Balancing Income and Bequest Goals in a DB/DC Hybrid Pension Plan

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Defined benefit (DB) plans can provide guaranteed income for life; however, there is no potential for wealth accumulation. Moreover, most DB plans offer little or no death benefit. On the other hand, defined contribution (DC) plans offer the potential for wealth accumulation; participants might retire quite comfortably and leave a generous bequest for their heirs. However, since the participant bears all of the investment and longevity risk in a DC plan, she also faces the possibility of outliving her accumulated wealth.

In this paper, we examine hybrids of DB and DC plans. We simulate investment returns and the time of death and we measure the hybrid plans' performance relative to income and bequest goals. Through this analysis, we quantify the trade-offs between the income security of a DB plan and the potential for wealth accumulation in a DC plan. In other words, we address the questions, "How much income security will I forfeit by focusing more on wealth accumulation?" and vice versa. In addition, we suggest allocations between DB and DC that perform particularly well relative to given metrics.

1. INTRODUCTION

Individuals face competing objectives in retirement planning: do they want a steady stream of guaranteed income? Do they want to accumulate wealth for healthcare costs, bequest, or unforeseen expenses? Or do they want some combination of the two? Defined benefit (DB) plans can provide guaranteed income for life; however, in general there is little to no potential for wealth accumulation. Moreover, in the United States, most DB plans primarily offer retirement benefits. As a rule, death benefits must be incidental to the primary purpose of the plan; those that offer non-annuity death benefits generally offer only a small burial benefit of $5,000 or a multiple of one or two times salary. (McGill et al., 1996) On the other hand, defined contribution (DC) plans offer the potential for wealth accumulation; participants might retire quite comfortably and leave a generous bequest for their heirs. However, since the participant bears all of the investment and longevity risk in a DC plan, she also faces the possibility of outliving her accumulated wealth.

The shift away from DB plans toward DC is well documented. Indeed, among wage and salary private-sector workers who participate in a retirement plan, in 2011, 69% had DC only, compared with 16% in 1979. Meanwhile, DB participation decreased from 62% in 1979 to just 7% in 2011. (EBRI, 2016) In his article, *The Crisis in Retirement Planning*, Robert Merton notes, “Once an add-on to traditional retirement planning, DC plans – epitomized by the ubiquitous 401(k) – have now become the main vehicles for private retirement saving.” (Merton, 2014) He observes further that the current emphasis on fund value, returns, and volatility is misguided; rather, individuals, employers, financial professionals, and regulators should be more attentive to investors’ primary goal of income security in retirement. According to a recent survey, 84% of respondents said that guaranteed monthly income for life is important to them; 48% said that this is the primary goal for a retirement plan. However, only 14% of respondents had purchased a life annuity. (TIAA-CREF, 2015)

Several researchers have examined the trade-offs between DB and DC plans. For example, Almeida and Fornier (2008) considered a hypothetical cohort of 1,000 newly-hired employees and calculated the contribution required to fund a target retirement benefit. They concluded that, because of the efficiencies of DB plans, “a DB plan can provide the same level of retirement income at almost half the cost of a DC plan. Hence, DB plans should remain a centerpiece of retirement income policy and practice…”

Poterba et al. (2007) used individual earnings histories and job tenure data from the Health and Retirement Study (HRS) and they simulated the distribution of retirement wealth under representative DB and DC plans. They found that DC plans produce higher average wealth accruals than private sector DC plans, but are more likely to produce very low retirement wealth accruals; in other words, on average, DC plans perform better than private sector DB plans, but the worst case scenarios are worse for DC. The comparison between DC and public sector DB plans is more ambiguous because public sector DB plans tend to be more generous.

Bodie et al. (1988) and McCarthy (2003) focused on workers' exposure to both wage risk and investment risk and examined the conditions under which risk-averse individuals would prefer a DB or DC plan. Bodie et al. concluded, "Neither plan can be said to wholly dominate the other from the perspective of employee welfare… Of interest for future research is the possibility of pension plan designs that combine the best attributes of DB and DC plans."

Minimizing the probability of lifetime ruin – that is, the probability of depleting one's wealth prior to death - is an increasingly important objective in retirement planning as more individuals are responsible for managing their retirement portfolios through DC plans. This is reflected in the retirement and annuity literature. For example, Young (2004) and Moore and Young (2006) considered a retiree who does not have sufficient wealth and income to fund her future expenses. They used stochastic optimal control to determine the allocation between riskless and risky assets that minimized the probability of lifetime ruin. Milevsky et al. (2006) considered a similar problem, but included life annuities in the financial market. Naturally, the ruin probability decreased when life annuities were included in the investment set.

In a different direction, Bayraktar et al. (2014, 2015) used stochastic optimal control to determine life insurance purchase strategies to maximize the probability of reaching a bequest goal. To the best of the authors' knowledge, these papers were the first to consider bequest as an objective.

In this paper, as Bodie et al. (2007) suggested, we consider hybrids of DB and DC plans. We examine how they perform in terms of both income and bequest goals; thus, we bridge the literature on ruin probability and bequest goals by considering both metrics.

More specifically, we consider an employee with a fixed bequest goal of M who can contribute to a DB plan that provides a small death benefit $M' < M$, a DC plan, or a weighted combination of the two.

We simulate the investment returns and the time of death and we examine the following outcomes for different combinations of DB and DC:

- Probability of depleting DC assets
- Age of depletion of the DC fund
- Probability of reaching the bequest goal
- Bequest amount, conditional on not depleting DC assets

- Mean, standard deviation, and coefficient of variation of discounted lifetime retirement income.

Through this analysis, we quantify the trade-offs between the income security of a DB plan and the potential for wealth accumulation in a DC plan. In other words, we address the questions, "How much income security will I forfeit by focusing more on wealth accumulation?" and vice versa.

The remainder of the paper is organized as follows. In Section 2, we describe our model and its underlying assumptions. In Sections 3 and 4, we present the results of various numerical experiments and sensitivity testing with our model. In Section 5, we present our conclusions.

2. THE MODEL

2.1 ASSUMPTIONS

Our model relies on the following basic assumptions.

- Each employee is hired at age 27 in the year 2015.
- No deaths or terminations of contract occur during employment.
- Each employee retires at age 67 in year 2055.
- Post-retirement mortality follows the Society of Actuaries RP-2014 Mortality Tables (male and female), with fully generational mortality improvement. (Society of Actuaries, 2014)
- Starting salary is $40,000.
- Salary increase rates are give below:

Years of Service	Wage Inflation Rate
0 -- 4	6.50%
5 -- 9	5.50%
10 -- 14	4.50%
>=15	3.00%

  Thus, the final salary is $181,445 and the 3-year final average compensation (FAC) is $176,212.
- Pre-retirement contributions and post-retirement payments are made mid-year.
- Defined benefit is based on the final three-year-average salary.
- Targeted replacement ratio is 70% of final salary, or $127,012
- We consider two values of the targeted death benefit (bequest goal): $M = 100{,}000$ and $M = 500{,}000$
- Death benefit from the DB plan is $M' = 10{,}000$. Note that this amount is small compared to the bequest goal.
- DB portfolio return is lognormally distributed with the following properties.
  - Expected return is 7.00%.
  - Standard deviation (volatility) is 11.50%.

  - Median return is 6.39%.
- DC portfolio return is lognormally distributed with the following properties.
  - Expected return is 6.00%.
  - Standard deviation (volatility) is 10.50%.
  - Median return is 5.48%.

We remark that the higher returns on DB funds is well documented; see, for example, Almeida and Fornia (2008).

2.2 METHODOLOGY

To calculate the DB normal cost and DC contribution rate, we started with pure DB and pure DC plans on a deterministic basis. The DB multiplier, DB normal cost, and the DC contribution rate calculated *a priori* so that, in a deterministic setting, they provide a 70% replacement ratio; in other words, so that the funds accumulate to provide a life annuity beginning at retirement of $127,012 per year. We detail these calculations in the next two subsections, respectively.

