s Bank of China. The minimum-size house $(h)$ is set according to the housing asset value at first quartile among all the homeowners in CHIP 2002. This value is roughly 2 times the average annual wage income among all the homeowners in CHIP 2002. The maximum loan-to-value ratio restriction is set at $\lambda_h = 0.60$, corresponding to a constant down payment requirement of 40%27. The maximum loan-to-future-income ratio restriction $\lambda_y$ is chosen to be 0.1028. The two borrowing requirements in the model together determine the overall tightness of credit conditions in the model, generating an aggregate loan-to-value ratio of 5% consistent with CHIP 2002.

Note that the mortgage option setup in the model allows households to draw on a home equity line of credit (subject to a loan to income criterion). Thus, loans are essentially payment-option mortgages with a required interest rate payment and a pre-approved home equity line of credit. In principle, this means that a borrower can choose to only cover mortgage interest payments but not to pay down principal every period. However, mortgage loans in China are all installment loans: equal installments of the loan must be paid each period before maturity, although early retirement is allowed. It turns out that

---

27 The minimum down payment requirement for first home purchases in China is strictly regulated by the People’s Bank of China. It has changed over time but within a narrow band between two levels: 30% or 40%. Mortgages to finance purchases of second homes can be subject to even higher down payment requirements.

28 In China, the most important factor considered by the bank for mortgage approval is usually the applicant’s income (no official credit record system exists until 2006). However, there is no clear-cut loan to income criterion universally set for all the banks. Guidelines from bank regulators usually require that a borrower’s ratio of monthly mortgage payment to income should be lower than 50%. For a comprehensive discussion of residential mortgages in China, see Fang et al. (2015).
this is not a problem for the model. Households usually do borrow when they make new housing purchases, but they pay back their debts gradually afterward. This is because unemployment shocks and sizable adjustment costs in housing assets induce households to desire a certain amount of liquid wealth buffer. Moreover, the tight loan to income criterion further dampens the effect of a home equity line of credit since the borrowing constraints become much tighter when households are unemployed.

The depreciation rate of the housing asset $\delta$ is set to 0.03, to approximate both the maintenance and depreciation cost of holding housing wealth. I choose $f_1 = 0.03$ to roughly approximate the amount of commissions for realtors when transacting houses, and I calibrated $f_2 = 0.12$ to target the housing transaction volume-over-GDP ratio in 2002 calculated according to data from National Bureau of Statistics of China. Overall, the adjustment cost function is used to approximate various realistic costs associated with transacting houses in a simplified way. The setup serves the purpose of generating lumpiness in housing that has a differential impact across wealth groups. This feature is essential for the mechanism in the paper.

### 3.5 Steady-state properties

In this section, I discuss the steady-state properties of the model. I pay special attention to the distribution of wealth (including its composition) and cross-sectional housing adjustment behaviors, since they are central to the main mechanism of the paper.

#### Wealth distribution

Table 3.3 examines the steady-state wealth distribution produced by the model. Compared to the data, the model matches almost all the moments.

<table>
<thead>
<tr>
<th>% of wealth held by top</th>
<th>Fraction with wealth&lt;0</th>
<th>Gini coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% 5% 10% 20% 30%</td>
<td>1%</td>
<td>0.48</td>
</tr>
<tr>
<td>Model (baseline)</td>
<td>10% 24% 36% 55% 66%</td>
<td></td>
</tr>
<tr>
<td>Data (CHIP 2002)</td>
<td>8% 23% 35% 53% 64%</td>
<td>2% 0.48</td>
</tr>
</tbody>
</table>

Notes: The wealth distribution in the data is calculated from CHIP 2002. Appendix C contains more details about the calculation procedure.

Note that, when compared to the US data, wealth concentration is at a much lower level in China. Although this might be a reflection of the true inequality level difference.

---

29 There is no direct source to calculate the depreciation rate of housing in China. Previous literature routinely assumes a depreciation rate of 2.5%-3%. See Wu et al. (2012) and Chivakul et al. (2015), for example.

30 There are more than 20 different fees and taxes associated with housing transactions in China. Moreover, fees and taxes differ substantially according to the type of transaction and ownership rights (houses in China have different types of ownership rights due to institutional reasons). Details of those fees and taxes can be found at [http://esf.fang.com/zt/201403/esfjysfmx.html](http://esf.fang.com/zt/201403/esfjysfmx.html) (in Chinese, provided by a NYSE listed private realtor company SouFunHoldings).

31 According to the SCF 2001, the net worth shares of the top 1 and 5 percent wealthy households in the US are 32% and 57%, respectively.
between the two economies, it might also be due, at least in part, to the underrepresentation of the top richest households in CHIP 2002\textsuperscript{32} However, those at the top who are missing from the survey data are unlikely to hold mainly housing in their portfolios since they tend to have access to assets with better returns\textsuperscript{33} Thus, overlooking the richest households here is unlikely to affect the results in this paper.

Figure 3.3: Comparison of cross-sectional asset composition

![Figure 3.3: Comparison of cross-sectional asset composition](image)

(a) Housing-wealth-over-income ratio  
(b) Bond wealth share among homeowners

Notes: Panel (a) plots the distributions of net housing-wealth-over-income ratio both in the model and in the data. Each distribution is normalized by dividing its maximum value, so that the range is between 0 and 1. Panel (b) plots the distributions of bond wealth share in total wealth among homeowners. To be consistent with the model, bond wealth in the data is defined as fixed bank deposits, and total wealth is defined as bank deposits plus net housing wealth. The two data distributions in both panels are obtained from CHIP 2002.

**Cross-sectional asset composition** Since the difference in cross-sectional portfolio choices is central to the paper’s mechanism, it is important to examine how well the model matches with the real portfolio composition in the data. Panel (a) in Figure 3.3 shows that the model does a good job of capturing the fact that most households own similar housing wealth, a manifestation of the minimum size friction. Panel (b) in Figure 3.3 shows that liquidity conditions vary a lot among homeowners, both in the data and in the model. Households in the model ideally desire to hold a balanced composition between liquid wealth and housing wealth. Although housing assets earn a higher return, liquid

\textsuperscript{32}Xie and Jin (forthcoming) find that, after supplementing CFPS 2012 with data from the China Rich List reports, wealth concentration in China is comparable to that of the United States, at least according to measures of top wealth shares.

\textsuperscript{33}The rate of return to capital has stayed comfortably over 20\% throughout 1993 to 2005, according to Bai et al. (2006). Although these high corporate returns are not available to normal households, the wealthiest do have access to them.
wealth is more suitable for the purpose of smoothing temporary income shocks. The model can produce a wide range of liquidity conditions because, due to the adjustment costs, households opt for depleting liquid wealth when making new housing purchases, and gradually accumulating it back to the ideal level afterward.

**Housing adjustments** As discussed in the basic model in Section 2.3, the cross-sectional difference in housing demand responsiveness is the key to generating an interaction between housing market outcomes and inequality. Therefore, I show next to what extent the quantitative model features this differential responsiveness in housing demand.

I first discuss the housing decision for non-homeowners. As in the basic model, housing has a higher financial rate of return than bonds. Therefore, housing decisions for non-homeowners are standard: there is a threshold amount (depending on the exact household type) of liquid assets such that, if assets exceed the threshold, non-homeowners become homeowners. Also, the larger the initial liquid assets are, the less likely a household is to borrow for the purpose of financing a housing purchase.

As for homeowners, Figure 3.4 plots their housing adjustment decisions as a function of initial housing and liquid wealth, across 4 different types, employed or unemployed, top earners or bottom earners (medium patience level). A homeowner can stay put, upgrade, downsize, or completely sell off her housing stock (thus exiting the housing market).

