Repression Works
(just not in moderation)

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Abstract

Why does government violence deter political challengers in one context, but inflame them in the next? This paper argues that repression increases opposition activity at low and moderate levels, but decreases it in the extreme. There is a threshold level of violence, where the opposition becomes unable to recruit new members, and the rebellion unravels – even if the government is responsible for more civilian suffering overall. I show this result theoretically, with a mathematical model of coercion and popular support, and empirically, with micro-level data from Chechnya and a meta-analysis of sub-national conflict dynamics in 145 countries. The data suggest that such a threshold exists, but the level of violence needed to reach it varies. Many governments, thankfully, are unable or unwilling to go that far. I explore conditions under which this threshold may be higher or lower, and highlight a fundamental trade-off between reducing government violence and preserving civil liberties.

DRAFT

Repression is violence that governments use to stay in power. When confronting behavioral challenges to their authority, governments often respond by threatening, detaining and killing suspected dissidents and rebels. The coercive purpose of these actions is to compel challengers to stop their fight, and to deter others from joining it. The intensity of repression can vary greatly. To reestablish control in Chechnya after 1999, for example, the Russian government used a range of methods, from targeted killings to shelling and indiscriminate sweeps. Rebels' responses ranged from peaceful acquiescence in one village to violent escalation in the next.

Why does government violence sometimes deter political challengers, but other times inflame them? The dominant view in political science is that violent efforts to maintain power can create grievances that embolden the regime's opponents.[1] Others disagree, noting that repression can deter rebellion by making it unacceptably costly.[2]

This article maintains that both perspectives are wrong, and both are correct. What rebels do depends on how much violence the government uses: repression inflames opposition activity at low and moderate levels, but deters it in the extreme. There is a threshold level of violence, at which repression outpaces the opposition's ability to recover its losses. If the government can escalate violence past this threshold, civilians will believe that supporting the opposition is costlier than supporting the government, and will generally not rebel – even if the government is responsible for more civilian suffering overall. If the level of repression falls short, government violence will only invite new and more aggressive behavioral challenges.

I show this result logically, with a mathematical model of coercion and

[1] This perspective is particularly dominant among social scientists studying civil war and terrorism (Gurr and Lichbach, 1986; Mason and Krane, 1989; Francisco, 1995, 2004; Mason, 1996; Lichbach, 1987; Heath et al., 2000; Arreguìn-Toft, 2001, 2003; Carr, 2002; Abrahms, 2006; Findley and Young, 2007; Saxton and Benson, 2008). Skepticism of repression – especially indiscriminate repression – is also a central theme of the "population-centric" school of counterinsurgency policy research (as exemplified by Galula 1964; Thompson 1966; Kitson 1971; Nagl 2002; Smith 2007 and Kilcullen 2009), and is embedded in U.S. counterinsurgency doctrine (Headquarters, Department of the Army, 2014).

[2] Notable examples include Langer (1969); Hibbs (1973); Tilly (1978); Trinquier (1961); Opp and Roehl (1990); Rasler (1996); Nepstad and Bob (2006); Lyall (2009); Downes and Cochran (2010); Beissinger (2007); Weyland (2009, 2010). Some have argued that coercive effectiveness depends on the type of repression used: overt vs. covert (Davenport, 2014), or selective vs. indiscriminate (Lyall, 2010).

rebel recruitment, and empirically, with a disaggregated analysis of violence in Russia's Chechnya region. I examine Chechnya due to its prominence in recent literature on political violence (Lyall, 2009, 2010; Toft and Zhukov, 2015), and its geopolitical significance as a 'test case,' whose lessons other governments have sought to learn (e.g. Ukraine, Kazarin 2014) and even emulate (e.g. Syria, Hill 2013). To ensure that the threshold is not unique to Chechnya, I evaluate the generalizability of these results with a meta-analysis of sub-national conflict data from 145 countries.

The idea of a threshold effect is not new. A rich literature on contentious politics and social movements has hypothesized an 'inverted-U' relationship between repression and dissent (Bwy, 1968; Gurr, 1970; Feierabend, Feierabend and Gurr, 1972; Muller and Seligson, 1987; Muller and Weede, 1990; Olivier, 1991; Khawaja, 1993; DeNardo, 2014). The scope of this research, however, has been mainly on protests and forms of resistance short of armed conflict. Most empirical tests, moreover, have relied on macro-level data, and indirect measures of coercion. This article's contribution is to unpack the theoretical logic behind the threshold effect, and to conduct the most comprehensive empirical test yet fielded in the literature. I demonstrate that the threshold effect holds at the sub-national level, and is robust across multiple estimation strategies, countries and data sources.

The logic of two-sided coercion pushes governments to repress massively, or not at all. This dilemma – between inaction and mass murder – creates strong incentives for governments to restrict civil liberties and may explain the emergence and retrenchment of autocracy after civil war.

The logic of coercion in civil conflict

The current section introduces a theory of government repression.[3] The scope of this inquiry is on the dynamics of political violence during armed conflict.[4] It does not seek to explain the original causes and triggers of

[3] I use the terms 'repression' and 'government violence' interchangeably below.

[4] I define an armed civil conflict as the sustained use of organized violence, by at least two groups of actors, toward the pursuit (or maintenance) of political power. This definition includes most civil wars, insurgencies, revolutions, and anti-occupational uprisings. It excludes one-sided political violence (e.g. government crackdowns on protesters), unorganized violence (e.g. spontaneous riots), and non-political violence (e.g. turf wars

rebellion (ala Hegre et al. 2001; Fearon and Laitin 2003; Collier and Hoeffler 2004). Rather, it examines the subsequent violent interaction between armed groups, and their competitive efforts to build a base of support. The narrative begins after government forces and rebels fail to reach a bargain that both prefer to warfare (Fearon, 1995; Reiter, 2003; Powell, 2006).[5] The narrative ends when one of the two sides re-establishes a monopoly, either through the other party's cessation of violence, or through neutralization of their ability to generate it (Tilly, 1997, 7:5).

Using a dynamical model of coercion and popular support, I prove the existence of a violence threshold, at which repression outpaces the opposition's ability to recruit and recover its losses. The theoretical discussion proceeds in several steps. I use a system of ordinary differential equations to describe a scenario where combatants compete for the support of a security-seeking population.[6] I show that such a system will converge to either a government or a rebel monopoly, depending on the relative costs the combatants inflict on each other's supporters. I then use a simple ascending bid game to study the mutual escalation and predict how much violence each side will use in equilibrium.[7] I show that the size of

between street gangs).

[5] Following Gates (2002); Azam and Hoeffler (2002), I assume that both sides have already overcome some of the collective action problems associated with organizing an armed group. "Core" rebel and government supporters already exist, and remaining collective action problems pertain to the retention and recruitment of personnel.

[6] In my attempt to model the dynamics of irregular warfare, I build most closely on the work of Atkinson and Kress (2012); Kress and Szechtman (2009); Deitchman (1962); Schaffer (1968); MacKay (2013), who adapt various extensions of Lanchester (1916) and Richardson (1919, 1935) combat and arms race models to an asymmetric setting (see Kress 2012 for a recent review of this literature). Unlike these and related operations research efforts, my model more explicitly accommodates the role of civilian agency and two-sided information problems, and drops the Lanchester framework in favor of a simpler approach based on models of biological and ecological systems.

[7] Although traditional dynamical models of combat (Richardson, 1919; Lanchester, 1916) and population ecology (Kermack and McKendrick, 1927; May and Nowak, 1995) take behavioral choice to be exogenous, my model considers optimizing strategic behavior on the part of the players. To accommodate some of these features, I adopt a hybrid approach that uses solution concepts from game theory to further explore combatant behavior in the dynamical system. Specifically, I employ a dynamical model to derive stability conditions for government and rebel monopoly equilibria, and use these stability conditions to determine expected payoffs under various types of strategic interactions. I then use a simple auction game to predict how much coercion the two combatants are

a coercive bid is decreasing in selectivity – the greater a combatant's informational disadvantage, the more violence it takes to meet the threshold. I close with a discussion of the model's observable implications.

To develop a theoretical benchmark, I make several simplifying assumptions, each of which I will subsequently loosen to allow for a richer parameterization and a more realistic conflict environment. First, I assume that the quality of the intelligence each combatant uses to punish her opponents is fixed, and that any subsequent improvement or deterioration in intelligence collection capacity comes too late to exert a substantive effect on the selectivity of violence. Second, I assume that combatants receive all of their support from the local population, and none from external sources.[8]

Summary of the argument

Consider a stylized conflict zone populated by two combatants – government forces and rebels – and a group of neutral civilians. Sovereignty is divided between the combatants, each of whom seeks to establish a monopoly on the use of force – locally, regionally or country-wide. They pursue this goal by extracting the resources needed to maintain military operations and establish a viable state – principally taxes, intelligence, supplies and manpower – while denying these same resources to their opponent. The civilians – whose cooperation both sides need to collect these resources – are interested in security above all else, and will cooperate with one of the sides or remain neutral – whichever is least costly.

To deter civilians from supporting the opponent, each combatant needs to make collaboration as costly as possible – by killing and capturing more of the opponent's supporters than the opponent can of one's own. The opponent, in turn, has strong incentives to reciprocate. Absent any constraints on the use of force, equilibrium behavior becomes one of mutual escalation, as each side attempts to "outbid" the opponent's use of coercion. Yet mass violence requires significant resources to implement, and their mobilization is subject to constraints, in the form of societal norms,

likely to use, and how information asymmetry shapes best response strategies.

[8] Although too restrictive to accurately convey the complexities of real-world combat, these assumptions are common ones in the civil war and counterinsurgency literatures (Hammes, 2006; Headquarters, Department of the Army, 2014; Kalyvas, 2006; Balcells, 2010). I impose them here for parsimony and conceptual clarity.

restrictive rules of engagement, or even a simple lack of ammunition.

To establish a monopoly on the use force, each combatant needs to escalate to the point where the opponent's response would require more resources than it is able to extract. This dynamic implies the existence of a threshold of violence, at which one side is unable to replace its losses with new recruits, and can no longer sustain the fight.

Coercion as a dynamical system

Let G_t and R_t denote the sizes of government and rebel forces at time t. Let C_t denote the size of the neutral civilian population at time t. Let $\pi_G(\mathbf{s}) = \frac{G_{eq}}{G_{eq}+R_{eq}} \in [0,1]$ denote the government's payoff from strategy set $\mathbf{s} = \{s_G, s_R, s_C\}$, or the government's share of public support at equilibrium. Similarly, let $\pi_R(\mathbf{s}) = \frac{R_{eq}}{G_{eq}+R_{eq}} \in [0,1]$ denote the rebels' payoff. An equilibrium outcome with $\pi_G = 1, \pi_R = 0$ is a case of *government victory*, in which the rebel population converges to zero and the government establishes a monopoly on the use of force. An outcome with $\pi_G = 0, \pi_R = 1$ is a *rebel victory*, similarly defined. Let $\pi_C\{\mathbf{s}\} = -\kappa \in (-\infty, 0]$ be the civilians' payoffs, defined as the costs inflicted on civilians by combatants.

The combatants $i \in \{G, R\}$ maximize their equilibrium shares of popular support by increasing the costs of cooperation with the opponents' group. Let $s_R : \rho_R > 0$ be the intensity of rebel military operations against government forces and $s_G : \rho_G > 0$ be the intensity of government operations against the rebels. As the relative intensity of violence inflicted against a group increases, cooperation with that group becomes more costly. However, combatants are unable to inflict these costs against their opponents with perfect accuracy.

Let $\theta_i \in (0,1)$ denote the selectivity of a combatant's coercive force, such that $\rho_i\theta_i$ is the proportion of punishment that i correctly inflicts against her opponent, and $\rho_i(1-\theta_i)$ is the share that erroneously befalls neutral civilians. Where selectivity is high, punishment is based on individual criteria (e.g. "target is a known rebel"). Where selectivity is low, punishment relies on collective criteria (e.g. "targets live where rebels are thought to be active"). The availability of information depends on exogenous barriers to intelligence collection, like ethno-linguistic differences and rough terrain, as well as the population's willingness to provide information.

