Marriage, marital investments, and divorce:
Theory and evidence on policy non neutrality

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Abstract

In the traditional marriage market literature, policies affecting partners' property rights do not affect who marries whom. In this paper I build a theory that shows that this neutrality result breaks down if we consider marital investments which returns are unverifiable to courts and accumulate in the private account of one of the spouses. I develop an equilibrium model of marriage, household specialization, and divorce in which working spouses with stay-at-home partners accumulate relatively more human capital (a feature verified in the data). In this environment, I consider a policy change that decreases the commitment of ex-spouses to share the returns from the human capital accumulated during the marriage. I show that such a policy gives rise to an equilibrium with higher incidence of two earner households (even when specialization is efficient) and higher spousal assortative matching in human capital, relative to the pre-reform equilibrium. This prediction is supported by empirical evidence showing that the introduction of unilateral divorce in the US (a regime that reduced the enforcement of transfers among ex-spouses) is associated with higher sorting in education, parental education, and pre-marital labor earnings among newlyweds.

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1 Introduction

In this paper I build a theory of marriage, marital investments, and endogenous dissolution and show that when spouses make non-contractible marital investments during marriage, changes in divorce laws affect the types of families that form. Potential spouses find each other attractive in the marriage market because of the expected welfare they can generate together, but also because of the share each one accrues of such value. The marriage market literature has heavily relied on the transferable utility structure of marital welfare. Transferable utility implies that the way in which spouses share their marital output does not change its value. The classical results by Becker (1973) and Shapley and Shubik (1971) show that under transferable utility the equilibrium in the marriage market will be the *optimal matching*, that is, the set of couples and singles that maximizes the total sum of marital output. Another classical result in the literature follows, the so called Becker-Coase theorem, according to which policies shifting the share of marital value among spouses do not change who matches with whom. The reason is simple. The transferable utility structure prevents the shift in sharing rules from changing the value of marriages, so the configuration of couples and singles that maximizes total marital value will be the same before and after the policy. Of course, the individual gains from marriage may change as a result of the policy, but who marries whom will be unchanged.

However, the set of assumptions that generate those results are very restrictive. Chiappori, Iyigun, and Weiss (2015) show that utility must be transferable *both* in marriage and divorce for divorce laws to be neutral with respect to marriage and divorce patterns. In this paper I show that transferable utility breaks down if we consider marital investments which returns are unverifiable to courts and difficult to assign in the event of a divorce. The leading example of such type of *non-contractible* marital investment is spousal support through the accumulation of career capital of the partner. I model this by allowing human capital accumulation within the marriage to depend on the work behavior of the partner. Specifically, working spouses whose partners do not work accumulate more human capital relative to working spouses in two-earners households. In such environments, policies affecting the distribution of property rights among spouses impact the equilibrium in the marriage market. In line with observed policy changes worldwide, I consider a change in divorce laws that decreases the commitment of ex-spouses to share the returns to the human capital accumulated during the marriage. I show that such a policy reduces the willingness of lower earning spouses to specialize in home production, even when specialization is efficient for the household. As a result, the policy change causes a reduction in the value of marriage for some potential couples and, hence, affects the marriage market equilibrium. In the new divorce regime, spouses match more assortatively in initial human capital and fewer households specialize.

The idea that lower barriers to divorce may affect the accumulation of marital specific capital and, in turn, impact on selection into marriage and partner choice is present in previous papers (Wickelgren (2009), Stevenson (2007), Peters (1986), Parkman (1992), and Zelder (1993)). I contribute to this literature by developing a new framework that allows me to characterize marriage sorting patterns under both the baseline and the new divorce regimes and study how making divorce easier changes who marries whom.

In the model (introduced in section 2 and solved in section 3), individuals match on their discrete level of initial human capital (low or high), live together for one period, and continue together or divorce during the second period. The lifetime value of a marriage to an individual is the sum of his or her share of the welfare generated while married and while divorced (if divorce occurs). The marriage market is competitive: the lifetime welfare of an individual is a *price* that potential partners take as given and must commit to deliver at the moment of matching for the couple to get married. In period one, couples that got married decide if the spouse with the lower earning capacity will specialize in market or household labor. Housework is a marital investment that increases the working partner's earnings in period two. This return is unverifiable to courts. If the couple divorces, ex-spouses pool and share their second period earnings only if divorce requires a mutually agreed settlement. Otherwise, they live off their individual earnings.

I first solve the model when divorce requires the mutual consent of both spouses (section 3.2). This means that, in the event of a divorce, spouses negotiate over a division of the joint value generated by the spouses in the divorce state. I show that in this environment, couples make efficient marital investments (proposition 1). Intuitively, in the event that household specialization is efficient for the couple, spouses with comparative advantage in home production can expect to receive a share of their working partner's second period increased earnings in the event of a divorce. This implies that the marriage market matching game exhibits a transferable utility structure. I next show that the marriage market equilibrium under mutual consent divorce is optimal (proposition 2) and characterize it (proposition 8). When the gains from specialization are sufficiently low, the optimal matching is positive assortative and, in all types of couples, both spouses work in the labor market. When the gains from specialization are sufficiently high, the optimal matching is also positive assortative but only couples of high skilled females and high skilled males specialize. For intermediate levels of the gain from specialization, matching is negative assortative and only couples with high skilled males and low skilled females specialize.

I then solve the model when divorce is unilateral and divorce settlements are not enforced (section 3.4). This means that working spouses are not forced to share the returns to household specialization with their investing ex-partners after divorce. First, I show that, under certain circumstances, this may induce lower earning spouses to inefficiently work in the labor market as they may secure a higher share of the (lower) marital output (proposition 3). Hence, the value of marriages under unilateral divorce depends on how this value is split. That is, the marriage market matching game exhibits an imperfectly transferable utility (ITU) structure. To find equilibria in this ITU environment, I use a modified version of the salary adjustment process proposed by Kelso and Crawford (1982). To characterize equilibria in terms of sorting patterns, I follow Legros and Newman (2007) and show that the equilibria delivered by the salary adjustment process is positive assortative whenever high types on one side of the market are more able to make transfers to high types on the other side of the market, relative to low types (proposition 5).

In section 4 I present the main result of the paper: a change from mutual consent to unilateral divorce weakly increases assortativeness in the marriage market (proposition 6). To illustrate this result, I provide numerical simulations of the model for different interesting sets of parameters. Moreover, I discuss novel evidence introduced in Reynoso (2019) showing that newlyweds that marry in unilateral divorce states match more assortatively in various measures of human capital (including education, parental education, and pre-marital labor income), relative to newlyweds in the baseline mutual consent regime, supporting the main result of this paper.

In developing this model I build on two strands of the literature. First, I source on the literature that considers marriage and divorce in an equilibrium framework (Chiappori, Iyigun, Lafortune, and Weiss (2016), Chiappori, Iyigun, and Weiss (2009a), Chiappori, Iyigun, and Weiss (2009b)). This literature is embedded in the Becker-Coase transferable utility environment, in which divorce laws have no impact on matching patterns. To study the impact of divorce laws on who marries whom when matched couples can endogenously divorce, I incorporate the decision to make non-contractible marital investments in marriage. Second, in combining the matching of couples with their marriage life, I build on Chiappori, Dias, and Meghir (2018), who develop an equilibrium model of pre-marital investments, marriage, savings, and labor supply over the life cycle. Unlike them, I consider the possibility of divorce during the couple's life and relax the assumption that couples can commit to a constant allocation of resources throughout their marriage. By departing from the Becker-Coase framework, my model is closest in spirit to the work by Rasul (2006), who develops a search model of marriage to show that unilateral divorce increased the probability of remaining single (a prediction that the author confirms in the data).

I also make a contribution to the vast literature that studies how divorce and other family laws impact on household organization (Voena (2015), Bayot and Voena (2015), Fernández and Wong (2011), Oreffice (2007), Chiappori, Fortin, and Lacroix (2002), Fernández and Wong (2017)). This literature focuses on the impacts of divorce laws on *already formed* couples. My focus, instead, is on the impact of divorce laws on the *equilibrium* types of families that form.

The failure of divorce laws neutrality in the marriage market shown in this paper imply that changes in divorce institutions may have unintended consequences related to the changes in the types of families that form. The main contribution of this paper is to develop a framework that departs from the Becker-Coase environment and is suitable for analyzing these policy issues from a general equilibrium perspective.

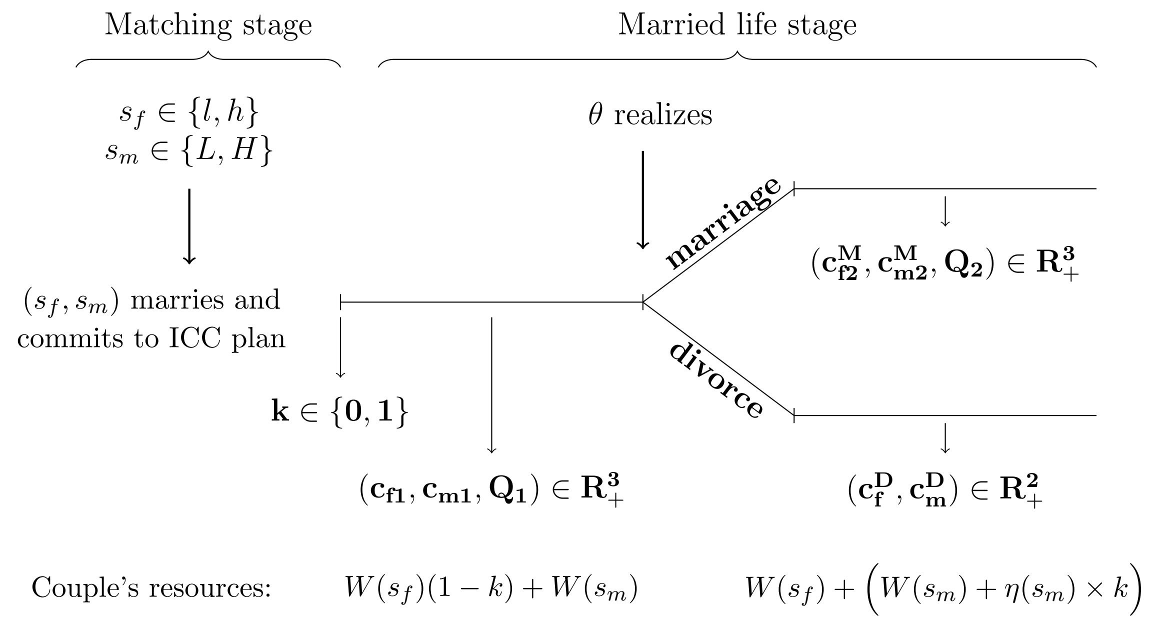
2 The model

2.1 Timing and choice sets

The economy is populated by a continuum of females, $f \in \mathcal{X}$ of mass $\mu_{\mathcal{X}}$, and a continuum of males, $m \in \mathcal{Y}$ of mass $\mu_{\mathcal{Y}}$. Within gender sets, individuals are distinguished by their exogenous discrete level of initial human capital that can be low or high and denoted by $s_f \in \{l, h\}$ for females and by $s_m \in \{L, H\}$ for males. The mass of individuals of type s is denoted by μ_s.

Individuals live for three periods, denoted $t = \{0, 1, 2\}$, grouped in two stages. Figure 1 illustrates the timeline of the model.

Figure 1: The life cycle of individuals



Period $t = 0$ corresponds to the *matching stage* when males and females meet in a competitive heterosexual marriage market and match based on their initial human capital levels. Periods $t = 1$ and $t = 2$ correspond to the *married life stage*. In period one, individuals that formed couples in $t = 0$ live as married, consume private goods, c, and household public goods, Q. In addition, wives in the first period can either supply housework ($k = 1$) or else work in an exogenous labor market ($k = 0$). At the end of period one, each individual draws a couple specific match quality shock from a symmetric mean zero uniform distribution: $\theta \sim U[\underline{\theta}, \overline{\theta}]$. This shock shifts the value of marriage for individuals. After observing their match quality draw, married individuals in the second period decide whether they wish to continue the marriage or to divorce. Individuals in the second period consume private goods (either in marriage or in divorce) and public goods if they remain married.

During every period in the *married life stage*, married or divorced couples have resources that depend on individuals' participation in the labor market. Earnings depend on initial human capital, and are denoted by $W(s_f)$ for a female type s_f and by $W(s_m)$ for a male type

s_m. It is assumed that $W(s)$ is increasing in s. Additionally, the working behavior of the wife in the first period determines the couple's resources in the second period: if $k = 1$, the couple's resources in period two are increased by a term $\eta(s_m) \geq 0$ that depends on the human capital of the husband. This term reflects the only gain from household specialization in the model. I make two assumptions:

Assumption 1 *The gain from household specialization, $\eta(s_m)$,*

- *is the private property of the working spouse, and*
- *is only positive for high skilled males:*

$$\eta(L) = 0 \text{ and}$$
$$\eta(H) = \eta \geq 0$$

Potential spouses are utility-price takers in the marriage market: suitors observe a given vector of utility prices that any potential partner requires to accept the marriage. These prices only depend on the initial human capital of individuals and are, hence, denoted by $v_f(s_f)$ for a woman of human capital s_f and $v_m(s_m)$ for a man of human capital s_m. If a couple marries in $t = 0$ both spouses deliver the given utility prices $(v_f(s_f), v_m(s_m))$, in expectation, by committing to a feasible, incentive compatible, and contingent-upon-θ plan to allocate value during periods one and two. The agreed upon contingent plan prescribes the allocation of female housework supply in the first period (k), the expenditure on local public goods (Q_t) and the allocation of private consumption among spouses (c_{ft} and c_{mt}) in both periods $t = \{1, 2\}$, and the divorce decision (D) at the beginning of period two. By committing to a contingent-upon-θ plan, the couple guarantees that the expected lifetime value that spouses derive from the prescribed allocations adds up to their marriage market prices $(v_f(s_f), v_m(s_m))$.

2.2 The marriage market

How do households form in this environment? Let the matrix of female and male types utility prices posted in the marriage market be denoted by

$$\Upsilon = \Big\{ v_f(s_f), v_m(s_m) \Big\}_{s_f \in \{h,l\}, s_m \in \{H,L\}}$$

Let $v_m = \varphi(s_f, s_m, v_f)$ be the function that captures the relationship between potential partners of types (s_f, s_m) utility levels. For now, let us consider this a fixed object. I normalize the value of being single to zero.[1] In this model, then, every individual desires to get married.

The partner choice problem male of type s_m is to choose the type of the female that maximizes his gain from joining the marriage (females' problems are analogous):

$$v_m = \max_{s_f \in \{l,h\}} \varphi(s_f, s_m, v_f(s_f)) \tag{1}$$

An equilibrium in the marriage market is a matrix of prices and an assignment of female types to male types, $\left\{\Upsilon, \mu\right\}$ such that at the given prices, there is no excess demand for any couple type. In equilibrium, $\left\{\Upsilon, \mu\right\}$ is a *stable matching*:

Definition 1 **[Stable matching]** $\left\{\Upsilon, \mu\right\}$ *is a stable matching if for all couple types* $(s_f, s_m) \in \{l, h\} \times \{L, H\}$,

$$\varphi(s_f, s_m, v_f) = v_m; \quad \forall s_f, s_m \in Spt(\mu)$$
$$\varphi(s_f, s_m, v_f) \leq v_m; \quad otherwise$$

Hence, an equilibrium set of couples is one such that all individuals are married to their most preferred potential partner among the set of potential partners that chooses them.

2.3 The ex ante Pareto frontier, $\varphi(s_f, s_m, v_f)$

The relationship between female and male (expected) utility prices is actually endogenous in this model, and determined by the decisions couples make during the two periods in the *married life stage*. Interestingly, in this model, the shape of $\varphi(s_f, s_m, v_f)$ depends on the exogenous divorce regime.