Across all types of households, larger liquid assets increase the chance of upgrading one’s housing stock, and transaction costs create a region of inaction where the housing stock is constant. If liquid wealth falls below a certain level, the household either downsizes or exits the housing market. Moreover, when a household does adjust her housing stock, the size of the adjustment also depends on her liquid wealth level.

One important feature of the model is that bottom earners have much larger inaction zones – i.e., unresponsive housing demand; while top earners adjust their housing stock much more frequently – i.e., responsive housing demand. The unresponsive housing demand from the bottom earners is also reflected in the feature that when the amount of liquid assets is small, the housing tenure decision depends on the initial level of housing wealth non-monotonically. Consider, for instance, an unemployed homeowner with liquid assets equal to about thirty percent of annual income (i.e., a bottom earner). If the initial housing stock size is really small, the homeowner pays the adjustment cost and, because of his low liquid assets, completely quits the housing market. If the initial housing stock size is medium, the homeowner does not change house size since, given the modest size, quitting the housing market is too big an adjustment, while downsizing a bit is not economical given the sizable fixed adjustment costs. If the initial house size is large, there is enough room to optimally downsize the housing stock instead of completely selling it off.
Figure 3.4: Housing adjustment functions

Notes: The figure illustrates the equilibrium optimal housing adjustment policy as functions of initial housing wealth and bond wealth. The plots in Panel (a) and Panel (c) correspond to a medium patient bottom earner ($\tilde{\beta} = \tilde{\beta}_m$).

4 Transition experiment

China’s fast house price appreciation in the last decade has been frequently labeled as a bubble by commentators. However, as noted by Fang et al. (2015) and Wu et al. (2015), it is an challenging task to reliably identify whether the enormous house price appreciation in China is a bubble since one would need a systematic framework that can take into account special features of the Chinese economy, such as fast economic growth, large-scale urbanization, and severe financial frictions.
The calibrated model in Section 3.4 can take this task seriously. With special features of the Chinese economy built in, the model not only captures the wealth distribution of Chinese households in 2002, it also matches well the distribution of household portfolio composition. Thus, the calibrated model is well suited to study the impact of changes in macroeconomic factors on equilibrium house prices in China after 2002.

In this section, I conduct the following transition experiment: starting from the calibrated model, and introducing income growth of the same magnitude as in the data, I analyze the response of house prices. Moreover, the transition exercise will also take into account another factor, changes in income inequality, which has not been discussed much in the literature focusing on house prices in China. The main goal of this transition exercise is to quantitatively evaluate how much the proposed mechanism in this paper can account for the dynamics of house prices in China. The model will also speak to the dynamics of aggregate saving as well as wealth inequality.

4.1 Transition problem

In this section, I formulate the transition exercise mathematically. I will specify the shocks to the model, define the transition problem, and briefly discuss the solution methods.

In the main exercise below, I engineer an increase in both income inequality and income growth of the same magnitude as in the data. In particular, I assume that the entire wage inequality increase is generated by an increase in the top income share over time. By keeping the individual income process and all other parameters constant during the transition, I fix the earnings uncertainty faced by households during the whole transition exercise so that there is no changes in precautionary saving motives. This allows the model to isolate the effect of changes in cross-section inequality from changes in individual earnings uncertainty.

To keep things simple, I assume a linear increase in both the top income share and total income between the initial steady state (2002) and period 10 of the transition (2012). From period 11 (2013) onward, I assume that both the top income share and total income stay constant at their final steady-state values (the same as the 2012 value). Note that I set the top income share and total income to be constant from 2013 onward, in order to cleanly illustrate the strength of the mechanism in generating a house price run-up with only a temporary growth shock. If households in the model are assumed to expect the income growth to continue after 2013, the resulting house price response would be stronger. I leave the detailed investigation of how different expectations of economic growth can affect the dynamics of house prices to future sensitivity analyses.
\[ x + b' + P_t h' + \Phi(h', h) = \varepsilon y^i_t + (1 - \varepsilon) \omega y^i_t + (1 + r)b + R_th + (1 - \delta)P_t h \]
\[ b' \geq B(\lambda_h, h', \lambda_y, \varepsilon) \]
\[ h' \in \{0, [h, +\infty)\}, \]

for \( i \in \{B, T\} \). Here \( \Lambda_0 \) corresponds to the calibrated initial steady state distribution, and \( T \) is the ending period of the transition when \( \Lambda_T \) will stay constant at the new steady state level after \( t \geq T \). Note that \( T \) is an endogenous equilibrium object.

I consider a perfect foresight transition path to the new steady state (no aggregate uncertainty); i.e., the whole sequence of \( \{Y_t, \pi_t\}_{t=1}^{\infty} \) is known at \( t = 1 \). In particular,

\[
\pi_t = \begin{cases} 
\pi_{t-1} + 0.01 & \text{if } t < 10 \\
\pi_{t-1} & \text{otherwise}
\end{cases}, \quad
Y_t = \begin{cases} 
1.08 \times Y_{t-1} & \text{if } t < 10 \\
Y_{t-1} & \text{otherwise}
\end{cases}.
\]

Markets are required to clear at all times during the transition, as in the steady-state equilibrium. Note that solving the household optimization problem along the transition path requires adding time to the state variables listed in the steady-state problem described earlier in the paper because both current-period states and future states affect households’ optimal decisions. The computation is still done by a fixed point iteration procedure. Unlike in the steady-state equilibrium, instead of iterating over a set of stationary aggregate variables, the algorithm now iterates over the sequences of prices \( P_t \), bond savings \( A_t \), transaction volumes at \( t \), and transaction frequencies at \( t \) along which the optimal decisions of households clear all the three markets. Given a sequence of those aggregate variables, the dynamic programming problem can be solved recursively, moving backward in time from time period \( T \). Note that I also need to solve for the new steady-state equilibrium with different top income shares \( \pi_T \) and output level \( Y_T \). Details of the computational procedure can be found in Appendix D.2.

### 4.2 Results

In this section, I study the economy’s transition from 2002 until 2012, and ultimately towards the new steady state. I particularly focus on the outcomes of three aggregate variables: house price, wealth inequality and aggregate saving rate.

In the three panels I present below in Figure 4.1, the starting point is the initial 2002 steady state; the solid line denotes the transition path outcomes of the model, and the dotted line with circles denotes the data. Note that I only present the transitional dynamics until 2020, since the economy is pretty much stabilized by then and the transition to new steady state afterward is very gradual.\(^{35}\)

\(^{35}\)The steady-state aggregate variables are close to their levels in the year 2020 although the full transition will take about 20 more periods forward.
Notes: The raw data for Panel (a) is constructed from the same sources as those in Panel (b) of Figure 3.1. The trend is constructed by HP filtering the raw data with frequency parameter $\lambda = 6.25$. Note that since the price index is available only after 2004, the comparison in Panel (a) is done by “pretending” the starting year in the model is 2004. The data in Panel (b) is calculated by author from the same sources as those in Panel (a) of Figure 3.1. See Appendix C for more details. The data for Panel (c) is taken from Yang et al. (2013), where the savings rates are calculated from Urban Household Surveys conducted by National Bureau of Statistics of China. Note that the level of savings rate in the model is not directly comparable to that of the data; we only compare the relative changes here.
The main object of interest is the evolution of house prices. In particular, the central question of the paper is whether the large-scale housing price appreciation can be accounted for by the relatively parsimonious model setup within a rational expectation regime. Panel (a) in Figure 4.1 shows the model’s prediction for house price dynamics along the transition path (solid line), presented as the aggregate price-over-income ratio relative to the initial steady state. Note that aggregate income in the model is assumed to grow at 8% annually between 2002 and 2012, and at 0% afterward in the baseline setting.