2.2.1 Setting the DB Multiplier and Normal Cost: Deterministic Returns

First, we project the salary using wage inflation rates listed in Section 2.1. The starting salary is $40,000, the final salary is $181,445, and the 3-year final average compensation (FAC) is $176,212. The DB multiplier is set to meet a target replacement ratio of 70%; i.e., it satisfies

$$\text{Multiplier} = \frac{\text{Final Salary} \times \text{Replacement Ratio}}{\text{Years of Service} \times \text{FAC}} = \frac{181{,}445 \times 70\%}{40 \times 176{,}212} = 1.80\%.$$

Thus, the annual defined benefit amount at retirement is, therefore, given by

$$\text{FAC} \times \text{Years of Service} \times \text{Multiplier} = \$176{,}212 \times 40 \times 1.80\% = \$127{,}012.$$

We determined the normal cost rate based on the entry age normal cost method. (Winkevoss, 1993) The normal cost rate required to fund an annual benefit payment of $127,012 is 10.58% of salary for males and 11.34% for females. Normal cost rates are higher for female employees because they have a longer life expectancy. DB assets begin with $0, and we assume that the balance at the beginning of each year grows at the median DB portfolio return of 6.39%. Then, we add normal contributions during employment and subtract benefit payments starting at age 67; both are made at mid-year and, therefore, credited or debited with a half-year of interest to bring the balance to the beginning of the next year. There is no unfunded liability in this deterministic setting.

2.2.2 Setting the DC Contribution Rate: Deterministic Returns

The DC contribution rates are calculated so that in the deterministic setting, the contributions will achieve the 70% replacement ratio; i.e., so that the accumulated fund will provide a life annuity of \$127,012 per year. The resulting contribution rates are 13.92% for males and 14.66% for females. As for the DB plan, to fund a given replacement ratio, female employees contribute more because they have longer life expectancy. Also, as in the pure DB case, DC assets start with a balance of \$0. The beginning balance grows at the median DC portfolio return of 5.48%. Then, we add employee contributions during employment and subtract benefit payments starting at age 67 both are made at mid-year and, therefore, credited or debited with a half-year of interest to bring the balance to the beginning of the next year. DC annuity factors are calculated separately for males and females by setting the annuity conversion rate to the median DC return. Thus, the annual benefit payment is given by

$$\frac{\text{Account Balance at Retirement}}{\text{Annuity Factor}} = \$127{,}012.$$

Note that this amount equals the annual benefit amount for the DB plan, as by design, and both equal 70% of final salary \$181,445.

2.2.3 The DB / DC Hybrid

Now, consider an employee who can choose how much to contribute to the DB and DC plans. Let $\alpha \in [0,1]$ be the allocation to the DB plan. Thus, for example, a female employee could allocate 11.34α% of her salary to the DB plan and $14.66(1-\alpha)\%$ to the DC plan, for a total contribution of $0.1134\alpha + 0.1466(1-\alpha) = \ 0.1466 - 0.0332\alpha$. We scale the DB multiplier by α and the DC contribution rate by $1-\alpha$, so that the total benefit payments from DB and DC plans add to \$127,012.

Accumulation Phase

We consider DB and DC funds in which annual contributions are invested at the rates described in Section 2.2.3. We simulate random returns each year for the funds using a lognormal distribution with the parameters given in Section 2.1. Due to the variability of returns, there usually exists unfunded liability in DB plans. DC payments stop if the fund value reaches 0.

Decumulation Phase

We assume that the retiree receives a life annuity of \$127,012α from the DB plan. Note that this payment is independent of market returns; all investment risk is borne by the plan sponsor. We assume further that the retiree withdraws \$127,012 (1-α) from the DC plan until the fund is depleted, or until the time of her death. In other words, we assume that she self-annuitizes the DC fund to achieve the target replacement salary of \$127,012.

At the time of death, the bequest amount is $M'\alpha = 10{,}000\alpha$ from the DB plan, plus the DC fund balance.

In the numerical experiments in Section 3, we examine whether the DC fund is depleted and whether the individual achieves her bequest goal of M.

2.2.4 Mortality

In our model, we assume all employees live to retirement. Starting at age 67, the probability of dying is simulated by Monte Carlo method, using q_x from RP-2014 table (Society of Actuaries, 2014) with fully generational mortality improvement. The male and female morality rates appear in Appendix A. We also assume a uniform distribution of deaths between integer ages.

Exhibits 1 and 2 summarize the distribution of the simulated age at death of our retirees. Based on 15,000 trials, we obtained 89 and 92 for the average age at death for males and females, respectively. These results are quite close to the life expectancy in the RP-2014 report with generational mortality improvement: 88 for males and 91 for females. The median ages at death (50th percentile) are slightly greater than the means because the distributions are negatively skewed, that is, they are skewed towards smaller ages of death.

[Exhibits 1 and 2 here]

3. RESULTS

We tested five DB weights: $\alpha = 0, 0.25, 0.5, 0.75$, and 1, to examine how the death benefit and lifetime payout vary with α. The following results are based on 15,000 simulations for each value of α.

3.1 PROBABILITY OF DEPLETING THE DC FUND; AGE AT DEPLETION

We first calculated the probability of outliving one's DC savings; see Exhibit 3. In each simulation, both females and males have just under a 50% probability of outliving their savings because we used the median rate of return to compute the DC contribution rate. We remark that the probability of depletion should be independent of α, since both the contributions and payouts are multiplied by α. We ran 15,000 scenarios for each value of α. From Exhibit 3, we see that there is little variation in the probability of depletion as we vary α.

[Exhibit 3 here]

Of the retirees whose DC assets deplete, we calculated the depletion age, the age at which a retiree's DC fund value hits zero. Exhibit 4 shows the mean, the median, and various percentiles of the retiree's depletion age. Note that these quantities are calculated based on retirees whose DC assets deplete before death.

[Exhibit 4 here]

We remark that the median depletion age is 86 for males and females, which is considerably lower than the median ages at death of 91 and 93.

3.2 BEQUEST

Suppose the retiree has a bequest goal of M, but the pure DB plan has a death benefit of M', which is small compared to M. In our model, we set M equal \$100,000 and M' equal \$10,000. In this case, the retiree needs to invest some money into the DC plan to increase her probability of reaching the bequest goal. As a result, the death benefit from the DB plan equals $\$10{,}000\alpha$, and the death benefit from the DC plan is the balance in the DC account at death. If the DC fund depletes before death occurs, the death benefit from the DC plan is \$0. Thus, the total death benefit from the α-hybrid DB and DC plan is given by

$$\$10{,}000\alpha + \max\{0, \text{DC assets at death}\}.$$

Here, the DC assets are weight-adjusted by the factor $1 - \alpha$ because the DC contribution rate scales by that factor. In the remainder of the paper, we will call the event of reaching the bequest goal a "success," with probability p. Then, the probability of "failure" is $q = 1 - p$. Exhibit 5 shows how the probability of success varies with α.

[Exhibit 5 here]

Clearly, because $M' < M$, there is a 0% probability of success for the pure DB plan ($\alpha = 1$).

There are two events that lead to failure:

1. The retiree outlives her DC savings, so that $\text{DC Assets at Death} = 0$.
2. The retiree has remaining assets in her DC account at death, but the assets are inadequate to reach the bequest goal; i.e., $\$0 < \text{DC Assets at Death} < M - \alpha M'$.

If the first event occurs (with probabilities listed in Exhibit 3), the retiree gets \$0 death benefit from her DC plan, and the DB death benefit is small compared to the bequest goal.

If the second event occurs, the DC account balance is nonzero, but the assets are not enough to make up for the difference between her bequest goal M and the death benefit $\alpha M'$from the DB plan. Let us now compute the probability of the second event. We know that the probability of failure contains the probability of outliving DC savings. Thus, the probability of the second event occurring can be calculated by subtracting the probabilities in Exhibit 3 from the probabilities of failure q, that is, from 1 minus the probabilities in Exhibit 5; see Exhibit 6 for the results. By comparing the results in Exhibit 3 and Exhibit 6, we notice that the major reason why retirees fail to reach their bequest goal is that they outlive their DC savings.

[Exhibit 6 here]

An attractive feature of DC plans is the potential for significant wealth accumulation. In Exhibits 7 and 8, we examine the distribution of the total death benefit (from both the DB and DC plans) for those retirees who do not deplete their DC savings. For example, we found that over 8% of males who are enrolled in the pure DC plan (α=0) reach a bequest of over $5 million.

