Robustly Optimal Auctions
with Unknown Resale Opportunities

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We consider revenue-maximizing auctions when bidders are a priori asymmetric. It is well-known that the optimal auction generally implements an inefficient allocation that is biased against “stronger” bidders (Myerson 1981, McAfee and McMillan 1989). However, one might suspect that the benefits of such misallocation would be undermined if a strong bidder has the ability to acquire the object from the auction’s winner in the resale market instead of winning it in the auction. For example, Ausubel and Cramton (1999, p. 19) suggest that “the possibility of resale undermines the seller’s ability to gain by misassigning the good. The best that the seller can do is to conduct an efficient auction, perhaps withholding ... the good.” Motivated by this intuition, Ausubel and Cramton (2004) propose a class of auctions, called “Vickrey auctions with (bidder-specific) reserve prices,” which allocate the object efficiently provided that at least one of the bidders has beaten his reserve price. These auctions induce an ex-post Nash equilibrium in which bidders bid truthfully and there is no resale.

However, contrary to this intuition about the desirability of resale-proof auctions are the findings of Zheng (2002) and Calzolari and Pavan (2006). These papers assume that no private information is leaked before resale (other than information about auction bids that is disclosed by the designer), in which case the possibility of resale can only reduce the auctioneer’s optimal revenues: indeed, the equilibrium allocation of any auction mechanism followed by resale must be incentive compatible and therefore would be feasible for the designer if resale were impossible. Nevertheless, these papers find that the optimal auction in the presence of resale typically implements a biased allocation, after which resale may occur in equilibrium. For a simple example due to Zheng (2002), suppose one bidder is known to have a zero value for the good, but to be able to design a revenue-maximizing mechanism for resale. Then the auctioneer would find it optimal to sell to this bidder, at a price equal to Myerson’s second-best revenue, which is what the bidder expects to get in resale.
Theorem 4 of Ausubel and Cramton (1999) attempted to formalize the optimality of Vickrey auctions with reserves, starting from the assumption of “perfect resale.” However, as their paper acknowledges, it is unclear what this assumption means. On the one hand, by the theorem of Myerson and Satterthwaite (1983), efficient resale is impossible if no private information is revealed before resale. On the other hand, if we assume that resale takes place under symmetric information, by positing that the parties’ values exogenously become public before resale, then typically, an auctioneer who anticipates the resale procedure would again find it optimal to misallocate the object and induce resale in equilibrium. For example, if we consider the example in the previous paragraph but assume that all private values are exogenously revealed before resale, the optimal mechanism would extract full first-best surplus by selling the object at this price to the zero-value bidder, who would then extract the highest-bidder’s value in resale.

Our paper shows that the intuition of Ausubel and Cramton could actually be validated if the designer is uncertain about the resale procedure (including possible revelation of private information before resale) and wants revenue maximization to be robust to this uncertainty. Namely, we show that Vickrey auctions with reserves maximize the designer’s worst-case expected revenue, where the expectation is taken over buyers’ independent private values and the worst case is over the possible resale procedures.

Our conclusion is conceptually similar to prior work on foundations for strategy-proof auctions: this literature shows that when the designer is ignorant about bidders’ beliefs about each other’s values (Chung and Ely 2007) or about each other’s strategies (Yamashita 2015), she would optimally choose to use a strategy-proof mechanism, even though she is not restricted to such mechanisms a priori. Similarly, in our case, revenue maximization that is robust to resale makes it optimal to use resale-proof mechanisms. However, the proof techniques are quite different.

We begin our analysis with the simple case in which the auctioneer is restricted to always sell the object (Section 3 of the full paper). For this case, we show that the simple Vickrey auction (second-price sealed-bid auction) with no reserve price is optimal. To do this, we guess a “worst-case” resale procedure, in which the highest-value bidder learns other bidders’ values and has full bargaining power in resale. With this resale procedure, in any auction that always sells the object, the highest-value bidder would be able to extract at least his marginal contribution to the total surplus by bidding low to let another bidder win and then buying from the winner. Given that the designer is unable to reduce bidders’ information rents below their expected marginal contributions to the total surplus, she could do no better than the simple Vickrey auction with no reserve. Since this auction sustains truthful bidding as an ex post equilibrium under any resale procedure, it is robustly optimal with unknown resale.

We then proceed to the more complex setting in which the designer can withhold the object, for simplicity starting with the two-bidder case (Section 4). We continue with the same guess for the worst-case resale procedure, and derive bidders’ reduced-form utilities from auction allocations under this resale procedure. Note that these reduced-form utilities exhibit both externalities and interdependent values, since a bidder who does not win cares whether the other bidder wins and, if so, what the other bidder’s value
is. Nevertheless, the usual envelope-theorem approach to local (first-order) incentive compatibility constraints yields a simple expression for bidders’ information rents, which allows us to express the expected revenue as the expectation of an appropriately defined virtual surplus. If we were to ignore all other incentive constraints and solve the resulting relaxed problem by maximizing the virtual surplus state-by-state, the solution would always allocate the object between the bidders efficiently. More specifically, it would sell to the efficient bidder if and only if his value exceeds the optimal reserve price for him (and otherwise withhold the good).

Unfortunately, the solution to the relaxed problem violates non-local incentive constraints: In some cases, the object is left unsold, but the “strong” bidder can, by reducing his bid, make the object instead go to the “weak” bidder; this gives the strong bidder an incentive to underbid and then buy in resale. We guess that the correct solution to the auctioneer’s full problem is instead the Vickrey auction with reserves described by Ausubel and Cramton (1999, 2004). This auction satisfies many non-local incentive constraints with equality under our worst-case resale, and, as suggested in the previous paragraph, these constraints need to be explicitly taken into account, since ignoring them leads to an incorrect solution. To establish that Vickrey with reserves is indeed optimal, we construct Lagrange multipliers on these binding non-local downward incentive constraints, such that maximization of the Lagrangian yields the solution. Since there is an incentive constraint for each type and each possible misreport, and a double continuum of such incentive constraints is binding, our Lagrange multipliers are defined by a measure over this double continuum. The Lagrangian, being a linear functional of an allocation rule, can be written as the expected value of a function that we label “modified virtual surplus.” In the two-bidder case, under appropriate regularity conditions, we construct a product measure of Lagrange multipliers on the binding incentive constraints that works for us, i.e., maximization of the resulting modified virtual surplus yields the optimal Vickrey auction with reserves.

In Section 5 we extend the approach to the case of many bidders. Some additional complications arise because it becomes necessary to consider binding non-local downward incentive constraints both from a given type (when this type is the highest-value bidder but may underreport to buy from another bidder in resale) and to the same type (when this type is reported by some higher type so as to concede the object to another bidder and buy it from him in resale). This necessitates a somewhat more complex construction of Lagrange multipliers. Under appropriate regularity assumptions on value distributions we can construct nonnegative Lagrange multipliers that yield a Vickrey auction with reserves as an optimal auction. In Section 7, we formally state the result that the Vickrey auction with reserves sustains truthful bidding as an ex post equilibrium under any resale procedure, and so it is robustly optimal with unknown resale; this result is essentially due to Ausubel and Cramton but we include it for completeness.

Our approach also yields an iterative construction of the optimal bidder-specific reserve prices in $n$-bidder Vickrey auctions with reserves. We illustrate this (in Section 6) for the case where bidders’ values are distributed uniformly with different upper limits.
References