The results show that the amplification mechanism of the model is sufficient for rationalizing a large run-up of house prices. In particular, the growth rate of house prices in the model is significantly higher than the growth rate of income during early periods of the transition and then gradually drops below income growth and even becomes negative toward the end of the time period examined. The existing data show a similar price-over-income trend in the first phase of the model outcome although the data trend begins less abruptly and is sustained over a longer period than that in the model. As will be discussed in the next section, this difference might be reconcilable by incorporating more realistic features such as continued income growth or urbanization.

Panel (b) and Panel (c) in Figure 4.1 show that the hump-shaped house price-over-income dynamics is accompanied by similar patterns in the evolutions of wealth inequality and the aggregate savings rate. Again, the model outcomes are a bit more dramatic than the data counterpart, for similar reasons as argued above. Overall, the results in Figure 4.1 show that the model can roughly account for both the trend dynamics and the changes in magnitude for all the three aggregate variables in consideration.

In the following, I investigate the mechanism behind the model outcome. As discussed in earlier parts of the paper, one key feature of the model is the differential responsiveness in housing demand across wealth groups. The differential responsiveness turns out to be crucial for the transitional analysis, too. In particular, it implies an endogenous feedback loop between house prices and wealth inequality. It is exactly this feedback loop that causes house prices to overreact to temporary shocks. The general intuition about the mechanism is the following.

The feedback mechanism is initiated by expected income growth, which will ultimately lead to house price appreciation. Due to potential capital gains, a period of high premium in housing is thus anticipated among households. In response, the most responsive wealthy households stock up on housing wealth initially, causing a surge in house prices. Due to the skewed housing wealth distribution, the price surge causes a higher dispersion in capital income and thus higher wealth inequality follows. Higher wealth inequality, in turn, helps sustain the growth in house prices. This is because, while housing demand

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36I focus on the data trend here since the raw data is volatile, especially due to the interruption of the 2008 economic crisis, while the model outcomes are smooth since I only consider a one-time period of anticipated temporary shocks.

37Wealth inequality only falls a bit after the early surge and remains at a much higher level than the initial steady state afterwards. This is not the case for savings rate and price-over-income ratio. The reason for this difference is the model’s assumption that the increase in top income share is permanent.
from wealthy households saturates, demand from households with modest wealth starts to pick up after a short delay. Even with a high premium, transaction frictions and liquidity constraints still limit less-wealthy households’ response in the housing market. Accordingly, the inequality gap between wealthy housing investors and the rest continues to widen as house prices keep creeping up. This creates a self-reinforcing feedback loop between house price and wealth inequality.

The loop ends eventually, when most of the capital gains (due to income growth) in the housing market are realized. The return premium sinks close to its steady-state level, wealthy households begin to draw down their over-sized housing stock, and at about the same time, households that initially had meager resources are able to enter the housing market after continued saving. Housing wealth becomes more dispersed as house prices adjust back to trend.

Figure 4.2: Housing premium and buyer distribution during transition

Notes: The housing premium in period \( t \) is calculated as \( \frac{\rho_t}{\pi_t} + \frac{\rho_t}{\pi_{t-1}} - \delta - \tau \). Each tertile buyer share in period \( t \) is obtained by accruing all the households who purchased housing assets (either by upgrading or making first-time purchases) in period \( t \) according to wealth tertile in period \( t \).
Figure 4.2 plots the transitional dynamics for the housing premium and the buyer distribution, to help confirm the above intuition. According to the top graph in Figure 4.2, the housing premium stays high only for early periods when there are high capital gains in housing. Although the high premium dwarfs the transaction costs in housing, early buyers who are able to enjoy the whole high return periods are still mostly households from the top wealth tertile. This is because tight borrowing limits and a high entry barrier prevents households with low wealth from responding quickly. Therefore, most of the initial housing demand comes from a portfolio shift from bonds to housing among the responsive wealthy. Households with relatively plentiful resources can tap into some of the high returns in the housing market during later periods of the high return phase, with modest saving efforts. Households with initial low resources (mostly non-homeowners) are forced to save more and wait longer to enter the housing market.

The transition dynamics of house prices and the buyer distribution make it clear that aggregate saving can increase due to a strong saving motive for housing purchases from households with few resources. However, the initial sharp rise in aggregate saving is also partly due to the portfolio re-balancing behavior of the wealthy. As explained earlier, households in the model desire a certain level of liquid wealth. After the initial portfolio shift toward housing wealth, while waiting for the capital gain in housing asset to fully realize, the wealthy need to save up more liquid wealth since it is still the better form of buffer against idiosyncratic income shocks.

In sum, the model rationalizes a house price run-up relative to income growth and its adjustment back to trend in a perfect foresight equilibrium with one-time shocks. It also explains increased wealth concentration and a rising aggregate saving rate. The mechanism links portfolio choices of the wealthy and the poor with house price dynamics and expectations, stressing the role of responsive investment behavior among wealthy households.

A final note is that the strength of the mechanism described above crucially depends on the exogenous inequality level in the economy, which is largely controlled by the top income share in the model. Thus, the inequality shocks fed into the model are not only empirically relevant but also important for amplifying the extent of the housing market responses and matching them with the observed magnitude of house price run-ups. I will investigate more of the role played by inequality shocks in future sensitivity analyses.

4.3 Discussion

In this section, I discuss some of the missing elements in the baseline transition exercise, additional sensitivity analyses, and potential extensions.

Other demand factors  The baseline transition exercise only considers one-time anticipated income shocks (both growth and inequality), abstracting from changes in many other demand factors. Although this choice makes our results clean for interpretation,
other demand factors might play important roles in the housing boom. We briefly discuss the implications of other demand factors here.

In particular, this paper does not consider changes in credit conditions. This choice is consistent with the empirical evidence. Although the amount of mortgage borrowing did increase over the years in China, the regulation has not changed much, in the sense that both the mortgage interest rates and down payment requirements vary within a relatively small range (see Fang et al., 2015 for more details). One might worry that borrowing from the shadow banking sector might mean that some lenders actions may work independently from official regulations. However, according to the two sources of household survey data analyzed in this paper (CHIP 2002 and CFPS 2012), the loan-to-value ratio at the household level remained low in the two survey waves, about 5% in both 2002 and 2012, which suggests that shadow financing is not a significant concern. If we allowed a controlled relaxation in credit conditions during the transition exercise, the model would be able to produce larger housing booms.

The large-scale urbanization process, rural-urban migration in particular, is a more relevant exogenous demand factor. According to the two latest National Population Censuses, the urban population in China increased at an average annual rate of about 4%. Chen and Song (2014) find that urbanization accounts for 80.4% of the total urban population growth. Moreover, among the urbanized population, rural-urban migration accounts for more than half. This means that including urbanization factors would introduce a large number of poor households who need to save up a long time for housing purchases to the model. Accordingly, the mechanism in the paper would get strengthened. In particular, this would likely cause the housing boom to last longer.

This paper does not consider demographic factors either. In particular, life-cycle components are not explicitly modeled. The intergenerational link is strong in China in the sense that parents are usually able and willing to provide financial support to their children’s home purchases. This means that the “actual” housing demand in China might present less of a life-cycle pattern than a standard life-cycle housing demand model would imply. This is partly supported by the empirical observation that the saving behavior does not vary significantly along the life-cycle dimension. Thus life-cycle does not seem to be a factor of the first order for housing market outcomes in the short and medium run.