At the time of marriage, given any potential vector of partner prices (v_f, v_m), potential

[1] That is, $v_i(s_i) = \varphi(s_i, \emptyset, v(\emptyset)) = 0$. This assumption is innocuous and facilitates the solution of the model under unilateral divorce, so it will become relevant later. I will then come back to discussing it.

couples commit to a feasible, incentive compatible, and contingent-upon-θ plan,

$$\begin{aligned} x(\theta) &= \Big\{x_1 = \{c_{f1}, c_{m1}, Q_1, k\}, x_2(\theta) = \{c_{f2}(\theta), c_{m2}(\theta), Q_2(\theta), D(k,\theta)\}\Big\} \\ &= \Big\{x_1, x_2(\theta)\Big\} \in \Big\{R^3_+ \times \{0,1\}\Big\}^2 \end{aligned}$$

with the objective of maximizing the expected lifetime value of the husband subject to the wife achieving an expected lifetime value of at least her posted price, $v(s_f)$, and to a set of budget and participation constraints.

Formally, let $u_i(x)$ denote the per period valuation that individual i derives from plan x. The couple chooses $x(\theta)$ so as to solve the following Pareto problem:

$$\begin{aligned} v_m(s_m) = \max_{x(\theta)} & & & u_m(x_1) + E_\theta u_m(x_2(\theta)) \\ s.t. & & [pc_f]: & \quad u_f(x_1) + E_\theta u_f(x_2(\theta)) \geq v_f(s_f) \\ & & [bc_1]: & \quad c_{f1} + c_{m1} + Q_1 = W(s_f)(1-k) + W(s_m) \\ \forall\theta : D(k,\theta) = 0: & & [pc_i^M]: & \quad c_{i2} \in \mathcal{C}^M(\mathcal{D}, k, \theta), \quad \forall i = \{f, m\} \\ & & [bc_2^M]: & \quad c_{f2} + c_{m2} = W(s_f) + W(s_m) + \eta k - Q_2 \\ \forall\theta : D(k,\theta) = 1: & & [pc_i^D]: & \quad c_i^D \in \mathcal{C}^D(\mathcal{D}, k, \theta), \quad \forall i = \{f, m\} \\ & & [bc_2^D]: & \quad c_f^D + c_m^D = W(s_f) + W(s_m) + \eta k \\ = & & & \quad \varphi(s_f, s_m, v_f(s_f)) \end{aligned} \tag{2}$$

The expected lifetime utility of spouses is the sum of the first period value and the expected value (over the distribution of θ) of the contingent plan in the second period. Note that this expected value includes the value obtained by individuals in the event of divorce in the second period. The first constraint in the couple's problem, $[pc_f]$, is the participation constraint of the wife at the time of marriage. This constraint restricts the plan to give the wife a lifetime expected utility of at least her posted price $v_f(s_f)$. The following five constraints restrict the choice of $x(\theta)$ to satisfy the budget constraint in each period and the participation constraints in marriage and divorce in the second period. These constraints reflect distinctive features of this model and, hence, I describe them next, separately. The value of couple's problem (2) defines the Pareto frontier, $\varphi(s_f, s_m, v_f(s_f))$.

Feasibility of plan $x(\theta)$

The contingent plan is feasible if it satisfies the couple's budget constraint at every period and for every realization of θ. Savings are assumed away in this model.

The budget constraint in period 1, $[bc_1]$, indicates that total expenditures in private and public goods do not exceed the sum of spouses earnings in that period. Household specialization, k, affects the wife's earnings in period one. If the couples $k = 1$, the couple foregoes female earnings and total resources equal male earnings. In period two, the couple will decide to continue the marriage for some values of θ or to divorce for other values of θ. In both cases, the budget constraints $[bc_2^M]$ and $[bc_2^D]$ indicate that total expenditures do not exceed the sum of spouses or ex-spouses resources. As was noted before, the choice of k in period one affects working spouse's earnings in period two.

Divorce laws and incentive compatibility constraints

The last set of constraints to the couple's problem contains the individuals' participation constraints in marriage $[pc_i^M]$ and in divorce $[pc_i^D]$. Restrictions $[pc_i^M]$ constrain $x(\theta)$ to be such that all the allocations of private consumption among spouses who continue their marriage in period two belong to a set $\mathcal{C}^M$. Similarly, $[pc_i^D]$ constrain $x(\theta)$ to be such that the allocations of private consumption among (ex) spouses in divorce belong to a set $\mathcal{C}^D$.

The sets $\mathcal{C}^M$ and $\mathcal{C}^D$ depend on the exogenous laws governing divorce that are in force at the moment of marriage ($\mathcal{D}$), the couples' choice of k, and the realization of θ. These sets guarantee that $x(\theta)$ is incentive compatible with respect to the decision to continue or end the marriage, given the divorce regime. I distinguish between two regimes, in accordance to the history of US divorce laws, $\mathcal{D} \in \{MDC, UD\}$:

Mutual consent divorce. A *mutual consent divorce* regime, henceforth MCD, is the legal framework in which divorce can only occur if both spouses agree on the terms of the divorce. Hence, a couple can only divorce if there exists a divorce settlement that makes both spouses better off than in marriage. In practice, this implies that any spouse that wishes to obtain a divorce, must shift resources to her or his partner in order to convince them to agree to divorce. In the model, this defines the set of incentive compatible sharing rules in divorce, $\mathcal{C}^D$.

Unilateral divorce. A *unilateral divorce* regime, henceforth, UD, is the legal framework in which any spouse can file and obtain a divorce without the agreement of the partner. Hence, a couple can only continue to be married if there exists an allocation of value *in marriage* that makes both spouses better off than in divorce. In practice, this implies that any spouse that wishes to continue the marriage, must shift resources to her or his partner in order to convince them not to file for divorce. In the model, this defines the set of incentive compatible sharing rules in marriage, $\mathcal{C}^M$.

2.4 Specification of instantaneous utilities

Married individual i derives utility from the consumption of a private and public goods according to functions:

$$u_{it}(c_{it}, Q_t) = \begin{cases} c_{it}Q_t & \textit{if } t = 1 \\ c_{it}Q_t + \theta & \textit{if } t = 2 \end{cases}$$

Divorcees in period $t = 2$ only derive utility from private goods (hence losing the benefit from economies of scale in the consumption of public goods in marriage):

$$u_{i2}(c_{i2}) = c_{i2}$$

The shape of these preferences imply that utility is transferable *within period* both in marriage and divorce.[2] This means that the way spouses or ex spouses allocate the joint value generated within each period among partners does not impact the value jointly produced.

2.5 Model discussion

Two points are worth emphasizing. First, divorce laws affect the value of marriages through their influence on the participation constraints of problem (2). I formally solve problem (2) when the divorce regime is MCD in section 3.2 and when it is UD in section 3.4.

Second, the main driving force of the impact of divorce laws on the equilibrium in the marriage market is the existence of a husband's private return, $\eta(s_m)$, to the decision about k.

The gain from specialization, $\eta(s_m)$, is new in the related literature of matching with career investments and divorce. $\eta(s_m)$ is a reduced form parameter capturing many effects of household specialization on spousal human capital. One story is that spouses with stay-at-home

[2] A model satisfies the transferable utility property if for each agent there exists a cardinal representation of utilities such that the Pareto frontier is a straight line with slope -1 (Chiappori, 2010).

partners can focus on increasing their productivity in the labor market without compromising the production of public goods within the household. For example, working spouses with non working partners can travel for work, work longer hours, take up temporary job positions out of town, and avoid absenteeism in case of sickness of children. Another story is that non working partners can directly work to improve the career prospects of their working partners by, for example, hosting social and professional meetings, or taking care of their partner's health (by cooking nutritious meals, etc). Finally, $\eta(s_m)$ may capture the effect of spousal support through the completion of professional degrees. Even when in the model the choice is between participating or not in the labor market, the gains from specialization can be reinterpreted as the gain from having one spouse employed in a low risk, flat career capital accumulation job to support their partner through professional school. All in all, $\eta(s_m)$ reflects a transfer of human capital investment opportunities from one spouse to the other.

Investing in the career of the spouse with the highest earning ability over the life cycle might be efficient for the couple if the gains $\eta(s_m)$ are high relative to the foregone earnings of the supporting spouse (in this model, $W(s_f)$). Under household specialization, in effect, there is more joint value to share in the second period, both if the marriage continues or ends. However, the fact that $\eta(s_m)$ is the private property of the working spouse might deter opportunities to make efficient spousal support investments. The main insight from this model is: when divorce laws reduce the commitment of ex spouses to sharing the value of the human capital raised during the marriage, couples may not be able to sustain efficient household specialization as part of an incentive compatible plan.

3 Model solution

I now turn to solving the formed couples' problem (2) and the partner choice problem (1) for the two divorce regimes, MCD and UD. First, I reformulate problem (2) following a *three stage formulation* (Chiappori and Mazzocco, 2015) and arrive at an equivalent problem of choosing k under different divorce regimes (section 3.1). I then solve for k under MCD and show the existence of the marriage market equilibrium under this regime (sections 3.2 and 3.3). Finally, I do the analogous under UD (sections 3.4 and 3.5).

3.1 The three stages formulation

To solve problem (2) I take a *three stage formulation* as described by Chiappori and Mazzocco (2015). The solution in three steps leads to the same solution to problem (2). Note that state variables at the beginning of period two, for any couple of type (s_f, s_m), consist of the history of female housework supply and the realization of the match quality shock. The set of states variables is, then, $\{(k, \theta) \in \{0, 1\} \times [\underline{\theta}, \overline{\theta}]\}$.

The first two stages consist of solving two static problems for every period and state. The first stage corresponds to the *intrahousehold allocation stage*. In this step, we take any arbitrary level of total expenditures in private and public goods, C and Q (respectively), and solve for the spouses' allocation of total private consumption among themselves, c_f and c_m. The second stage is the *resource allocation stage*. In this stage, we take as given the sharing rule (c_f, c_m) and any amount of lifetime resources allocated to each period and state, and solve for the couple's optimal allocation of the period's resources between $C_t(\theta)$ and $Q_t(\theta)$. In the third stage, we take the previous policy rules as given and solve for the optimal allocation of lifetime resources between periods and states. In this model, the only variable that affects periods resources is k, so this stage consists of solving for the optimal female housework supply, k.

3.1.1 The value of the intrahousehold and resource allocation problems

The detailed solution of the static problems of the first two stages are presented in appendix A. Here, I outline the resulting value functions. An important feature of this model that I demonstrate in the appendix is that the sharing rules within period and state are indeterminate: there are many optimal ways to allocate private consumption among spouses. This is a consequence of the *within period* transferable utility structure of the model. To pin down the sharing rules, hence, we need to solve for the equilibrium in the marriage market. This will reduce the set of feasible sharing rules that satisfy the participation constraints that define the couple's problem (2).

The value of the household problem of allocating the period resources to private and public consumptions are presented next. Note that these values will be conditional on the intertemporal choice of k.

The household value in the first period is

$$V_1^M(k) = \Big(\frac{W(s_f)(1-k) + W(s_m)}{2}\Big)^2$$

The values that each spouse receives in period one results in

$$v_{f1}(c_{f1}, k) = c_{f1}\Big(\frac{W(s_f)(1-k) + W(s_m)}{2}\Big) = (C_1 - c_{m1})\Big(\frac{W(s_f)(1-k) + W(s_m)}{2}\Big)$$

for the wife and

$$v_{m1}(c_{m1}, k) = c_{m1}\Big(\frac{W(s_f)(1-k) + W(s_m)}{2}\Big) = (C_1 - c_{f1})\Big(\frac{W(s_f)(1-k) + W(s_m)}{2}\Big)$$

for the husband.

Note, hence, that the sum of spouses' values add up to the household value,

$$v_{f1}(c_{f1}, k) + v_{m1}(c_{m1}, k) = V_1^M(k)$$

reflecting the transferable utility structure of the within period problem. Hence, the period relationship between the value of the husband and that of the wife is,

$$\begin{aligned} v_{m1}(c_{m1}, k) &= V_1^M(k) - v_{f1}(c_{f1}, k) \\ s.t. \quad & v_{m1}(c_{m1}, k) \geq 0 \end{aligned} \tag{3}$$

The problems in period two depend upon the realization of θ and the decision on k. For any vector of state variables (k, θ), the household value if the couple remains married is

$$V_2^M(k, \theta) = \Big(\frac{W(s_f) + W(s_m) + \eta(s_m)k}{2}\Big)^2 + 2\theta$$

With similar derivations, one can show that the relationship between spouses' values if the marriage continues in period two is, for every (k, θ),

$$\begin{aligned} v_{m2}(c_{m2}, k, \theta) &= V_2^M(k, \theta) - v_{f2}(c_{f2}, k, \theta) \\ s.t. \quad & v_{m2}(c_{m2}, k, \theta) \geq 0 \\ & [pc_i^M]: \;\; c_{i2} \in \mathcal{C}^M(\mathcal{D}, k, \theta) \quad \forall i = \{f, m\} \end{aligned} \tag{4}$$

Note that, in period two, the exogenous laws governing divorce impose restrictions to the sharing rule (c_{f2}, c_{m2}) that is otherwise only restricted by non negativity constraints.

The couple's problems in divorce are very similar. At any vector of state variables (k, θ) such that the household divorces, the household value is

$$V^D(k) = W(s_f) + W(s_m) + \eta(s_m)k$$

The relationship between ex spouses' values is

$$\begin{aligned} v_m^D(c_m^D, k) \quad &= \quad V^D(k) - v_m^D(c_f^D, k) \\ s.t. \quad & \quad v_m^D(c_m^D, k) \geq 0 \\ & [pc_i^D]: \;\; c_i^D \in \mathcal{C}^D(\mathcal{D}, k, \theta) \quad \forall i = \{f, m\} \end{aligned} \tag{5}$$

Again, note that the exogenous divorce laws impose restrictions to the sharing rule (c_f^D, c_m^D).

