Robust Strong Nash Implementation

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Abstract: We introduce coalitional structure into the belief-free approach of Bergemann and Morris (2005, 2011, etc.). Specifically, we provide necessary and sufficient conditions for robust full implementation of a social choice set as an interim strong Nash equilibrium. This enables us to implement new social choice sets.

Keywords: Bayesian implementation, robust implementation, interim strong Nash equilibrium.

1 Introduction

In implementation theory, if a mechanism can be designed such that all its equilibria coincide with an exogenous social choice set, then the set is said to be fully implementable. Under incomplete information, agents' private information is traditionally modeled by a commonly known type space. However, some details of the type space (e.g., agents' beliefs) may not be available to the mechanism designer in practice. Therefore, it is suggested to weaken the common knowledge assumption for a more robust conclusion (Wilson, 1987). To relax the common knowledge assumption, Bergemann and Morris (2005, 2011, etc.) adopt a belief-free approach to study robust mechanism design. Consistent with their spirit of pursuing robustness, we investigate the problem of robust full implementation as an alternative solution concept, the interim strong Nash equilibrium.

We establish that robust coalitional incentive compatibility and robust coalitional monotonicity are necessary and almost sufficient for robust strong Nash implementation. In another word, these two conditions are key for existence of an

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mechanism that implements the social choice set as an interim strong Nash equilibrium under all type spaces. Moreover, the robust implementation framework enables us to go beyond Bayesian implementation, and thus one can include more general preferences, e.g., the maximin expected utility of Gilboa-Schmeidler (1989). Our modeling provides new insights for social choice sets that may not be robustly Nash implementable (e.g., the fine core).

The interim strong Nash equilibrium is immune to all coalitional deviations, and therefore it is more stable than the interim Nash equilibrium. Stability is a concern in many fields of economics, such as matching and network. Excluding all collusion possibilities may be impractical and undesirable. For example, it may be the case that a social choice set is not interim (robust) Nash implementable, but it is interim (robust) strong Nash implementable, and thus the mechanism designer may wish to facilitate agents' communication in order to obtain her objective. Under complete information, the problem of strong Nash implementation has been studied by Maskin (1979) and Dutta and Sen (1991) among others. Under the Bayesian framework, Hahn and Yannelis (2001) consider a related notion in exchange economies.

Our framework to address robust strong Nash implementation provides insights on other implementation concepts, such as interim Nash implementation, pairwise stable Nash implementation, and simultaneous implementation as an interim Nash and strong Nash equilibrium. In fact, by restricting the deviating coalitions to be singletons, our interim strong Nash implementation concept reduces to interim Nash implementation. Similarly, by restricting the deviating coalitions to have cardinality no more than two, our equilibrium has the flavor of pairwise stable Nash equilibrium, which has been used widely in the network literature. Slightly modifying our mechanism, we can obtain one for robust simultaneous implementation as an interim Nash and strong Nash equilibrium, under which a robust coalitional incentive compatible and robust monotonic social choice set can be guaranteed as an outcome, even when the mechanism designer does not know which coalitions could be formed. Hence, if future researchers were to adopt a different implementation concept with certain stability requirements, the methodology of our paper could be adapted.

From a technical point of view, we construct new arguments in order to prove our robust strong Nash implementation results. Our basic idea follows the lottery construction of Bergemann and Morris (2011), but their mechanism cannot prevent profitable coalitional deviations. Therefore, the focus on coalitional structure requires non-trivial modifications.

The interim strong Nash equilibrium is a refinement of the interim Nash equilibrium. This does not mean that our robust strong Nash implementation is stronger than robust Nash implementation because full implementation requires the equilibria to coincide with the social choice set. On the partial implementation side, robust strong Nash implementation implies robust Nash implementation. However, the ro-
bust coalitional monotonicity condition is weaker than the robust monotonicity condi-
tion of Bergemann and Morris (2011), which is necessary and almost sufficient for
robust Nash implementation. This allows us to implement some social choice sets
that are not robustly Nash implementable.

Several applications of our results are provided. The first application is the fine
core. We show that it is robustly strong Nash implementable. This complements the
work of Palfrey and Srivastava (1987), which shows that several core notions fail to be
Bayesian Nash implementable. The second application involves a benevolent social
planner allocating a unit of private good efficiently. The efficient rule is robustly
strong Nash implementable, but not robustly Nash implementable. Then we revisit
the public good example of Bergemann and Morris (2009) and show that their efficient
social choice function is robustly strong Nash implementable if and only if agents have
a common value. This example shows that robust Nash implementation doesn’t imply
robust strong Nash implementation and vice versa. Finally, we robustly strong Nash
implement the rational expectations equilibrium.
References


