On the Foundations of Ex Post Incentive Compatible Mechanisms (Extended Abstract)

TAKURO YAMASHITA and SHUGUANG ZHU, Toulouse School of Economics

1 Introduction

Motivation. The recent literature on mechanism design provides a series of studies on robustness issues, motivated by the idea that a desirable mechanism should not rely too heavily on the agents’ common knowledge structure (Wilson, 1985). One approach is to adopt stronger solution concepts that are insensitive to various common knowledge assumptions, such as dominant-strategy incentive compatibility (e.g., Segal, 2003) and ex post incentive compatibility (e.g., Dasgupta and Maskin, 2000).

However, a mechanism that achieves desired outcomes without the agents’ common knowledge assumption does not immediately imply dominant-strategy or ex post incentive compatibility. In revenue maximization in private-value auction, Chung and Ely (2007) fill in this gap by establishing the maxmin and Bayesian foundation of the optimal dominant-strategy mechanism. Consider a situation where the seller in an auction (principal) only knows a joint distribution of the bidders’ (agents) valuation profile for the auctioned object, but does not have reliable information about the bidders’ knowledge of each other’s value. Then dominant-strategy mechanisms have a maxmin foundation if, for any non-dominant-strategy mechanism, there is a possible belief of the seller with which the optimal dominant-strategy mechanism achieves (weakly) higher expected revenue than the non-dominant-strategy mechanism. As a stronger concept, if the same belief can be found for any non-dominant-strategy mechanism with which the optimal dominant-strategy mechanism achieves (weakly) higher expected revenue, then there is a Bayesian foundation.

Our contributions. We examine the existence of such foundations for ex post incentive compatible mechanisms in interdependent-value environments. First, we show that similar foundation results hold true as long as each agent’s payoff type is one-dimensional. The key observation is that interdependence only exists in a cardinal sense; that is, a bidder’s valuation is always higher if he has one payoff type than another, while his precise valuation varies with the other bidders’ payoff types.

On the other hand, multi-dimensional payoff-type spaces may exhibit ordinal interdependence — that is, a bidder’s preference over two payoff types gets reversed as the other bidders’ payoff types change — with which ex post mechanisms may not have foundation. More precisely, based on a simple “bundling” idea, we can construct a
mechanism which achieves \textit{strictly higher} expected revenue than the optimal ex post incentive compatible mechanism, regardless of the agents’ (high-order) belief structure.

\textbf{Related work.} Chen and Li (2016) show that with the uniform-shortest-path-tree property, the foundation results for dominant-strategy mechanisms in Chung and Ely (2007) can be extended to a general class of private-value environments where agents have multi-dimensional payoff types.

Regarding the no-foundation results, there are several papers that provide examples without foundations for dominant-strategy or ex post incentive compatible mechanisms (e.g., Bergemann and Morris, 2005; Jehiel, Meyer-ter-Vehn, Moldovanu, and Zame, 2006; Chen and Li, 2016). Our work contributes to this line of research by providing a general class of environments with a no-foundation result, and the economic intuition based on the “bundling” idea for this no-foundation result.

Bergemann and Morris (2005) provide another sort of foundation for ex post incentive compatible mechanisms by showing that, any \textit{separable} social choice correspondence that is implementable given any (high-order) belief structure of the agents must satisfy ex post incentive compatibility. Our work is complementary to theirs because we consider revenue maximization, which is excluded from the principal’s objective in their paper.

Börgers (2013) constructs a non-dominant-strategy mechanism that yields weakly higher expected revenue than the optimal dominant-strategy mechanism for any agents’ belief, but \textit{strictly} higher expected revenue for some belief structures. Our no-foundation result is stronger in that it provides a \textit{strict} improvement in expected revenue for any (high-order) belief structure, though under stronger conditions on the environment.

\section{Model and Results}

\textbf{Payoff environment.} We consider a digital goods environment (Goldberg, Hartline, Karlin, Saks, and Wright, 2006) where the principal can make arbitrarily many copies of the indivisible goods at no cost. (In the paper, we discuss possibilities of showing the same sort of results in various other environments, including a single-good auction.) There is a finite set of risk-neutral agents, 1, 2, \ldots, I, and each agent has a single-unit demand for the good. Agent \(i\)’s privately-known payoff type is \(\theta_i \in \Theta_i \subseteq \mathbb{R}^d\), where \(|\Theta_i| = N\) for all \(i\). (Potential extensions to cases with continuous payoff type spaces are also discussed in the paper, which we omit here.) The principal’s prior belief for \(\theta = (\theta_1, \ldots, \theta_I)\) is \(f \in \Delta(\Theta)\), where \(f(\theta) > 0\) for all \(\theta \in \Theta\). Each agent \(i\)’s valuation for the good is \(v_i : \Theta \rightarrow \mathbb{R}\), which is increasing in \(\theta_i\). The binary relation \(\succeq_i\) over \(\Theta_i\) satisfies \(\theta_i \succeq_i \theta_i'\) if and only if \(v_i(\theta_i, \theta_{-i}) \leq v_i(\theta_i', \theta_{-i})\). We assume the single-crossing condition on \(v\) and \(f\) as in Chung and Ely (2007).