Table 1: Notation Table

Symbol	Description
Population parameters	
$C_t \in [0, \infty)$	total neutral civilians at time t
$R_t \in [0, \infty)$	total rebel supporters at time t
$G_t \in [0, \infty)$	total government supporters at time t
Strategy choices	
$\rho_R \in (0, \infty)$	rebels' rate of punishment (s_R)
$\rho_G \in (0, \infty)$	government's rate of punishment (s_{G1})
Exogenous parameters	
$\theta_G \in (0, 1)$	government's selectivity
$\theta_R \in (0, 1)$	rebels' selectivity
$k \in (0, \infty)$	constant civilian immigration rate
$u \in (0, \infty)$	constant population death rate
Endogenous parameters	
$\mu_i = 1 - \frac{\rho_{-i}\theta_{-i}}{\rho_{-i}+\rho_i}$	rate of civilian cooperation with combatant $i \in \{G, R\}$ (s_C)
Objective functions	
$\pi_C(\mathbf{s}) = -\kappa(i)$	minimize costs associated with membership in group $i \in \{G, R, C\}$ $\kappa(G) = \rho_R\theta_R, \kappa(R) = \rho_G\theta_G$, and $\kappa(C) = \rho_R(1-\theta_R) + \rho_G(1-\theta_G)$
$\pi_G(\mathbf{s}) = \frac{G_{eq}}{G_{eq}+R_{eq}}$	maximize equilibrium share of popular support
$\pi_R(\mathbf{s}) = \frac{R_{eq}}{G_{eq}+R_{eq}}$	maximize equilibrium share of popular support

Coercion with low selectivity is inefficient not only because it inflicts fewer costs on the opponents ($\rho_i\theta_i < \rho_i$), but also because it inflicts harm on non-combatants. Civilians minimize the costs they expect to incur over the course of the conflict by staying neutral or choosing sides. If civilians join G or R, they will accrue costs at rates proportional to levels of selective violence inflicted against that group. If civilians stay neutral, they accrue costs in proportion to overall indiscriminate violence directed at civilians.

Lemma 1. *If selectivity is imperfect ($\theta_i < 1 \ \forall i \in \{G, R\}$), it is always more costly to remain neutral than to cooperate with one of the combatants.*

Proof. Appendix A.1 □

Lemma 1 states that the use of indiscriminate violence partially solves the combatants' collective action problem by rendering "free-riding" (i.e.

staying neutral) more costly than cooperation (Kalyvas and Kocher, 2007). Because civilians absorb damage from both government and rebel violence, staying neutral will always be strictly costlier than cooperating with the combatants – each of whom only absorbs damage inflicted by one side.

Let $s_C : \mu_i \in [0, \infty)$ be the rate of civilian cooperation with group i. Assuming this choice is primarily security-driven, civilians will cooperate with G and R in proportion to rates of survival in each group. μ_i must then be non-negative, and monotonically decreasing in costs inflicted against i. A simple formulation that meets these conditions is:

$$\mu_R = 1 - \frac{\rho_G \theta_G}{\rho_G + \rho_R} \tag{1}$$

$$\mu_G = 1 - \frac{\rho_R \theta_R}{\rho_G + \rho_R} \tag{2}$$

If G can inflict more selective violence against R than R can against G ($\rho_G \theta_G > \rho_R \theta_R$), then C will cooperate with G at a higher rate.

Taken together, the conflict dynamics comprise a system of ordinary differential equations

$$\frac{\delta C}{\delta t} = k - (\mu_R R_t + \mu_G G_t - \rho_R(1 - \theta_R) - \rho_G(1 - \theta_G) - u)\, C_t \tag{3}$$

$$\frac{\delta G}{\delta t} = (\mu_G C_t - \rho_R \theta_R - u) G_t \tag{4}$$

$$\frac{\delta R}{\delta t} = (\mu_R C_t - \rho_G \theta_G - u) R_t \tag{5}$$

where $\frac{\delta i}{\delta t}$ is the rate of change in the size of group i over time, k is an immigration parameter and u is a natural death rate, interpreted as losses due to disease, natural disasters and other exogenous factors that afflict civilians and combatants equally.[9]

As the fighting unfolds over time, the system in (3-5) will converge to one of two equilibria of primary interest: government monopoly or rebel monopoly.[10] The stability conditions for these equilibria depend on the strategic choices of combatants and civilians, and initial informational en-

[9] The immigration-death process is traditionally used in mathematical epidemiology to ensure a stable, non-negative population (Nowak and May, 1994; May and Nowak, 1995).

[10] I use the terms 'monopoly' and 'victory' interchangeably below.

dowments. Since the model is symmetrical, I focus the following discussion on conditions for government victory.

Proposition 1 ("Coercive advantage"). *A government monopoly is stable if and only if the government's rate of selective violence is greater than the rebels'.*

Proof. Appendix A.2 □

Proposition 1 states that – if combatants rely exclusively on coercion to attract support – victory is sustainable if and only if civilians expect cooperation with the opponent to be more costly. This result is in and of itself unsurprising, but it reveals an important threshold in the system.

To achieve a stable monopoly, each combatant must "outbid" the opponent's use of coercion by choosing a ρ_i above a minimum *stalemate threshold*, where each combatant matches the other's intensity of violence, scaled by the initial balance of selectivity between them:

$$\rho_i^* = \rho_{-i}\frac{\theta_{-i}}{\theta_i} \tag{6}$$

Where the government is unable to reach this threshold ($\rho_G < \rho_G^*$), its efforts become unsustainable and the system converges to a rebel monopoly. Where neither side has a coercive advantage ($\rho_G = \rho_G^*$), a stalemate occurs, with active support evenly split between the combatants.

Without constraints, equilibrium behavior becomes one of mutual escalation. If combatant i coerces at level $\rho_i > \rho_{-i}\frac{\theta_{-i}}{\theta_i}$, the opponent will respond by escalating ρ_{-i} to a level that meets or exceeds $\rho_i\frac{\theta_i}{\theta_{-i}}$. However, the two sides do not escalate equally. Where rebels enjoy an advantage in selectivity ($\theta_R > \theta_G$), government forces will need to employ a higher level of force to break even. Where rebel selectivity is overwhelming ($\theta_R \gg \theta_G$), government violence has to be even more overwhelming ($\rho_G \gg \rho_R$).

Such escalation is perilous for two reasons. The first is that constraints on the use of force often do exist, in the form of societal norms, restrictive rules of engagement, or even a lack of ammunition. Let $\bar{\rho}_i$ be the maximum level of force that i can employ, such that $\rho_i \in (0, \bar{\rho}_i]$. If $\rho_G^* > \bar{\rho_G}$, then the government will be unable to outbid the rebels. Second, escalation by the more indiscriminate side makes it increasingly costly for civilians to remain neutral. If $\theta_G < \theta_R$ and $\theta_G + \theta_R = 1$, then $\rho_G(1 - \theta_G) > \rho_G\theta_G$. If

the government fails to exceed the threshold $\rho_G \geq \rho_G^*$, this increased flow of popular support will go overwhelmingly to the other side.

To explore the role of constraints more fully, consider the combatants' strategic interaction in the context of a simple ascending bid game. At the outset of fighting, Nature specifies a profile $\bar{\rho} = (\bar{\rho_G}, \bar{\rho_R})$ of upper bounds on the use of force. Each combatant observes only her own upper bound $\bar{\rho}_i$ and chooses a coercive bid $b_i(\bar{\rho}_i) = \rho_i \in (0, \bar{\rho}_i]$. A combatant achieves victory ($\pi_i = 1$) if her bid is strictly higher than the stalemate threshold ρ_i^*. Defeat ($\pi_i = 0$) occurs if the bid is below this threshold, and stalemate ($\pi_i = 1/2$) occurs if it just matches it. We will assume that the combatants prefer a victory achieved by minimum force, and must pay an additional cost $\rho_{-i}\theta_{-i}$, determined by the rate of selective violence used by their opponent against them. The net benefits of fighting are then $\pi_i(\bar{\rho}_i - \rho_i - \rho_{-i}\theta_{-i})$. Because $\pi_i = \frac{x_i}{\sum_i^n x_i}$ at equilibrium, i receives $\bar{\rho}_i - \rho_i - \rho_{-i}\theta_{-i}$ if she wins and nothing if she loses.

Proposition 2 ("Coercive outbidding"). *If $\bar{\rho}_i$ are uniformly distributed on $[0, 1]$, the unique Bayesian Nash Equilibrium is $s_i(\bar{\rho}_i) = \frac{1}{1+\theta_i}\bar{\rho}_i/2 \; \forall i \in \{G, R\}$.*

Proof. Appendix A.3 □

Proposition 2 states that – even if violence is subject to an exogenous constraint on the use of force – the information problem creates strong incentives for escalation. In equilibrium, combatants will employ levels of punishment well below their respective limits ($\bar{\rho}_i$). How close they approach these limits depends on how easily they can identify and selectively punish their opponents. Where a combatant has very poor selectivity ($\theta_i \to 0$), her equilibrium coercive bid will be up to twice as high as where her selectivity is almost perfect ($\theta_i \to 1$).

Observable implications

What does a physical manifestation of violence actually mean in the context of the model? What we observe on the battlefield – and what later appears as integers in our spreadsheets – is not "strategy" as such (e.g. high ρ_R vs. low ρ_R), but the implementation of that strategy by people (e.g. high $\rho_R R$ vs. low $\rho_R R$). As Clausewitz (1832/1984, Book 1, Ch. 1) observed, the power to wage war is "the product of two factors... namely,

the sum of available means and the strength of the will." For a violent event to occur in a particular space and time, combatant i must choose a punishment level above zero ($\rho_i > 0$), *and* members of i's group must be physically present to implement that strategy. Best response dynamics may well call for a strategy of "kill a thousand soldiers for every rebel dead," but if the rebels have no forces in the area, they will do little killing.

A low level of violence may then indicate either a peaceful strategy being implemented by a large force, or a belligerent strategy implemented by a tiny force. One of the advantages of the dynamical model is that it accommodates predictions about both – including how force numbers might change over time as a result of the chosen strategy.

Observed violence is then a function of punishment and local group size:

$$y_{G,t} = \rho_G^* G_t \tag{7}$$

$$y_{R,t} = \rho_R^* R_t \tag{8}$$

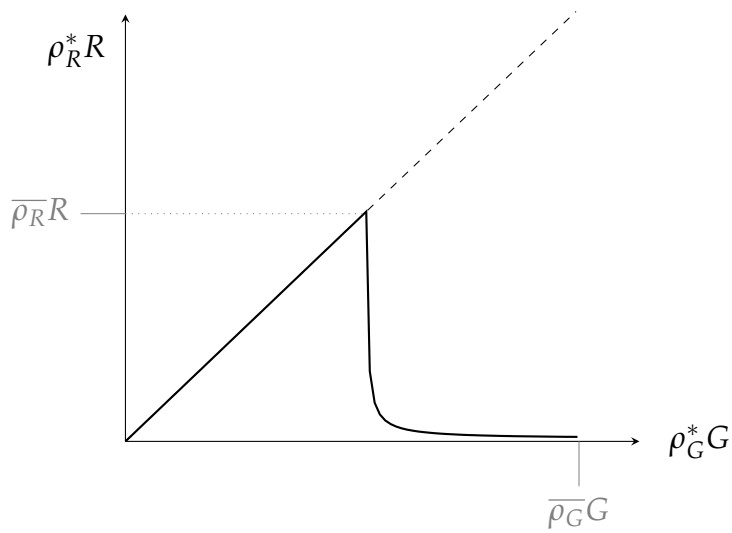
where $y_{i,t}$ denotes total violence by group i at time t, ρ_i^* is i's equilibrium level of punishment, and G_t, R_t are local group sizes at t. Because group sizes are time-variant and endogenous, they cannot be ascertained from an analysis of closed-form equilibrium solutions. But we can use numerical integration to obtain estimates at specific intervals of time, or cumulative measures of violence over a conflict's full history.