[Exhibits 7 and 8 here]

The conditional expected value of total death benefit increases significantly as α decreases. For the pure DC plan, the conditional expected death benefit is $2,993,902, which is almost 300 times greater than the death benefit from the pure DB plan, although the median is less than $2,000,000 because the conditional distribution of death benefits is skewed heavily to the right. If the retiree is willing to take more risk by investing more in the DC plan (smaller α), she might end up with a large bequest. However, such an investment strategy is quite risky since there is large variability in the death benefit amount, as we see from Exhibits 7 and 8. Remember that these figures show the death benefits for those who do not deplete their DC plans before dying. Thus, in addition to the variability shown in these figures, there is also a large proportion (47%) who completely deplete their DC plans, so that their bequest equals only $\$10{,}000\alpha$.

3.3 LIFETIME PAYOUT

In this section, we examine the expected discounted lifetime payout at retirement. We designed the annual benefit payment to be $127,012 regardless of α. Let simulated age of death be n, and DC depletion age be n'. Then the present value of lifetime payout can be calculated by:

$$\alpha \times \$127{,}012 \times a_{\overline{n-67+0.5}|0.07} + (1-\alpha)\times \$127{,}012 \times a_{\overline{\min\{n,n'\}-67+0.5}|0.06},$$

where $a_{\overline{m}|i}$ denotes the present value at rate i of $1 per year for m years. Unlike our deterministic models, here we set the discounting factor to the mean portfolio return (7% for DB, 6% for DC). Based on the mid-year payment assumption, we discount the benefit payment by another half year.

In Exhibits 9 and 10, we see that expected discounted lifetime payout at retirement increases with α. This occurs because, in DC plans, people have the risk of not receiving benefit payment due to assets depleting, while in DB plans such risk is borne by the employer. Values of R^2 close to 1 also suggest there is a strong, positive linear relation between lifetime payout and DB weight.

[Exhibits 9 and 10 here]

On the other hand, the standard deviation of discounted lifetime payout is not monotonic; see Exhibits 11 and 12.This non-monotonicity is caused by the different influences of investment and longevity risks on the retiree's payout. The lifetime payout from the pure DC and any other hybrid plans are affected by both the real market return and the individual's longevity. By contrast, with the lifetime payout from the pure DB plan, the individual faces no investment risk. Its standard deviation is only due to the variability in longevity. Moving from the pure DB to the pure DC plan, the investment risk for the individual increases from zero. An individual's longevity has an asymmetric influence on her lifetime payout for any hybrid plan, that is, $0 < \alpha < 1$. Indeed, those who have longer life expectancies get more DB payout on average, but their DC assets face more investment risk for a longer period.

[Exhibits 11 and 12 here]

To have a better sense of the risk-return tradeoff, we consider another measurement. Specifically, consider the reciprocal of coefficient of variation (CV), given by

$$\frac{1}{CV} = \frac{\mu}{\sigma},$$

in which μ is the expected discounted lifetime payout and σ is the corresponding standard deviation. In our context, the reciprocal of coefficient of variation measures how many dollars of expected discounted lifetime payout one receives per dollar of risk. In Exhibits 13 and 14, we see that the payout-risk ratio increases with α until α reaches approximately 0.75 for both female and male. Also, note that the payout per unit risk for the pure DB plan is not much less than when $\alpha = 0.75$, so if an individual cares more about consumption than any bequest goal, she might choose $\alpha = 0.75$ or even $\alpha = 1$.

[Exhibits 13 and 14 here]

4. SENSITIVITY TESTING

In this section, we test the sensitivity of the results to the choice of bequest goal *M* by increasing the value of *M* from \$100,000 to \$500,000. We recall that in Section 3.2, we defined a success to be meeting the bequest goal of *M*, and we denoted the probability of success by *p*. The values of

p when $M = 100{,}000$ are given in Exhibit 5. Observe that p decreases as the allocation to the DB plan α increases, but the slope is small between α=0 and α=0.5. One could interpret this as follows: as one increases her allocation to the DB plan from α=0 to α=0.5, she gains income security from the higher DB payout with only a small decrease in the probability of reaching her bequest goal.

It is natural to ask if this result holds for a larger bequest goal. To address this question, we set M=500,000 and let p' denote the probability of success for this value of M. The DB death benefit, $M' = 10{,}000$, is unchanged.

From Exhibits 15 and 16, we see that p' decreases more sharply as α increases. Thus, when the bequest goal M is larger, if one increases α to gain more income security, she must sacrifice more probability of reaching her bequest goal. Moreover, the difference between p and p' grows as $\alpha < 1$ increases. The probability of success decreases by about 5% for the pure DC plan ($\alpha = 0$), and it decreases by 26% for $\alpha = 0.75$. Thus, the probability of success is more sensitive to the bequest goal as one places less weight on the DC plan and more weight on the DB plan.

[Exhibits 15 and 16 here]

Another sensitivity test is to compare the causations of failure to meet one's bequest goal. As we discussed in Section 3.2, we know there are two situations that lead to failure: (1) depleted DC assets at death, and (2) positive but inadequate DC assets at death. We calculate the *conditional* probability of these two causations by dividing their respective unconditional probabilities by $q = 1 - p$. The conditional probability tells us the percentage of the failures that are caused by inadequate DC assets (event 2) versus ruin (event 1). From Exhibits 17 and 18, we notice a significant increase in the conditional probability of inadequate DC assets when we raise the bequest goal. For example, when $\alpha = 0.5$, inadequate DC assets accounts for only about 10% of failure when the bequest goal is \$100,000, while this percentage increases to about 38% for the larger bequest goal.

[Exhibits 17 and 18 here]

5. CONCLUSIONS

If an individual values only income security and is not interested in bequest or wealth accumulation, she should invest 100% in a DB plan. Similarly, if she values only wealth accumulation and bequest and has no concern about income security, she should allocate all of her resources to a DC plan. Naturally, most individuals are somewhere in the middle and have both income and bequest goals. In this paper, we examined the trade-offs between defined benefit (DB) and defined contribution (DC) retirement plans; our model *quantifies* the gain and loss behind different strategies and helps a retiree find a rational way to allocate between pension plans.

We summarize our main results below.

- In our model, the probability that a retiree outlives her DC account was about 47%, regardless of the allocation to the DC plan.

- When the difference between the DB death benefit and the bequest goal is smaller (in our case, $M' = 10{,}000$ versus $M = 100{,}000$), one could increase the DB allocation from $\alpha = 0$ to $\alpha = 0.5$ without a significant reduction in the probability of meeting the bequest goal. So, if the individual wants to ensure a good probability of meeting a bequest goal of \$100,000 while guaranteeing some minimum level of income, then $\alpha = 0.5$ might be a good choice. With $\alpha = 0.5$, the probability of reaching the bequest goal is about 47% (compared to a maximum of 51% when $\alpha = 0$), and the probability of depleting the DC assets is about 47%, after which one would continue to receive $\$127{,}012\alpha = \$63{,}506$ each year from the DB plan.

- When the difference between the DB death benefit and the bequest goal is larger (for example, $M' = 10{,}000$ versus $M = 500{,}000$), the probability of reaching the bequest goal declines more steeply as one allocates more to the DB plan. In other words, as one allocates more to the DB plan to achieve income security, one sacrifices more bequest potential.

- The expected discounted value of lifetime income increases linearly (or almost linearly) with α. The payout-risk ratio is not strictly increasing with DB weight because of the interaction between the mortality and investment risk. The payout-risk ratio was maximized at $\alpha = 0.75$

- Our simulation results show that the pure DC plan gives probabilities of 51% and 46% of reaching bequest goals of \$100,000 and \$500,000, respectively. Moreover, there is potential for significant wealth accumulation; for the pure DC plan, the expected total death benefit is about \$3 million, conditional on not depleting the DC fund.

- Our calculations recognize two reasons for the failure to reach the bequest: (1) DC assets deplete before the retiree dies; and (2) remaining DC assets at death are not sufficient to make up for the difference between the bequest goal and the death benefit from the DB plan. For a bequest goal of \$100,000, most failures are caused by the first event, but for the larger bequest goal, a more significant proportion of the failures are caused by the second.

