Constrained Optimization Approaches to Solving Real World School Choice Problems

Aaron L. Bodoh-Creed
University of California, Berkeley

1 Abstract

Matching markets have been the subject of intensive mechanism design efforts over the past 15 years with school choice being a frequent application, which has led Boston and other large cities employ centralized matching procedures to allocate students to public schools. One common recommendation of market designers is to build a school assignment mechanism in two steps. First, given the school system’s public policy goals, choose a priority structure that gives certain students higher priorities at certain schools than other students. For example, to encourage a student to attend her neighborhood school, the Boston Public Schools (BPS) until recently gave higher priority for seats at a school to students that live close to that school. Second, given the priority structure chosen, solicit student preferences over the schools and run a version of the Gale-Shapley algorithm to generate the school assignment.

There is much to recommend this advice. First, the outcome of the Gale-Shapley algorithm is incentive compatible, so the students have no motive to try and manipulate the assignment system. Second, the outcome is stable, which reflects a notion of fairness that requires that an agent with a high priority for a good be allocated that good before a lower priority agent is provided the good. However, it is far from obvious that encoding policy goals into the priority structure and then running the Gale-Shapley algorithm is the most effective method of implementing goals such as encouraging neighborhood schools. In addition, it is not clear what the trade-offs are between goals such as student welfare and encouraging neighborhood schools.

I argue in this paper that matching problems in general, and school choice problems in particular, can be posed as computationally tractable constrained optimization problems. The constrained optimization approach has the advantage of allowing the market designer to explicitly describe the policy goals in the objective function and encode required properties such as stability and incentive compatibility in the constraints. Using my approach I can compute the global optimum for various desiderata (e.g., the student welfare maximizing school assignment) or assess trade-offs between the goals (e.g., encouraging neighborhood schools and school diversity). The optimization approach also allows me to easily assess the benefits of weakening the constraints of the problem by, for example, adding capacity to a popular school.

Treating a school choice problem as a constrained optimization problem is not a novel theoretical idea. However, I combine this idea with the more recent approach of modeling large school choice problems as a match between a continuum of seats at a finite set of schools and a continuum of each of a finite set of student types. Since each student has a negligible effect on the aggregate outcome, I can write the incentive compatibility conditions in a computationally tractable form. Although the resulting optimization problem
has a large number of constraints, modern techniques for solving large linear and quadratic programs allow me to solve these problems in a matter of minutes even with modest computational resources.

I use my framework to solve for the optimal school assignment using data from the Boston Public Schools (BPS) high school match for the 2011-2012 school year. During this period, BPS used the student-proposing Gale-Shapley algorithm to assign students to schools. Students report a rank-ordered list of up to 10 school programs to BPS, and students are assigned a priority at each school based on whether or not they live within a school’s walk-zone. My data includes the preference lists the students submitted to the mechanism, the priority of each student for assignment to each school, and demographic information such as the student’s ethnicity and whether the student participated in a free school lunch program.

I consider three potential objectives: student welfare, student body diversity, and encouraging neighborhood schools. My optimization approach allows me to compare the status quo outcome generated by the Gale-Shapley algorithm with the global optimum found for the relevant optimization problem. In addition, I study the tensions between these goals by, for example, determining to what extent fostering diverse schools impairs my ability to achieve high student welfare.

Although my analysis focuses on a particular year of the BPS school assignment problem, my goal is to make three broader contributions to the market design and economics of education literature. First, while welfare objectives are clearly important in any school choice setting, there are distributional goals, such as the encouragement of neighborhood schools or building ethnically and socioeconomically diverse student bodies, that warrant consideration. My optimization framework provides a method for assessing existing school assignments and designing new school assignments to achieve these distributional goals. Second, the analysis of the BPS data shows that there is room to improve on the Gale-Shapley algorithm especially when it comes to distributional goals such as student diversity and encouraging neighborhood schools. Third, my analysis of the BPS school assignment problem serves as a proof of concept for applying the constrained optimization approach to real-world school assignment settings.

First I consider the objective of maximizing student welfare subject to the capacity, stability, and incentive compatibility constraints. Although the Gale-Shapley algorithm is known to satisfy these constraints, the random tie-breakers used in Boston’s Gale-Shapley algorithm can result in a welfare loss (Erdil and Ergin [3], Kesten [4]). I define a student’s welfare as her assigned school’s rank in the student’s preference list, and the welfare generated by a school assignment is defined as the average welfare across the population of students. The Gale-Shapley algorithm generates roughly half of the welfare gain from moving from the worst to the best stable and incentive compatible match. I also find that the range of welfare values possible in a match is restricted by the incentive compatibility constraints rather than the stability constraints. Finally, I use the shadow prices on the capacity constraints to identify the school program with the highest marginal value for an extra seat, which suggests where resources ought to be directed when given an opportunity to expand school programs.

My second goal is to assess the potential tension between student welfare and encouraging ethnically and socioeconomically diverse student bodies. I measure the diversity of a school by how closely the composition of a school’s student body matches the demographics of the entire BPS student population, and the average diversity of a match is then the average diversity across all of the schools. In other words, a school is diverse if the demographics of its student body closely resembles the demographics of all the students in the BPS system, and a school assignment is diverse if the schools are diverse on average.