Figure 1 plots the expected relationship between government and rebel violence. The solid black curve shows cumulative levels of rebel violence ($\rho_R^* R$, vertical axis) associated with each hypothetical level of government violence ($\rho_G^* G$, horizontal axis). The figure assumes that both sides punish at stalemate level $\rho_i^* = \rho_{-i}\frac{\theta_{-i}}{\theta_i}$, subject to the constraint $\rho_i^* \leq \overline{\rho_i}$. The dashed, diagonal line shows the rebels' response curve without this constraint.[11]

The threshold effect is readily visible in Figure 1. Rebel violence first

[11] The curve is based on numerical integration of the system in Equations 3-5. Numerical integration was necessary to obtain values for G_t, R_t at each t. I assume that G and R have the same share of initial supporters at $t = 0$ ($G_0 = R_0 = .01$) and most of the population is initially neutral ($C_0 = .8$). I also assume that R has superior selectivity ($\theta_G = .075, \theta_R = .925$), but G has a higher upper bound on punishment ($\overline{\rho_G} = 100, \overline{\rho_G} = 50$). The natural immigration and death parameters are constant at $k = \frac{(\rho_R\theta_R + u - \alpha_G)(\rho_G(1-\theta_G)+\rho_R(1-\theta_R)+u)}{\mu_G} + c$ (see Proof to Proposition 1), $u = 1$. To produce the curve, I modified the level of government punishment ρ_G^* over the interval $(1, \overline{\rho_G})$, with

Figure 1: Stalemate threshold. Government violence on the horizontal axis ($\rho_G^* G$) and rebel violence on the vertical ($\rho_R^* R$). The solid black curve shows the coordinate pairs ($\rho_G^* G, \rho_R^* R$), with $\rho_i^* = \rho_{-i} \frac{\theta_{-i}}{\theta_i}$ and $\rho_i * \leq \overline{\rho_i}$. The dashed, diagonal line shows the same response curve without the upper bound on ρ_i^*.



rises in response to increases in government violence, and then drops exponentially. This change occurs where the government escalates to $\rho_G = \overline{\rho_R} \frac{\theta_R}{\theta_G}$, forcing the rebels to produce violence at maximum capacity. Because the rebels cannot employ a level of punishment greater than $\overline{\rho_R}$, they can no longer maintain a stalemate if the government escalates further. When the government does so, rebels start taking disproportionately high losses, security-seeking civilians begin cooperating with the government at higher rates, and the system starts to converge toward a government monopoly.

both sides playing the strategy $\rho_i^* = \begin{cases} \rho_{-i} \frac{\theta_{-i}}{\theta_i} & \text{if } \rho_i^* \leq \overline{\rho_i} \\ \overline{\rho_i} & \text{if } \rho_i^* > \overline{\rho_i} \end{cases}$. I then evaluated the behavior of the system at each strategy profile (ρ_G^*, ρ_R^*), and found coordinate pairs ($\rho_G^* G, \rho_R^* G$) by integrating the expressions in 7-8 over (t_0, t_{max}), with $t_0 = 0, t_{max} = 100$.

Empirical test

The central claim of the theoretical model is that, to suppress a rebellion, a government must escalate coercion beyond a stalemate threshold, inflicting more costs on the rebels than the rebels can against the government (Proposition 1). The current section tests this proposition with disaggregated data on government and opposition violence in Russia's Chechnya region (2000-2012).[12] I then examine the generalizability of these results with disaggregated data on hundreds of civil conflicts since 1979, from multiple event datasets covering Africa, Asia, Latin America and Europe.

To analyze the conflict in Chechnya, I use Toft and Zhukov (2015)'s' dataset on violence in the North Caucasus, which relies on incident reports from Memorial, a Russian human rights NGO. Within the territory of the Chechen Republic, this dataset includes 35,130 incidents of government violence and 9451 incidents of rebel violence. I aggregated these individual events to monthly indicators at the district (*rayon*) level, yielding an overall sample size of 2016 (14 districts $\times$ 144 months between May 2000 and March 2012).[13] For robustness, I replicated all analyses with regular grid cells (.5$\times$.5 degree PRIO-grid) as geographic units of analysis.

The outcome variable, rebel violence ($Y_{it} = \widehat{\rho_R R}$), is a local monthly event count, ranging from 0 to 64, with a mean of 5.[14] This measure includes any act of violence (e.g. firefight, ambush, hit-and-run attack, terrorist attack, hostage-taking, bombing) by armed opposition groups, such as the nationalist Chechen Republic of Ichkeriya and the Islamist Caucasus Emirate. The treatment variable, government violence ($T_{it} = \widehat{\rho_G G}$), is similarly distributed, from 0 to 346, with a mean of 17, and includes any act of violence (e.g. arrests, assassinations, sweeps, search and destroy missions, artillery shelling, air strikes) by Russian security forces and their local allies, like the pro-Moscow government of Chechen President Ramzan Kadyrov.[15] Figure 2 shows the geographic and temporal distribution of these data.

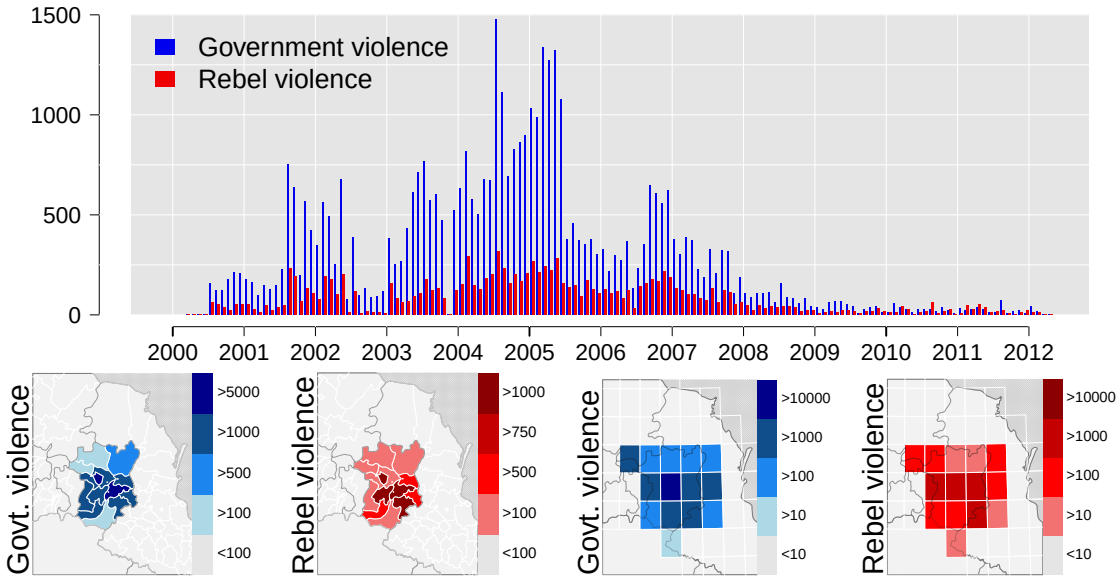
[12] This date range excludes the conventional phase of the Second Chechen War in August 1999 - April 2000, and begins after Russian forces re-established a presence in major population centers.

[13] The sample size is the same in grid-cell level data.

[14] In grid-cell level data, the range is 0 to 149, with a mean of 6.

[15] In grid-cell level data, the range is 0 to 714, with a mean of 21.

Figure 2: Geographic and temporal distribution of Chechnya data. Height of bars in time plot and shading in maps represent the intensity of violence per (a) district or (b) grid cell, over 2000-2012. T = government violence, Y = rebel violence.



(a) Districts (b) PRIO grid cells

Estimation strategy

To asses whether the expected threshold effect – graphically depicted in Figure 1 – aligns with real-world conflict dynamics in Chechnya, I estimate a dose–response function (DRF). The DRF represents the conditional expectation of the outcome (i.e. local intensity of rebel violence), given each level of treatment (i.e.recent exposure to government violence):

$$\psi(t) = E[Y_{it}(\tau)] \tag{9}$$

where $Y_{it}(\tau)$ is the potential outcome at $T = \tau$.

A challenge in estimating the DRF is that governments do not repress at random. Situations where the government represses at a high level are likely to be systematically different from situations where it does not. For example, the theoretical model predicts that incentives for escalation are

greatest where selectivity is low – due to poor information about one's enemies – and constraints on the use of force are limited (Proposition 2). The resulting bias makes it difficult to asses whether variation in rebellion is the outcome of government efforts to suppress it, or of other confounding factors that may have preceded the government's actions.

To adjust for this covariate imbalance – while accounting for the continuous nature of the treatment – I employ inverse generalized propensity score (GPS) weights, of the form $w_{it} = f(T_{it})/f(T_{it}|X)$, where $f(T_{it}|X)$ is the conditional density of treatment given covariates X and $f(T_{it})$ is a stabilizing factor based on the marginal probability of treatment (Robins, Hernan and Brumback, 2000). The logic of this approach is to create a re-weighted version of the dataset, in which more common government actions receive less weight (e.g. higher repression in rugged, difficult-to-control areas), and the level of treatment is weakly unconfounded by observable pre-treatment factors (Imbens, 2000).[16]

The estimation strategy proceeds in three steps. First, I find the GPS using several estimators, including Generalized Linear Models (Hirano and Imbens, 2004; Guardabascio and Ventura, 2013), covariate-balancing generalized propensity scores (CBGPS) Imai and Ratkovic (2014), and their non-parametric variants (npGBPS) (Fong, Hazlett and Imai, 2017).[17] I restrict this analysis to observations that overlap and have common support.

In the second step, I use each of the GPS estimates to calculate inverse probability weights $w_{it} = f(T_{it})/f(T_{it}|X)$, and model the conditional expectation of rebel violence. Here, I begin with a second-order polynomial approximation:

$$\ln(y_{it+1}) = \beta_1 T_{it} + \beta_2 T_{it}^2 + \gamma X_{it-1} + \alpha_i + \nu_t + u_{it} \tag{10}$$

[16] If $Y_i(\tau)$ is the potential outcome associated with treatment level τ and $D_i(\tau)$ is an indicator of receiving treatment τ, then weak unconfoundedness implies pairwise independence of treatment with each potential outcome, $D(\tau) \perp Y(\tau)|X$.

[17] The GLM estimator uses a Negative Binomial distribution for the treatment given the covariates, $T_{it}|X_{it-1} \sim NB(\theta X_{it-1} + \iota_i + \zeta_t, k)$, where X is a matrix of pre-treatment covariates, ι_i are local fixed effects, ζ_t are time fixed effects, and k is a dispersion parameter. The CBGPS relies on a generalized method-of-moments framework and uses a Normal conditional distribution for $T_{it}|X_{it-1} \sim N(\theta X_{it-1} + \omega_i + \zeta_t, \sigma^2)$. Following Fong, Hazlett and Imai (2017), I use a Box-Cox transformation on the treatment variable for CBGPS. The npCBGPS makes no distributional assumptions about T.

where y_{it+1} is the local intensity of rebel violence at $t+1$, T_{it} is the local intensity of government violence at t, and X_{it-1} is a matrix of local pre-treatment covariates, including terrain, population density, temperature, rainfall and previous levels of rebel violence.[18] To account for time-invariant local factors, I include district-level (or grid cell-level) fixed effects α_i. To account for common temporal shocks across all units, I include time fixed effects ν_t.

To ensure that the quadratic functional form in (10) is not driving the results, I also model the outcome with a threshold regression,

$$\ln(y_{it+1}) = \beta_1 T_{it}\, 1\{q_{it} \leq \tau^*\} + \beta_2 T_{it}\, 1\{q_{it} > \tau^*\} + \gamma X_{it-1} + \alpha_i + \upsilon_t + u_{it} \tag{11}$$

where $q_{it} = q(T_{it})$ is a threshold variable, and τ^* is the threshold value, estimated by $\widehat{\tau^*} = \arg\min_{\tau^* \in [\tau_0^*, \tau_1^*]} SSE(\tau^*)$. While the polynomial regression assumes that $E[Y_{it}(\tau)]$ is a continuous function of T, the threshold regression allows the DRF to be discontinuous around the threshold value.

In the last step, I use the results of (10,11) to obtain estimates of the full dose-response function, $\widehat{E[Y(\tau)]}$, by estimating the average potential outcome at each level of the treatment. As a benchmark, I also report results with an unweighted version of the data.