- As a result of our study, individuals who care only about consumption during retirement should invest in the pure DB plan to maximize their post-retirement income with the lowest contribution rate. On the other hand, individuals with bequest motives should invest in a hybrid plan to increase the chance of reaching their bequest goals. As for pension plan allocation, there is no "best"α– the choice of α depends on the retiree's preference between leaving assets to her heirs or guaranteeing more income during retirement.

- For many people, $\alpha = 0.5$ might serve as a good rule of thumb; as we observed in Section 4, for a bequest goal of $100,000, that particular value of α results in probability of success that is close to the probability for a pure DC plan while ensuring income of 50% of the pure DB payout. Furthermore, by moving α from 0.25 to 0.5, say, the probability of reaching the bequest goal decreases by less than 3%, while the expected discounted lifetime payout at retirement increases by around $35,000.

There are many ways in which our model may be extended and improved. For example, we do not allow diversities in hiring and retirement ages, nor do we consider early termination of contract that could affect the accumulation of assets. Also, we have not allowed non-level contribution rate during employment. In real life, employees usually have the ability to adjust their contribution rates annually. Often employees make small contributions early in their careers and gradually increase their contributions as they near retirement. A non-level contribution rate will affect the accumulation of assets in the DC plan and thereby affect the post-retirement benefit payment. Also, we have not considered the risk in DB benefit payments due to corporate changes in pension arrangements, such as the risk of reduced benefit payment or unexpected plan termination. Addressing these issues would enhance our model; these are possible directions for future work.

Exhibit 1

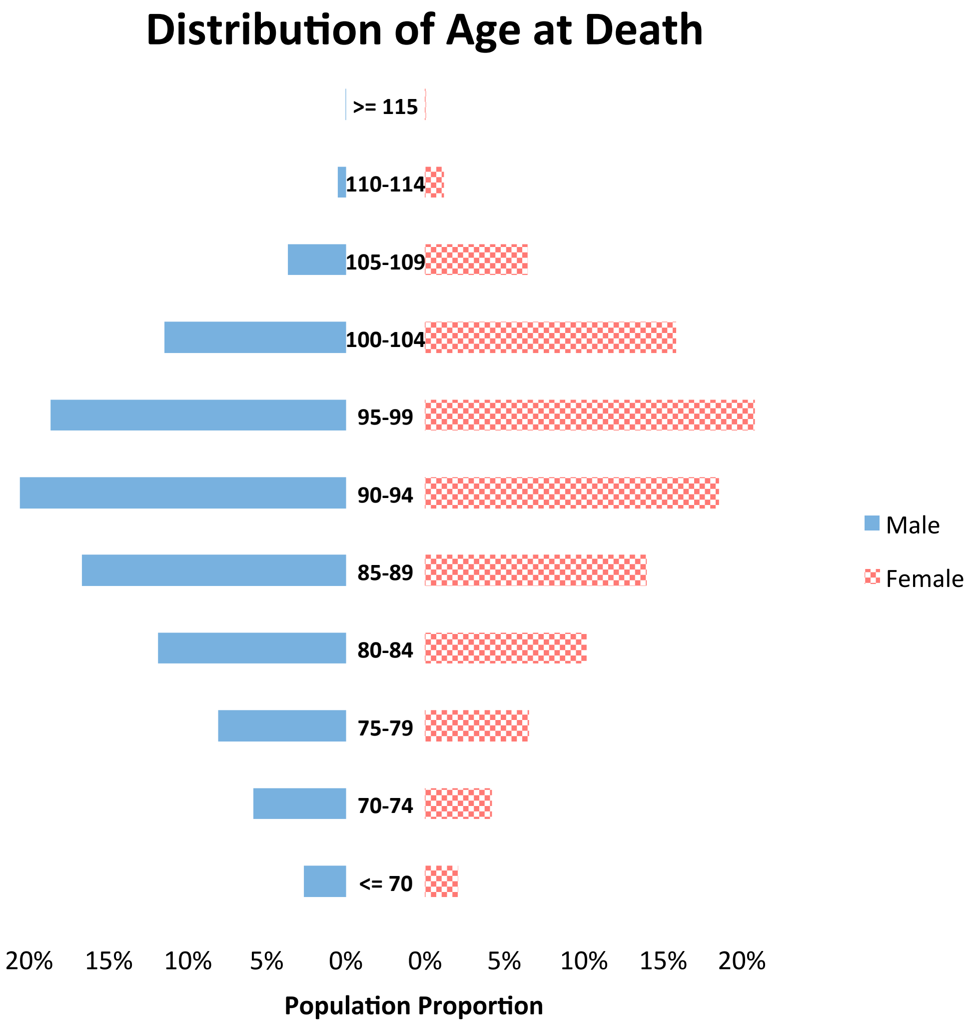

Distribution of Age at Death
>= 115
110-114
105-109
100-104
95-99
90-94
85-89
80-84
75-79
70-74
<= 70
Male
Female
20%
15%
10%
5%
0%
0%
5%
10%
15%
20%
Population Proportion

Exhibit 2

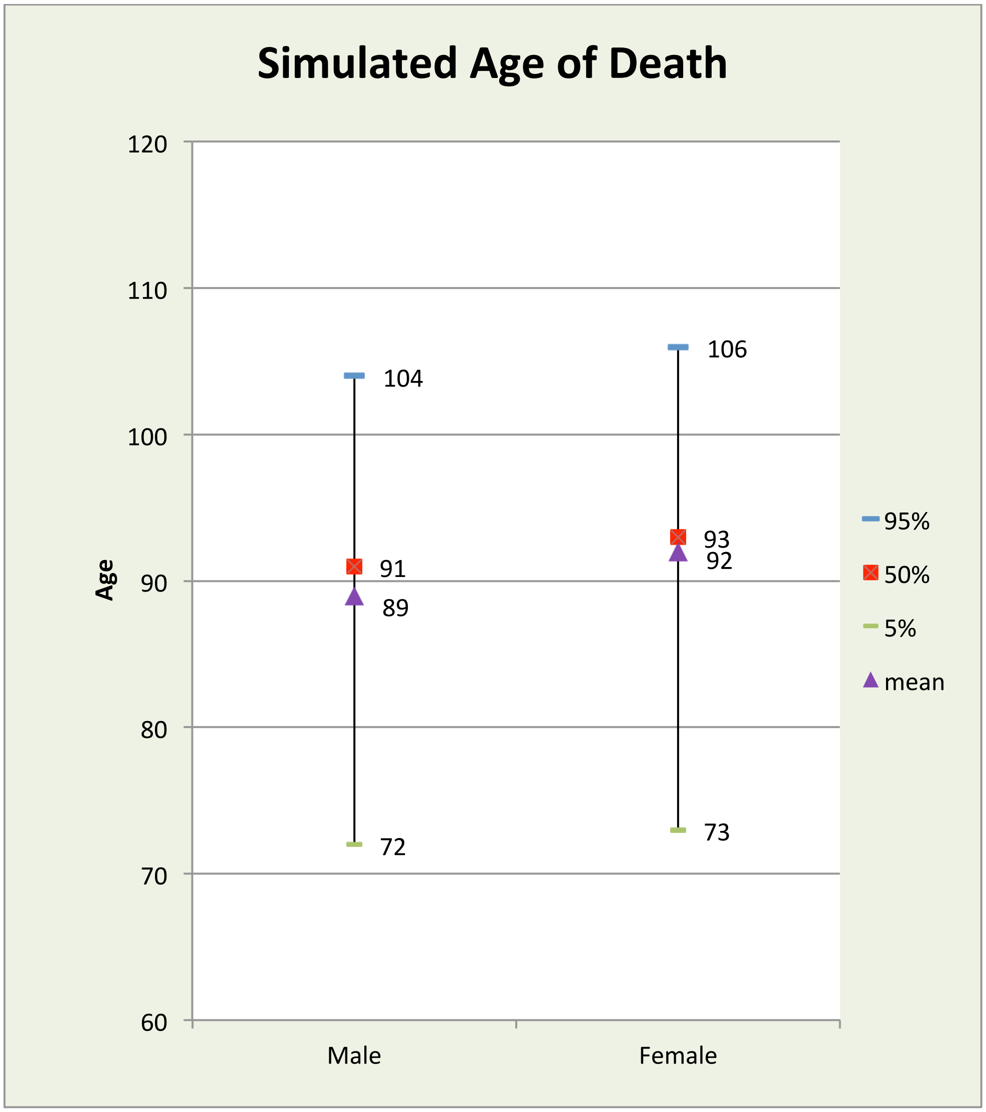


Exhibit 3

Probability of Depletion

DB Weight α:	0	0.25	0.5	0.75	1
Probability of Depletion (female)	47.21%	48.23%	48.12%	47.57%	N/A
Probability of Depletion (male)	47.43%	47.09%	46.80%	47.15%	N/A