The couple's divorce decision is made by comparing the values that spouses would obtain in marriage and in divorce. In general, the optimal divorce decision will depend on divorce laws. In this model, however, because utility is transferable, both in marriage and in divorce, divorce will occur whenever the household value in divorce exceeds the household value in marriage in the second period, *irrespective of the divorce regime* (Chiappori, Iyigun, and Weiss, 2015). Hence,

$$D(k, \theta) = 1 \Leftrightarrow V^D(k) > V_2^M(k, \theta)$$

3.1.2 The intertemporal stage

The last stage in the solution of program (2) corresponds to the *intertemporal stage*, where the couple decides how to allocate lifetime resources to each period. In this model, the only decision variable that changes lifetime and within period resources is female housework supply. Let $\Pi(k)$ be the ex ante probability of divorce (that is, the expected probabiliy of divorce over the distribution of θ from the point of view of the moment of marriage) that is conditional on k.[3] Once the solutions from the intrahousehold and resource allocation stages are taken into

[3]

$$\Pi(k) = P_\theta\Big(\theta < \underbrace{\frac{1}{2}\Big[W(s_f) + W(s_m) + \eta(s_m)k - \Big(\frac{W(s_f) + W(s_m) + \eta(s_m)k}{2}\Big)^2\Big]}_{\theta^*(s_f, s_m, \eta)}\Big) = \frac{\theta^*(s_f, s_m, \eta) - \underline{\theta}}{\overline{\theta} - \underline{\theta}} \tag{6}$$

account, problem (2) reduces to choosing k so as to maximize

$$\begin{aligned}
v_m = \max_{k\in\{0,1\}} \quad & v_{m1}(c_{m1},k)+ \\
& +(1-\Pi(k))E_{\theta|D(k,\theta)=0}v_{m2}(c_{m2},k,\theta)+\Pi(k)E_{\theta|D(k,\theta)=1}v_m^D(c_m^D,k,\theta) \\
s.t. \quad [pc_f]: \quad & v_{f1}(c_{f1},k)+ \\
& +(1-\Pi(k))E_{\theta|D(k,\theta)=0}v_{f2}(c_{f2},k,\theta)+\Pi(k)E_{\theta|D(k,\theta)=1}v_f^D(c_f^D,k,\theta)\geq v_f(s_f) \\
[ira]: \quad & v_{m1}(c_{m1},k)=V_1^M(k)-v_{f1}(c_{f1},k) \\
& v_{m2}(c_{m2},k,\theta)=V_2^M(k,\theta)-v_{f2}(c_{f2},k,\theta), \quad \forall\theta \\
& v_m^D(c_m^D,k,\theta)=V^D(k,\theta)-v_f^D(c_f^D,k,\theta), \quad \forall\theta \\
[pc_i^M]: \quad & c_{i2}\in\mathcal{C}^M(\mathcal{D},k,\theta), \quad \forall i=\{f,m\}, \quad \forall\theta: D(k,\theta)=0 \\
[pc_i^D]: \quad & c_i^D\in\mathcal{C}^D(\mathcal{D},k,\theta), \quad \forall i=\{f,m\}, \quad \forall\theta: D(k,\theta)=1
\end{aligned}$$

where the three constraints denoted by $[ira]$ reflect the relationships between the per period values of spouses that result from solving the intrahousehold and resource allocation problems (expressions (3), (4), and (5)). Because the relationship between lifetime utilities is negative and preferences are monotone, the female participation constraint will hold with equality at the solution. We can, hence, rewrite the problem as

$$\begin{aligned}
v_m = \max_{k\in\{0,1\}} \quad & V_1^M(k)+(1-\Pi(k))E_{\theta|D(k,\theta)=0}V_2^M(k,\theta)+\Pi(k)E_{\theta|D(k,\theta)=1}V^D(k,\theta)-v_f(s_f) \\
s.t. \quad [pc_i^M]: \quad & c_{i2}\in\mathcal{C}^M(\mathcal{D},k,\theta), \quad \forall i=\{f,m\}, \quad \forall\theta: D(k,\theta)=0 \\
[pc_i^D]: \quad & c_i^D\in\mathcal{C}^D(\mathcal{D},k,\theta), \quad \forall i=\{f,m\}, \quad \forall\theta: D(k,\theta)=1 \\
& =\varphi(s_f,s_m,v_f(s_f)) \qquad (7)
\end{aligned}$$

The household constrained efficient choice of k has the aim of locating the couple at the Pareto frontier subject to the participation constraints in marriage and divorce imposed by the exogenous divorce laws. I next study the shape of the Pareto frontier and the resulting marriage market equilibria under each divorce regime.

3.1.3 The marital surplus

Before moving forward, let us define the lifetime total value produced by couple type (s_f, s_m), in the absence of transfers among spouses. This is a well defined object called the *marital surplus*:

Definition 2 **[Marital surplus]** *Is the following function of population types* (s_f, s_m)*:*

$$\varphi(s_f, s_m) = \max_k \quad \varphi(s_f, s_m, k)$$

In this model,

$$\varphi(s_f, s_m) = \max_k \Big\{ V_1^M(k) + (1 - \Pi(k)) E_{\theta|D(k,\theta)=0} V_2^M(k, \theta) + \Pi(k) E_{\theta|D(k,\theta)=1} V^D(k) \Big\}$$

3.2 Ex ante Pareto frontier under MCD

In this section I solve problem (7) when the divorce regime is MCD. The main result of this section is that under MCD, couples optimally choose the efficient level of female housework supply, k.

3.2.1 Participation constraints under MCD

Under MCD, the set of second period sharing rules in marriage and divorce, $\mathcal{C}^M$ and $\mathcal{C}^D$, respectively, are for all θ:

$$\mathcal{C}^M(MCD, k, \theta) = \{(c_{f2}, c_{m2}) : \forall i = \{f, m\}, v_{i2}(c_{i2}, k, \theta) \in [0, V_2^M(k, \theta)] \wedge v_{f2} + v_{m2} = V_2^M\}$$
$$\mathcal{C}^D(MCD, k, \theta) = \{(c_f^D, c_m^D) : \forall i = \{f, m\}, v_i^D(c_i^D, k, \theta) \in [v_{i2}^M(c_{i2}, k, \theta), V^D(k)] \wedge v_f^D + v_m^D = V^D\}$$

That is, when divorce requires the consent of both spouses, divorce only happens when both spouses find it optimal. While this situation puts no restriction on the sharing of value in marriage, it restricts the sharing rule in divorce. Divorce is optimal in state (k, θ) for both spouses when the sharing of private consumption in divorce is such that both spouses are at least as well off as if the marriage had continued in that state (k, θ).

3.2.2 Marital investment decision under MCD

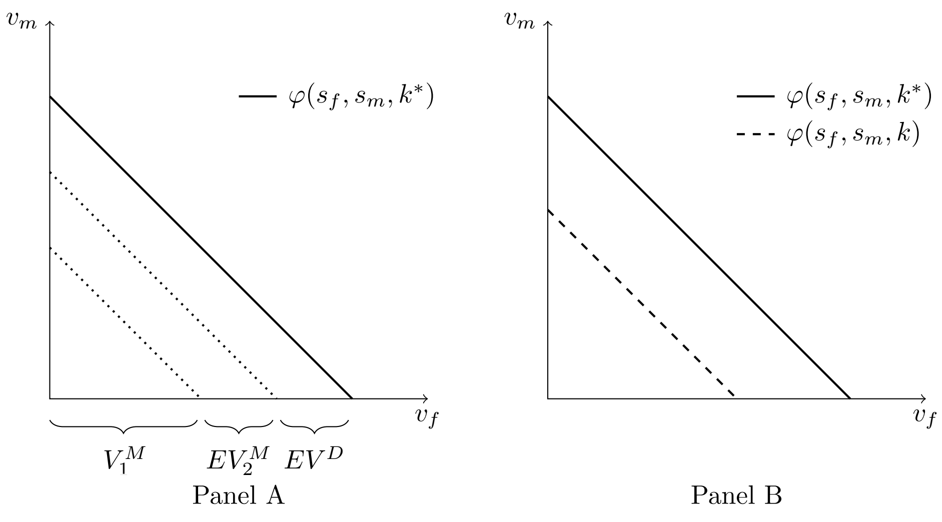
It is instructive to explore the relationship between potential spouses' utilities (7) for each possible choice of $k \in \{0,1\}$. Under the MCD participation constraints, the value of problem (7) becomes:

$$v_m(k) = \varphi(s_f, s_m, k) - v_f(s_f)$$

$$\forall (v_f, v_m) : \begin{cases} v_{i2}^M \in [0, V_2^M(k^*, \theta)] \wedge v_{f2}^M + v_{m2}^M = V_2^M(k^*, \theta) & \forall i = \{f, m\}, \quad \forall \theta : D(k, \theta) = 0 \\ v_i^D \in [v_{i2}^M, V^D(k^*)] \wedge v_f^D + v_m^D = V^D(k^*) & \forall i = \{f, m\}, \quad \forall \theta : D(k, \theta) = 1 \end{cases}$$

Panel A in figure 2 illustrates this relationship. The lifetime utility possibility frontier, denoted $\varphi(s_f, s_m, k^*)$, is the solid line with slope -1. This frontier is the sum of the per period expected utility possibility frontiers: $\varphi(s_f, s_m, k^*) = V_1^M + EV_2^M + EV^D$, where I shorten notation by setting $V_1^M = V_1^M(k^*)$, $EV_2^M = (1 - \Pi(k^*))E_{\theta|D(k^*,\theta)=0}V_2^M(k^*, \theta)$, and $EV^D = \Pi(k^*)E_{\theta|D(k^*,\theta)=1}V^D(k^*)$.

Figure 2: Conditional utility possibility frontiers for couple type (s_f, s_m).



Participation constraints under MCD only impose the restriction that the sharing rule in divorce make both spouses prefer their allocation in divorce than their allocation in marriage. However, the MCD regime puts no restriction on the allocations in marriage. Hence, from the perspective of the moment of matching, when committing to a contingent plan on k, any lifetime individual utility price $v_i \in [0, \varphi(s_f, s_m, k^*)]$ such that $v_f + v_m = \varphi(s_f, s_m, k^*)$ can be attained with a feasible marriage contract satisfying the MCD participation constraints. In effect, whenever the household value in divorce exceeds the household value in marriage in the second period (equivalently, whenever divorce occurs), there will be at least one allocation in divorce that satisfies the $[pc^D]$ constraint. It is just a matter of allocating the excess value of divorce among spouses on top of their division of the value in marriage.

Note, also, that there are infinitely many combinations of *within period* sharing rules that deliver the same values v_m and v_f, reflecting the within period transferable utility property of the problem. Any contingent marriage contract planned at the time of marriage (contingent on the realizations of the marital shocks), that delivers per period values $\{v_{m1}, v_{m2}(\theta), v_m^D(\theta)\}$ such that $v_{m1} + Ev_{m2}(\theta) + Ev_m^D(\theta) = v_m$ is a payoff equivalent feasible marriage contract.

Panel B of figure 2 illustrates another feature of the problem under MCD: the utility possibility frontier associated with the first period investment decision that maximizes the marital surplus, k^*, contains the utility possibility frontier associated with taking any other inefficient action k. The next proposition proves this assertion.

Proposition 1 *At any given female utility price, v_f, the couple's optimal choice of k is the one that maximizes the marital surplus as defined in (2):*

$$\arg\max_k \quad \{\varphi(s_f, s_m, k) - v_f\} \quad = \quad \arg\max_k \quad \varphi(s_f, s_m, k)$$

Proof 1 *See appendix C.1* ■

All in all, proposition 1 states that the lifetime total value generated by any potential marriage is separable from the sharing of that lifetime value among potential spouses:

$$v_m(s_m) \quad = \quad \varphi(s_f, s_m) - v_f(s_f) \tag{8}$$

Expression (8) describes the Pareto frontier for couple (s_f, s_m) under MCD.

3.3 Existence of equilibrium under MCD

The problem of individuals at the matching stage is to chose the partner that, at the given prices, maximizes their own expected gain from marriage subject to being accepted by this partner. As the next remark highlights, when divorce requires mutual consent, the transferability of utility *within period* translates into ex ante transferability of lifetime utilities at the time of marriage.[4]

Remark 1 (Transferable lifetime surplus) *The utility possibility frontier under mutual consent (8) is additive separable in prices v_f and v_m, hence, the problem of couples at the moment of matching has a transferable utility structure.*

Transferable utility at the time of marriage means that, when choosing a partner, the sharing rule to split the lifetime marital surplus ((v_f, v_m), taken as given in the market) does not influence the lifetime production of marital value.

Under transferable utility at the time of marriage, Becker (1973) and Shapley and Shubik (1971) have shown that the equilibrium in the marriage market will be the matching that maximizes total surplus among all possible matchings. Proposition 2 formalizes this.

Proposition 2 *The marriage market equilibrium is optimal and efficient.*

Proof 2 *Because the marriage market has a transferable utility structure, by the classical results by Becker (1973) and Shapley and Shubik (1971), the equilibrium stable matching maximizes total social marital surplus. Hence, it is optimal. Efficiency follows from proposition 1: all potential couples agree on first period investments that maximize their marital output, conditional on getting married.* ■

The results stated in propositions 1 and 2 generalize to any model of marriage, marital investments, and divorce with two features. First, models where utility is transferable in marriage and divorce. Second, models where there are no restrictions to within period sharing rules. These two features together imply that the marital surplus is transferable, as stated in remark 1. A convenient feature of the transferable utility matching model is that searching for stable matchings reduces to solving for the set of optimal assignments, which is usually a simple linear programming problem. Another convenient feature of transferable utility models is that equilibrium matching patterns can be characterized by exploring the properties of the surplus function $\varphi(s_f, s_m)$. I turn to this in section 4.2.

[4] A matching market described by frontier (7) exhibits a transferable utility structure (Chiappori, Dias, and Meghir, 2018) when there exists a monotone transformation of utilities, $g(v_i)$, such that

$$g(v_f) = g(\varphi(s_f, s_m)) - g(v_m)$$

3.4 Ex ante Pareto frontier under UD

To characterize problem (7) under UD, I introduce the structure of this divorce institution in the following definition and assumption.

Definition 3 **[Autarky value]** *Let the autarky allocation* (c_f^A, c_m^A) *be the allocation ex spouses would obtain in divorce if they left their marriage without negotiating a divorce settlement with their partners. In this model, the autarky allocation is the individual income in period two. The autarky value is the value derived in the divorce state from the autarky allocation. Because divorcees only derive utility from private consumption:*

$$v_i^D(c_f^A(k)) = c_f^A(k) = W(s_f)$$
$$v_i^D(c_m^A(k)) = c_m^A(k) = W(s_m) + \eta(s_m)k$$

Assumption 2 *Under UD, individuals have the property right over their autarky value in the event of a divorce.*

This assumption implies that divorce under the UD regime requires no settlement among ex spouses. This captures instances in which transfers among ex spouses are difficult to enforce, as is the case for the value of human capital.

3.4.1 Participation constraints under UD

Assumption 2, together with the definition of UD imply that under UD, a marriage can only continue if both spouses prefer their allocation in marriage to their autarky allocation. Hence, the allocation of private consumption in marriage in the second period is restricted for both spouses to deliver a value of at least the autarky value. Formally, the set of second period sharing rules in marriage and divorce, $\mathcal{C}^M$ and $\mathcal{C}^D$, respectively, for all θ are:

$$\mathcal{C}^M(UD, k, \theta) = \{(c_{f2}, c_{m2}) : \forall i = \{f, m\}, v_{i2}(c_{i2}, k, \theta) \in [v_i^D(c_i^A(k)), V_2^M(k, \theta)] \wedge v_{f2} + v_{m2} = V_2^M\}$$
$$\mathcal{C}^D(UD, k, \theta) = \{(c_f^A(k), c_m^A(k))\}$$

3.4.2 Marital investment decision under UD

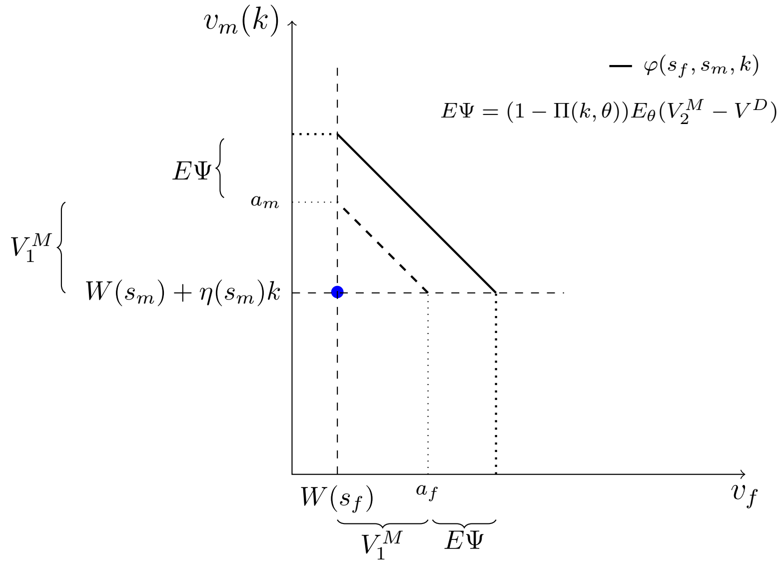
When divorce is UD, the sharing rule of private consumption in the second period is such that the value of marriage for both spouses in the second period is at least the value of the autarky allocation, $v_i^D(c_i^A(k)) = c_i^A(k)$. Hence, the allocation of value over the lifetime is restricted by the value of autarky. Under the UD participation constraints, the value of problem (7) for each possible choice of $k \in \{0, 1\}$ becomes:

$$v_m(k) = \varphi(s_f, s_m, k) - v_f(s_f), \quad \forall (v_f, v_m) : \begin{cases} v_f(s_f) \geq W(s_f) \\ \varphi(s_f, s_m, k) - v_f(s_f) \geq W(s_m) + \eta(s_m)k \end{cases} \tag{9}$$

Figure 3 depicts this relationship between $v_m(k)$ and $v_f(s_f)$. In the figure, the vertical axis displays the set of feasible male utility prices, v_m, and the horizontal axis the set of feasible female utility prices, v_f. Not every feasible marriage market price v_f or v_m can be delivered at the time of marriage in this environment, because the unilateral divorce restrictions $[pc_i^M]$ and $[pc_i^D]$ in problem (7) constrain the sharing of marital lifetime value, by constraining the second period allocations to give each spouse at least their value of autarky.