38Recent literature explaining the early 2000s US housing booms focuses on credit expansions (see, for example, Kiyotaki et al., 2011, Sommer et al., 2013, and Favilukis et al., forthcoming). Housing market outcomes are sensitive to credit conditions in those models because house prices are mainly driven by consumption demand among the majority poor agents. Due to the very little cross-sectional variation in wealth across low income agents, small changes in credit availability can cause large groups of identical, low income, low wealth households to move between renting and homeownership. However, the large swings in homeownership often produced by those models contradict the data.

39This is partly because of the Chinese tradition that children support parents in their old age (and sometimes even live in the same house).

40According to Chamon and Prasad (2010), the age profile of savings among Chinese households has an unusual pattern in recent years, with younger and older households having relatively high savings rates.

41Curtis et al. (2015) show that long-run demographic change in China can significantly affect the aggre-
Aggregate uncertainty  The paper does not model aggregate uncertainty. Due to the modeling approach of this paper (which treats housing as a lumpy financial asset), adding aggregate uncertainty can be expected to strengthen the mechanism in the model. Since housing becomes a risky asset in the presence of aggregate uncertainty, it can be expected that housing would be compensated with a risk premium in addition to the illiquidity premium. The lumpiness of housing would thus increase. This might amplify the differential responsiveness in housing demand. Taking aggregate uncertainties into account would also help the model in matching the volatility in house prices in the data.

Supply side factors  Finally, housing supply is fixed in the model. Allowing for housing production is unlikely to affect the qualitative result of this paper either, especially since land is an important factor in housing construction. As discussed earlier, according to Wu et al. (2015), house price growth in China is driven by rising land values, not by construction costs. However, incorporating housing production can be an important future avenue to explore. The cross-sectional variation in housing supply elasticities could be an important factor in accounting for the regional variations in house price growth.

In sum, the qualitative results from the baseline transition experiment are expected to hold even considering various additional elements.

5 Conclusion

This paper develops a heterogeneous agent equilibrium model to study the interaction of inequality, income growth, and house price dynamics. Important features of the housing market environment are explicitly captured: housing delivers returns both in terms of rental dividends and capital gains; market frictions make housing adjustment lumpy. House prices and rents (and therefore the financial rate of return on housing) are jointly determined in equilibrium. The modeling approach of the paper allows housing purchases to be purely financial decisions. Thus, housing market outcomes are closely linked to cross-sectional household portfolio saving decisions: prices and inequality endogenously interact.

The model can account for the positive empirical relationship between inequality and house prices. More importantly, a transitional equilibrium analysis of the Chinese economy can rationalize its observed house price growth accompanied by a rise in the private savings rate. Key mechanisms behind the results are based on the differential responsiveness in housing demand across wealth groups, which is due to the differential impact of the lumpy housing.

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42 Combined with the fixed population mass, the model essentially assumes exogenous changes in housing demand are met by the supply side. This turns out not to be a bad assumption. According to Wu et al. (2015), in the aggregate, the growth in households demanding housing units driven by fundamentals roughly matches with new housing construction in urban China, although with considerable regional variation.
This work contributes to the housing literature by illustrating a new price formation mechanism, which highlights the role played by the investment motive among the wealthy. Another important contribution is that the paper provides a tractable quantitative model that are suitable to study housing market outcomes in China. Moreover, the theory in this paper shows that market frictions can have differential impact cross-sectionally, increasing inequality. Inequality can in turn amplify market frictions. This adds to our understanding of a broader topic: how inequality and macroeconomic forces can interact.

At its current stage, this research only focuses on explaining the mechanisms behind certain housing market outcomes. Important welfare and policy implications are the next step. For example, the interaction between inequality and return premium in the housing market directly speaks to the efficient provision of housing services in a steady state. If housing supply is explicitly modeled, increased inequality might boost housing production and ultimately improve welfare. Moreover, in the short run, low-income households can be priced out of the housing market due to the feedback loop between house prices and wealth inequality. This highlights the potentially large welfare cost to low-income households during fast economic growth.

Possible extensions of this work may have applications beyond explaining housing market outcomes. For example, one could develop a related model to explain the cross-sectional pattern in debt-to-income ratios and its implication on aggregate default risk. In an environment in which heterogeneous households face investment opportunities with different risk-return perspectives and different investment costs, wealthy households can afford to load on riskier and costlier assets with a low debt-to-income ratio, earning high returns. While the wealthy households bid up the prices for high-return assets, the poor households are either “trapped” in risk-free assets with meager returns or forced to borrow a lot (relative to income) and bear higher default risks. The mechanism can potentially explain the cross-sectional debt-to-income ratio patterns in the United States and shed light on the development of financial crises.

This mechanism might be an important force behind the recent housing boom in the United States: evidence from the SCF shows that the housing assets owned by wealthy households increased much more dramatically than the rest of the housing stock during the housing price run-up in the early 2000s.
References


A Additional stylized facts about inequality and house prices

Figure 1.1 in Section 1 presents the raw correlation between top income share growth and the appreciation of house prices both across countries and across states in the United States. In this section, I explain more details of the data source and variable construction. I also conduct some robustness checks on these correlations by controlling for some factors, such as income and population growth and housing supply elasticity. Moreover, I show that rents do not co-move with top income shares. But due to limitations on data availability, the analysis with rental data is limited only to the US cross-state level. In the end, I discuss potential directions for future empirical work.

The choice of house price indexes Every house price index is representative in its own way and has different strengths and weaknesses. Even within the same geographic area, house price indexes from different sources can differ significantly, depending on construction methodology, sample composition, and data aggregation. Therefore, sometimes it is crucial to choose the appropriate index according to specific purposes.

The focus of this paper is on the relationship between house prices and inequality. One of the paper’s central empirical and theoretical arguments is that the investment motives of wealthy households play important roles in driving house price dynamics. Since wealthy households tend to purchase more expensive houses than the rest of the population, using a value-weighted index, in which expensive houses have a greater influence on estimated price changes, will naturally bias the results of the analysis. This is especially true if more expensive houses have different price dynamics than less expensive ones. Therefore, for this study, it is important to choose a house price index that weights price trends equally for all residential properties. To this end, I choose the all-transactions index produced by the Federal Housing Finance Agency (FHFA) for the US cross-state analysis.

The FHFA indexes are available from 1976 to the present for all US states; this makes them great for cross-section analysis. Each state-level index is constructed using a weighted, repeat sales method that compares transaction prices of the same property over time. This method is preferred because it avoids composition biases from quality changes in the stock of houses in transaction.

For the cross-country analysis, I try to use house price indexes that are consistent with the US FHFA index methodology. These types of indexes are identified and compiled for many countries in the Federal Reserve Bank of Dallas’s International House Price Database as described in Mack and Martínez-García (2011). For the purpose of this study, I use the house price indexes expressed in real terms to control for potential correlation between changes in inequality and the overall price level in one country.

44In an ongoing project, from various data sources, my coauthor and I document that the price dynamics of houses from different price ranges does differ. In particular, more expensive houses tend to appreciate faster than the rest of the houses.
**Robustness checks with other data sources**  
The weakness of the FHFA index is that its underlying sample composites only houses purchased with conforming mortgages, and this might understate the sensitivity of house prices to alternative credit linking to investment purposes, especially those cash transactions. Moreover, in specific regions and during certain times, housing purchases with sub-prime, jumbo, and other non-conforming loans can contribute significantly to changes in house prices as shown by some researchers. To address these issues, I repeat the same analysis using an alternative index: the Zillow Home Value Index (ZHVI), which is also available across all US states, although extending back only to 1996. ZHVI also uses repeat-sales methods to address composition biases, making it conceptually similar to the FHFA index, with the additional advantage of a broader sample coverage. The results of the analyses are similar when using either the FHFA index or the ZHVI for the same time periods. Similarly, for the cross-country analysis, the robustness of the result is verified by using an alternative set of comparable house price indexes from the BIS Residential Property Price database.