Exhibit 4

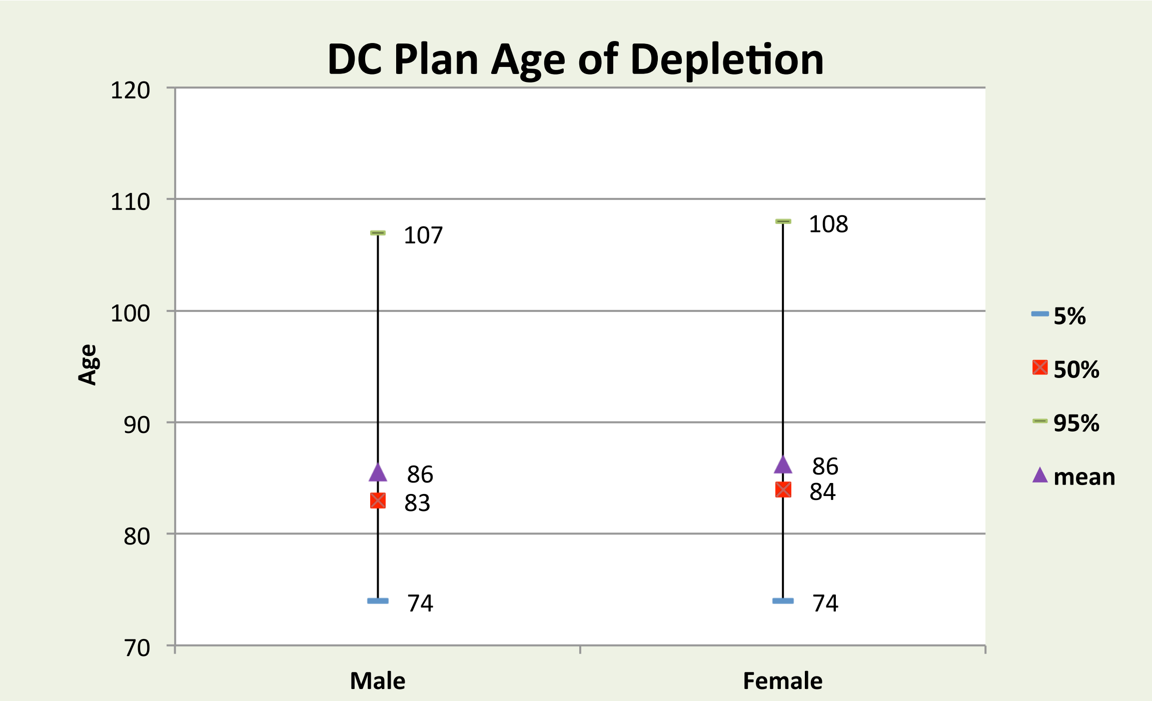


Exhibit 5

Probability of Reaching Bequest Goal

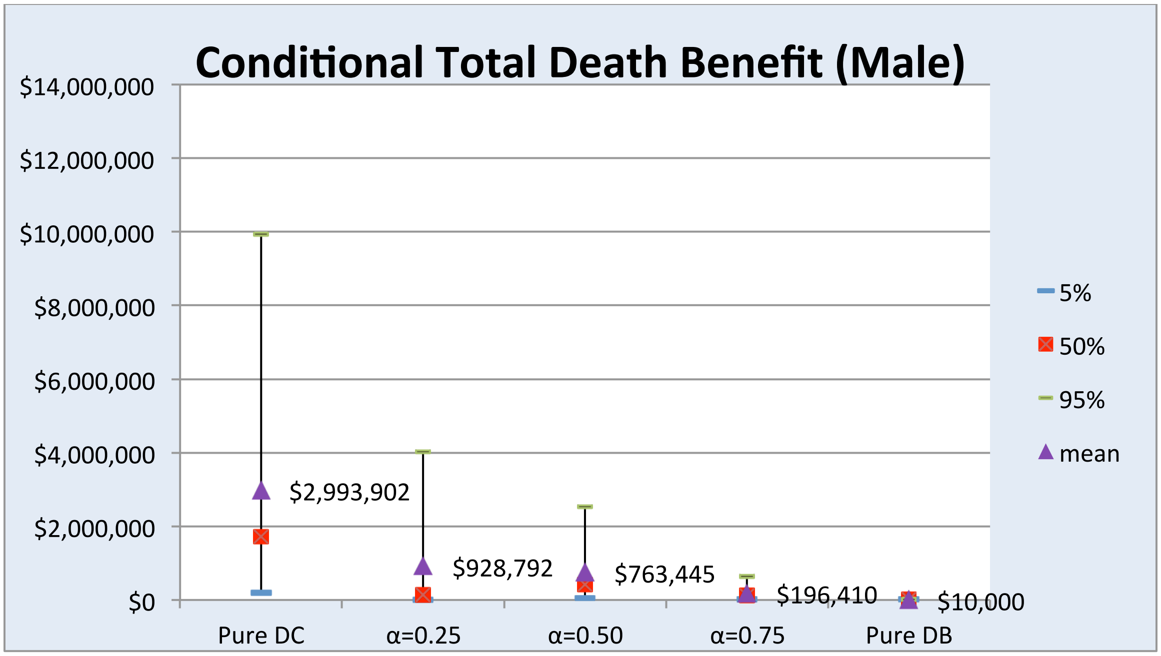
DB Weight α:	0	0.25	0.5	0.75	1
Probability of Success (female)	51.48%	49.43%	46.75%	31.63%	0.00%
Probability of Success (male)	51.25%	50.09%	47.25%	29.37%	0.00%

Exhibit 6

Probability of Positive but Insufficient Bequest

DB Weight α:	0	0. 25	0. 5	0. 75	1
Prob{0<DC Assets at Death<M-αM'} (female)	1.31%	2.33%	5.13%	20.81%	N/A
Prob{0<DC Assets at Death<M-αM'} (male)	1.33%	2.81%	5.95%	23.48%	N/A

Exhibit 7



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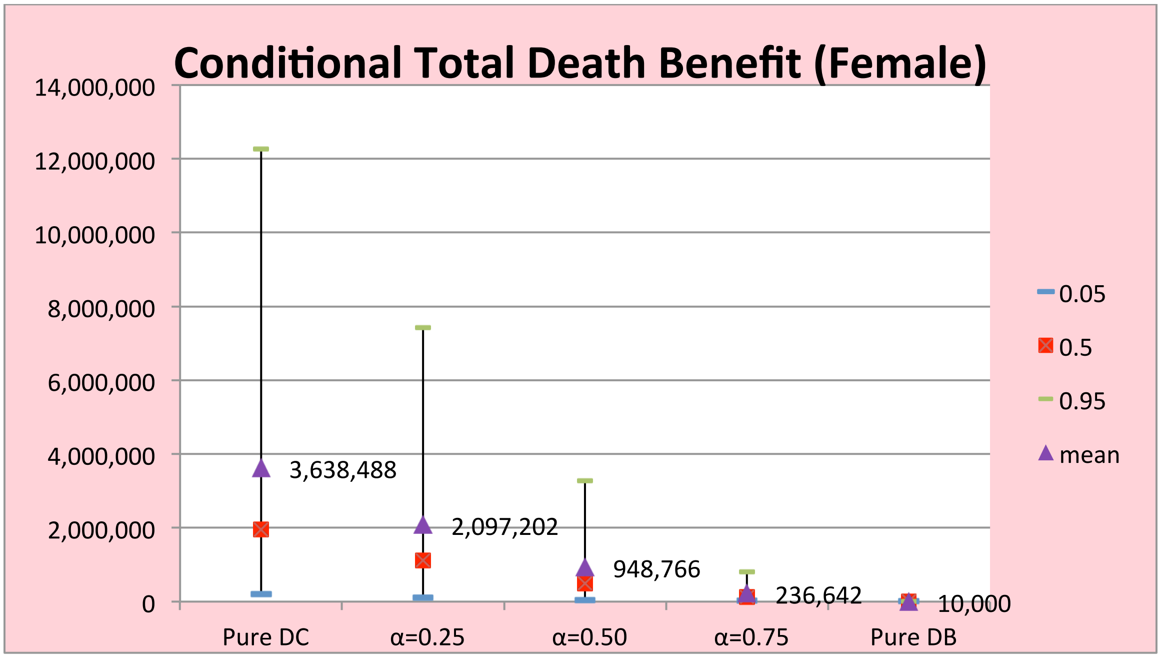