Boston has a complicated history of school desegregation beginning with the 1974 “Garrity Decision.” In the case, Tallulah Morgan et al. v. James Hemigan et al., Federal Judge Arthur Garrity determined
that the Boston School Committee had intentionally segregated the schools by race. The ruling mandated that a school busing program be used to desegregate the schools, and violent protests broke out against busing on several occasions. The federal mandate was lifted in 1987, but BPS continued to set aside seats at the prestigious examination schools for minority students. This race-based admissions system was ruled unconstitutional in 1998 by the U.S. Court of Appeals for the First Circuit. In 1999 BPS began using the Boston mechanism, which does not take ethnic or socioeconomic background into account when assigning students to schools. There is concern that the reforms to the BPS school assignment mechanism in the 1990s has led to a resegregated school system. Barbara Fields, a former equity officer for BPS, has said, “Some of us fear we’re going to return to a very segregated school system.” This history may suggest to the reader that there is a significant tension between student welfare and diversity in school assignment. Because incentive compatibility limits what can be achieved in terms of student welfare, the reader might assume that there is little that can be done to encourage diverse schools. Fortunately, my analysis shows that both of these intuitions are incorrect. There are stable and incentive compatible school assignments that simultaneously perform better on both student welfare and diversity than the outcome generated by the Gale-Shapley algorithm. Moreover, one can increase the diversity of the average school significantly with only modest reductions to student welfare.

My third analysis studies the cost of encouraging neighborhood schools in terms of both student welfare and school diversity. In a 2012 speech, Boston Mayor Thomas Menino articulated the following externality based logic for encouraging neighborhood schools:

“Something stands in the way of taking our [public school] system to the next level: a student assignment process that ships our kids to schools across our city. Pick any street. A dozen children probably attend a dozen different schools. Parents might not know each other; children might not play together. They can’t carpool, or study for the same tests. [. . . ] Boston will have a radically different school assignment process, one that puts priority on children attending schools closer to their homes.”

A school assignment encourages neighborhood schools if a high fraction of each school’s student body is drawn from the school’s walk-zone. The demographics of Boston differ widely from zip code to zip code. While some zip codes do not exhibit significant segregation, there are many zip codes that have either a very low (e.g., 1.3% in the 02126 zip code) or a very high (85.2% in the 01906 zip code) fraction of African-Americans among their school-age residents. One might assume that an assignment that encourages neighborhood schools would require that the schools reflect the potentially segregated zip codes around each school.

I study the tension between encouraging neighborhood schools and other desiderata by maximizing a linear combination of the average percentage of students drawn from each school’s walk-zone and my metric for the other goal. First, I find stable and incentive compatible school assignments that perform significantly better than the Gale-Shapley outcome in terms of both diversity and encouraging neighborhood schools. Second, I show that there are stable and incentive compatible school assignments that do significantly better than the status-quo outcome in terms of both welfare and encouraging neighborhood schools. Third, there exist stable and incentive compatible school assignments that draw a high percentage of the average school’s student body from the school’s walk-zone, with at most, a modest decrease in either diversity or student welfare.

In summary, my analysis suggests that one can significantly improve upon the status quo BPS school assignment generated by the Gale-Shapley algorithm in terms of student welfare, the diversity of the schools,
and the percentage of students drawn from the schools’ walk-zones. Since there seems to be at most only weak tensions between these goals, it is possible to find a school assignment that simultaneously achieves nearly the maximal level of diversity, student welfare, and school cohesion as possible, while still satisfying stability and incentive compatibility.

As I mentioned above, the use of a continuum model is absolutely crucial to the tractability of my constrained optimization approach. In the finite model, deviations from truthfulness can have a large effect on the structure of the match since a long sequence of agents can be affected by a single agent’s deviation. The negligible aggregate impact of an agent’s deviation in the continuum model makes it possible to write down the full set of feasibility, stability, and incentive compatibility constraints in a computationally tractable form.

Before accepting the results of my analyses using the continuum model, it is reasonable to ask what connection, if any, my analyses have to the real-world in which a finite set of students is assigned to a finite set of school seats. I provide two answers.

My first approach uses my constrained optimization formulation directly. By “directly,” I mean that I study a mechanism that receives a distribution of student preferences as an input, solves a constrained optimization problem, and provides the solution in the form of an assignment for each student. The primary theoretical issue is whether such a mechanism is incentive compatible. As a first step, I prove that the stable set is generically continuous in the distribution of student-types. I use this result to prove that if there are many participants, then this mechanism is exactly feasible and stable, approximately incentive compatible for all students, and exactly incentive compatible for most students. The downside of this mechanism is that explaining how it functions to a layperson is likely to be difficult.

The second approach uses Proposition 1 of Shi [5] to reformulate a particular solution to one of my constrained optimization problems into a lottery-plus-cutoff mechanism played by a continuum of student-types. Azevedo and Leshno [2] proves that a finite version of this mechanism is exactly incentive compatible, approximately stable, and approximately feasible. This structure also has the benefit of being easy to explain to a person with a limited mathematical background.

References