**The choice of inequality measures**  
The inequality measure in the empirical part of this paper focuses on households at the upper end of the income distribution. According to the theory of the paper, the more relevant dimension to look at should be household wealth. This mismatch happens because there is insufficient comparable wealth data at the cross-section over time (see Saez and Zucman, 2014 for an account of the series of wealth distribution only within the United States using capitalized income tax data). Cross-sectional data on income distribution has been better documented; thanks to the recent effort by Alvaredo et al. (The World Top Incomes Database) and Frank et al. (2015). Thus, the measurement for inequality in this paper is consistently the top income (excluding capital incomes) shares.

**Control for average income growth and housing supply elasticity**  
Some important factors that affect the dynamics of house prices might also affect changes in inequality. Two obvious ones are average income and housing supply elasticity. Figure A.1 conducts the same correlation analysis as in Figure 1.1 in Section 1 with additional elements to control for those factors. In particular, I add cross-sectional series on personal disposable income per capita to control for changes in average income and population growth. Consistent with the cross-country house prices data, cross-country disposable income per capita series are expressed in real terms. Moreover, I also try to control for cross-sectional differences in housing supply elasticity, although in very crude ways. For both cross-country and cross-state analysis, the supply elasticity measures I have are only for a cross section and thus could not account for changes in supply elasticity itself over

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time. This is only going to be problematic if changes in the supply elasticity correlate with changes in inequality. Potential corrections of this issue will be addressed in future work.

Figure A.1: Top income share growth and appreciation of house prices, with controls

Notes: House prices and inequality measures are exactly the same as those in Figure 1.1 in Section 1. Income is measured by personal disposable income per capita, with the same panel structure as the price and inequality data. The average growth rates of income are obtained by averaging the yearly growth rates over the entire sample periods. Supply elasticity measures are available only as a cross-section. In Panel (a), cross-country personal disposable income per capita (real) series are taken from Federal Reserve Bank of Dallas’s [International House Price Database] as described in Mack and Martínez-García (2011); cross-country housing supply elasticities are cross-sectional estimates from Caldera and Johansson (2013). In Panel (b), US cross-state (nominal) personal disposable income per capita series are from [BEA Regional Economic Accounts]; US cross-state housing supply elasticities are approximated by the cross-sectional Wharton Land-Use Regulatory Index (WRLURI) from Gyourko et al. (2008).

As can be seen in Figure A.1, the positive association between top income share growth and appreciation of house prices remains significant after those controls. It even gets stronger at the cross-country level. This evidence provides us with more confidence in the causal relationship between inequality and house prices.

US cross-state rents and inequality The theory in this paper suggests that rents depend mostly on average resources instead of the inequality level. It is important to verify this result to justify the model setup in this paper. However, at the cross-country level we lack a long enough series of rents data, especially if we want them to be comparable. Thus, the analysis using rental data is limited to only the US cross-state level. Figure A.2 plots the correlation between growth in the top 5 percent income share and rental prices across states in the United States, both with and without controls for average income and
housing supply elasticity.

Figure A.2: Top income shares growth and rents appreciation

Notes: In both panels, inequality measures, income measures, and supply elasticity measures are exactly the same as those in Panel (b) of Figure A.1. In Panel (b), US cross-state (nominal) rents are taken from Historical Census of Housing Tables (decennial), provided by the US Census Bureau, Housing and Household Economic Statistics Division. Gross rent is the monthly amount of rent plus the estimated average monthly cost of utilities and fuels. Monthly rents were computed for specified renter-occupied units paying cash rent. This category excludes one-family houses on ten or more acres.

As can be seen in Figure A.2, rents do not co-move with top income shares (there is a hint of a positive correlation between those two in the raw data in Panel [a], which is gone after we add the controls in Panel [b]). This observation is consistent with the theory posited in this paper.

Future work To empirically establish a causal relationship between movements in house prices and in inequality, the analysis in this paper is clearly inadequate. Although the focus of this paper is to understand the theoretical interactions between housing market outcomes and inequality dynamics, further empirical analysis is relevant both in its own right and because it might provide validation to the mechanisms in the paper. In particular, it is of interest to expand the US cross-state exercise to a rigorous panel analysis, and to extend the data to a more disaggregated level, such as across US Metro areas or counties. Some preliminary analysis suggests that the correlation between growth in inequality and house prices is robust at different aggregation levels when controlling for certain factors. A more detailed analysis will not only inform causality, but will also help identify contributing factors and mechanisms behind this correlation.
B  Equilibrium characterization of the basic model

Before characterizing the equilibrium, we first recast the definition of equilibrium in more detail in the following.

**Definition of equilibrium** The steady-state stationary equilibrium associated with \( \mu, \pi, Y, r, \lambda_h, H \) and \( \bar{h} \) is defined as: a set of prices \( R \) and \( P \), a set of policy functions \( b(y, R, P), h(y, R, P) \) and \( x(y, R, P) \) for young households, as well as a stationary distribution \( \lambda(y, b, h) \), such that

1. Given \( R \) and \( P \), \( b(y, R, P), h(y, R, P) \) and \( x(y, R, P) \) solve the young household’s problem defined in (2.1).
2. The stationary distribution \( \lambda(y, b, h) \) is induced by \( \mu, \phi, Y, r, H \) and \( \bar{h} \) and \( b(y, R, P), h(y, R, P), x(y, R, P) \).
   
   (a) \( \int \int \lambda(y, b, h) h(y, b, h) = H \)
   
   (b) \( \int \int \lambda(y, b, h) s(y, b, h) = H \)

B.1 Equilibrium with \( \frac{R}{P} = r \)

Although housing has a minimum size friction than bonds, equilibrium with \( \frac{R}{P} = r \) does exit, providing the condition that the risk free interest rate is sufficiently high and the saving motive is sufficiently strong. For equilibrium with \( \frac{R}{P} = r \), there is no strict portfolio choice problem. Regardless of the income level \( y \), the solution to problem (2.1) is simply

\[
x = \frac{1}{1 + \beta} y, \quad b + Ph = \frac{\beta}{1 + \beta} y,
\]

subject to constraints (2.2) and (2.3).

Combining the above optimal consumption decision with the rental market clearing condition (3b), we can recover \( R \); i.e.,

\[
\frac{1}{2} \frac{1}{1 + \beta} \varphi Y + \frac{1}{2} \frac{(1 + r)\beta}{1 + \beta} \varphi Y = RH \Rightarrow R = (1 + \frac{\beta r}{1 + \beta}) \frac{\varphi Y}{2H}.
\]

Note that in the above derivation we used the condition that \( Rs = \varphi x \). \( R \) increases with \( r \), due to a pure wealth effect from the older generation. But the increase is less than one for one, since there is still a substitution effect from the younger generation.

House price simply follows from the no arbitrage condition between bonds and housing:

\[
P = (\frac{1}{r} + \frac{\beta}{1 + \beta}) \frac{\varphi Y}{2H}.
\]
$P$ decreases with $r$ due to the fact that equilibrium $R$ grows less than one for one with the growth of $r$. This makes housing less attractive than bonds. Note that in this case income inequality does not affect either $P$ or $R$. This is the result of the log utility specification: the marginal propensity to consume does not change along the wealth dimension.