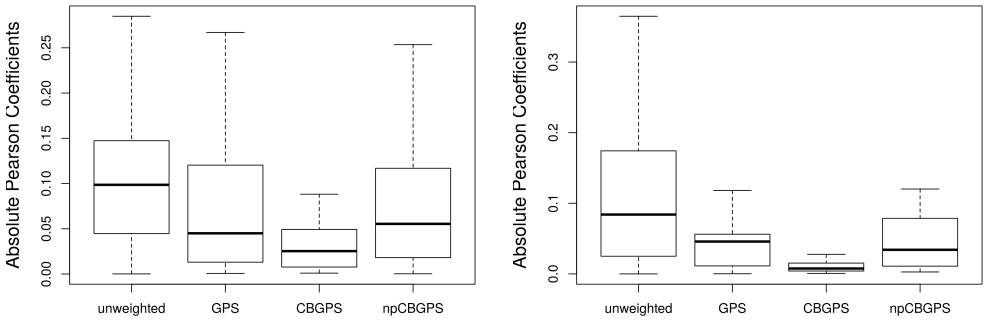
Figure 3 visualizes the degree of covariate balance achieved by each GPS method. The plot reports the distribution of absolute Pearson correlation coefficients between the treatment and each covariate, before and after weighting, with districts (a) and grid cells (b) as the geographic units of aggregation. Weighting by CBGPS improves covariate balance the most, particularly in grid cell-level data. Due to space limitations, I discuss mainly the CBGPS results below, although the other estimators yielded similar findings.

Evidence from Russia's operations in Chechnya, 2000-2012

The dynamics of violence in Chechnya provide strong evidence of a threshold effect. Figure 4 reports the average dose-response function for government and rebel violence in Chechnya, as estimated with both the polyno-

[18] Because Y is highly skewed, I use a logarithmic transformation.

Figure 3: Covariate imbalance. Box plots represent the distribution of absolute Pearson correlation between the treatment and each covariate after weighting. Whiskers indicate maximum and minimum values, boxes indicate upper and lower quartiles, thick lines indicates median values.



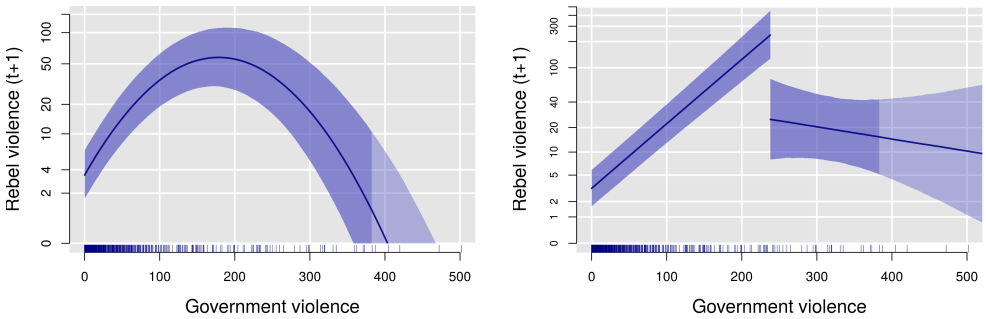
(a) District-month (b) PRIO grid cell-month

mial model (4a) and threshold regression (4b). Similarly to the predicted threshold effect in Figure 1, the DRF shows that intermediate levels of repression increase rebel violence, but higher levels decrease it.

The quadratic model (Figure 4a) finds an 'upside-down U' relationship between government and rebel violence. In an average locality, 4 rebel attacks occurred in months following no use of government violence, 50 attacks took place if the government escalated to 200 operations per month. But this number dropped to less than 2 attacks per month where the government was more extreme, at 400 operations per month (or 13 per day).

Similar patterns are apparent from the threshold regression (Figure 4b). Here, the relationship between repression and rebellion is initially strongly inflammatory: less than 5 rebel attacks following months with no government violence, but over 100 attacks after 200 government operations. Once repression exceeds 234 operations per month, however, the model predicts a sharp drop-off in rebel violence, from over 200 attacks the following month, to under 40. After that point, the relationship between repression and rebel violence leans negative.

Figure 4: Dose-response function, violence in Chechnya. Dark line represents conditional expectation of rebel violence (vertical axis) in the month following each level of government repression (horizontal axis). Shaded area is 95% confidence interval. Darker shade delineates the area of common support used for estimation. Short dashes indicate empirical distribution of treatment variable. Inverse probability of treatment weights estimated with CBGPS. Vertical axis on logarithmic scale.



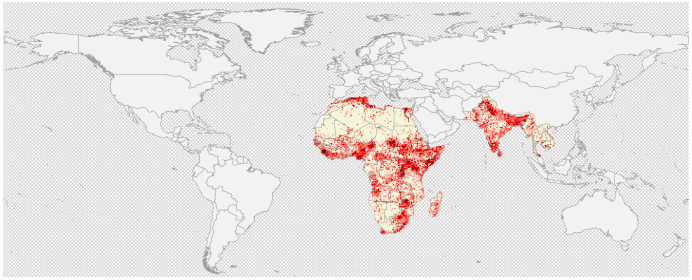
(a) Polynomial regression (b) Threshold regression

Evidence from conflicts in 145 other countries, 1964-2016

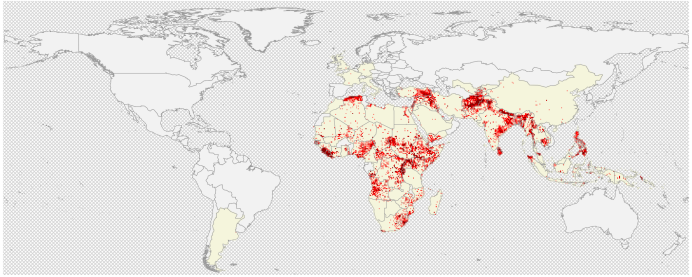
Is the threshold effect unique to Chechnya, or part of a broader trend? While the empirical patterns uncovered in this case align closely with theoretical expectations, one may worry that Chechnya is an idiosyncratic outlier, where relatively isolated rebels have confronted an unusually powerful government, one with few material or normative constraints on the use of force. To evaluate the generalizability of the Chechen case, I conducted a meta-analysis of sub-national conflict trends around the globe.

The meta-analysis seeks to replicate the Chechen results with armed conflict data on 145 countries, from four well-known multi-national event datasets: Armed Conflict Location and Event Data Project (ACLED) (Raleigh et al., 2010), the UCDP Georeferenced Event Dataset (UCDP-GED) (Sundberg and Melander, 2013), Political Instability Task Force (PITF) Worldwide Atrocities Dataset (Schrodt and Ulfelder, 2016), and Social Conflict Analysis Database (SCAD) (Salehyan et al., 2012). Figure 5 summarizes the geographic and temporal scope of the four datasets, along with the number of events each contains.

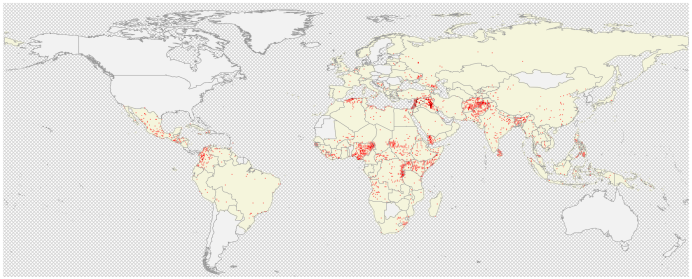
Figure 5: Geographic scope of data used in meta-analysis. Colors denote number of events per PRIO grid cell: 0, <10, <50, <100, >100.



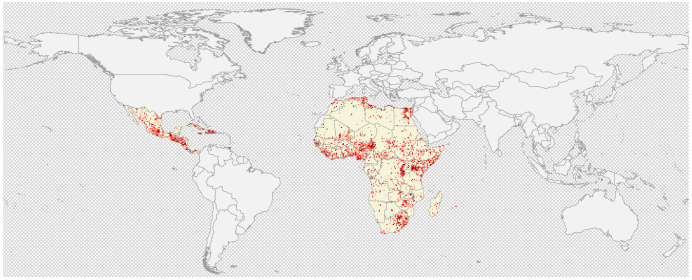
(a) ACLED: 175,518 events, 59 countries, 1997-2016



(b) GED: 419,959 events, 81 countries, 1989-2014



(c) PITF: 11,435 events, 123 countries, 1994-2016



(d) SCAD: 198,297 events, 60 countries, 1964-2015

To assemble these conflict events into consistent categories and units of analyses, I obtained pre-processed versions of the four datasets from the xSub data portal (Zhukov, Davenport and Kostyuk, 2017). For consistency, I used the same spatio-temporal scale as for Chechnya (PRIO grid cell, month). The outcome variable Y_{ikt+1} is the number of rebel attacks in grid cell i, county k during month $t+1$, and the treatment T_{it} is the number of government operations in the same location the previous month.

To examine variation in the DRF across countries as well as within them, I estimate a varying slope and intercept model, with a quadratic term:

$$\ln(y_{ikt+1}) = \beta_{k1} T_{ikt} + \beta_{k2} T_{ikt}^2 + \kappa_k + \gamma X_{ikt-1} + v_t + u_{ikt} \tag{12}$$

where each country k has a unique baseline level of violence (κ_k) and a uniquely-shaped relationship between repression and rebellion (β_k). To estimate these country-specific coefficients, the model utilizes sub-national variation in violence (Y_{ikt+1}, T_{ikt}) and pre-treatment covariates (X_{ikt-1}). I ran this model separately for each of the four datasets, and used the $\hat{\kappa}_k, \hat{\beta}_k$ parameters to estimate country-specific DRFs, $E[Y_{ikt}(\tau)]$.

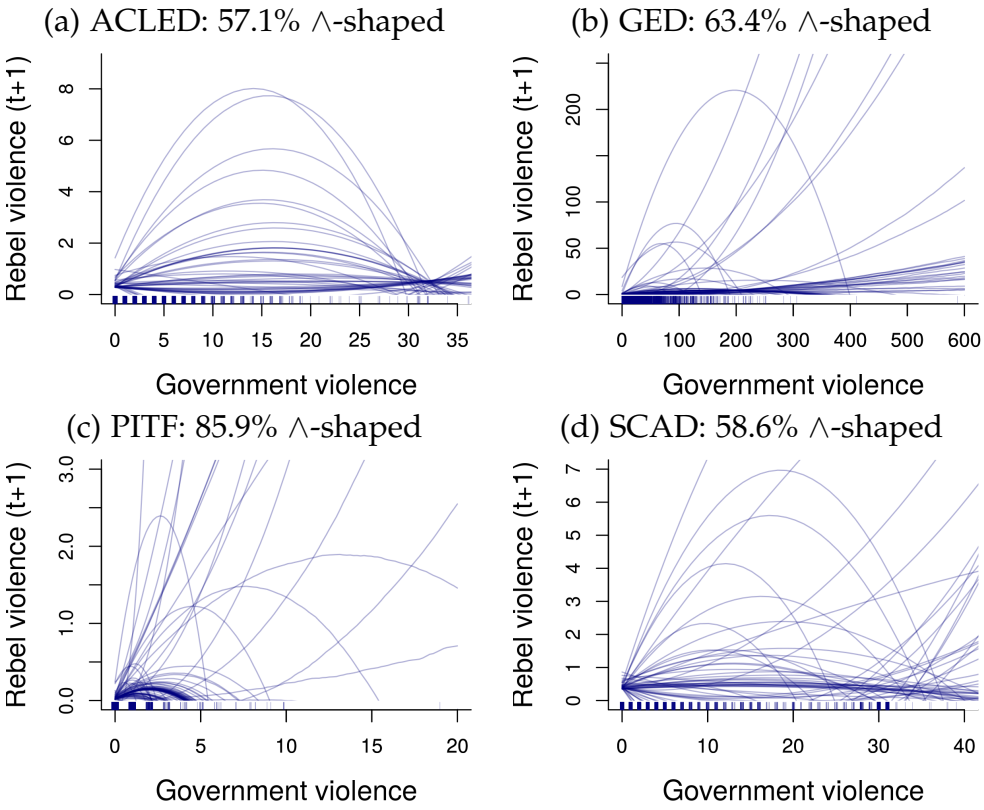
The global meta-analysis largely corroborates the evidence from Chechnya. Figure 6 reports the results, with each line representing the estimated DRF for a single country. For most countries in each dataset, the shape of the DRF is a concave, 'upside-down U'. This apparent threshold effect appears in 57.1% of the conflicts in ACLED, 63.4% in GED, 85.9% in PITF and 58.6% in SCAD. The scale of violence, however, varies greatly from country to country, as does the level of repression needed to reach the threshold.

Also clear from Figure 6 is that there are many cases where repression is only inflammatory, and never decreases rebellion. The DRF is strictly positive in 40.8% of conflicts in ACLED, 33.8% in GED, 12.1% in PITF, and 41.4%. It is strictly negative in less than 3% of cases, across all datasets.