Autarky values are indicated with the bullet point $(W(s_f), W(s_m) + \eta(s_m)k)$ in figure 3. Once spouses guarantee each other these autarky values, they freely share two values. First, the value produced in the first period, V_1^M. In the figure, the different ways of sharing this value is indicated by the dashed line with slope -1. If the male appropriates all this value, he would be assigned value $a_m = W(s_m) + \eta(s_m)k + V_1^M$; if the female appropriates all this first period value, she would be assigned $a_f = W(s_f) + V_1^M$; if both spouses get a strictly positive share of V_1^M, they would be assigned values in interior of the dashed segment. On top of this, spouses share a second value, namely, the expected surplus of marital value over divorce value conditional on the state being such that the couple stays married in the second period $(E\Psi)$. The solid line labeled $\varphi(k)$ indicates the resulting total value of the marriage associated with investment k.

Figure 3: Expected value of taking action k^* for female f and male m.

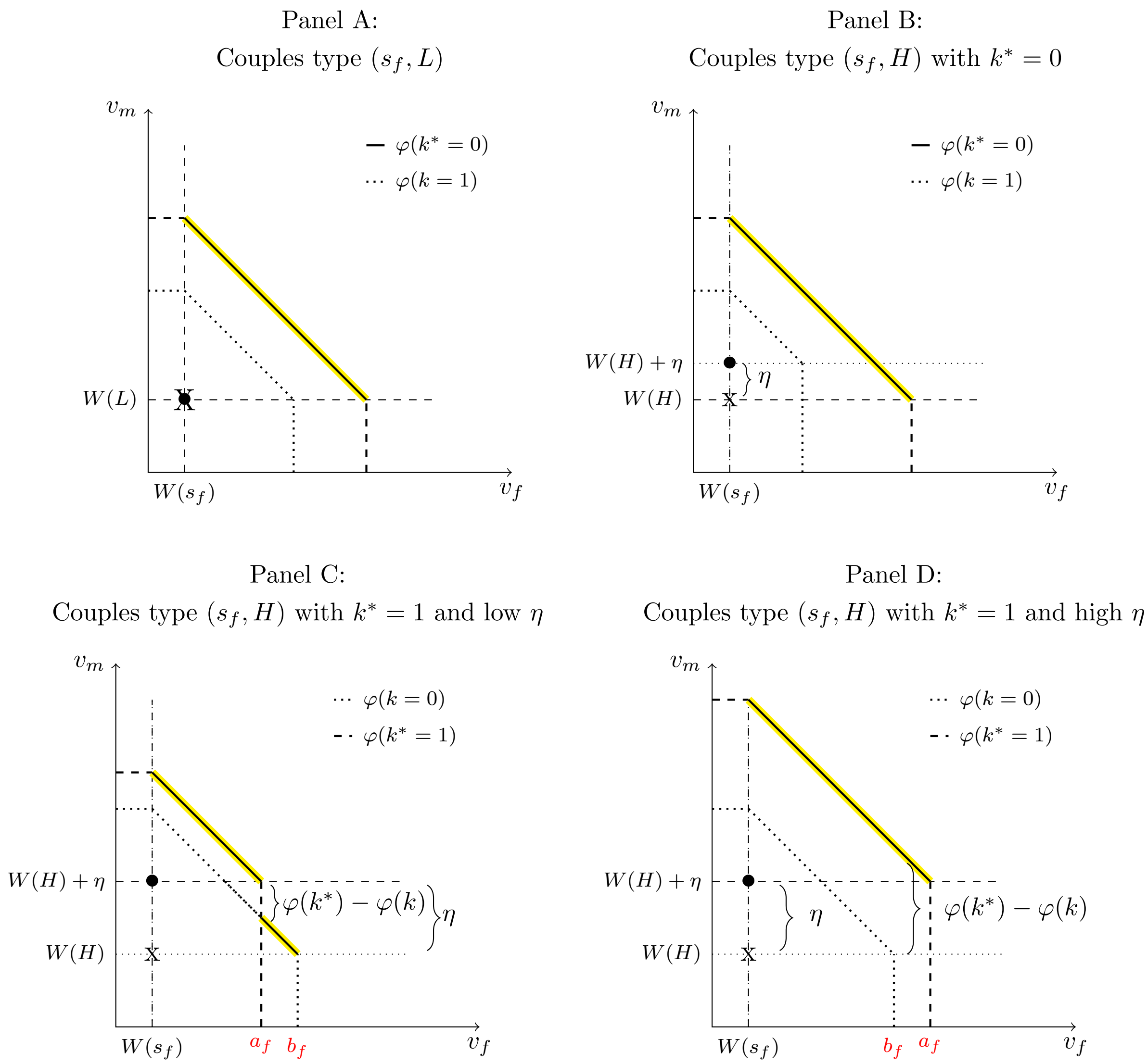


Note that this figure is analogue to the plot in panel A in figure 2 that captured the gains from marriage associated with action k^* when the regime is MCD. The value of marriage conditional on action k, $\varphi(s_f, s_m, k)$, is constructed the same way under both regimes, with the exception that under unilateral divorce the marital value is kinked at the autarky values. The kinks in figure 3 appear because when divorce does not require settlement, in the second period any spouse can unilaterally end the relationship and take their second period value of autarky with them. This implies that any given female or male prices falling below the autarky values cannot be delivered.

However, note that conditional on marriage market prices being feasible and above autarky values, there are infinite combinations of *within period* sharing rules that deliver the same values $v_m(s_m)$ and $v_f(s_f)$.

To study the constrained efficient choice of k, figure 4 depicts the utility possibility frontiers (relationship (9)) associated with choices $k = 0$ and $k = 1$ for different types of couples and different values of $\eta(s_m) = \eta$. In this model there are four possible scenarios:

Figure 4: Ex ante Pareto frontier for different types of couples and η values.



Panel A plots this relationship for couples with any type of woman $s_f = \{l, h\}$ and L-type males. For these couples, the efficient action for the household is $k = 0$ (proposition 7). Panels B through C depict the case for couples with any type of woman $s_f = \{l, h\}$ and H-type males. For these couples, whether $k = 0$ or $k = 1$ is the efficient household investment depends on the values of the gains from specialization, η (proposition 7). Panel B shows the case where η is low enough so that the efficient action for the household is $k = 0$. Panel C depicts the case where the efficient household investment is $k = 1$ but the household gains from specialization, $\varphi(s_f, s_m, 1) - \varphi(s_f, s_m, 0)$, are lower than the male gains from specialization, η. Finally, Panel D shows the case where the efficient household investment is $k = 1$ and the household gains from specialization, $\varphi(s_f, s_m, 1) - \varphi(s_f, s_m, 0)$, are higher than the male gains from specialization, η.

In all panels, the vertical axis represents the male price, v_m, and the horizontal axis represents the female price, v_f. All panels plot relationship (9) for decisions $k = 1$ and $k = 0$ together. In all panels, the two relationships are kinked at the corresponding value of autarky for both spouses, which depend on the choice of k. If the household does not specialize ($k = 0$), the kink is at point $(W(s_f), W(s_m))$, indicated with an "X" in the figures. If the household specializes, males accumulate human capital, so the kink is at point $(W(s_f), W(s_m) + \eta(s_m))$ indicated with a bullet point "$\bullet$" in the figures.

In Panel A, couples of wives of human capital $s_f = \{l, h\}$ and L-type husbands, maximize husband's utility given any feasible female price $v_f \in [W(s_f), \varphi(s_f, L, 0) - W(L)]$ by taking the household efficient action $k^* = 0$.

Panel B shows a similar picture. Couples of wives of human capital $s_f = \{l, h\}$ and H-type husbands, maximize husband's utility given any feasible female price $v_f \in [W(s_f), \varphi(s_f, L, 0) - W(L) - \eta]$ by taking the household efficient action $k^* = 0$.

In Panels C and D couples (s_f, H) maximize their joint marital value by specializing in the first period ($k = 1$). In both panels, the outer dashed kinked lifetime utility frontier corresponds to the frontier conditional on taking the efficient action, $k^* = 1$. The inner dotted kinked lifetime utility frontier corresponds to the frontier conditional on the couple's taking the inefficient action, $k = 0$. Because the efficient action is to specialize, the outer frontier is kinked in the horizontal dimension at a value of female share, a_f, that secures the husband his second period earning plus the boost in earnings coming from having specialized in the first period $(W(H) + \eta)$. On the contrary, the inefficient frontier is kinked in the horizontal dimension at a female share, b_f, that secures a lower second period outside option for the male $(W(H))$.

In Panel C, couples of wives of human capital $s_f = \{l, h\}$ and H-type husbands, do not always maximize husband's utility given any feasible female price by taking the household efficient action $k^* = 1$. In particular, for feasible female prices $v_f \in [\varphi(s_f, H, 1) - W(H) - \eta, \varphi(s_f, H, 0) - W(H)]$, the couple maximizes the husband's lifetime value by choosing the constrained efficient action $k = 0$.

In Panel D, couples of wives of human capital $s_f = \{l, h\}$ and H-type husbands, maximize husband's utility given any feasible female price $v_f \in [W(s_f), \varphi(s_f, H, 1) - W(H) - \eta]$ by taking the household efficient action $k^* = 1$.

This illustration is formally stated and proved in the next proposition. Proposition 3 is the analogue in the unilateral divorce regime of proposition 1 in the mutual consent divorce regime. The result in proposition 1 stated that independently of the lifetime allocation of value among spouses, the couple in the MCD regime always finds it optimal to take the action $k = \{0, 1\}$ that maximizes household output. In contrast, proposition 3 below demonstrates that in the UD regime, couples with H-type males may find optimal an action that is inefficient for the household.

Proposition 3 *Under UD, the choice of k that maximizes the couple's problem (7) depends on the given female marriage market utility price. In particular:*

1. *For couples (s_f, s_m) such that $\varphi(s_f, s_m, k = 0) > \varphi(s_f, s_m, k = 1)$, husband's utility is maximized by choosing $k = 0$ for any feasible female price.*

2. *For couples (s_f, H) such that $\varphi(s_f, H, k = 1) > \varphi(s_f, H, k = 0)$, the couple's choice is $k = 1$ for feasible female prices $v_f \in [W(s_f), \varphi(s_f, H, 1) - W(H) - \eta]$; and $k = 0$ for feasible female prices $v_f \in [\varphi(s_f, H, 1) - W(H) - \eta, \varphi(s_f, H, 0) - W(H)]$.*

Proof 3 *See appendix C.4.* ■

All in all, proposition 3 implies that the total lifetime value produced by a marriage depends on the sharing of this value. First, the value of female price v_f determines the optimal choice of k. Second, and conversely, the choice of k determines the set of values (v_f, v_m) that can be delivered at the time of marriage because k influences the value of autarky. Hence, under UD we can no longer separate the problem of choosing k from the problem of maximizing the husband's utility given the wife price. We, therefore, lose the transferability of the lifetime marital surplus that characterized the MCD regime.

The utility possibility frontiers (captured by relationship (9) for all types of couples and model parameters) are described below:

$$v_m(k) = \varphi(s_f, s_m, k) - v_f(s_f), \quad \forall (v_f, v_m) : \begin{cases} v_f(s_f) \geq W(s_f) \\ \varphi(s_f, s_m, k) - v_f(s_f) \geq W(s_m) + \eta(s_m)k \end{cases}$$

- For couples $(s_f, s_m) \in \{h, l\} \times \{L, H\}$ such that $\varphi(s_f, s_m, 0) > \varphi(s_f, s_m, 1)$,

$$v_m = \varphi(s_f, s_m, 0) - v_f(s_f), \quad \forall (v_f, v_m) : \begin{cases} v_f(s_f) \geq W(s_f) \\ \varphi(s_f, s_m, 0) - v_f(s_f) \geq W(s_m) \end{cases}$$

- For couples $(s_f, H) \in \{h, l\} \times \{H\}$ such that $\varphi(s_f, H, 1) > \varphi(s_f, H, 0)$ and $\varphi(s_f, H, 1) - \varphi(s_f, H, 0) < \eta$,

$$v_m = \begin{cases} \varphi(s_f, H, 0) - v_f(s_f), \quad \forall (v_f, v_m) : \begin{cases} v_f(s_f) \geq a_f \\ \varphi(s_f, H, 0) - v_f(s_f) \geq W(H) \end{cases} \\ \varphi(s_f, H, 1) - v_f(s_f), \quad \forall (v_f, v_m) : \begin{cases} v_f(s_f) \geq W(s_f) \\ \varphi(s_f, H, 1) - v_f(s_f) \geq W(H) + \eta \end{cases} \end{cases}$$

  where $a_f = \varphi(s_f, H, 1) - (W(H) + \eta)$

- For couples $(s_f, H) \in \{h, l\} \times \{H\}$ such that $\varphi(s_f, H, 1) > \varphi(s_f, H, 0)$ and $\varphi(s_f, H, 1) - \varphi(s_f, H, 0) \geq \eta$,

$$v_m = \varphi(s_f, H, 1) - v_f(s_f), \quad \forall (v_f, v_m) : \begin{cases} v_f(s_f) \geq W(s_f) \\ \varphi(s_f, H, 1) - v_f(s_f) \geq W(s_m) + \eta \end{cases}$$

An aspect of this model that is worth emphasizing is that there exist values of the gains from specialization η, such that couples of type (s_f, H) face a discontinuous utility possibility frontier, as depicted in panel C of figure 4. As the graph in panel C of figure 4 indicates, such structure of the model occurs at values of η for which $k^* = 1$ and $\varphi(s_f, H, 1) - \varphi(s_f, H, 0) - \eta < 0$. Lemma 8 in appendix C.6 shows that the difference $\varphi(s_f, H, 1) - \varphi(s_f, H, 0) - \eta$ is negative for the lowest values of η, it increases as η increases, and becomes zero at a threshold $\eta^{**}_{s_f, H}$. At this threshold, the couple efficient action is still $k^* = 1$ but the the utility possibility frontier becomes continuous as depicted in panel D of figure 4.

3.5 Existence of equilibria under UD

In this section I demonstrate the existence of the equilibrium under unilateral divorce with no enforcement of divorce settlements. I first remark that the model is of imperfectly transferable utility (henceforth, ITU). Then, I propose an algorithm to find equilibria.

Remark 2 *When divorce is unilateral and no divorce settlement is enforced, there exist parameters of the model such that the utility possibility frontier for couples* (s_f, s_m), $Ev_f = \varphi(s_f, s_m, Ev_m)$ *is not separable in* Ev_m. *Hence, the lifetime marital surplus is non transferable.*

Remark 2 reflects that the sharing rule influences the size of the value to be shared.