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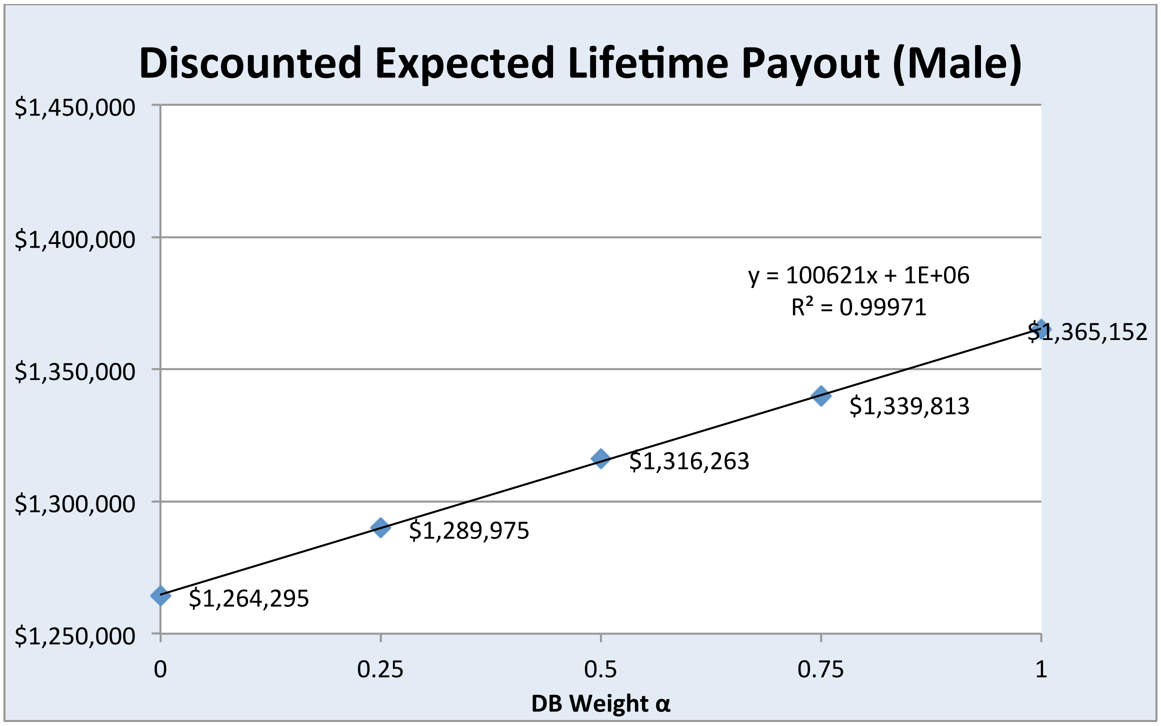
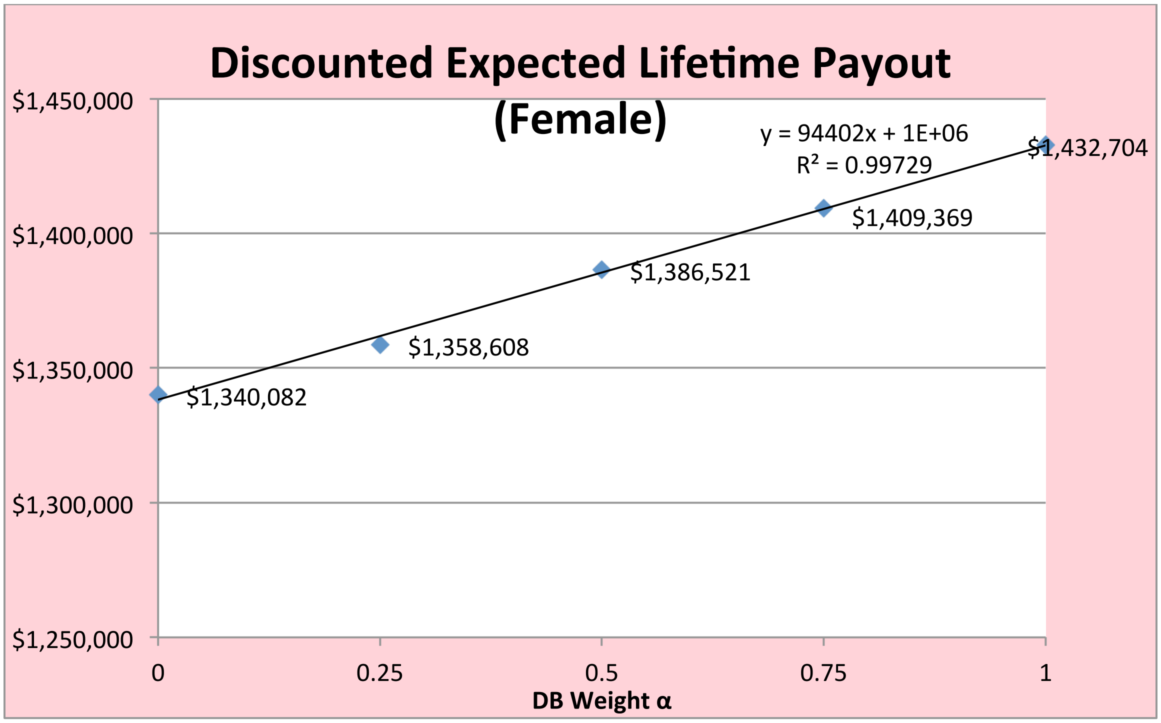


Exhibit 10



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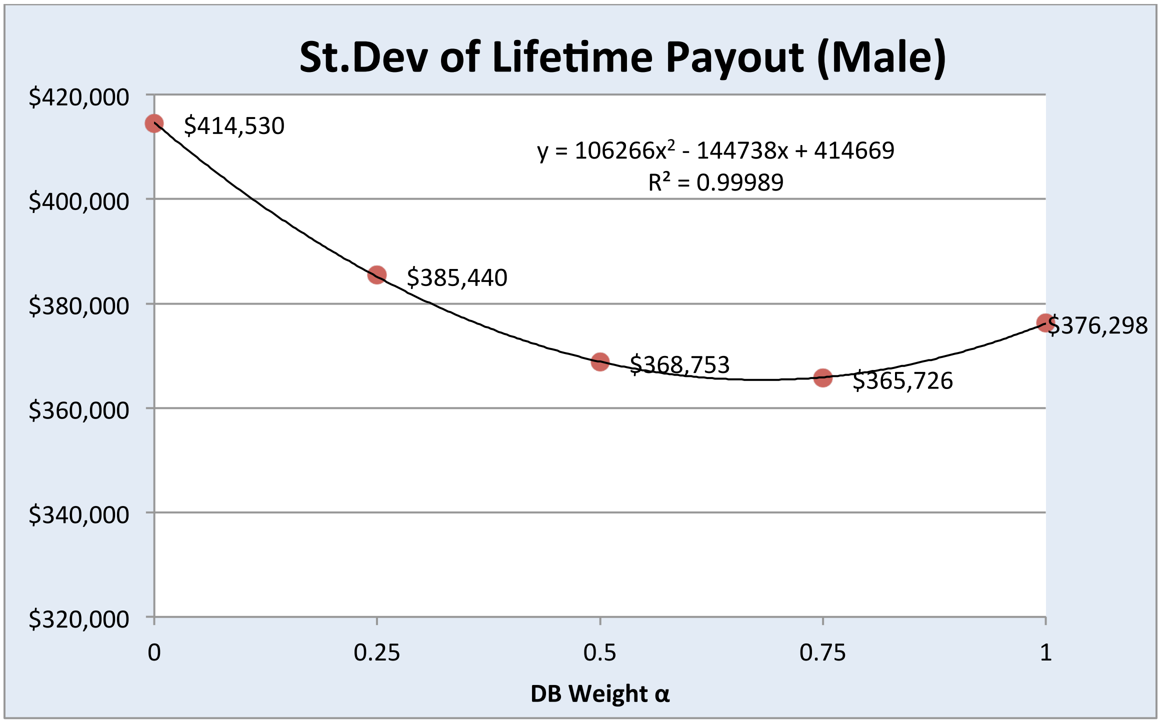