How high the threshold?

While generally supportive of the theory, the empirical analysis raises two important questions. First, why does the relationship between repression and rebellion sometimes resemble a threshold (or 'upside-down U') but other times is strictly inflammatory? Second, why is the threshold high

Figure 6: Country-specific dose-response functions. Each line represents the estimated DRF for a single country in the dataset.



in some cases, and low in others? The second of these question may offer a potential answer to the first: the higher the threshold, the less likely a government is to reach it.

As is clear even in Chechnya (Figure 4), the empirical distribution of the treatment (intensity of repression) is highly skewed, with most observations falling on the left tail of the dose-response curve. Much of what we observe in practice, therefore, may be cases where the government uses an intermediate level of violence, which in turn only inflames the opposition. This censoring issue may also explain why repression often appears counterproductive in observational data: thankfully, we don't always witness cases where the government truly 'goes all out'.

If true, the theoretical implications are quite severe: backlash happens not because governments use repression, but because they sometimes do not repress enough. How much repression is 'enough,' however, varies from case to case. The most efficient coercion is one that hardly requires any violence at all. In this sense, a government that can deter with just one arrest is more 'efficient' than one who can only do so after arresting a thousand. Both governments are seeking the same objective, but the second can only reach it at a much higher cost – to itself, the rebels and civilians.

Why do some governments reach the threshold at a low level of violence, while others escalate to the extreme? I now return to the theoratical model, and relax some of its more restrictive assumptions. In doing so, I examine several ways in which the government might reduce the level of violence needed to reach the threshold: improving surveillance, cutting off opponents' external support, and "buying" support through private goods.

From a normative standpoint, none of these measures is inherently preferable to the others. The first two make coercion more efficient by restricting civil liberties. The third, while more clientelistic than autocratic, is only effective under a limited set of conditions. I summarize the logic of these model extensions here, and provide a formal discussion in the appendix.

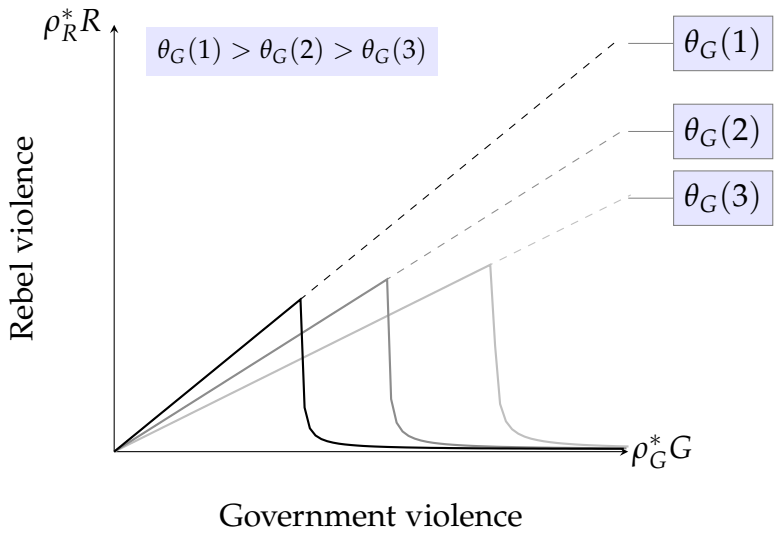
Mass surveillance

One way to lower the threshold is to collect more information about the opponent's supporters. If the government has a better grasp of who these supporters are and where they hide, its use of force can become less indiscriminate and more efficient. Figure 7 shows the same response curve as in Figure 1, with different values of the selectivity parameter θ_i. The threshold occurs at a lower level of government violence when selectivity is high ($\theta_G(1)$), and a higher level of violence when selectivity is low ($\theta_G(3)$).[19] As we should expect, higher selectivity increases the slope of the rebels' response curve. When the government is better able to distinguish rebels from civilians ($\theta_G(1)$), rebels need to use a higher level of punishment to keep up – causing them to hit their upper limit sooner.

How can the government improve its information, and make violence

[19] Numerical values were $\theta_G(1) = .06, \theta_G(2) = .075, \theta_G(3) = .095$ and $\theta_R(1) = .94, \theta_R(2) = .925, \theta_R(3) = .905$. All other parameters were the same as in in Figure 1.

Figure 7: Stalemate threshold and selectivity. Government violence on the horizontal axis ($\rho_G^* G$) and rebel violence on the vertical ($\rho_R^* R$). The solid black curves show the coordinate pairs ($\rho_G^* G, \rho_R^* R$), with $\rho_i^* = \rho_{-i} \frac{\theta_{-i}}{\theta_i}$ and $\rho_i * \leq \overline{\rho_i}$. Each curve assumes a different level of government selectivity θ_G, with $\theta_G(1) > \theta_G(2) > \theta_G(3)$.



more selective? Option one is to enhance human intelligence through a network of local informants. Recruitment of these informants, however, is subject to the same challenges as recruitment of supporters more generally: few will cooperate if it is not safe for them to do so. To explore these dynamics more directly, I consider an extension of the theoretical model, where selectivity is an endogenous, time-varying parameter (θ_{it}), which rises with the proportion of a combatant's supporters in the local population. As I show in Appendix B.1, however, the model's results are mostly unchanged; the system just converges the same equilibria more slowly.

Option two is to invest in intelligence capabilities that are less dependent on local support, such as surveillance. Solutions here can range from low-tech (e.g. taking a census, clearing forests to improve visibility) to high-tech (e.g. electronic intercepts, CCTV cameras, facial recognition). Surveillance is not a perfect substitute for human intelligence. The information it reveals is more plentiful, but also noisier. Yet by allowing the government

to passively monitor the population's movements, contacts and activities – at least those which are most readily visible – surveillance can improve selectivity on the margins. This new flow of information can enable the government to target rebels with higher precision, especially where it is very costly for informants to provide tips.

External support

A second way to lower the threshold is to isolate one's opponents, and cut them off from outside aid and resources. While the baseline model assumed that combatants rely exclusively on support from the local population, the availability of external support can complicate the armed struggle in non-trivial ways. In Appendix B.2, I consider an extension of the model, where one or both sides receives part of its resources (e.g. fighters, financial support) from outside the conflict zone. Unlike local support, which requires interaction with the local population, these additional resources do not depend on civilian cooperation.

This diversification of resources elevates the conditions needed for victory. While external support for the government can compensate for shortcomings due to poor information, abundant external support for rebels has the opposite effect, and can prevent a government monopoly even where the government otherwise has a decisive coercive advantage. This dynamic creates incentives for the government to further escalate violence. Local repression deters only locals from supporting the rebels. If rebels can offset local losses with resources from outside, they will be able to sustain themselves even where it is too costly for locals to support them.

If external support for rebels creates incentives for more government violence, then cutting off this support should have the opposite effect. By closing borders, setting up roadblocks and otherwise restricting population mobility, the government can reduce the flow of outside goods and personnel. This isolation makes the rebels more reliant on local sources of support, and hence more vulnerable to the government's coercive pressure. As with mass surveillance, however, these measures are most effective when they place significant restrictions on civil liberties.

Private goods

A potential alternative – to both coercive violence and totalitarian rule – is for a government to simply "buy" popular support, by offering private goods, like cash payments, political appointments, land, loot or other material incentives (Berman, Shapiro and Felter, 2011; Cooper, 2013; Weinstein, 2005, 2007). In Appendix B.3, I consider another extension of the model, where civilians are not solely security-driven, and where combatants can attract support by offering rewards as well as punishment. Even assuming that the government can exclude non-supporters from receiving these private goods, however, such an approach can only reduce the threshold under a limited set of circumstances.

If civilians respond to positive as well as negative inducements in deciding whom to support, a government offering sufficiently generous rewards can achieve a monopoly at a lower level of coercion. By the same token, however, a government offering too few rewards can easily lose, even if its level of violence is quite high. Much depends on how competitive the rebels' reward package appears to be. As I show in Appendix B.3, private goods can only compensate for a lack of coercive leverage if the gap between inducements offered by the two sides is rather vast.

In this sense, rewards do not negate the importance of punishment. At best, they offer a substitute. The greater the government's coercive disadvantage (i.e. the worse its information, the higher its stalemate threshold), the larger its rewards package must be. Yet even where the government has a coercive advantage, the weaker rebels can attract more civilian cooperation by offering the more compelling set of positive incentives: a more lucrative package of private goods, a more appealing ideological platform, or more charismatic political and military leadership. For this reason, we should expect repression to be particularly desperate and brutal where rebels have more to offer the population.

Conclusion

The central finding of this article is grim: repression works, just not in moderation. Government violence can suppress rebellion, but only if that violence is sufficiently high to convince civilians that supporting the rebels

is more costly than supporting the government. If the government is unable or unwilling to escalate to this point, it will only succeed in provoking reciprocal escalation by the rebels. This non-monotonic relationship holds in dozens of modern civil conflicts, across multiple datasets, and is robust to multiple estimation strategies.

Rather than asking why states repress, we may wonder why they don't repress more. The theoretical model offers several potential explanations. In many cases, mass repression does not occur because it is infeasible. A government may simply lack the resources to do it: the intensity of violence needed to reach the threshold exceeds what a government is capable and willing to produce. In such instances, repression is strictly inflammatory, and never achieves its intended deterrent effect.

In other cases, mass repression does not occur because it is avoidable or unnecessary. If the government has highly-accurate information on rebels' identities and whereabouts, it does not need to resort to indiscriminate tactics. If the government can isolate the rebels from sources of external support, the rebels become much more sensitive to coercion. If the government can offer a competitive package of private goods to its supporters, it can potentially 'buy' loyalty rather than obtain it through force. Under each of these scenarios, the government can reach its threshold at a lower level of violence. Yet these 'solutions' all come at a price: coercion becomes less lethal, but the population also becomes less free, and the government less publicly accountable.

If we conceive of war as part of the state-making enterprise (Tilly, 1985), the logic of repression may help us understand the wartime origins of autocracy and clientelism. To be effective, surveillance and curfews do not require cooperation from the general public: they are designed to control the population, not to earn its support. A citizen under constant monitoring, with no freedom of movement, is a citizen with few opportunities to rebel. Such a citizen may feel a strong motivation to oppose her government, and many of her compatriots may agree. Yet if she cannot organize and maintain an armed struggle, she cannot rebel.

Governments who defeat their challengers through repression and side-payments are likely to govern by these same means. A central implication of the theoretical model's equilibrium stability conditions is that – for a government monopoly to be stable – the policies used to achieve it must

remain in place indefinitely. The dismantling of surveillance, the lifting of roadblocks, the halting of payoffs to loyalists are all actions that risk upsetting this fragile equilibrium, should an opportunistic challenger arrive. This result explains why violence in Chechnya re-emerged in the 1990s despite two centuries of Soviet and Russian efforts to suppress it, from forcible disarmament to mass deportation.

Some may question the need to rationalize practices of such wanton cruelty and destructiveness. Many of these activities seem so cynical and callous as to defy explanation. It is tempting to dismiss humanity's darkest moments by citing the idiosyncrasies of political ideology, errors of judgment, or the personal whims of leaders. It is also tempting to dismiss the empirical basis for these theoretical claims as historical aberrations and regional peculiarities. It would also be a mistake. The data reveal that such patterns are common not only in Chechnya, but in hundreds of conflicts around the globe. The empirical regularity and persistence of repression oblige us to explain this phenomenon more fully.

The purpose of such research, needless to say, is not to advise dictators on how to repress their own people. They do not need such advice. As I have shown, political actors of many stripes already act in a manner consistent with the theory's predictions. If we are to understand why these acts of unspeakable cruelty happen, it is necessary to examine the incentives their perpetrators face, and how their targets are likely to react.