\textbf{Type space.} We work on the universal type space, \(\mathcal{T} = (\hat{T}_i, \hat{\theta}_i, \hat{\pi}_i)_{i=1}^I\), which is a collection of a measurable space of types \(\hat{T}_i\) for each agent \(i\), a measurable function \(\hat{\theta}_i : T_i \rightarrow \Theta_i\) that describes the agent’s payoff type, and a measurable function \(\hat{\pi}_i : T_i \rightarrow \Delta(T_{-i})\) that describes his belief about the others’ types. Let \(\mathcal{M} \subseteq \Delta(\mathcal{T})\) represent the
set of principal’s all possible prior beliefs, \( \mu \in \Delta(T) \), whose marginal distribution over \( \Theta \) coincide with \( f \).

**Mechanism.** A mechanism is denoted by \( \Gamma = (M, q, p) \), where \( M_i \) represents a message set for each agent \( i \), \( M = M_1 \times \ldots \times M_I \), \( q : M \to Q = [0, 1]^I \) is an allocation rule, and \( p : M \to \mathbb{R}^I \) is a payment function. The optimal EPIC mechanism, \((\Theta, q_{EP}, p_{EP})\), is defined in the standard way and achieves expected revenue \( R_{EP}^f \). For any non-EPIC mechanism \( \Gamma = (M, q, p) \) and \( \mu \in M \), the expected revenue obtained in the truth-telling Bayesian equilibrium, \( \sigma^* \), is \( R_{\mu}(\Gamma) = \int_{t \in T} \sum_i p_i(\sigma^*(t))d\mu \). Then, the condition for EPIC mechanisms to have a maxmin (or Bayesian) foundation is given by

- **Maxmin foundation:** \( R_{EP}^f = \sup_{\Gamma} \inf_{\mu \in M} R_{\mu}(\Gamma) \);
- **Bayesian foundation:** \( \exists \mu \in M, \text{ s.t. } R_{EP}^f = \sup_{\Gamma} R_{\mu}(\Gamma) \).

Our main results are summarized in the following two theorems.

**Theorem 1.** With \( d = 1 \) and the single-crossing condition, there exists a Bayesian (and hence a maxmin) foundation for EPIC mechanisms.

**Theorem 2.** With \( d > 1 \) and additional assumptions (omitted here), EPIC mechanisms do not have the maxmin foundation (nor the Bayesian foundation). That is, there exists a mechanism and an equilibrium (given \( T \)) such that, for any belief \( \mu \in M \) of the principal, the mechanism achieves a strictly higher expected revenue than the optimal EPIC mechanism.

Theorem 1 is a direct extension of Chung and Ely (2007) in the private-value environment to the interdependent-value environment, because one-dimensional payoff types \((d = 1)\), together with monotonicity of \( v_i \) in \( \theta_i \), implies \( \leq_{\theta_i} \leq_{\theta_i}' \), for all \( \theta_i, \theta_i' \in \Theta_i \), which induces exactly the same ordinal structure of agents’ preferences as in the private-value setting.

The basic idea for revenue improvement in Theorem 2 is to use “bundle”. Consider the following example where agent 1’s valuation changes depending on agent 2’s binary payoff type. The optimal EPIC mechanism for agent 1 is to set a price separately for each of the “two rights”: 2 for the right to buy the good when agent 2 reports \( \theta_2 \), and 2 for another right to buy the good when agent 2 reports \( \theta_2' \). As is in the bundling literature (e.g., McAfee, McMillan, and Whinston, 1989), the seller can get additional revenue by setting a different price for the “bundle” (i.e., in case agent 1 buys given both payoff types of agent 2) rather than simply combining the prices for two individual rights. (This example is for illustrative purposes only. To prove Theorem 2 we also need side-betting to get strict revenue improvement for non-full-support belief structure.)

<table>
<thead>
<tr>
<th>( v_1(\theta) )</th>
<th>( (q_{11}^{EP}, p_{11}^{EP}) )</th>
<th>( \theta_2 )</th>
<th>( \theta_2' )</th>
<th>( (q_{11}^{new}, p_{11}^{new}) )</th>
<th>( \theta_2 )</th>
<th>( \theta_2' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_1 )</td>
<td>3, (1,2)</td>
<td>3, (1,2)</td>
<td>( y(t_1) \geq \frac{1}{2} )</td>
<td>1, (1,2)</td>
<td>1, (1,2)</td>
<td>( y(t_1) \leq \frac{1}{2} )</td>
</tr>
<tr>
<td>( \theta_1' )</td>
<td>2, (1,2)</td>
<td>1, (0,0)</td>
<td>( y(t_1) )</td>
<td>1, (0,0)</td>
<td>1, (0,0)</td>
<td>( y(t_1) )</td>
</tr>
<tr>
<td>( \theta_1'' )</td>
<td>1, (0,0)</td>
<td>2, (1,2)</td>
<td>( \theta_1' )</td>
<td>(1,2)</td>
<td>(0,0)</td>
<td>( \theta_1'' )</td>
</tr>
</tbody>
</table>

where \( y(t_1) := \hat{\pi}_1(t_1) | \theta_2 \) stands for agent 1’s belief that agent 2 has payoff type \( \theta_2 \).
References