Existence of equilibria in these types of models cannot, then, rely on total surplus maximization. In fact, the concept of "marital surplus" is no longer well defined in the ITU context since the value of the sum of partner's utilities changes depending on how this total welfare is shared among spouses. To find stable outcomes and prove their existence, I then rely on the well known algorithm proposed by Crawford and Knoer (1981) and Kelso and Crawford (1982), namely, the *salary adjustment process.* This algorithm is an extension of the Gale-Shapley deferred acceptance algorithm (Gale and Shapley, 1962) to the case where utility can be transferred imperfectly between the two sides of the market. In the algorithm, individuals in one side of the market (say, the females) make marital proposals to their preferred individuals in the other side of the market (say, the males) successively until they are tentatively accepted. Marital proposals consist of a feasible utility payment. Each round male receive utility offers and keep on hold the best offers and rejects the others. If at any round a woman is rejected by a man she finds desirable, in the subsequent round, the woman offers this man a higher utility if (at this increased price) the man continues to be desirable. The process stops when no rejections are issued and males accept their offers on hold. Note that in this model individuals differ only by their skill type ($s_f = \{l, h\}$ for females and $s_m = \{L, H\}$ for males). Hence, the stable outcome must satisfy *equal treatment* (Legros and Newman, 2007), that is, equilibrium utility levels for all individuals within the same type must be equal. Otherwise, all individuals would want to marry the person of their preferred type in the other side of the market that has the lowest price, implying the assignment would not be an equilibrium one.

Hence, I adapt the Kelso and Crawford (1982) salary adjustment algorithm to the case where individuals only differ by a discrete type and the marriage game is monogamous (one-to-one).

The female (male) proposing utility adjustment algorithm is as follows:

1. Females (males) begin facing a set of "permitted" utility values to offer males (females).
   - Initial permitted utility values are the minimum utility levels that have to be granted to a potential partner for this partner not to prefer to remain single. Since in this model the value of being single is zero, the initial permitted value is the value of divorce.
2. Females (males) start by offering the initial permitted utility value to a male (female) of their preferred type. For example, females choose to offer $v^0_{s_m} = \{W(s_m), W(s_m) + \eta(s_m)\}$ to a potential partner of type s_m such that:

$$\max_{s_m \in \{L,H\}} \varphi(s_f, s_m) - v^0_{s_m}$$

   The proposing side may break ties with any arbitrary rule.
3. Each male (female) who receives at least one offer puts on holds his (her) preferred one. The receiving side may break ties with any arbitrary rule.
4. Offers not rejected remain in force. Offers rejected are updated the following way:
   - Define a male (female) to be *available at round* n^{th} if at the end of round n^{th} they are of the preferred type (at the price offered at round n^{th}) and are holding an offer from a previous round $\tau < n^{th}$ or are holding no offer.
   - If there are *available* males (females), at round $n^{th} + 1$ repeat the n^{th} round offer to one of the available males (females) of the preferred type. This is the round $n + 1$ current permitted utility. Note that repeating offers from previous rounds to individuals within a type ensures the *equal treatment* condition that must hold in a stable outcome.
   - If there are no *available* males (females), at round $n^{th} + 1$ offers are increased by an arbitrarily small number, ϵ. For example, if a female was rejected by a male of type s_m at round n, the female updates her offer to $v^{n+1}_{s_m} = v^n_{s_m} + \epsilon$. This is the round $n+1$ current permitted utility.
   - At any round r^{th}, rejected females (males) continue to make offers to their preferred potential partner, taking into account the current permitted utility values. For exam-

ple, rejected females make offers to a potential partner of type s_m such that:

$$\max_{s_m \in \{L,H\}} \varphi(s_f, s_m) - v^r_{s_m}$$

5. The process stops at the round when no rejections are issued. Males (females) accept the offers they have on hold.

The algorithm described above is analogous to an algorithm in which *types* on one side of the market make proposals to *types* on the other side of the market, and the matching between individuals is random. This algorithm satisfies all the assumptions made by Crawford and Knoer (1981) and Kelso and Crawford (1982). Hence, existence of stable outcomes in this model is guaranteed by existence of stable outcomes in the Crawford and Knoer (1981) and Kelso and Crawford (1982) environments. The following proposition states this formally.

Proposition 4 *The female (male) proposing salary adjustment process converges to a stable matching.*

Proof 4 *Crawford and Knoer (1981) and Kelso and Crawford (1982) show that the process converges to discrete or continuous core allocation. Because the matching model in this paper is one-to-one, the core allocations coincide with the set of stable matchings.* ■

4 The impact of shifting from MCD to UD

4.1 Outline

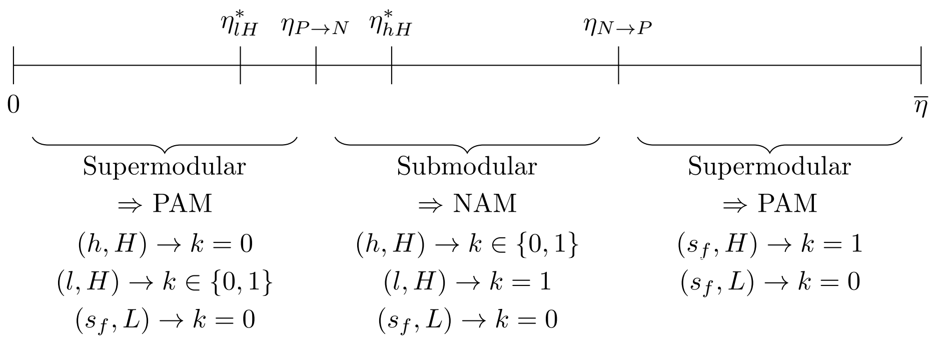
In this section I prove the main result of the paper: the introduction of UD causes an increment in assortativeness in the marriage market. To do this, I first characterize matching patterns under the two divorce regimes. In section 4.2 I characterize the optimal matching, that is, the equilibrium under MCD. In section 4.3 I characterize the equilibrium delivered by the female proposing mechanism under UD. I then derive conditions under which matching patterns under both regimes differ. Finally, I present three numerical exercises to illustrate the main result.

4.2 Characterization of the optimal efficient matching

We are interested in predicting what type of females match to what type of males in a marriage market where the divorce regime is of mutual consent. In a transferable utility environment, Becker (1973) and Shapley and Shubik (1971) showed that in order to characterize the matching patterns it suffices to study the modularity of the surplus function, $\varphi(s_f, s_m)$ in this context. When the surplus function is supermodular,[5] the optimal matching is *positive assortative matching* (henceforth, PAM): high types match together and low types match together. When the surplus function is submodular,[6] the optimal matching is *negative assortative matching* (henceforth, NAM): high types match low types.

In this transferable utility environment, the value that different types of marriages produce depend on their choice of marital investment $k = \{0, 1\}$. Hence, in appendix B I study the patterns of efficient choice of $k = \{0, 1\}$ and the implied modularity of the surplus as a function of the male gains from specialization, η. First, I present the closed form structure of the surplus (section B.1) and make further assumptions for tractability (section B.2). The main results from the analisis under this structure are illustrated in figure 5.

Figure 5: Equilibrium sorting and household specialization.



In section B.3 I characterize the efficient marital investment decisions. First, note that in this model couples with L-type males always find it efficient to choose $k = 0$, given that they draw no gains from specialization ($\eta(L) = 0$). This is not the case for couples with H-type males for whom the returns from choosing $k = 1$ are strctily increasing in the value of η.

[5]The surplus function $\varphi(s_f, s_m)$ is supermodular if $\varphi(h, H) + \varphi(l, L) > \varphi(l, H) + \varphi(h, L)$. This means that high female (male) types are more valuable to high male (female) types than to low male (female) types.

[6]The surplus function is submodular if $\varphi(h, H) + \varphi(l, L) < \varphi(l, H) + \varphi(h, L)$. This means that high female (male) types are more valuable to low male (female) types than to high male (female) types.

Proposition 7 shows that there exist thresholds $\eta^*_{l,H}$ and $\eta^*_{h,H}$ below which couples type (l, H) and (h, H) efficiently choose $k = 0$ and above which they efficiently choose $k = 1$, respectively. Hence, when the male gains from specialization, η, are low, no couple chooses to specialize. As the gains from specialization increase, the marital output under specialization approaches, and eventually offsets, the marital output under no specialization for couples with H-type males.

This characterization allows me obtain the closed form structure of the marital surplus for each couple type (s_f, s_m) at each possible value of η. In figure 5, underbraces at the bottom indicate the modularity of the surplus function at different values of η. The figure illustrates the statement that is formally proved in proposition 8: there exist two thresholds, $\eta_{P\to N}$ and $\eta_{N\to P}$, such that at values of η below $\eta_{P\to N}$ and above $\eta_{N\to P}$ the optimal matching is PAM, whereas at values of η in between these thresholds the optimal matching is NAM. Intuitively, at low levels of η, no couple would choose to specialize. Hence, complementarity in spousal earnings imply that H-type males gain more from marrying a woman of a higher type than L-type males. As η increase and couples with H-type males start choosing $k = 1$, a trade off arises between female earnings in the first period and male boost in earnings in the second period. At the lowest values of η such that specialization is efficient, l-females complement H-males more than they complement L-males: by marrying low skill females, H-type males obtain the same gains from specialization as marrying high skill females, but forego a lower value of first period female earnings. At the highest values of η, on the contrary, H-type males can afford to lose a higher value of female earnings, hence making them able to outbid L-type males to marry the h-type females.

4.3 Characterization of marriage market equilibria under UD

In the remainder of the paper, I focus on studying the properties and implications from the equilibrium arising from the female proposing salary adjustment algorithm. There are two reasons for this. First, census data suggests that females are in the short side of the market at the age of first marriage (see, for example, figure 2 in Choo and Siow (2006)). Second, in the model, marital investments are ultimately a choice of the female (whether to work or not). Being in excess demand and making the decision about k, hence, justifies the exploration of the equilibria that arise from a mechanism that gives females the highest potential to extract rents (with respect to singlehood) from matching.

Under what conditions does the female proposing mechanism deliver an equilibrium that exhibits positive or negative assortative matching? In this section I provide sufficient conditions. To simplify exposition, I make the following assumption:

Assumption 3 *The mass of females is higher than the mass of males of the H-type and the L-type. That is, $\mu_{\mathcal{X}} \geq \mu_H$ and $\mu_{\mathcal{X}} \geq \mu_L$.*

As Legros and Newman (2007) show for general ITU models, sorting patterns in this model depend on the ability of different types to make offers to their preferred type on the other side of the market. Intuitively, if high types are more able to outbid low types when both make proposals to high types, the structure of the equilibrium will have males and females of similar types matched together: the high types will be able to marry high types and matching will move down the ranking of the proposing side. The next definition formalizes this idea.

Definition 4 [Positive transfer complementarity] *Let $\overline{v}_{H\leftarrow s_f}$ be the offer that makes females of type s_f indifferent between paying $\overline{v}_{H\leftarrow s_f}$ to H-type males and paying the minimum acceptable price to L-type males ($W(L)$):*

$$\overline{v}_{H\leftarrow s_f} : max_k\{\varphi(s_f, H, k)\} - \overline{v}_{H\leftarrow s_f} = \varphi(s_f, L, 0) - W(L) \tag{10}$$

*A model exhibits **positive transfer complementarity (PTC)** if $\overline{v}_{H\leftarrow l} < \overline{v}_{H\leftarrow h}$.*

In words, suppose both types of females are competing in a female proposing mechanism for the H-type males. PTC is a condition that specifies that high type females are able to make offers to high type males for more rounds than low type females. In effect, when PTC is satisfied, low type females "give up" at a round before high type females when they are updating H-type males prices. The next proposition relates PTC to sorting patterns delivered by the female proposing mechanism.

Proposition 5 *PTC on the female side is sufficient for the female proposing mechanism to deliver an equilibrium that exhibits positive assortative matching.*

Proof 5 *See appendix C.5* ■

4.4 Characterization of change in equilibria

In this section, I compare marriage market equilibria under a unilateral divorce regime to the optimal assignment (that is, the equilibrium under mutual consent divorce regime). The main result from this section is that a change from MC to UD can only increase assortativeness in the marriage market. As I show below, the stable matching under unilateral divorce is either optimal (and hence it does not change with respect to the equilibrium under mutual consent regime) or suboptimal and positive assortative (and hence it changes with respect to the optimal negative assortative matching equilibrium under mutual consent).

Whether equilibria are different across divorce regimes depends on two features of the structure of the marriage market: the modularity of the surplus (undestanding *surplus* as in definition 2), and the distance between the marital gains from household specialization and the male earnings gain from specialization, $\varphi(s_f, H, k = 1) - \varphi(s_f, H, k = 0) - \eta$. In this section I show that whenever the surplus is supermodular, the female proposing mechanism delivers an equilibrium that exhibits positive assortative matching. Hence, equilibria under both divorce regimes exhibit the same sorting patterns. Contrary to this, when the surplus is submodular, the female proposing algorithm may converge to an outcome that exhibits negative or positive sorting, depending on the distance $\varphi(s_f, H, k = 1) - \varphi(s_f, H, k = 0) - \eta$. In the first case, equilibria under both regimes show the same matching patterns: they are both NAM. In the second case, sorting increases from NAM under mutual consent to PAM under unilateral. The next proposition formally proves this assertion.

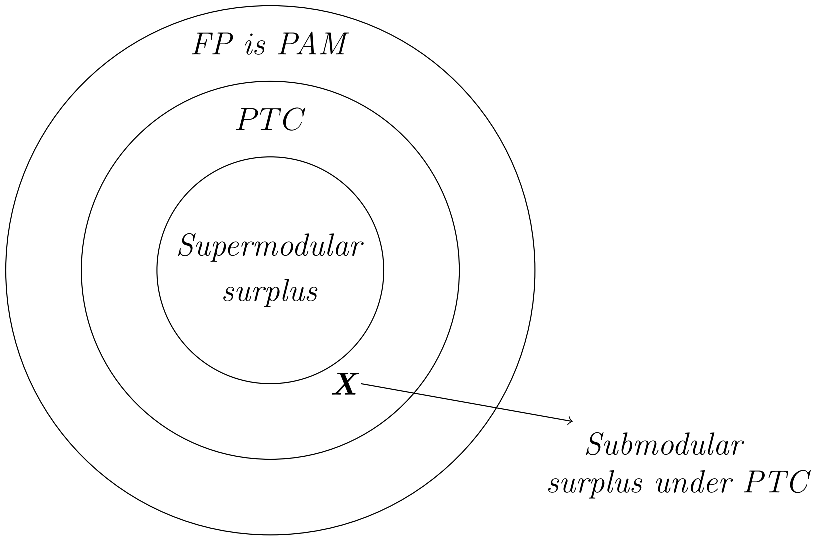
Proposition 6 *A change in the divorce regime from mutual consent divorce to unilateral divorce results in a weakly increment in assortativeness in initial human capital.*

Proof 6 *See appendix C.6* ■

Figure 6 illustrates the proof of this statement. Each Venn diagram represents the set of model parameters such that the structure of the model satisfies the property indicated in each diagram. The inclusion relation illustrates the implication relation. I next present a sketch of the proof.

Sketch of proof:

Figure 6: Increment in assortativeness after a change from MC to UD



1. I start by showing that under any values of the parameters of the model, both types of females start making offers to H-type males in a female proposing algorithm (lemma 9).
2. I then show that whenever the marital surplus is supermodular, PTC is satisfied (lemma 10).
   - In figure 6, the set of parameter values for which the surplus is supermodular is included in the set of parameters for which the model satisfies PTC.
   - This means that females of both types start making successively increasing offers to H-type males until they reach a round at which l-type females “give up” and start offering to L-type males.
3. Hence, whenever matching is positive assortative under the mutual consent divorce regime, it will be positive assortative when females propose in a unilateral divorce regime.
   - By proposition 5, under a supermodular surplus the equilibrium delivered by the female proposing mechanism is PAM. In figure 6, the set of parameter values for which the model satisfies PTC is included in the set of parameter values for which the female proposing algorithm delivers a PAM outcome.
   - Because the surplus is supermodular, the optimal matching is also PAM.
4. Finally, I show that when the surplus is submodular, PTC may or may not hold (lemma 11).