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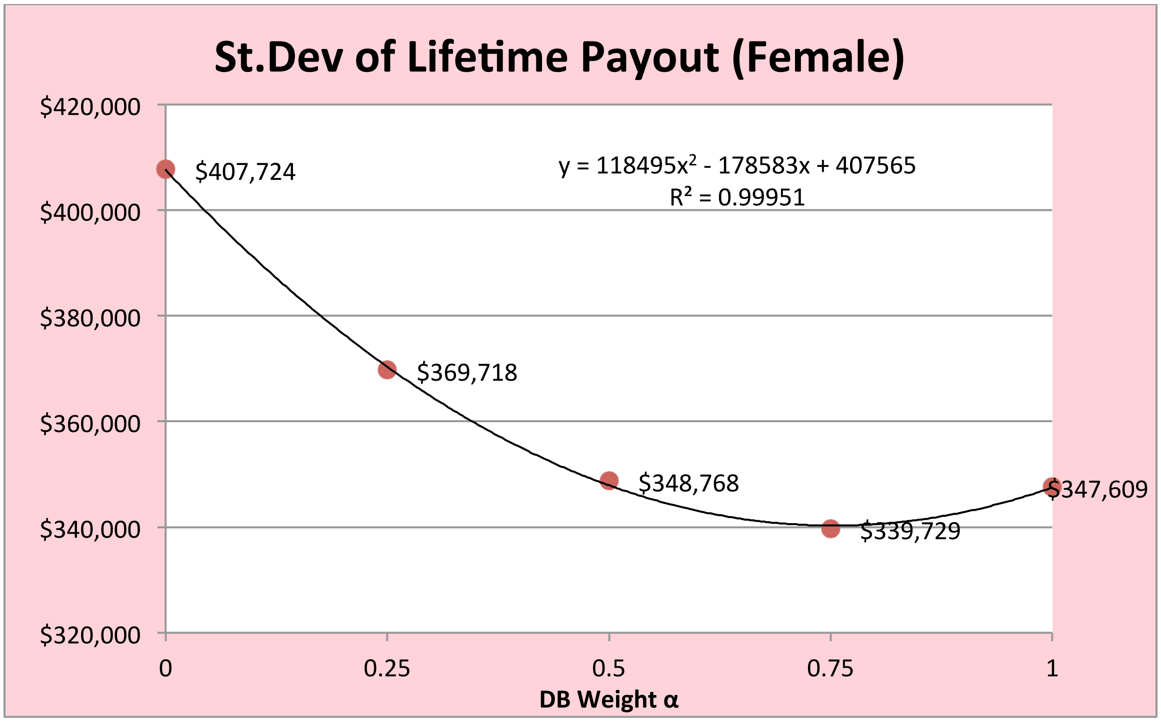

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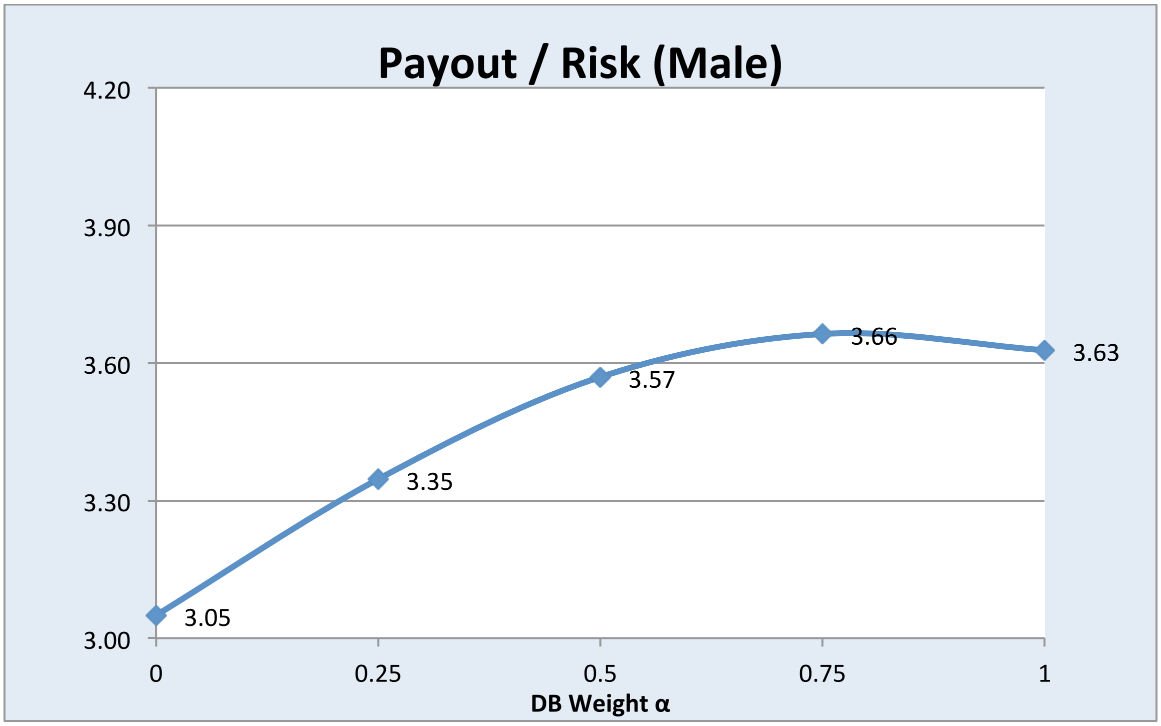

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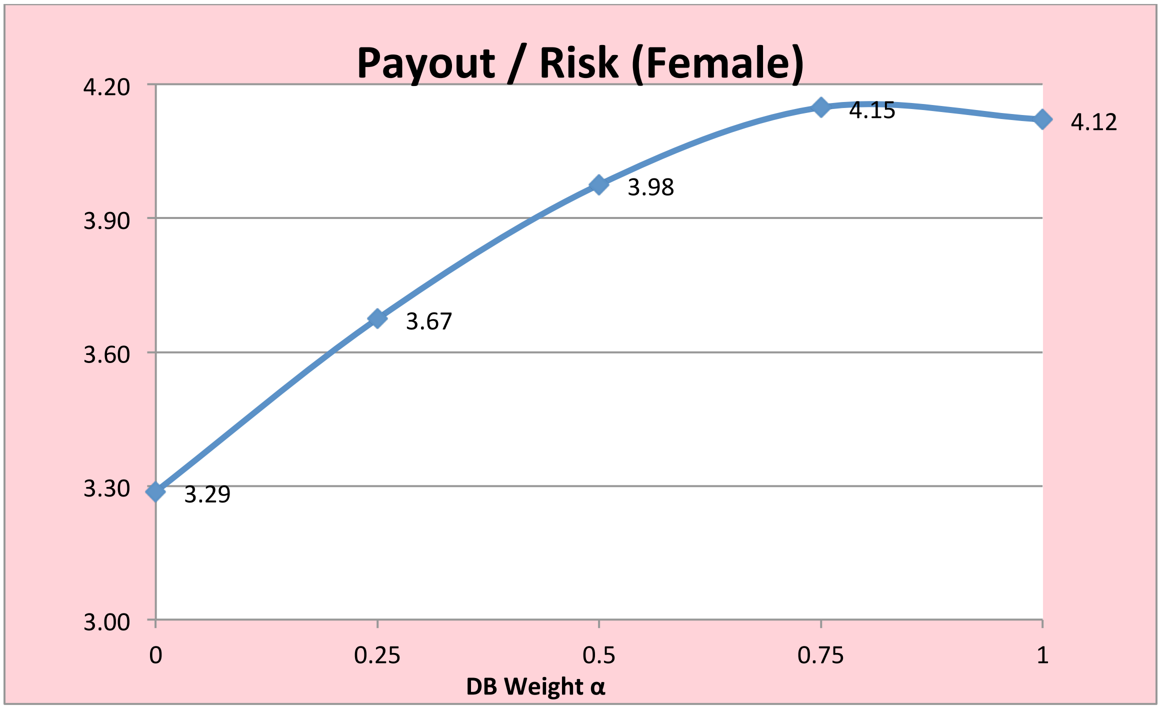