The equilibrium existence condition further requires that housing asset market clearing condition (3a), constraint (2.2), and (2.3) are satisfied. Those conditions imply:

$$\lambda_h P_h \leq \frac{\beta}{1+\beta} \frac{\pi Y}{\mu} \text{ and } \lambda_h PH \leq \left\{ \begin{array}{ll} \frac{\beta}{1+\beta} \frac{\pi Y}{2} & \text{if } \lambda_h P_h > \frac{\beta}{1+\beta} \frac{(1-\pi)Y}{(1-\mu)} \\ \frac{\beta}{1+\beta} \frac{Y}{2} & \text{otherwise} \end{array} \right.$$

Plugging the above conditions into the equilibrium pricing functions, we have the following two cases.

**Case One:** both bottom and top earners can afford the minimum house when the parameters satisfy the following condition:

$$K > \varphi \lambda_h \text{ and } r \geq \max \left\{ \frac{1+\beta}{\beta} \frac{\varphi \lambda_h}{K-\varphi \lambda_h}, \frac{1+\beta}{\beta} \frac{\varphi \lambda_h}{1-\varphi \lambda_h} \right\},$$

where $K \equiv 2\frac{(1-\pi)H}{(1-\mu)H}$, which measures the strength of inequality and market friction. This is the case where both inequality and market friction are small.

**Case Two:** only top earners can afford the minimum house when the set of parameters either satisfies the following condition:

$$K < \varphi \lambda_h \text{ and } r \geq \frac{1+\beta}{\beta} \frac{\varphi \lambda_h}{\pi-\varphi \lambda_h},$$

or satisfies the following condition:

$$K > \varphi \lambda_h \text{ and } \frac{1+\beta}{\beta} \frac{\varphi \lambda_h}{K-\varphi \lambda} \geq r \geq \frac{1+\beta}{\beta} \frac{\varphi \lambda_h}{\pi-\varphi \lambda_h}.$$

Note that in one situation bottom earners could not afford the minimum house because of the high levels of inequality and market friction; in the other situation, the reason is due to the relatively low interest rate level.

**B.2 Equilibrium with $\frac{R}{P} > r$**

For equilibrium with $\frac{R}{P} > r$, due to the minimum size friction in housing ownership and the borrowing constraint, optimal portfolio choice depends on income level $y$. Moreover, whenever the optimal portfolio consists of non-zero housing, constraint (2.2) will bind; i.e., it is in the household’s interest to fully utilize the collateralized borrowing opportunity. In the following, we first derive the policy functions for the situation where $\frac{R}{P} > r$. 

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Policy functions  First, note that constraint \( [2.3] \) necessarily binds for households with sufficiently low income. In particular, let’s denote

\[
y \equiv \lambda h P h
\]

as the income level at which the minimum house is affordable only with zero consumption. Then for households with an income level lower than \( y \), they are forced to save in terms of bonds, and the solution to problem \( (2.1) \) for \( 0 < y \leq y \) is simply

\[
x = \frac{1}{1 + \beta} y, \quad h = 0, \quad b = \frac{\beta}{1 + \beta} y.
\]

Second, for households with high enough income, constraint \( [2.3] \) will not bind. Due to the assumed log preference, the cutoff income level can be conveniently identified as

\[
\bar{y} \equiv \frac{1 + \beta}{\beta} y.
\]

Thus, the solution to problem \( (2.1) \) for \( y \geq \bar{y} \) is simply

\[
x = \frac{1}{1 + \beta} y, \quad h = \frac{\beta y}{1 + \beta} \frac{1}{\lambda h P}, \quad b = (1 - \lambda h) P h.
\]

For households with an income level in the middle range \( \bar{y} \leq y \leq \bar{y} \), they are unable to optimally choose more than \( h \) units of housing assets. Moreover, due to the log preference, their optimal decision must be either choosing exactly \( h \) units of housing or opting for zero housing.

If a household decides to choose \( h \) units of housing, its current consumption will suffer from its over-saving, but the household can enjoy a higher return from its savings. In the end, the household’s lifetime utility level will be

\[
\ln(y - \bar{y}) + \beta \ln(\bar{y}(1 + \rho)),
\]

where

\[
\rho \equiv r + \frac{1}{\lambda h} \left( \frac{R}{P} - r \right)
\]

denotes the net financial rate of return from holding the housing asset.

If a household chooses zero housing, the optimal consumption-saving decision would be the same as for those households with an income lower than \( \bar{y} \), and the associate utility level would be

\[
\ln(\frac{1}{1 + \beta} y) + \beta \ln\left(\frac{\beta(1 + r)}{1 + \beta} y\right).
\]

Denote \( d(y) \) as the difference between the two utility levels, or, in simplified form,

\[
d(y) = \ln\left(\frac{(1 + \beta) y - \bar{y}}{y}\right) - \beta \ln\left(\frac{\beta(1 + r)y}{(1 + \beta)\bar{y}(1 + \rho)}\right).
\]
It can be shown that \( d(y) \) monotonically increases with \( y \) when \( y \in [\bar{y}, \hat{y}] \). Moreover, \( d(\bar{y}) = -\infty \) and \( d(\hat{y}) > 0 \). This ensures that the solution to the equation \( d(y^*) = 0 \) is unique, and \( d(y) < 0 \) if and only if \( y < y^* \). Therefore, it’s optimal to purchase \( h \) units of housing for households with income levels between \( y^* \) and \( \hat{y} \), while households save only in bonds when they have an income level below \( y^* \); i.e.,

\[
\begin{align*}
    h(y, R, P) &= \begin{cases} 
        \frac{\beta}{1+\beta} \frac{y}{\lambda_h P} & \text{if } y \geq \hat{y} \\
        \frac{1}{h} & \text{if } y^* < y < \hat{y} \\
        0 & \text{otherwise}
    \end{cases}, \quad
    b(y, R, P) = \begin{cases} 
        -\frac{\beta}{1+\beta} \frac{(1-\lambda_h)y}{\lambda_h} & \text{if } y \geq \hat{y} \\
        -(1-\lambda_h)P_h & \text{if } y^* < y < \hat{y} \\
        \frac{\beta}{1+\beta} y & \text{otherwise}
    \end{cases}.
\end{align*}
\]

**Characterizing the equilibrium**  With the policy functions identified above, equilibrium prices can be derived using market clearing conditions (3a) and (3b). In the following, we discuss three different equilibrium outcomes case by case.