- In figure 6, there exists a set of parameter values, denoted with "X", such that the surplus is submodular but PTC holds.
- I show that there exists a threshold $\overline{\eta^{nn}}$ such that for values of $\eta < \overline{\eta^{nn}}$, PTC holds in a submodular area. That is, PTC holds for η such that

$$\eta_{P\to N} < \eta < \overline{\eta^{nn}},$$

where $\overline{\eta^{nn}}$:

$$\varphi(l, H, 1) - \varphi(l, L, 0) - (W(H) - W(L)) - \eta = 0 \tag{11}$$

- Threshold $\overline{\eta^{nn}}$ defines a *non neutrality* area (hence, the superscript "nn"), where the optimal matching is negative assortative, but the equilibrium delivered by the female proposing mechanism is PAM.

5. Hence, a change from mutual consent to unilateral divorce increases assortativeness.

4.5 Numerical examples

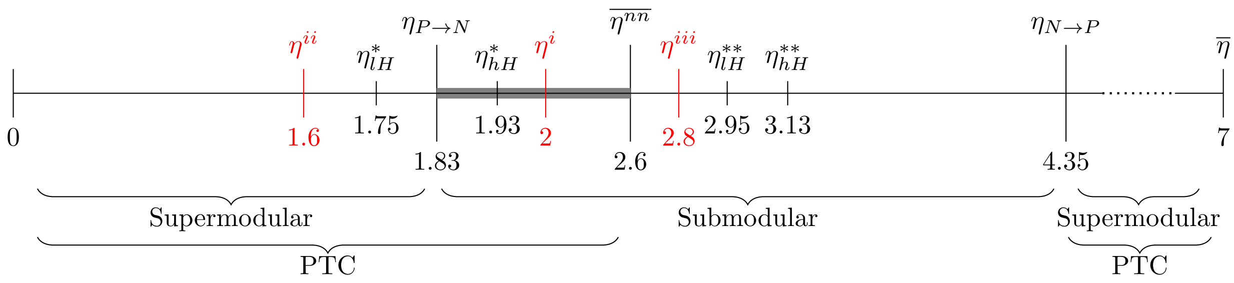
In this section I simulate the solution of the model for a fixed set of structural parameters and illustrate the statement that a change in the divorce regime from mutual consent divorce to unilateral divorce increases assortativeness in the marriage market. The fixed structure of the model is as follows:

- the match quality is drawn from the uniform distribution in the range $[-10, 10]$: $\theta \sim U[-10, 10]$, and
- the labor market structure is $\mathcal{W} = (W(l), W(h), W(L), W(H)) = (2, 2.2, 2.1, 2.4)$.

Figure 7 depicts the relevant thresholds in the range of possible values of the male gains from specialization, η, associated with this fixed structure of the model.

For this structure, the upper bound in the gains from specialization, $\overline{\eta}$, is approximately 7. As proposition 8 shows, low levels of η correspond to an area in which the surplus is supermodular. At $\eta_{P\to N} = 1.83$ the marital surplus becomes submodular. The submodularity area expands until $\eta_{N\to P} = 4.35$, the value at which the surplus becomes supermodular and continues to be so in the η range. The threshold $\overline{\eta^{nn}} = 2.6$ indicates the maximum value of

Figure 7: Relevant η thresholds when $\theta \sim U[-10, 10]$ and $\mathcal{W} = (2, 2.2, 2.1, 2.4)$



η in the submodular area for which condition PTC is satisfied. Hence, the shaded segment in the figure corresponds to values of $\eta \in [1.83, 2.6]$ such that a change in the divorce regime resulting in reduced commitment to sharing among ex spouses, increments assortativeness in the marriage market. Other relevant thresholds are $\eta^*_{l,H}$ and $\eta^*_{h,H}$, that indicate the value of η at which $k = 1$ becomes the efficient household investment for couples (l, H) and (h, H), respectively. The last thresholds to note are $\eta^{**}_{l,H}$ and $\eta^{**}_{h,H}$, indicating the value of η at which the utility possibility frontier for couples (l, H) and (h, H), respectively, becomes continuous when the efficient household investment is $k = 1$. That is, at values of η such that $\eta^*_{s_f,H} < \eta < \eta^{**}_{s_f,H}$, the utility possibility frontier for couple (s_f, H) is discontinuous as depicted in panel C of figure 4, whereas at values of $\eta \geq \eta^{**}_{s_f,H}$ the utility possibility frontier for couple (s_f, H) is continuous as depicted in panel D of figure 4.[7]

Finally, the figure indicates three values of the gains from specialization, η^i, η^{ii}, and η^{iii} in three different relevant areas. These values will be imputed to solve the model for the three cases described next.

Case I illustrates an economy with gains from specialization $\eta^i = 2$ for which the surplus is submodular and PTC holds. Moreover, since $\eta^i > \eta^*_{hH} > \eta^*_{lH}$, couples with H-type husbands produce more by specializing $(k = 1)$. Recall that at any value $\eta \in [0, \overline{\eta}]$ couples with L-type males produce more by not specializing $(k = 0)$. Figure 8 shows the conditional utility possibility frontiers associated with each type of potential couple. For couples with H-type males, there exist male prices such that females secure a higher marital share by inefficiently choosing $k = 0$. The theory predicts that the optimal matching is NAM, but that the equilibrium under unilateral divorce is PAM. In effect, this is what happens when females propose.

[7]Existence of these thresholds is established in lemma 8 in appendix C.6.

Different rounds of the female utility proposing mechanism are displayed in the graph.[8] In the first round, females propose the minimum acceptable value to their preferred male. When they consider L-type males, the minimum they can offer is $W(L)$. When females consider H-type males, the minimum they can offer is $W(H)$ and choosing $k = 0$. Both types of females, $s_f = l$ and $s_f = h$ find H-type males most preferable when they offer the minimum acceptable price (this is true in this example and in general, as lemma 9 in appendix C.6 shows). That is,

$$\forall s_f = \{l, h\} : \quad H = \arg \max_{s_m \in \{L,H\}} \varphi(s_f, H) - W(s_m)$$

Hence, after the first round, males of type H have multiple and equal offers from l-type and h-type females. This is indicated with bullet points "1^{st}" in the graph. On the contrary, L-type males receive no offer. The H-type males break the ties however they like. In the second round, rejected females increment their offer to H-type males by ϵ and evaluate whether they still prefer to marry an H-type male. If they propose to the H-type male, a rejected woman of type s_f would gain

$$v_{s_f}^2 = \varphi(s_f, H) - W(H) - \epsilon$$

If, instead, rejected female of type s_f proposes to the L-type male, she would gain

$$v_{s_f}^2 = \varphi(s_f, H) - W(L)$$

In this example, both low and high type females prefer to keep increasing their offer to H-type males. Hence, in the second round, women who were rejected in the first round update their offer to the H-type males. H-type males, hence, reject the first round offer kept on hold. This horse race between l and h-type females to win the H-type males continues until one type of female becomes indifferent between H and L-type males. Because under these values of parameters, the model satisfies PTC, l-type females give up first. This happens at a certain round n^{th}, indicated in the graph. This is the round where l-type females become indifferent between H and L-type males. That is, after n rounds of updating the offer to the H-type males

[8]For ease of exposition, I consider a round to be an instance in which there are no more *available* males of each type (in the sense defined in the algorithm presented in section 3.5). Hence, offers to available males occur in "sub-rounds".

by ϵ,

$$\begin{aligned} v_l^n &= \varphi(l,H) - W(H) - n\epsilon \\ &= \varphi(l,H) - \overline{v}_{H\leftarrow l} = \varphi(l,L) - W(L) \end{aligned}$$

At this offer to H-type males, on the contrary, the h-type females still prefer to propose to the H-type males:

$$v_h^n = \varphi(h,H) - W(H) - n\epsilon > \varphi(h,L) - W(L)$$

At this round, hence, l-type females propose to the L-type males and h-type females continue to propose to the H-type males. Whether the algorithm stops at this round or not depends on the relative measure of females to males within skill types.

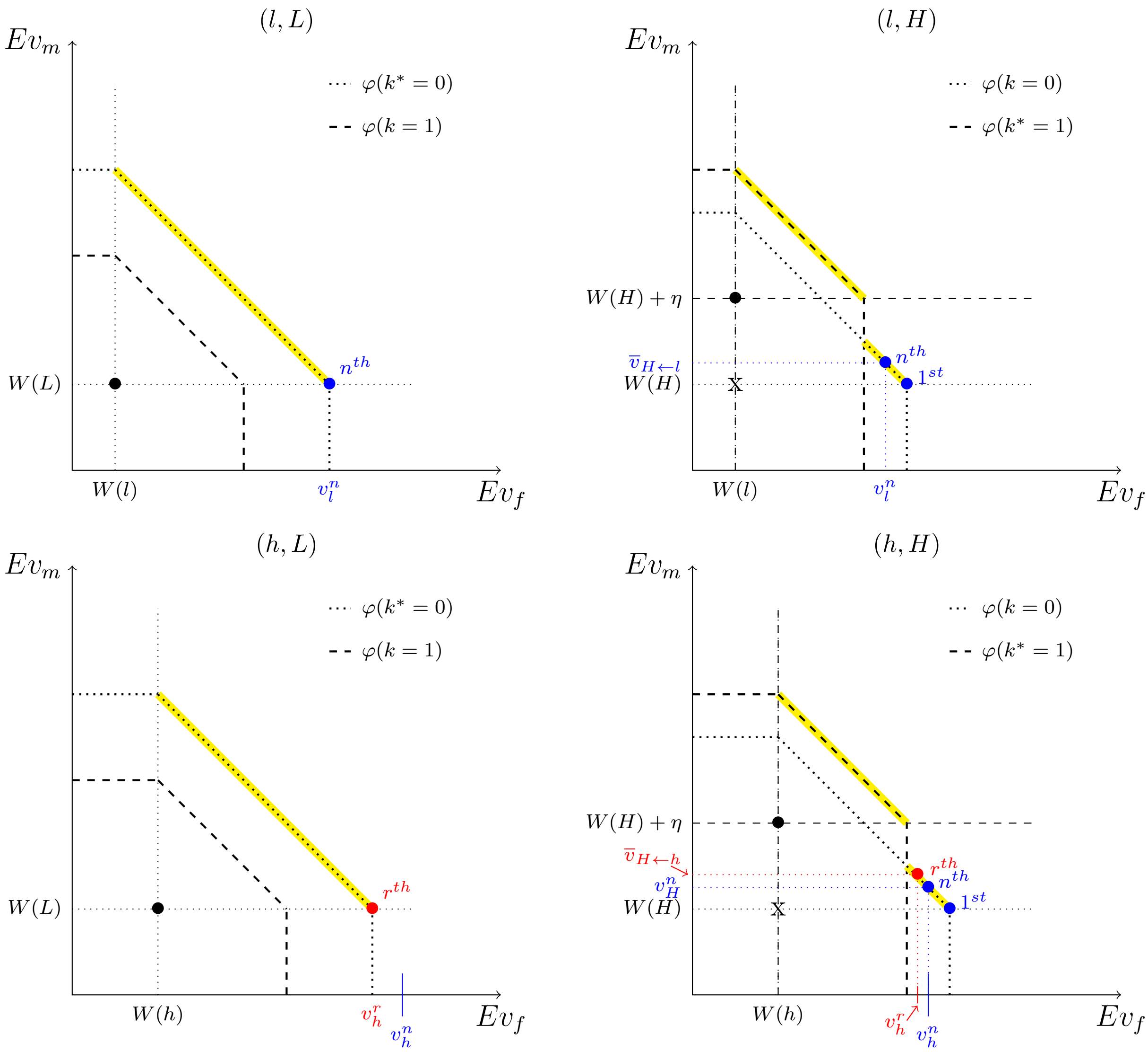
If the mass of h-type females is less than or equal to the mass of H-type males ($\mu_h \leq \mu_H$), the algorithm stops at round n. At this round, all h-type females marry H-type males, some l-type females marry the excedent of H-type males, and some l-type females marry L-type males. The matching pattern is positive assortative.

If, on the contrary, the mass of h-type females is greater than the mass of H-type males ($\mu_h > \mu_H$), the algorithm will not stop at round n. First, note that since there are more males than females in the market, the amount of l-type females is less than the amount of L-type males. Hence, at round n all L-type males hold offers from l-type females. In turn, h-type females continue to compete among each other for the H-type males, by successively increasing the utility offer. This horse race between h-type females for marrying H-type males continues until h-type women become indifferent between males of type H and L. This happens at a certain round r^{th}, indicated in the graph. At the round r^{th} prices,

$$\begin{aligned} v_h^r &= \varphi(h,H) - W(H) - r\epsilon \\ &= \varphi(h,H) - \overline{v}_{H\leftarrow h} = \varphi(h,L) - W(L) \end{aligned}$$

At this round, some h-type women will offer $W(L)$ to L-type males and some will offer $W(H)+r\epsilon$ to H-type males. Since L-type males receive the same offer from any type of woman, no

Figure 8: Case I: submodular surplus under PTC.



rejections are issued at round r^{th}, and males accept the offers they have on hold. At this round, some h-type females marry H-type males, the excedent h-type females marry L-type males, and all l-type females marry L-type males. The matching pattern is positive assortative.

All in all, the female proposing equilibrium is positive assortative, different from the optimal matching that is negative assortative given that η falls in the submodular area. The key feature that makes the equilibrium change in this example is that the low type females reach the indifference between the two types of males before high type females.

Case II illustrates an economy with lower gains from specialization, $\eta^{ii} = 1.6$, for which the surplus is supermodular (implying that PTC holds). Moreover, since $\eta^{ii} < \eta^*_{lH} < \eta^*_{hH}$, couples with H-type husbands produce more by not specializing ($k = 0$). Figure 9 shows the conditional utility possibility frontiers associated with each type of couple. The figure reveals that for all types of couples, the frontier associated with the efficient action $k^* = 0$ is outside the frontier associated with the inefficient action $k = 1$. The theory predicts that the optimal matching is PAM and that the introduction of unilateral divorce will not change those sorting patterns. In effect, this is what the simulation shows. The rounds of female proposing algorithm are illustrated in figure 9.

Women of both types start competing for the H-type males, increasing the offers successively, until a round n^{th} is reached at which l-type females become indifferent between marrying an H-type male and transferring him the n^{th} round offer, and marrying an L-type male and transferring the minimum acceptable price, $W(L)$. Again, sorting patterns depend on the within type sex ratio.