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Theoretical Appendix

Proof of Lemma 1

Proof. Let $\kappa(i)$ denote the expected costs associated with membership in group $i \in \{G, R, C\}$, with $\kappa(G) = \rho_R\theta_R, \kappa(R) = \rho_G\theta_G$, and $\kappa(C) = \rho_R(1-\theta_R) + \rho_G(1-\theta_G)$. The statement $[\kappa(C) < \kappa(G)] \wedge [\kappa(C) < \kappa(R)]$ ("staying neutral is less costly than joining either combatant") is never true for any $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1]$ and $\theta_G + \theta_R = 1$. The statement $[\kappa(C) < \kappa(G)] \wedge [\kappa(C) > \kappa(R)]$ ("staying neutral is less costly than joining G but more costly than joining R") is true if and only if $[\rho_G < \rho_R] \wedge \left[0 \leq \theta_G < \frac{\rho_R - \rho_G}{2\rho_R - \rho_G}\right]$, and $[\kappa(C) > \kappa(G)] \wedge [\kappa(C) < \kappa(R)]$ ("staying neutral is more costly than joining G but less costly than joining R") is true if and only if $[\rho_G > \rho_R] \wedge \left[\frac{\rho_G}{2\rho_G - \rho_R} < \theta_G \leq 1\right]$. The statement $[\kappa(C) > \kappa(G)] \wedge [\kappa(C) > \kappa(R)]$ ("staying neutral is more costly than joining G or R") is true in all other cases: (1) $[\rho_G > \rho_R] \wedge \left[0 \leq \theta_G < \frac{\rho_G}{2\rho_G - \rho_R}\right]$, (2) $[\rho_G < \rho_R] \wedge \left[\frac{\rho_R - \rho_G}{2\rho_R - \rho_G} < \theta_G \leq 1\right]$. □

Proof of Proposition 1

Proposition 1 depends on the following Lemma:

Lemma 2. *There exist three equilibrium solutions to (3-5) in which the outcome of the fighting does not depend on the initial balance of forces: government victory, rebel victory and mutual destruction.*

Proof. Define a *government victory equilibrium* of (3-5) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0,\infty), G_{eq} \in [0,\infty), R_{eq} \in [0,\infty)$ and $\pi_G(\mathbf{s}) = 1, \pi_R(\mathbf{s}) = 0$. These conditions hold at

$$C_{eq} = \frac{\rho_R\theta_R + u}{\mu_G} \tag{A.1}$$

$$G_{eq} = \frac{k}{\rho_R\theta_R + u} - \frac{\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u}{\mu_G} \tag{A.2}$$

$$R_{eq} = 0 \tag{A.3}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1], k \in (0,\infty), u \in (0,\infty)$, with μ_G, μ_R as defined in (1,2).

Define a *rebel victory equilibrium* of (3-5) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0,\infty), G_{eq} \in [0,\infty), R_{eq} \in [0,\infty)$ and $\pi_G(\mathbf{s}) = 0, \pi_R(\mathbf{s}) = 1$. These conditions hold at

$$C_{eq} = \frac{u + \rho_G \theta_G}{\mu_R} \tag{A.4}$$

$$G_{eq} = 0 \tag{A.5}$$

$$R_{eq} = \frac{k}{\rho_G \theta_G + u} - \frac{\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u}{\mu_R} \tag{A.6}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1], k \in (0,\infty), u \in (0,\infty)$, with μ_G, μ_R as defined in (1,2).

Define a *mutual destruction equilibrium* of (3-5) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0,\infty), G_{eq} \in [0,\infty), R_{eq} \in [0,\infty)$ and $\pi_G(\mathbf{s}) = 0, \pi_R(\mathbf{s}) = 0$. These conditions hold at

$$C_{eq} = \frac{k}{\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u} \tag{A.7}$$

$$G_{eq} = 0 \tag{A.8}$$

$$R_{eq} = 0 \tag{A.9}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1], k \in (0,\infty), u \in (0,\infty)$, with μ_G, μ_R as defined in (1,2). □

I now turn to the main proof of Proposition 1.

Proof. The stability of this equilibrium can be shown through linearization. Assume $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1]$, with μ_i as defined in (1,2). To ensure non-negative population values in equilibrium, we impose a lower bound on immigration parameter
$k > \frac{(\rho_R\theta_R+u)(\rho_G(1-\theta_G)+\rho_R(1-\theta_R)+u)}{\mu_G}$.

Let $\mathbf{J}$ be the Jacobian of the system in (3-5), evaluated at fixed point (A.1-

A.3).

$$\mathbf{J} = \begin{pmatrix} -\frac{k\mu_G}{\rho_R\theta_R+u} & -\rho_R\theta_R - u & -\frac{\mu_R(\rho_R\theta_R+u)}{\mu_G} \\ 0 & 0 & \frac{\mu_R(\rho_R\theta_R+u)}{\mu_G} - \rho_G\theta_G - u \\ \frac{k\mu_G-(\rho_R\theta_R+u)(\rho_G(1-\theta_G)+\rho_R(1-\theta_R)+u)}{\rho_R\theta_R+u} & 0 & 0 \end{pmatrix} \tag{A.10}$$

The determinant and trace of $\mathbf{J}$ are

$$\det(\mathbf{J}) = \frac{(-\rho_R\theta_R - u)\left(\frac{\mu_R(\rho_R\theta_R+u)}{\mu_G} - \rho_G\theta_G - u\right)(k\mu_G - (\rho_R\theta_R + u)(\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u))}{\rho_R\theta_R + u} \tag{A.11}$$

$$\mathrm{tr}(\mathbf{J}) = -\frac{k\mu_G}{\rho_R\theta_R + u} \tag{A.12}$$

The equilibrium point (A.1-A.3) is stable if all the eigenvalues of $\mathbf{J}$ have negative real parts, or $\det(\mathbf{J}) > 0, \mathrm{tr}(\mathbf{J}) < 0$. These conditions hold if and only if $\frac{\rho_G\theta_G}{\rho_R\theta_R} > 1$. □

Proof of Proposition 2

Proof. We assume that $\bar{\rho}_i$ is private information, but the distribution of $\bar{\rho} \sim U(0,1)$ is not. Let $u_i(\rho_i, \rho_{-i})$ be i's net payoffs from fighting,

$$u_i(\rho_i, \rho_{-i}) = \begin{cases} \bar{\rho}_i - \rho_i - \rho_{-i}\theta_{-i} & \text{if } \rho_i > \rho_i^* \\ \frac{1}{2}(\bar{\rho}_i - \rho_i - \rho_{-i}\theta_{-i}) & \text{if } \rho_i = \rho_i^* \\ 0 & \text{if } \rho_i < \rho_i^* \end{cases} \tag{A.13}$$

where $\rho_i^* = \rho_{-i}\frac{\theta_G}{\theta_R}$. Combatant i's expected utility is then

$$\begin{aligned} E[u_i(\cdot)] = &(\bar{\rho}_i - \rho_i - \rho_{-i}\theta_{-i})\, F\left(\rho_{-i}^*\right) \\ &+ \left(\frac{1}{2}(\bar{\rho}_i - \rho_i - \rho_{-i}\theta_{-i})\right) f(\rho_{-i}^*) + (0)\left(1 - F(\rho_{-i}^*)\right) \end{aligned} \tag{A.14}$$

We will assume that $b_i(\bar{\rho}_i)$ is strictly increasing, and ties occur with probability zero. From the CDF of $U(0,1)$, we obtain $F\left(\rho_{-i}^*\right) = \rho_{-i}^* = \rho_i\frac{\theta_i}{\theta_{-i}}$, and the objective function simplifies to

$$\max_{\rho_i}\ (\bar{\rho}_i - \rho_i - \rho_{-i}\theta_{-i})\left(\rho_i\frac{\theta_i}{\theta_{-i}}\right) \tag{A.15}$$

In a victory equilibrium the expression $\rho_{-i}\theta_{-i}$ has an upper bound of $\rho^*_{-i}\theta_{-i}$, or $\rho_i\theta_i$, which simplifies the function to

$$\max_{\rho_i} \; (\bar{\rho}_i - \rho_i(1+\theta_{-i}))\left(\rho_i \frac{\theta_i}{\theta_{-i}}\right) \tag{A.16}$$

from which we can obtain the first order conditions

$$\frac{\delta E[u_i]}{\delta \rho_i} = \frac{\theta_i}{\theta_{-i}}\left(\bar{\rho}_i - 2\rho_i(1+\theta_i)\right) \tag{A.17}$$

The FOC can be easily solved to find a symmetric BNE

$$\rho_i = \left(\frac{1}{1+\theta_i}\right)\frac{\bar{\rho}_i}{2} \tag{A.18}$$

□

Model extensions

Endogenous selectivity

Can gradual improvements in intelligence change combatants' incentives? The previous analysis rested on the assumption that combatants' ability to identify their opponents is a function of preexisting (i.e. prior to fighting) levels of control or popular support. Although such assumptions are common in the civil war literature, they are often violated in practice. The U.S. Army's counterinsurgency field manual, for instance, notes that the frequency and quality of reporting depends on the dynamics of fighting and recruitment: "Intelligence drives operations and successful operations generate additional intelligence" (Headquarters, Department of the Army, 2014, 3.25). As the number of rebel supporters in a conflict zone declines – due to attrition or defection – it becomes safer for civilians to cooperate with government forces. Meanwhile, if government operations alienate the populace – due to a lack of coercive leverage or any of the other reasons described above – it becomes less safe for civilians to offer information.

Let $\theta_{it} \in [0,1]$ be combatant i's selectivity at time t. Given a starting level of selectivity at time $t = 0$, this parameter changes over time as a function

of relative combatant support in the conflict zone:

$$\theta_{G,t+\Delta t} = \begin{cases} \theta_{G,0} & \text{if } t = 0 \\ f\left(\frac{G_t}{R_t+G_t}\right) & \text{if } t > 0 \end{cases} \tag{B.1}$$

$$\theta_{R,t+\Delta t} = \begin{cases} \theta_{R,0} & \text{if } t = 0 \\ f\left(\frac{R_t}{R_t+G_t}\right) & \text{if } t > 0 \end{cases} \tag{B.2}$$

where $\theta_{i,0} \in [0,1]$ is a constant initial value, $f(\cdot)$ is a monotone increasing, continuous function on $[0,1]$, G_t is the number of active government supporters, R_t is the number of active rebel supporters, and $R_t + G_t$ is the total combatant population in the conflict zone at time t.

By changing intelligence from a constant to a variable, we also induce changes to other parameters in the system, which depend directly or indirectly on θ. If we substitute $\theta_{i,t}$ for θ_i in the expressions for civilian cooperation (1,2), we obtain time-varying civilian strategies

$$\mu_{R,t+\Delta t} = 1 - \frac{\rho_G \theta_{G,t}}{\rho_G + \rho_R} \tag{B.3}$$

$$\mu_{G,t+\Delta t} = 1 - \frac{\rho_R \theta_{R,t}}{\rho_G + \rho_R} \tag{B.4}$$

and a more complicated system of equations

$$\frac{\delta C}{\delta t} = k - \left(\mu_{R,t} R_t + \mu_{G,t} G_t - \rho_{R,t}(1-\theta_{R,t}) - \rho_{G,t}(1-\theta_{G,t}) - u\right) C_t \tag{B.5}$$

$$\frac{\delta G}{\delta t} = (\mu_{G,t} C_t - \rho_{R,t}\theta_{R,t} - u) G_t \tag{B.6}$$

$$\frac{\delta R}{\delta t} = (\mu_{R,t} C_t - \rho_{G,t}\theta_{G,t} - u) R_t \tag{B.7}$$

Compared to (3-5), the system now includes several new sets of endogenous, time-varying parameters. The only static terms remaining in the model are the immigration and death rates k, u. Intelligence ($\theta_{i,t}$) changes as a function of R_t, G_t and civilian cooperation ($\mu_{i,t}$) changes as a function of $\theta_{i,t}$. We also allow coercive strategies ($\rho_{i,t}$) to adapt as new intelligence comes to light and opponents change their behavior.

These changes imply a new stalemate threshold

$$\rho^*_{i,t+\Delta t} = \rho_{-i,t} \frac{\theta_{-i,t}}{\theta_{i,t}} \tag{B.8}$$

To outbid her opponent, each combatant must determine an optimal level of punishment at the outset of the fighting ($\rho_{i,0}$) – based on the opponent's initial choice and the initial balance of selectivity – and then update it iteratively with new values of $\theta_{i,t}$ and $\rho_{-i,t}$.