**Case One:** When \( \frac{(1-\pi)Y}{(1-\mu)} \geq \hat{y} \), asset market clearing condition (3a) implies

\[
\frac{\beta Y}{2(1+\beta)\lambda_h} = PH \Rightarrow P = \frac{\beta Y}{2(1+\beta)\lambda_h H}.
\]

Rental market clearing condition (3b) implies

\[
\frac{\varphi Y}{2(1+\beta)} + \frac{\beta \varphi Y(1+\mu)}{2(1+\beta)} = RH \Rightarrow \frac{R}{P} = \frac{\varphi}{1-\varphi} \left( \frac{1}{\beta} + 1 + \frac{(\lambda_h - 1)\varphi}{\lambda_h} \right).
\]

Equilibrium rental price can then be derived as

\[
R = \frac{\varphi \beta Y}{2(1-\varphi)(1+\beta)H} \left( \frac{1}{\beta} + 1 + \frac{(\lambda_h - 1)\varphi}{\lambda_h} \right).
\]

Note that neither prices nor returns depend on any inequality measure in this case. However, for this type of equilibrium to exist, we need a relatively equal income distribution, small market friction \( h \), and a low interest rate. In particular, those conditions imply the following:

\[
K \geq 1 \quad \text{and} \quad r < \frac{\varphi \lambda_h (1+\beta)}{\beta(1-\lambda_h \varphi)}.
\]

**Case Two:** When \( \frac{\pi Y}{\mu} > \bar{y} > \frac{(1-\pi)Y}{(1-\mu)} \geq y^* \), asset market clearing condition (3a) implies

\[
\frac{\beta \pi Y}{2(1+\beta)\lambda_h} + \frac{(1-\mu)}{2} PH = PH \Rightarrow P = \frac{\beta \pi Y}{(1+\beta)\lambda_h (2H - (1-\mu)h)}.
\]

Rental market clearing condition (3b) implies

\[
\pi Y + \frac{\beta \pi Y}{1+\beta} (1-\pi)Y + (1-\mu)PHh = \frac{2RH}{\varphi} \Rightarrow \frac{R}{P} = \frac{\varphi}{1-m\varphi} \left( \frac{1}{\beta} + 1 + n(\lambda_h - 1)\varphi \right).
\]
Equilibrium rental price can then be derived as

\[ R = \frac{\varphi \beta Y}{(1 - m) \varphi (1 + \beta)(2H - (1 - \mu)H)} \left( \frac{1}{\beta} + 1 \right) n + \frac{(\lambda_h - 1) \rho m}{\lambda_h}, \]

where \( m \equiv \frac{1 + (1 - \mu) \frac{1}{1 - \lambda_h} \frac{h}{H}}{2} \) and \( n \equiv \frac{1 - (1 - \mu) \frac{h}{H}}{2}. \)

Note that in this case an increase in either the top earner’s income share \( \pi \) or the bottom earner’s population share \( 1 - \chi \) will cause house prices to increase. This is a case in which income inequality is modest compared to the amount of the housing market friction. The equilibrium outcome is that the bottom earners participate in the housing market but are liquidity constrained; i.e., \( \bar{y} > \frac{(1 - \pi)Y}{1 - \chi} \geq y^*. \) The equilibrium condition requires the following:

\[ \frac{\beta \pi}{1 - (1 - \mu)h/H/2} \leq K(1 + \beta) \leq \frac{\pi(1 + \beta)}{1 - (1 - \mu)h/H/2}, \quad r < \frac{\varphi \lambda_h (1 + \beta) n}{\beta (1 - \lambda_h \varphi m)} \quad \text{and} \quad \varphi m < 1. \]

Case Three: When \( \frac{\pi Y}{\mu} > \bar{y} > y^* > \frac{(1 - \pi)Y}{(1 - \mu)} \), asset market clearing condition (3a) implies

\[ \frac{\beta \pi Y}{2(1 + \beta)\lambda_h} = PH \Rightarrow P = \frac{\beta \pi Y}{2(1 + \beta)\lambda_h H}. \]

Rental market clearing condition (3b) implies

\[ \varphi Y + \beta \varphi \left( \pi Y(1 + \rho) + (1 - \pi)Y(1 + r) \right) = 2(1 + \beta)RH \Rightarrow \frac{R}{P} = \frac{\varphi}{1 - \varphi} \left( \lambda_h \left( \frac{1}{\beta} + \frac{1 + r}{\pi} \right) - r \right). \]

Equilibrium rental price can then be derived as

\[ R = \frac{\varphi \beta \pi Y}{2(1 - \varphi)(1 + \beta)H} \left( \frac{1}{\beta} + \frac{1 + r}{\pi} - \frac{r}{\lambda_h} \right). \]

Note that both prices and returns only depend on the inequality measure \( \pi. \) However, for this type of equilibrium to exist, we need a relatively unequal distribution and a big market friction \( h. \) In particular, the equilibrium conditions require the following:

\[ K \leq \frac{\beta \varphi y^*}{(1 + \beta)\Sigma} \quad \text{and} \quad r < \frac{\varphi(1 + \beta)}{\beta(\pi - l \varphi)}. \]

C Institutional background and data sources for China

C.1 Backgrounds

Before 1978 China had a centrally planned socialist economy in which the ownership of land (and housing) was nationalized and public rental housing was the predominate form of urban housing provision. Little has been changed about China’s housing welfare system during early 1980s, after China’s major reform in 1978. It was not until 1988 that the
Chinese constitution was amended to allow for land transactions, which set the legal stage for the privatization of housing in China.

In the mid-1990s, the Chinese central government identified the final goal of housing reform as the creation of a new urban housing system that suited socialist market economics: “commodity housing” was allowed for market transaction. Moreover, employees in the state sector were encouraged to purchase full or partial property rights of their current apartment units at subsidized prices. However, the housing reform of this period did not manage to completely shift the system away from state-provided housing. Commodity housing was still relatively rare during this period.

An important milestone in housing policy occurred in 1998, when the central government announced that welfare housing distribution in urban China would be abandoned at the end of 1998 and completely replaced by market provision. Market provision of housing surged after 1998: the share of private housing units among all completed housing units almost doubled in 4 years, increasing from about 30% in 1998 to over 50% in 2002 (Wu et al., 2014). Moreover, during the same period, the People’s Bank of China lowered the mortgage interest rate five times to encourage home purchases (Fang et al., 2015).

According to Walder and He (2014), in 2002, 78% of all households in urban China owned their homes with partial or full property rights and only 16% continued to live in rent-subsidized work unit housing: the housing privatization program was roughly complete. For more discussion about the housing market development in China after the early 2000s, readers are referred to Fang et al. (2015) and Wu et al. (2015).

C.2 Household income and wealth in China

The availability of household-level data from China is limited. One common data source being used in the literature to study questions related to cross-sectional income in China is the annual Urban Household Survey (UHS) conducted by the National Bureau of Statistics (for example, see Piketty and Qian, 2009; Chamon and Prasad, 2010; and Yang et al., 2013 for example). However, UHS does not adequately account for household wealth.

One of the few options for detailed information on both household income and household wealth during 1990s and the first decade of the 21st century is the Chinese Household Income Project (CHIP). Due to increasing interest in household finance and the wealth distribution in China, more efforts have been made in data collection, and several sources are now available after the end of the first decade of the 21st century. The China Family Panel Studies (CFPS), which this paper adopts, has arguably the best quality data for the purpose of this study.

In the following section, I describe the two data sources in more detail. In the interest
of saving space, this appendix does not present detailed calculation procedures for all of the distributional statistics used in the main text of this paper; however, these calculations are available upon request.

The Chinese Household Income Project (CHIP)  The Chinese Household Income Project, carried out by a team of international economists in collaboration with China’s National Bureau of Statistics, has conducted a series of national surveys covering the years 1988, 1995, 2002, and 2007. Each wave of CHIP contains both a rural survey and an urban survey, and the implementation is largely consistent across waves, with the 2007 wave as an exception. In this study, we only use the urban survey in each wave.

The urban surveys collect detailed information on household wealth and its components, including financial assets, market value of private housing, production assets, and value of durable consumer goods. In particular, the housing value in CHIP is estimated by asking households to assess the market value of their owned housing. This is similar to the procedure used by the Survey of Consumer Finance.

One major drawback of CHIP is that it does not contain sample weights. For more details about CHIP, such as its sample size, sampling procedure, and geographical coverage, readers are referred to Shi and Zhao (2007).