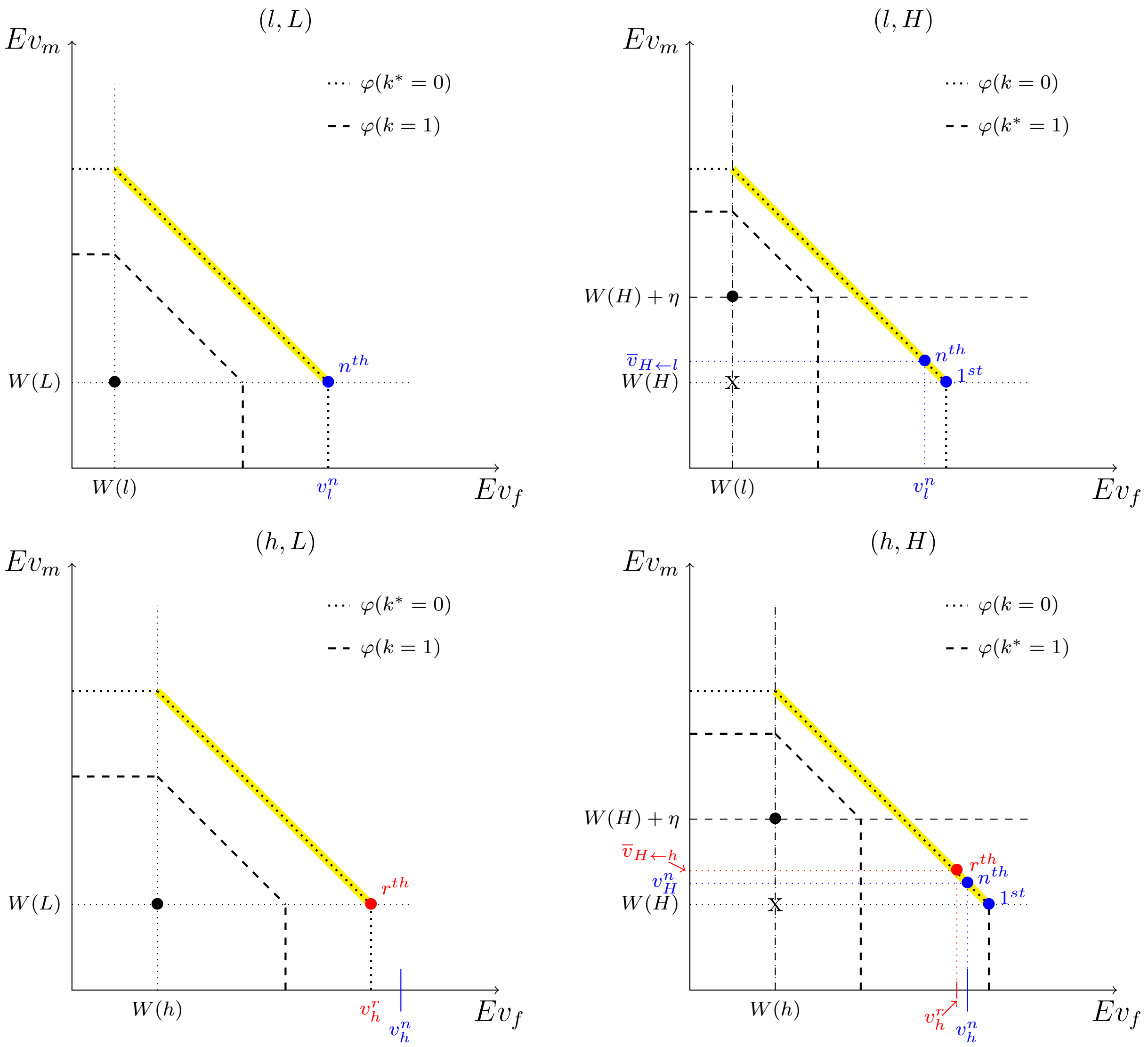
If the mass of h-type females is less than or equal to the mass of H-type males ($\mu_h \leq \mu_H$), the algorithm stops at round n, all h-type females marry H-type males, some l-type females marry the excedent of H-type males, and some l-type females marry L-type males. The matching pattern is positive assortative.

If, on the contrary, the mass of h-type females is greater than the mass of H-type males ($\mu_h > \mu_H$), by the end of round n some L-type males will be holding offers from all l-type females, but h-type females will continue to compete among each other for the H-type males, by successively increasing the utility offer. This horse race between h-type females for marrying H-type males continues until h-type women become indifferent between males of type H and L, at a certain round r^{th}, indicated in the graph. At this round, some h-type females marry H-type males, the excedent h-type females marry L-type males, and all l-type females marry L-type males. The matching pattern is positive assortative.

All in all, the female proposing equilibrium is positive assortative, just as the optimal matching.

Finally, **case III** illustrates an economy with higher gains from specialization, $\eta^{iii} = 2.8$, for which the surplus is submodular but PTC is not satisfied ($\eta^{iii} > \overline{\eta^{nn}}$). Intuitively, since for l-type females the gain from paying the minimum to L-type males is lower than paying the minimum to H-type males under the efficient household investment ($k = 1$), the female proposing rounds will start in the $k = 0$ segment but will jump to the $k = 1$ segment. When l

Figure 9: Case II: supermodular surplus under PTC.



and h type females compete for H-type males in the $k = 1$ segment, the surplus is submodular. Hence, h-type females are less able to transfer to H-type males and the matching ends up being negative assortative. Figure 10 illustrates this. At the offers of round n^{th}, h-type females become indifferent between H and L-type males while l-type women strictly prefer to marry the H-type males.

Once again, sorting patterns depend on the within type sex ratio. If the mass of l-type females is less than or equal to the mass of H-type males ($\mu_l \leq \mu_H$), the algorithm stops at round n^{th}. At this round, all l-type females marry H-type males, (some) h-type females marry L-type males, and the excedent of h-type females (if any) marry H-type males. The pattern is of negative assortativeness. If the mass of l-type females is higher than the mass of H-type males ($\mu_l > \mu_H$), the algorithm continues beyond round n^{th}. At round n^{th}, all h-type females are indifferent between H and L type males, but l-type females strictly prefer the H-type males. Since l-type females are in excess relative to H-type males, l-type females will continue to compete among each other until they reach the indifference between H and L males, at round r^{th}. At this round, all h-type females marry L-type males, some l-type females marry H-type males, and some l-type females marry L-type males. The matching pattern is negative assortative.

All in all, the female proposing algorithm delivers a stable matching that is negative assortative, just as the optimal matching.

Figure 10: Case III: submodular surplus under failure of PTC.

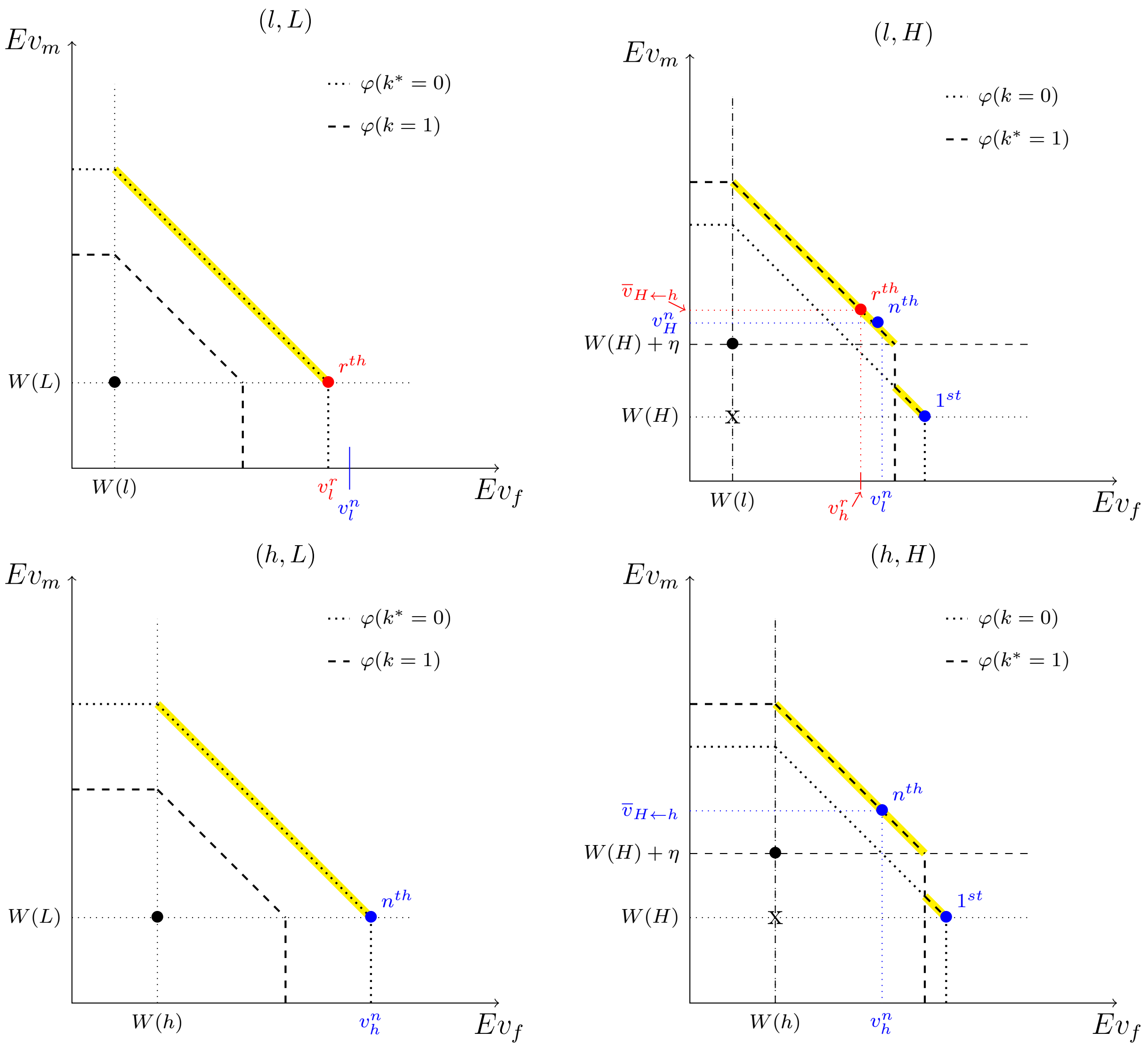
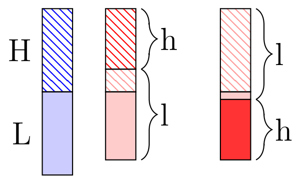
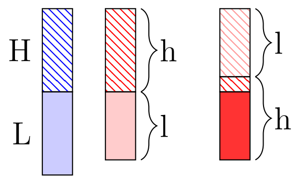
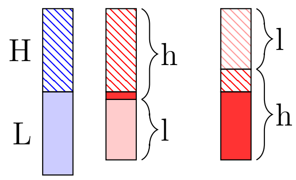


Table 1 summarizes the matching patterns associated with cases I, II, and III, under the two divorce regimes (MC and UD), and three different assumptions on the sex ratio within skill. In the table, the mass of females is normalized to 1 ($\mu_{\mathcal{X}} = 1$), the mass of males is assumed 1.1 ($\mu_{\mathcal{Y}} = 1.1$), and the mass of H-type males equals the mass of L-type males ($\mu_H = \mu_L = 0.55$). The first column shows the case where the percent of h-type females is 0.4, that is, $\mu_h < \mu_H$. The second column shows the case where the percent of h-type females is 0.55, that is, $\mu_h = \mu_H$. Finally, the third column shows the case where the percent of h-type females is 0.6, that is, $\mu_h > \mu_H$.

Table 1: Matching patterns and sex ratio

Populations measures	$\mu_{\mathcal{Y}} = 1.1, \quad \mu_{\mathcal{X}} = 1, \quad \mu_H = 0.55$					
	$\%h = 0.4$		$\%h = 0.55$		$\%h = 0.6$	
						
Pattern	PAM	NAM	PAM	NAM	PAM	NAM
Case I - MC		X		X		X
Case I - UD	X		X		X	
Case II - MC	X		X		X	
Case II - UD	X		X		X	
Case III - MC		X		X		X
Case III - UD		X		X		X

For each of the three cases, the first row depicts bars that illustrate population measures. The first bars plot the skill distribution of males, being half of H type and half of L type. The second bars display the skill distribution of females ordered from h at the top to l at the bottom, and the third bars depict the skill distribution of females in the opposite order. In both sides of the market, light colors represent low skilled individuals and dark colors represent high skilled individuals. Males are also differentiated by the patterns: high skilled males are indicated in a dashed dark (blue) pattern, and low skilled males are indicated in light (blue) filling pattern. Females are only differentiated by the color and, within each (color) type, different patterns match the patterns of the type of males they match. The row labeled *pattern* below each female skill distribution indicates the illustrated matching pattern. For example, when $\mu_h = 0.4 < 0.55 = \mu_H$, in a PAM assignment, all h-type females match H-type males (hence, h females are illustrated in a dashed dark- red- pattern), some l-type females marry the excedent H-type males (this l females being illustrated in a dashed light- red- pattern), and some l-type females marry L-type males (this l females being illustrated in a light- red- filled color pattern). Note that since there are more females than males in the market, some males will end up single for all within skill sex ratios. Finally, the last six rows indicate with an "X" mark the equilibrium matching pattern that results in each of the three cases simulated above, for the MC and the UD regime.

4.6 Empirical evidence

I present evidence in support of the the main prediction of my model that less commitment to sharing in divorce increases assortativeness in the marriage market in Reynoso (2019). I reproduce the main empirical results here.

I estimate the following model for a *newlywed* couple m, time t, and state s:[9]

$$\begin{aligned} HK^w_{mts} &= \alpha + \beta_1 UD_{ts} + \beta_2 HK^h_{mts} \times UD_{ts} + \\ &\quad + \beta_3(t) \times HK^h_{mts} + \beta_4(s) \times HK^h_{mts} + \beta_5(s) \times t + \delta_t + \delta_s + \epsilon_{mts} \end{aligned} \tag{12}$$

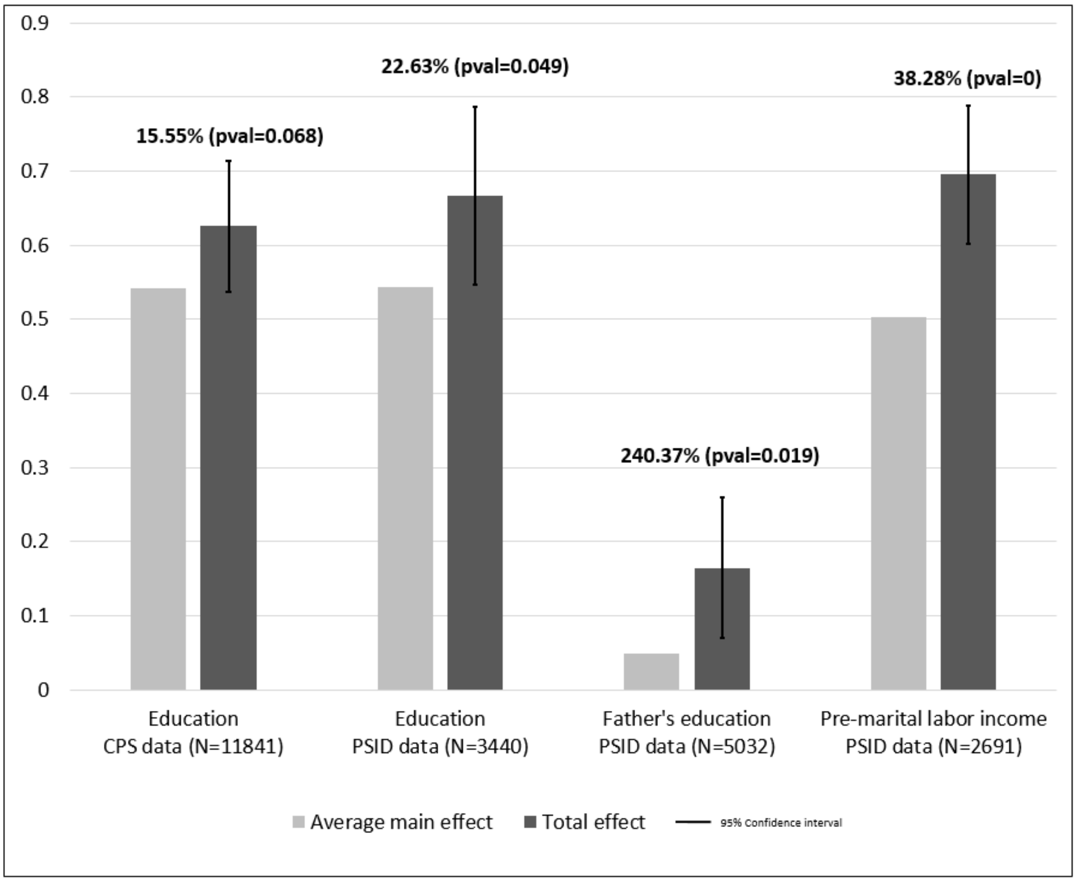
HK^w and HK^h denote a measure of human capital at the time of marriage of the wife and the husband (respectively); UD takes value one when UD is in place and zero when MCD is in place; δ_t are time dummies that control for general trends in female human capital at the time of marriage and δ_s are state dummies that control for permanent differences in female human capital across states. Identification is driven by states that shifted from MCD to UD. A positive relationship between HK^w and HK^h (allowed to vary by state and time in the specification) indicates positive assortative matching on education. Coefficient β_2 measures the extent to which UD changes these sorting patterns.