These modifications render the system in (B.5-B.7) too complex for a closed-form equilibrium solution. To gain analytical traction and describe the behavior of the dynamical system over time, I turn to numerical methods. Specifically, I use 4th and 5th order Runge-Kutta numerical integration to solve the differential equations.[20]

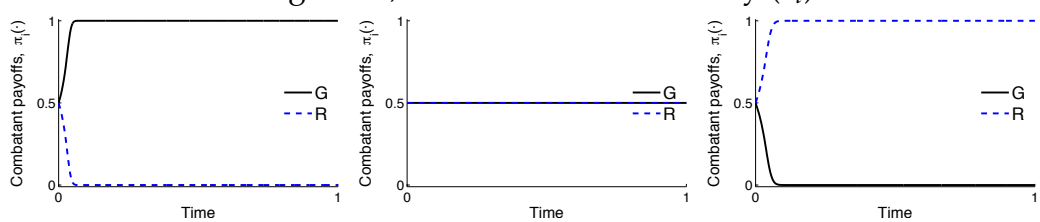
How do improvements or deteriorations in intelligence impact the dynamics of irregular war? As Figure B.1 suggests, the difference is one of duration rather than outcome. The equilibria reached (victory, stalemate, defeat) are the same as those in the exogenous selectivity case (top pane), which used the same starting values for all parameters. However, the system converges to these equilibria more slowly than before. Holding all else constant, the time needed to reach a government monopoly (Figure B.1d) is over 50 times longer than where intelligence is fixed (Figure B.1a).

What accounts for the longer duration? As the relative quality of intelligence changes over time (i.e. one combatant's ability to identify opponents improves, while the other's declines), one side's use of coercion consequently becomes more selective, while the other's becomes more indiscriminate. As a result, cooperation with the indiscriminate side becomes gradually more costly, and civilians respond to this change by cooperating at greater rates with the more selective combatant. The indiscriminate combatant responds to civilian defection by attempting to make cooperation with the opponent more costly – escalating violence, even if this violence is very inefficient. As civilian cooperation with the opponent slows down,

[20] For the purpose of the simulations, I assume a conflict zone that is at $t = 0$ evenly contested by government and rebel supporters, and populated predominantly by neutral civilians, with $C_0 = 100, R_0 = 5, G_0 = 5$. For simplicity, I assume that selectivity is at the outset of the fighting equal across the combatants, and the information problem is initially uniform, with $\theta_{G,0} = \theta_{R,0} = .5$. I take the intelligence gathering function $f(x) = fx$ to be linear, with $f = 1$. To ensure non-negative population values, I choose a k above the lower bound described in the proof to Proposition 1 ($k = 1000$), and take $u = 1$.

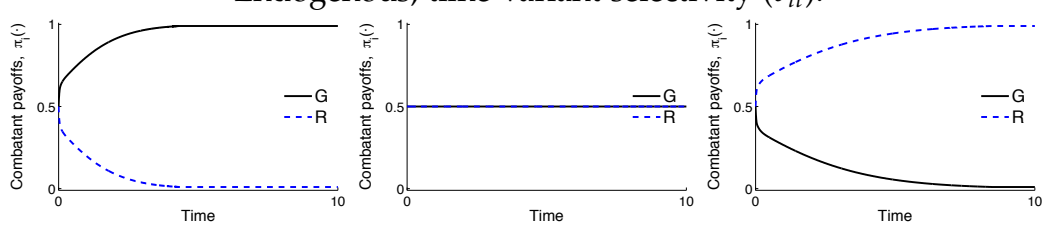
Figure B.1: Time to convergence. Vertical axis displays the combatants' payoffs at each t, with $\pi_G(\cdot) = \frac{G_t}{R_t+G_t}, \pi_R(\cdot) = \frac{R_t}{R_t+G_t}$. The government's payoffs are shown with a solid black line. A dashed blue line represents the rebels' payoffs. The horizontal axis shows the progression of time.

Exogenous, time-invariant selectivity (θ_i):



(a) $\rho_G > \rho_G^*$ (b) $\rho_G = \rho_G^*$ (c) $\rho_G < \rho_G^*$

Endogenous, time-variant selectivity (θ_{it}):



(d) $\rho_G > \rho_G^*$ (e) $\rho_G = \rho_G^*$ (f) $\rho_G < \rho_G^*$

and the latter now becomes starved of new intelligence, the opponent's violence in turn escalates and becomes more indiscriminate.

As a result, the stalemate threshold ρ_i^* increases exponentially over time, even if initial conditions do not favor either combatant. Adaptation to this escalatory dynamic tends to prolong the conflict, as both sides struggle to prevent civilian realignment by outbidding the other's coercive force. If no broad gap emerges between the relative costs of cooperation, it becomes more difficult for either combatant to rapidly consolidate civilian support.

External support

The preceding discussion assumed that both combatants rely exclusively on the local population for support. We will now loosen this assumption and take a deeper look at how external support affects conflict dynamics.

Let $\alpha_i \in [0, \infty)$ be the rate at which combatant i receives support from

outside the conflict zone. In addition to general necessities like water, food and ammunition, α_i may include some resources unique to each opponent. For the government, α_G may represent the ability to mobilize reserves, call up conscripts, and draw on any other sources of revenue and manpower that do not depend directly on the cooperation of local civilians. In a frontier, colonial or expeditionary conflict, such resources may be mobilized from regions closer to the state's administrative center, where the government's level of control is greater than in the periphery. For rebels, α_R may represent the ability to receive reinforcements, mobilize foreign fighters and units from sanctuary areas of neighboring states, or attract capital and labor from other regions, governments, charities, and ethnic diaspora elements located within or outside a country's borders.

To permit this diversification of combatants' sources of support, we modify the system of equations in (3-5) in the following manner:

$$\frac{\delta C}{\delta t} = k - (\mu_R R_t + \mu_G G_t - \rho_R(1-\theta_R) - \rho_G(1-\theta_G) - u)\, C_t \tag{B.9}$$

$$\frac{\delta G}{\delta t} = (\mu_G C_t + \alpha_G - \rho_R \theta_R - u) G_t \tag{B.10}$$

$$\frac{\delta R}{\delta t} = (\mu_R C_t + \alpha_R - \rho_G \theta_G - u) R_t \tag{B.11}$$

Note that unlike the flow of local support, which requires interaction with the local population ($\mu_i C_t$), external support (α_i) does not depend on any contact with civilians (C_t).

Proposition 3. *If the government has sufficient sources of external support, a coercive advantage is not necessary for victory.*

Proposition 3 depends on the following Lemma:

Lemma 3. *There exist three equilibrium solutions to (B.9-B.11) in which the outcome of the fighting does not depend on the initial balance of forces: government victory, rebel victory and mutual destruction.*

Proof. Define a *government victory equilibrium* of (B.9-B.11) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0, \infty), G_{eq} \in [0, \infty), R_{eq} \in [0, \infty)$

and $\pi_G(\mathbf{s}) = 1, \pi_R(\mathbf{s}) = 0$. These conditions hold at

$$C_{eq} = \frac{\rho_R \theta_R + u - \alpha_G}{\mu_G} \tag{B.12}$$

$$G_{eq} = \frac{k}{\rho_R \theta_R + u - \alpha_G} - \frac{\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u}{\mu_G} \tag{B.13}$$

$$R_{eq} = 0 \tag{B.14}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1], \alpha_G \in [0,\infty), \alpha_R \in [0,\infty), k \in (0,\infty), u \in (0,\infty)$, with μ_G, μ_R as defined in (1,2).

Define a *rebel victory equilibrium* of (B.9-B.11) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0,\infty), G_{eq} \in [0,\infty), R_{eq} \in [0,\infty)$ and $\pi_G(\mathbf{s}) = 0, \pi_R(\mathbf{s}) = 1$. These conditions hold at

$$C_{eq} = \frac{u + \rho_G \theta_G - \alpha_R}{\mu_R} \tag{B.15}$$

$$G_{eq} = 0 \tag{B.16}$$

$$R_{eq} = \frac{k}{\rho_G \theta_G + u - \alpha_R} - \frac{\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u}{\mu_R} \tag{B.17}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1], \alpha_G \in [0,\infty), \alpha_R \in [0,\infty), k \in (0,\infty), u \in (0,\infty)$, with μ_G, μ_R as defined in (1,2).

Define a *mutual destruction equilibrium* of (B.9-B.11) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0,\infty), G_{eq} \in [0,\infty), R_{eq} \in [0,\infty)$ and $\pi_G(\mathbf{s}) = 0, \pi_R(\mathbf{s}) = 0$. These conditions hold at

$$C_{eq} = \frac{k}{\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u} \tag{B.18}$$

$$G_{eq} = 0 \tag{B.19}$$

$$R_{eq} = 0 \tag{B.20}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1], \alpha_G \in [0,\infty), \alpha_R \in [0,\infty), k \in (0,\infty), u \in (0,\infty)$, with μ_G, μ_R as defined in (1,2). □

As Lemma 3 suggests, the addition of external support does not fun-

damentally change the range of possible outcomes in irregular war. The equilibrium solutions take forms nearly identical to the more restricted version of the model considered before. With the solutions in hand, I can now turn to the proof of Proposition 3.

Proof. Assume $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1], \alpha_G \in [0,\infty), \alpha_R \in [0,\infty), b \in [0,1]$. To ensure nonnegative population values in equilibrium, we impose a lower bound on the immigration parameter $k > \frac{(\rho_R\theta_R+u-\alpha_G)(\rho_G(1-\theta_G)+\rho_R(1-\theta_R)+u)}{\mu_G}$, with $\mu_G = 1 - \frac{\rho_R\theta_R}{\rho_R+\rho_G}$. By linearization, the government victory equilibrium is stable if all the eigenvalues of the Jacobian matrix of the system in (B.9-B.11), evaluated at fixed point (B.12-B.14), have negative real parts, or $\det(\mathbf{J}) > 0, \mathrm{tr}(\mathbf{J}) < 0$. These conditions hold if either (a) $\frac{\rho_G\theta_G}{\rho_R\theta_R} > 1$ and $\alpha_R < \overline{\alpha_R}$, where $\bar{\alpha_R} = \frac{\alpha_G(\rho_R+\rho_G(1-\theta_G))+(\rho_G+\rho_R+u)(\theta_G\rho_G-\theta_R\rho_R)}{(\rho_G+\rho_R(1-\theta_R))}$, or (b) $\frac{\rho_G\theta_G}{\rho_R\theta_R} < 1$, $\alpha_R < \overline{\alpha_R}$, and $\alpha_G > \underline{\alpha_G}$, where $\underline{\alpha_G} = \frac{(\rho_G+\rho_R+u)(\theta_R\rho_R-\theta_G\rho_G)}{(1-\theta_G)\rho_G+\rho_R}$.

□

Proposition 3 states that external support changes the conditions needed for government victory. Crucially, a coercive advantage ($\frac{\rho_G\theta_G}{\rho_R\theta_R} > 1$) is neither necessary for victory, nor is it sufficient. The dynamics also depend on critical values of government and rebel external support,

$$\underline{\alpha_G} = \frac{(\rho_G+\rho_R+u)(\theta_R\rho_R-\theta_G\rho_G)}{(1-\theta_G)\rho_G+\rho_R} \tag{B.21}$$

$$\overline{\alpha_R} = \frac{\alpha_G(\rho_R+\rho_G(1-\theta_G))+(\rho_G+\rho_R+u)(\theta_G\rho_G-\theta_R\rho_R)}{\rho_G+\rho_R(1-\theta_R)} \tag{B.22}$$

To evaluate the role of external support more intuitively, let us consider four scenarios, summarized in Table B.1. In the first (upper left), the government has an advantage in both selective violence and external support. Because $\alpha_R > \underline{\alpha_G}\ \forall \alpha_R \in [0,\infty)$, in this best-case scenario – and only in this scenario – a government victory equilibrium is always stable.

In the second scenario (upper right), the government retains a coercive advantage, but rebels have an advantage in external support. Here the government can sustain victory only if the rebels' rate of external support is below the critical value $\overline{\alpha_R}$. Conversely, this result suggests that abundant external support for rebels can prevent government victory even where the latter has a decisive coercive advantage. If the rebels' external support

advantage falls below this threshold, the government will lose the contest to the rebels – despite a more overwhelming use of coercion.

In the third scenario (lower left), the government has a disadvantage in selective violence, but an advantage in external support. Here, government access to external resources can compensate for a lack of coercive leverage, so long as $\alpha_R < \overline{\alpha_R}$ and $\alpha_G > \underline{\alpha_G}$. In the fourth and worst-case scenario (lower right), where the government has neither a coercive advantage, nor an external support advantage, a government monopoly is never stable.

Table B.1: Stability conditions for government monopoly, with external support. $\rho_G* = \rho_R \frac{\theta_R}{\theta_G}$ is the stalemate threshold.