Exhibit 15

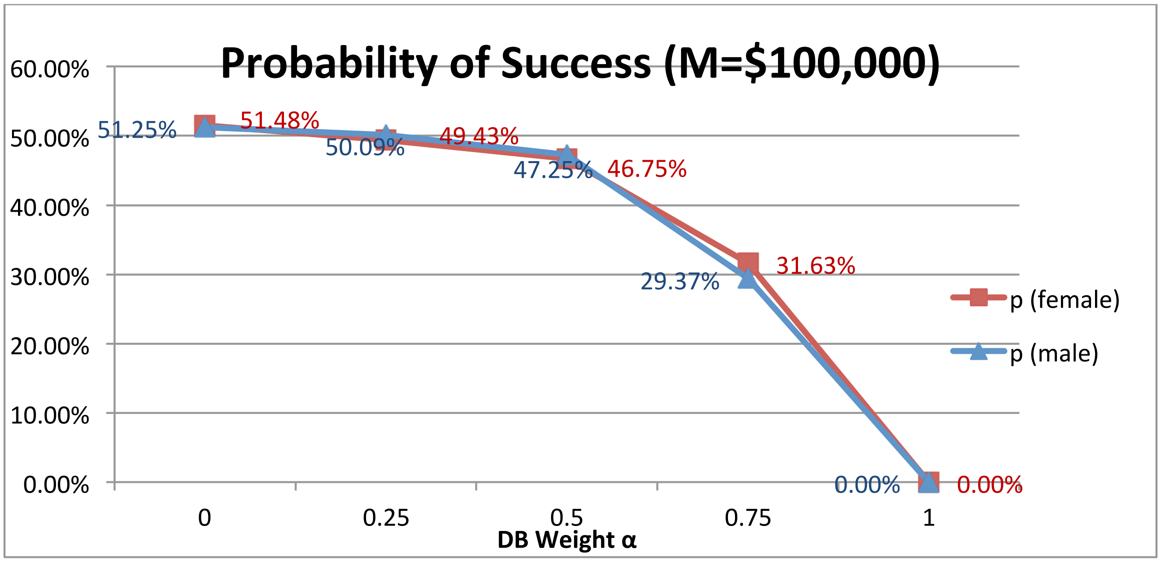

Exhibit 16

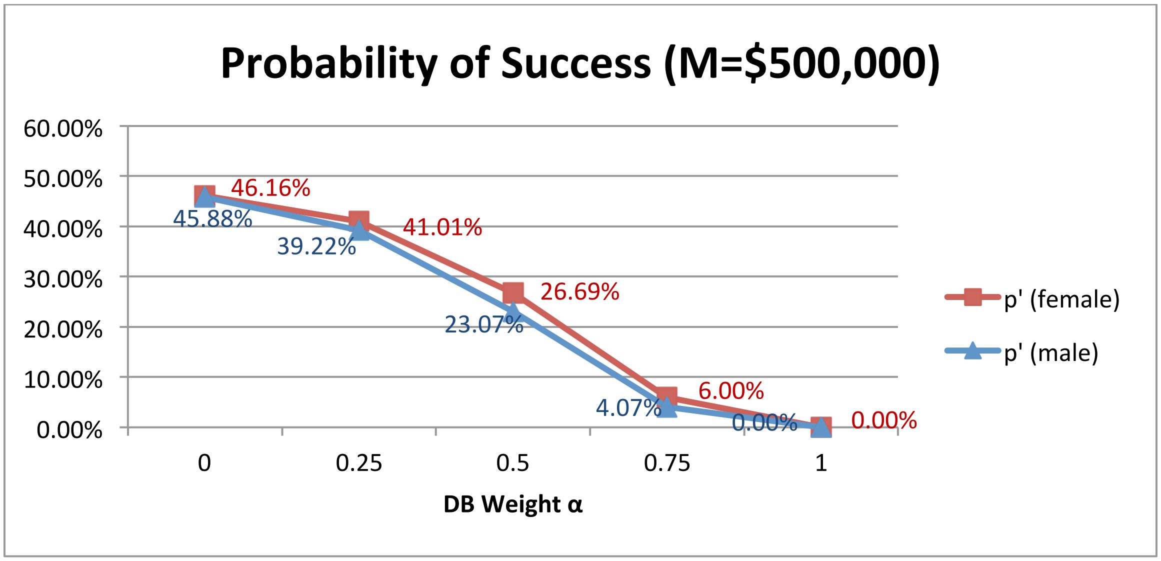

Exhibit 17 ($M = \$100,000$)

Conditional Probability of Positive but Insufficient Bequest when M=100,000

DB Weight α:	0	0. 25	0. 5	0. 75	1
Prob{0<DC Assets at Death<M-αM' |Failure} (female)	2.71%	4.61%	9.64%	30.43%	N/A
Prob{0<DC Assets at Death<M-αM' |Failure} (male)	2.72%	5.64%	11.27%	33.25%	N/A

Exhibit 18 ($M = \$500,000$)

Conditional Probability of Positive but Insufficient Bequest when M=500,000

DB Weight α:	0	0. 25	0. 5	0. 75	1
Prob{0<DC Assets at Death<M-αM' |Failure} (female)	13.05%	21.69%	36.96%	50.56%	N/A
Prob{0<DC Assets at Death<M-αM' |Failure} (male)	14.75%	23.42%	39.06%	50.87%	N/A

6. APPENDIX A. RP-2014 RATES WITH FULLY GENERATIONAL IMPROVEMENT

Age	Male Annuitant	Female Annuitant	Age
67	0.008441	0.006127	67
68	0.009061	0.006634	68
69	0.009738	0.007194	69
70	0.010482	0.007812	70
71	0.011297	0.008496	71
72	0.012199	0.009253	72
73	0.013198	0.010089	73
74	0.014303	0.011019	74
75	0.015541	0.012042	75
76	0.016926	0.013173	76
77	0.018489	0.014429	77
78	0.020243	0.015827	78
79	0.022219	0.017379	79
80	0.024442	0.019111	80
81	0.026939	0.021050	81
82	0.029745	0.023224	82
83	0.032893	0.025660	83
84	0.036413	0.028389	84
85	0.040340	0.031443	85
86	0.044975	0.035052	86
87	0.050420	0.039303	87
88	0.056273	0.043907	88
89	0.063144	0.049373	89
90	0.070537	0.055227	90
91	0.078932	0.061973	91
92	0.087344	0.068908	92
93	0.096666	0.076761	93
94	0.105839	0.084726	94
95	0.116065	0.093713	95
96	0.128411	0.104926	96
97	0.141681	0.117125	97
98	0.156915	0.13121	98
99	0.17244	0.145676	99

Age	Male Annuitant	Female Annuitant	Age
100	0.188982	0.161284	100
101	0.20659	0.178066	101
102	0.22654	0.197168	102
103	0.246009	0.216257	103
104	0.266424	0.236325	104
105	0.287546	0.257448	105
106	0.311501	0.281457	106
107	0.334271	0.304612	107
108	0.357676	0.328738	108
109	0.381929	0.353715	109
110	0.409722	0.382423	110
111	0.428832	0.409412	111
112	0.444266	0.437291	112
113	0.460664	0.459926	113
114	0.481497	0.481208	114
115	0.5	0.5	115
116	0.5	0.5	116
117	0.5	0.5	117
118	0.5	0.5	118
119	0.5	0.5	119
120	1	1	120

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