The China Family Panel Studies (CFPS)  The China Family Panel Studies (CFPS) are a nationally representative, annual longitudinal survey of Chinese communities, families, and individuals. The survey was launched in 2010 by the Institute of Social Science Survey of Peking University, China. The data on a follow-up survey conducted in 2012 are also available. Like CHIP, during each wave CFPS conducts two separate surveys for rural and urban areas. In this study, we only use the urban survey in each wave.

The CFPS dataset contains comprehensive measurements of assets, including housing assets, financial assets (which are further broken down into subcategories, such as fixed bank deposits, stocks, and mutual funds, etc.), business assets, detailed items of durable goods, and liabilities from housing and other sources. As in CHIP, in CFPS information on housing values is obtained by directly asking homeowners how much their houses are worth. Some missing housing values are imputed according to the size of the house and unit value for similar housing types. Dropping those imputed values do not affect any of the data moments used in this paper.

Detailed sampling weights are available for each wave of CFPS as well as for cross-sectional analysis between 2010 and 2012 waves. A detailed account of the advantage of CFPS in capturing Chinese household wealth can be found in Xie and Jin (forthcoming).
D Computation

D.1 Solving for the stationary equilibrium

The equilibrium is formally defined by the conditions presented in Section 3.3. Those conditions imply an economy with a stationary distribution \( \Lambda^* \) over \((\varepsilon, \tilde{\beta}, b, h, i)\) in which households behave optimally and markets always clear with prices \(\{P^*, R^*\}\). We approximate the distribution \( \Lambda^* \) by an economy with a finite number of households, each characterized by its individual type \((\varepsilon, \tilde{\beta}, b, h, i)\).

Note that, to solve their individual problems, households in this economy only need to know market prices and returns for both bonds and housing. Since \(P^*\) and \(R^*\) are sufficient statistics to determine the return on housing, the problem is reduced to finding the pair of prices \(P^*\) and \(R^*\) that can clear all the markets when households all behave optimally. However, directly searching for the equilibrium price pair \(\{P^*, R^*\}\) can be very time consuming because it involves repeatedly re-solving individual optimization problems over all the combinations of \(\{P, R\}\) and simulating data to check for market clearing conditions. This is especially the case since in the price searching process it is hard to find an updating rule that can make \(P\) and \(R\) monotonically converge due to their intercorrelation.

A more efficient approach is to use the goods market clearing condition (2iii) to identify the correlation between \(P\) and \(R\), along with other aggregate state variables: aggregate savings in bonds \(A\), housing transaction volume \(\Omega\), and the housing transaction frequency \(F\), and ultimately to eliminate one of the two prices. Here we choose to eliminate the rental price \(R\). By treating \(R\) as a residue from condition (2iii), and searching for a stationary combination of the other aggregate variables \(\{P^*, A^*, \Omega^*, F^*\}\), it is easy to find a monotonically converging price updating rule when utilizing the downward sloping demand property. The algorithm outlined in the following describes this computation procedure in detail:

Let \(\{P^k, A^k, \Omega^k, F^k\}\) represent the \(k\)th guess of the set of aggregate state variables.

**Step 1.** Guess a set of aggregate variables \(\{P^k, A^k, \Omega^k, F^k\}\).

**Step 2.** Given the set of aggregate variables, solve for the households’ optimal decision rules \(\{b^k_i, h^k_i\}_{i \in \{B,T\}}\). This step of the algorithm requires solving the households’ value functions \(V^k_i\). To find \(V^k_i\), I first approximate the household’s continuation value function with a set of interpolation grids for \((b, h)\). I then use a value function iteration procedure to solve the household’s parametric dynamic programming problem as defined in Section 3.3. Note that, by imposing \(\{P^k, A^k, \Omega^k, F^k\}\) in the household optimization problem, we can derive \(R^k\) according to the goods market clearing condition (2iii).

**Step 3.** Using the above obtained optimal decisions, we simulate the economy using \(M\) households and \(Z\) periods. \(M\) needs to be large enough and \(Z\) needs to be long enough.
such that the simulated economy becomes stationary within \( Z \) periods. In practice, \( M \) is chosen to be 100,000 and \( Z \) is set at 1,000.\(^{47}\)

**Step 4.** Calculate relevant aggregate statistics in the ending period of the simulation when the economy is stable. In particular, we collect total demand for housing assets and total demand for housing rentals in order to check equilibrium conditions.

**Step 5.** Check whether housing asset market clearing condition (2i) and housing rental market clearing condition (2ii) are satisfied.\(^{48}\) If not, update the guess for the set of aggregate variables \( \{P^k, A^k, \Omega^k, F^k\} \) and go back to **Step 1**. Note that the updating rule can be straightforwardly set according to the downward sloping demand property since we only have one price in the set of aggregate variables.

### D.2 Solving for the transition path equilibrium

This appendix describes the computation method to solve for the perfect foresight transition path equilibrium between two steady states, as described in Section 4.1.

As solving for the steady-state equilibrium, the computation is done by a fixed point iteration procedure. The main difference is that, instead of iterating over a set of stationary aggregate variables, we now need to iterate over a set of sequences of aggregate state variables.

Moreover, there is a setup stage when we need to get the following objects ready: a sample distribution \( \Lambda_0 \) over \((\varepsilon, \bar{\beta}_i, b, h, i)\) drawn from the initial steady state, an initial guess of the length for the transition period \( T \), and the value functions from the new steady-state equilibrium from period \( T \) onward.

After the setup stage, the algorithm begins by setting the relevant aggregate variables in periods \( t = 0 \) and \( t = T \) equal to their initial and final steady-state values. Next, a guess is made for the remaining sequences of aggregate state variables along the transition path. The transition path is found using the following algorithm:

Let \( \{P^k_t, A^k_t, \Omega^k_t, F^k_t\}_{t=1}^{T-1} \) represent the \( k \)th guess of the set of sequences of aggregate state variables along the transition path.

**Step 1.** Guess a set of aggregate variables \( \{P^k_t, A^k_t, \Omega^k_t, F^k_t\}_{t=1}^{T-1} \).

**Step 2.** Given the set of aggregate variables, solve for the households’ optimal decision rules at each time period \( t \leq T-1 \). Note that the decision rules can be solved backward, starting from period \( T-1 \), taking the sequence of aggregate state variables \( \{P^k_t, A^k_t, \Omega^k_t, F^k_t\}_{t=1}^{T-1} \) and ending period value functions as given. Similarly as in solving the steady-state

\(^{47}\)Note that we divide those \( M \) households into bottom earner group and top earner group, according to their population share. When \( M \) is set at 100,000, the economy usually stabilizes after about 100 periods, starting from reasonable initial conditions. We set \( Z = 1,000 \) for extra caution.

\(^{48}\)Note that the good market clearing condition (2ii) is automatically satisfied by the derivation of \( R^k \).
equilibrium, in each period, we first need to derive $R^k_t$ according to goods market clearing condition before we can solve for the households’ optimal decision rule.

**Step 3**, Using the above obtained sequences of optimal decisions, we simulate the initial distribution $\Lambda_0$ along the transition path from period $t = 1$ to period $t = T - 1$.

**Step 4**, Calculate relevant aggregate statistics in each period, based on the simulation results.

**Step 5**, Check whether market clearing conditions in each period are satisfied. If not, update the guess for the set of aggregate variables $\{P^k_t, A^k_t, \Omega^k_t, F^k_t\}_{t=1}^{T-1}$ and go back to **Step 1**.

Finally, we update the length of the transition period $T$ if necessary. We only need to do this if the converged aggregate state variables in period $t = T - 1$ do not match with those in period $T$, the new steady-state values. In practice, any $T \geq 50$ will result in the same computed equilibrium, within a given level of the convergence precision.