Coercion	External Support	
	G advantage ($\alpha_G > \alpha_R$)	R advantage ($\alpha_G < \alpha_R$)
G advantage $\left(\rho_G > \rho_G^*\right)$	Stable	Stable if $\alpha_R < \overline{\alpha_R}$
R advantage $\left(\rho_G > \rho_G^*\right)$	Stable if $\alpha_R < \overline{\alpha_R}$, $\alpha_G > \underline{\alpha_G}$	Unstable

What determines the threshold levels of external support needed for victory? As the expression in (B.22) shows, $\underline{\alpha_R}$ is monotonically increasing in government external support $\left(\frac{\delta \underline{\alpha_R}}{\delta \alpha_G} > 0\right)$, and in the size of the government's coercive advantage ($\frac{\delta \underline{\alpha_R}}{\delta \Phi} > 0$, where $\Phi = \rho_G \theta_G - \rho_R \theta_R$). In other words, rebels can only use external support to compensate for a lack of coercive leverage if they (a) face governments with meager external resources, or (b) face governments whose own coercive advantage is razor thin. Because $\frac{\delta \underline{\alpha_R}}{\delta \Phi} > 0$, governments will face an incentive to offset external support for the rebels by increasing punishment, such that the selective violence ratio becomes much greater than one ($\frac{\rho_G \theta_G}{\rho_R \theta_R} \gg 1$).

These conditions imply that external support for rebels can create additional incentives for the escalation of government violence. Let's assume that the government is relatively isolated from external sources of support, such that $\alpha_G < \alpha_R$, but enjoys a coercive advantage $\frac{\rho_G \theta_G}{\rho_R \theta_R} > 1$. If we solve B.22 for ρ_G and take a partial derivative with respect to α_R, we obtain $\frac{\delta \rho_G}{\delta \alpha_R} > 0$ as long as $\alpha_G < \rho_R \theta_R + u$ or $\alpha_G < u$. In other words, if a gov-

ernment's level of external support is by itself too low to offset her losses, she will need to compensate for this difference by using coercion to attract additional local support. Under these conditions, an influx of external support for the rebels only aggravates the government's supply problem in a relative sense, provoking higher levels of coercion.

Private goods

Can a combatant with a coercive disadvantage simply "buy" popular support? Let $\iota_i \in [0, \infty)$ be the size of a reward package combatant i offers to her supporters. I make three assumptions about ι_i. First, these rewards are financed by "free" resources – such as natural resource rents or foreign aid – the extraction of which does not require the population's cooperation (Bueno De Mesquita and Smith, 2010; Smith, 2008). Second, i's supporters receive their rewards in wartime, rather than only after their side's potential victory.[21] Finally, I assume that combatants are able to perfectly identify and reward their own supporters, and – unlike punishment – they can exclude neutral civilians from receiving these side payments.

If civilians respond to positive as well as negative inducements, we need a new expression for their cooperation strategy μ_i^*. This expression must be decreasing in the amount of punishment civilians expect to receive as a supporter of i, increasing in the expected rewards, and remain globally non-negative. A simple formulation that meets these conditions is

$$\mu_R^* = 1 - \frac{\rho_G \theta_G}{\rho_G + \rho_R} + \iota_R = \mu_R + \iota_R \tag{B.23}$$

$$\mu_G^* = 1 - \frac{\rho_R \theta_R}{\rho_G + \rho_R} + \iota_G = \mu_G + \iota_G \tag{B.24}$$

which we can substitute into (3-5) to yield:

$$\frac{\delta C}{\delta t} = k - (\mu_R^* R_t + \mu_G^* G_t - \rho_R(1 - \theta_R) - \rho_G(1 - \theta_G) - u)\, C_t \tag{B.25}$$

$$\frac{\delta G}{\delta t} = (\mu_G^* C_t - \rho_R \theta_R - u) G_t \tag{B.26}$$

[21] In other words, the combatant distributes private goods to her supporters in every time period with a probability of one, rather than when the system reaches a steady state in which one's side has a monopoly.

$$\frac{\delta R}{\delta t} = (\mu_R^* C_t - \rho_G \theta_G - u) R_t \tag{B.27}$$

Proposition 4. *If the government offers a sufficient rate of private goods, a coercive advantage is not necessary for victory.*

Proposition 4 depends on the following Lemma:

Lemma 4. *There exist three equilibrium solutions to (B.25-B.27) in which the outcome of the fighting does not depend on the initial balance of forces: government victory, rebel victory and mutual destruction.*

Proof. Define a *government victory equilibrium* of (B.25-B.27) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0, \infty), G_{eq} \in [0, \infty), R_{eq} \in [0, \infty)$ and $\pi_G(\mathbf{s}) = 1, \pi_R(\mathbf{s}) = 0$. These conditions hold at

$$C_{eq} = \frac{\rho_R \theta_R + u}{\mu_G + \iota_G} \tag{B.28}$$

$$G_{eq} = \frac{k}{\rho_R \theta_R + u} - \frac{\rho_G(1 - \theta_G) + \rho_R(1 - \theta_R) + u}{\mu_G + \iota_G} \tag{B.29}$$

$$R_{eq} = 0 \tag{B.30}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0, \infty), \rho_R \in (0, \infty), \theta_G \in [0, 1], \theta_R \in [0, 1], \iota_G \in [0, \infty), \iota_R \in [0, \infty), k \in (0, \infty), u \in (0, \infty)$, with μ_G, μ_R as defined in (1,2).

Define a *rebel victory equilibrium* of (B.9-B.11) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0, \infty), G_{eq} \in [0, \infty), R_{eq} \in [0, \infty)$ and $\pi_G(\mathbf{s}) = 0, \pi_R(\mathbf{s}) = 1$. These conditions hold at

$$C_{eq} = \frac{\rho_G \theta_G + u}{\mu_R + \iota_R} \tag{B.31}$$

$$G_{eq} = 0 \tag{B.32}$$

$$R_{eq} = \frac{k}{\rho_G \theta_G + u} - \frac{\rho_G(1 - \theta_G) + \rho_R(1 - \theta_R) + u}{\mu_R + \iota_R} \tag{B.33}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0, \infty), \rho_R \in (0, \infty), \theta_G \in [0, 1], \theta_R \in [0, 1], \iota_G \in [0, \infty), \iota_R \in [0, \infty), k \in (0, \infty), u \in (0, \infty)$, with μ_G, μ_R as defined in (1,2).

Define a *mutual destruction equilibrium* of (B.9-B.11) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0, \infty), G_{eq} \in [0, \infty), R_{eq} \in [0, \infty)$ and

$\pi_G(\mathbf{s}) = 0, \pi_R(\mathbf{s}) = 0$. These conditions hold at

$$C_{eq} = \frac{k}{\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u} \tag{B.34}$$

$$G_{eq} = 0 \tag{B.35}$$

$$R_{eq} = 0 \tag{B.36}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1], \iota_G \in [0,\infty), \iota_R \in [0,\infty), k \in (0,\infty), u \in (0,\infty)$, with μ_G, μ_R as defined in (1,2). □

I now turn to the main proof of Proposition 4.

Proof. Assume $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1], \iota_G \in [0,\infty), \iota_R \in [0,\infty)$. To ensure nonnegative population values, we also impose a lower bound on immigration parameter $k > \frac{(\rho_R\theta_R+u)(\rho_G(1-\theta_G)+\rho_R(1-\theta_R)+u)}{\mu_G+\iota_G}$, with $\mu_G = 1 - \frac{\rho_R\theta_R}{\rho_R+\rho_G}$. By linearization, a government victory equilibrium is stable if all the eigenvalues of the Jacobian matrix of the system in (B.25-B.27), evaluated at fixed point (B.28-B.30), have negative real parts, or $\det(\mathbf{J}) > 0, \mathrm{tr}(\mathbf{J}) < 0$. These conditions hold if (a) $\frac{\rho_G\theta_G}{\rho_R\theta_R} > 1$, $\iota_R < \bar{\iota_R}$, or (b) $\frac{\rho_G\theta_G}{\rho_R\theta_R} < 1$, $\iota_R < \bar{\iota_R}$, $\iota_G > \underline{\iota_G}$, where $\bar{\iota_R} = \frac{\rho_G\theta_G-\rho_R\theta_R}{\rho_R\theta_R+u}\left(\frac{\iota_G(\rho_G\theta_G+u)}{\rho_G\theta_G-\rho_R\theta_R} + \frac{u}{\rho_G+\rho_R} + 1\right)$ and $\underline{\iota_G} = \frac{(\rho_G+\rho_R+u)(\rho_R\theta_R-\rho_G\theta_G)}{(\rho_G\theta_G+u)(\rho_G+\rho_R)}$ □

If we drop the assumption that cooperation is driven solely by the pursuit of security, and allow the civilians' choices to depend on both damage limitation and profit maximization, then selective violence ceases to be an indispensable condition for victory (Proposition 4). A combatant offering sufficiently generous rewards to her supporters can win the contest despite a coercive disadvantage; a combatant who offers too few rewards can lose despite an abundance of coercive leverage. The mere possibility of a low-coercion victory, however, does not mean that it is easily attainable.

The result depends on two critical values for positive inducements,

$$\bar{\iota_R} = \frac{\rho_G\theta_G - \rho_R\theta_R}{\rho_R\theta_R + u}\left(\frac{\iota_G(\rho_G\theta_G + u)}{\rho_G\theta_G - \rho_R\theta_R} + \frac{u}{\rho_G + \rho_R} + 1\right) \tag{B.37}$$

$$\underline{\iota_G} = \frac{(\rho_G + \rho_R + u)(\rho_R\theta_R - \rho_G\theta_G)}{(\rho_G\theta_G + u)(\rho_G + \rho_R)} \tag{B.38}$$

where $\bar{\iota_R}$ is an upper bound on rebel rewards and $\underline{\iota_G}$ is a lower bound on government rewards.

Table B.2: Stability conditions for government victory equilibrium, with private goods.

Coercion	Private goods	
	G advantage ($\iota_G > \iota_R$)	R advantage ($\iota_G < \iota_R$)
G advantage $\left(\frac{\rho_G\theta_G}{\rho_R\theta_R} > 1\right)$	Stable	Stable if $\iota_R < \bar{\iota_R}$
R advantage $\left(\frac{\rho_G\theta_G}{\rho_R\theta_R} < 1\right)$	Stable if $\iota_R < \bar{\iota_R}, \iota_G > \underline{\iota_G}$	Unstable

Table B.2 summarizes the conditions under which a government victory equilibrium is stable, under four scenarios. In the first and best-case scenario (top left), the government generates more selective violence and offers greater positive inducements than the rebels. Here, a government monopoly is always stable.

In the second scenario (upper right), the rebels offer a superior reward package, but the government maintains its coercive advantage. Government victory remains stable here as long as rebel rewards are not too high, with $\iota_R < \underline{\iota_R}$. As B.37 shows, this critical value is monotonically increasing in ι_G. An increase in the government's rewards, in other words, raises the bar that rebels must clear in order to negate the former's coercive success.

In the third scenario (lower left), rebels have a coercive advantage but the government offers greater positive incentives. For government victory to be sustainable under these conditions, it is necessary not only for rebel rewards to be very low ($\iota_R < \bar{\iota_R}$), but for government rewards to also be quite high ($\iota_G > \underline{\iota_G}$). The threshold value $\underline{\iota_G}$ depends in part on the scope of the government's coercive disadvantage ($\rho_R\theta_R - \rho_G\theta_G$). The larger the coercive disadvantage, the greater the government's reward offer must be in absolute terms, beyond simply exceeding the rebels'. Note that $\bar{\iota_R}$ is a function of ι_G, and $\bar{\iota_R} = 0$ if $\iota_G = \underline{\iota_G}$. If the government fails to exceed this threshold, then, the rebels will achieve success at any $\iota_R \in [0, \infty)$.

In the fourth and final scenario (lower right), the government has disadvantages in both selective violence and rewards. Under these worst of circumstances, a government victory equilibrium is never stable.

Where an opponent can offer generous rewards, there are powerful incentives for escalation. If we solve B.37 for ρ_G and take a partial derivative with respect to ι_R, we obtain $\frac{\delta \rho_G}{\delta \iota_R} > 0$ for all nonnegative parameter values. A sudden increase in the rewards offered by rebels – all other things equal – compels the government to deter civilian realignment through an even higher level of punishment.