The data comes from the Current Population Survey (henceforth, CPS) for years 1965 to 1992 and the Panel Study of Income Dynamics (henceforth, PSID) for the years 1968 to 1992. Figure 11 plots in light grey columns the average (over states and times) effect of husband human capital on wife human capital at marriage (that is, the average of $\beta_3(t) + \beta_4(s)$). Dark grey columns show the sum of these average main effects and the additional effect of husband human capital on wife human capital when the couple marries in the onset of the introduction

[9] Newlyweds are couples formed within two years of the survey year. Restricting the analysis to newlyweds allows one to isolate the impact of UD on *matching* patterns from the effect of UD on the investment behavior of already married couples and selection bias driven by the duration of marriage.

of UD (that is, the average of $\beta_3(t) + \beta_4(s)$ plus β_2).

Figure 11: Newlyweds match more assortatively in human capital when UD is introduced (Reynoso, 2019)



Notes: Bars summarize the estimates from model $HK^w_{mts} = \alpha + \beta_1 UD_{ts} + \beta_2 HK^h_{mts} \times UD_{ts} + \beta_3(t) \times HK^h_{mts} + \beta_4(s) \times HK^h_{mts} + \beta_5(s) \times t + \delta_t + \delta_s + \epsilon_{mts}$, where HK^w and HK^h denote a measure of human capital at the time of marriage of the wife and the husband (respectively); UD takes value one when unilateral divorce is in place and zero when mutual consent divorce is in place; δ_t and δ_s are time dummies and state dummies, respectively. The sample consists of all newlyweds (couples married within two years of the survey year) in their first marriage. CPS stands for *Current Population Survey* and PSID stands for *Panel Study of Income Dynamics*. *Education* refers to years of completed education for husband and wife, respectively. *Father's education* is a dummy that takes value one if the father of the husband and wife went to college. *Average main effect* is the average of β_3 and β_4 across states and years. *Total* effect is the sum of *Average main effect* and β_2. *pval* refers to the p-value of the estimates of β_2.

The first two sets of bars (labeled *Education*) show results from estimating model (12) when HK is measured as years of education at the time of marriage.[10] In both the CPS and the PSID datasets, husbands with an additional year of education marry wives with an additional half year of education, evidence of strong positive spousal sorting on education at baseline (light grey bars in figure 11). Assortative matching in education increases between 15.55% and 22.63% among newlyweds when UD is introduced (in figure 11, this effect corresponds to the

[10]Because the measures of years of education consider up to a college degree, I restrict attention to couples that marry at or before the age of 25. The results are similar for all newlyweds. The PSID includes a variable for the education category of individuals, that specifies professional degrees. When the model is estimated using category of education the results remain valid for the whole sample of newlyweds.

difference between dark and light grey bars). Effects are statistically significant, with p-value between 0.049 and 0.068. All standard errors are clustered at the state level. The effects are robust to controlling for the average education of females between 18 and 27 years old by state and time.

The set of bars labeled *Father's education* and *Pre-marital labor income* similarly summarize estimation results when HK is measured as an indicator of whether spouses' fathers went to college and spouses' labor income before marriage, respectively. Similar conclusions hold: newlyweds that marry around the time UD is introduced match more assortatively on these measures of human capital at marriage relative to newlyweds in the baseline MCD regime.

All in all, this evidence rejects the null *neutrality* hypothesis that divorce laws do not affect marriage decisions and provides empirical support for the main prediction of this paper.

5 Conclusion

Divorce laws have been changing rapidly worldwide in the last few decades. The traditional laws favored agreement between ex spouses. Nowadays, ex spouses are very independent and transfers among them are not enforced. How does making divorce easier impact the sorting patterns in the marriage market? This paper provides a first step towards studying this question. Unlike previous papers, I propose a framework where changes in divorce laws are not neutral. I develop a model of marriage and divorce that incorporates spouses' decisions to specialize in household or market labor, one of the most recognized reasons for marriage. In such an environment, I show that making divorce easier reduces the willingness of lower earning spouses to specialize in household labor, even when it is efficient. Hence, the policy change causes the value of marriage to decrease for some potential couples, affecting the marriage market equilibrium. In the new divorce regime, spouses match more assortatively in human capital and fewer households specialize. The main prediction of the model is supported by the data.

The main contribution of this paper is to show that, under realistic conditions, the introduction of unilateral divorce may have affected the equilibrium in the marriage market. The previous literature either concentrates on the effect of introducing UD on already formed couples or adopts a Becker-Coase framework in which divorce laws have no effect on the matching equilibrium. The evidence introduced in Reynoso (2019) and reviewed in section 4.6 rejects this neutrality hypothesis and therefore suggests that divorce institutions may have unintended consequences through the changes in the marriage prospects of individuals. The model developed in this paper provides a tractable framework to think of these policy issues.

One of the takeaways from this framework is that whether divorce laws affect matching patterns or not crucially depends on the the structure of the marital surplus and on the value of the gains from specialization. Therefore, this paper motivates an empirical agenda to investigate the *equilibrium* effects of unilateral divorce on matching patterns and the gains from marriage. I answer that research question in Reynoso (2019), where I build an empirical life cycle equilibrium model of the US marriage market to estimate the effects of unilateral divorce on the equilibrium gains from marriage and quantify the associated change in assortative matching and equilibrium life cycle decisions of couples.

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Appendix A The solution to the intrahousehold and resource allocation stages

Conditional on the choice of k, in the first period the household chooses total private consumption, C_1, public consumption, Q_1, and the allocation of private consumption among spouses, (c_{f1}, c_{m1}). The couple chooses $x_1 = \{Q_1, c_{f1}, c_{m1}\}$ so as to solve the following Pareto problem:

$$\begin{aligned}
\max_{x_1} \quad & c_{m1}Q_1 \\
s.t. \quad & [pc_f] : c_{f1}Q_1 \geq v_{f1} \\
& [bc^M] : c_{f1} + c_{m1} = W(s_f)(1-k) + W(s_m) - Q_1
\end{aligned}$$

This problem satisfies the transferable utility property (that is, the Lagrange multiplier of constraint $[pc_f]$ is equal to one) and so it is equivalent to choosing x_1 to maximize the sum of utilities subject to the budget constraint in marriage:

$$\begin{aligned}
\max_{x_1} \quad & (c_{f1} + c_{m1})Q_1 \\
s.t. \quad & [bc^M] : c_{f1} + c_{m1} = W(s_f)(1-k) + W(s_m) - Q_1
\end{aligned} \tag{13}$$

We can now obtain the solution for any choice of female housework supply. The household efficient choice of expenditures in the public and the private goods are

$$Q_1 = \frac{W(s_f)(1-k) + W(s_m)}{2} = C_1$$

Hence, the household value of problem (13) is

$$V_1^M(k) = \Big(\frac{W(s_f)(1-k) + W(s_m)}{2}\Big)^2$$

Indirect utilities for $i = \{f, m\}$, conditional on the sharing rule that determines the allocation of private consumption among spouses are,

$$\begin{aligned}
v_{f1}(c_{f1}, k) &= c_{f1}\frac{W(s_f)(1-k) + W(s_m)}{2} + \theta \\
&= (C_1 - c_{m1})\frac{W(s_f)(1-k) + W(s_m)}{2} + \theta
\end{aligned} \tag{14}$$

for the wife and

$$\begin{aligned} v_{m1}(c_{m1},k) &= c_{m1}\frac{W(s_f)(1-k)+W(s_m)}{2}+\theta \\ &= (C_1-c_{f1})\frac{W(s_f)(1-k)+W(s_m)}{2}+\theta \end{aligned} \tag{15}$$

for the husband.

Remark 3 (TU in marriage) *The solution to program (13) does not depend on the conditional sharing rules ((c_{f1}, c_{m1})), reflecting the transferable utility (within marriage) structure of the program.*

In effect, the sum of indirect utilities, conditional on the sharing rule, does not depend on the sharing rule, for any choice of female household labor supply:

$$v_{f1}(c_{f1},k)+v_{m1}(c_{m1},k)=V_1(k)$$

This reflects the fact that the utility possibility frontier within marriage is a straight line with slope -1. Indirect utilities are then determined by the couple positioning at a point of this frontier, that is, for any choice of $k=\{0,1\}$,

$$v_f^M=V^M(k)-v_m^M \tag{16}$$

The exact location of wife and husband in this period frontier is determined endogenously as part of the equilibrium in the marriage market.

An analogous derivation leads to the conclusion that the problem of the couple in marriage in the second period also satisfies the transferable utility property.

Ex spouses in this model do not derive utility from public goods. Hence, the problem of ex spouses is to allocate total resources in divorce between the private consumption of the ex wife and the private consumption of the ex husband:

$$\begin{aligned} V^D = \max_{c_f^D,c_m^D} &\quad c_f^D+c_m^D \\ s.t &\quad [bc^D]: c_f^D+c_m^D=W(s_f)+W(s_m)+\eta(s_m)k \end{aligned} \tag{17}$$

Remark 4 (TU in divorce) *The solution to program (17) does not depend on the sharing rule in divorce, reflecting the transferable utility (in divorce) structure of the program.*

Hence, the second period problem of the couple, conditional on the choice of k, is to choose whether to remain married or to divorce ($D = \{0, 1\}$), the amount of the public good to purchase if the marriage continues (Q_2), and the allocation of private consumption in marriage and divorce (vectors (c_{f2}, c_{m2}) and (c_f^D, c_m^D), respectively) in order to maximize the period sum of utilities subject to the budget and participation constraints in period two:

$$\begin{aligned}
\max_{x_2} \quad & & (1-D)\Big[(c_{f2}^M + c_{m2}^M)Q_2 + 2\theta\Big] + D\Big[c_f^D + c_m^D\Big] \qquad (18)\\
s.t. \quad & D = 0: & [bc_2^M] : c_{f2} + c_{m2} = W(s_f) + W(s_m) + \eta(s_m)k - Q_2\\
& & [pc_i^M] : c_{i2} \in \mathcal{C}^M, \forall i = \{f, m\}\\
& D = 1: & [bc_2^D] : c_f^D + c_m^D = W(s_f) + W(s_m) + \eta(s_m)k\\
& & [pc_i^D] : c_i^D \in \mathcal{C}^D, \forall i = \{f, m\}
\end{aligned}$$

The solutions for total private consumption and expenditure in public goods in marriage are

$$Q_2 = \frac{W(s_f) + W(s_m) + \eta(s_m)k}{2} = C_2$$

The household decision on total consumption in divorce is trivial as all resources in divorce are devoted to private consumption.

Hence, the household value of problem (18) is

$$\begin{aligned}
V_1^M(k) &= (1 - D(k, \theta))\Big[\Big(\frac{W(s_f) + W(s_m) + \eta(s_m)k}{2}\Big)^2 + 2\theta\Big] + \\
&\quad + D(k, \theta)\Big(W(s_f) + W(s_m) + \eta(s_m)k\Big) \qquad (19)
\end{aligned}$$

Appendix B The optimal efficient matching

In this section I characterize matching and household specialization patterns in terms of the gains from specialization to H-type males, η, when the policy environment is such that all potential couples act efficiently and the marital lifetime output is transferable. I show that under reasonable conditions, the optimal matching is positive assortative for low and high values of η and is negative assortative for intermediate values of η.

B.1 The structure of the marital surplus

In this model, the marital surplus has a closed form structure that can be characterized in a very clean way. In effect, the marital surplus generated by a couple of initial human capital composition $(s_f, s_m) \in \{l, h\} \times \{L, H\}$ is the maximum value (over the choice of marital investment, k) of the sum of three values. First, the total value generated in the first period of marriage. Second, the total expected value generated in the second period if the marriage continues, weighted by the ex ante probability that the marriage continues. And third, the value generated in divorce, weighted by the ex ante probability of divorce. That is,

$$\varphi(s_f, s_m) = \max_{k \in \{0,1\}} \Big\{ \Big(\frac{W(s_f)(1-k) + W(s_m)}{2} \Big)^2 + \\ + \Big(1 - \frac{\theta^* - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\Big) \Big[\Big(\frac{W(s_f) + W(s_m) + \eta(s_m)k}{2} \Big)^2 + 2E[\theta \big| \theta > \theta^*] \Big] + \\ + \Big(\frac{\theta^* - \underline{\theta}}{\overline{\theta} - \underline{\theta}} \Big) (W(s_f) + W(s_m) + \eta(s_m)k) \Big\} \tag{20}$$

where θ^* is a short form for $\theta^*(W(s_f), W(s_m), \eta, k)$, the divorce threshold that was defined in expression (6).

Recall that this structure of the marital surplus (20) comes from solving for the per period problem of each potential couple (s_f, s_m). The per period problem consists of making consumption and (if in the second period) divorce decisions. Note, also, that the exogenous variable of the model are the initial human capital composition of the couple, (s_f, s_m), the labor market returns to initial human capital, $W(s_f)$ and $W(s_m)$, the range of match quality, $[\underline{\theta}, \overline{\theta}]$, and the male earnings gains from specialization, η.

Lets call the vector of labor market returns to skills by gender a *labor market structure*, and lets denote it $\mathcal{W} = (W(l), W(h), W(L), W(H))$. Take a particular labor market structure $\mathcal{W}$ as given. We are interested in predicting the investment decision that any potential couple $(s_f, s_m) \in \{l, h\} \times \{L, H\}$ would make for any value of the male gains from specialization

$\eta \geq 0$. This way, we would predict the value of the marital surplus $\varphi(s_f, s_m)$ generated by any potential couple for any value of η, given the exogenous parameters of the model.

B.2 Further assumptions

In order to characterize the investment decisions and marital surplus of each potential couple (s_f, s_m), I make some additional assumptions on the relationship between exogenous variables in the model.

Assumption 4 *Exogenous variables $W(l)$, $W(h)$, $W(L)$, $W(H)$, and $\underline{\theta}$ take values such that:*

1. *For all $s_f = \{l, h\}$, $W(s_f) > 0$. For all $s_m = \{L, H\}$, $W(s_m) > 0$.*
2. *For all $s_f = \{l, h\}$, all $s_m = \{L, H\}$, and all $\eta \geq 0$, the economic value of marriage in the second period exceeds the value of divorce:*
$$\Big(\frac{W(s_f) + W(s_m) + \eta}{2}\Big)^2 > W(s_f) + W(s_m) + \eta.$$
3. *$\overline{\eta}$ is high enough to yield*
   (a) for all $s_f = \{l, h\}$, $W(s_f) < \overline{\eta}$,
   (b) $\Big(\frac{\overline{\eta}}{2}\Big)^2 + \overline{\eta} > \Big(\frac{W(h) + W(H)}{2}\Big)^2$

Parts 1 and 2 are standard. Part 1 implies that, for any couple, the decision to specialize or not is not trivial. Because female earnings are strictly positive, by choosing $k = 1$, a couple trades off the second period boost in male earnings, η, against the first period loss of strictly positive female earnings. Part 2 of assumption 4 means that earnings and the gain from specialization are such that there is always an economic surplus of marriage over divorce. Note that since the assumption must hold for $\eta = 0$, the assumption is satisfied whenever $W(sf) + W(sm) > 4$, which is a very general condition. This part of the assumption makes sense of the modeling choice that couples that arrive married to the first period of the married life must remain married during that first period. The assumption assures that couples would find staying married optimal, if they had the chance to divorce in the first period.

An important additional implication of parts 1 and 2 of assumption 4 is that the match quality threshold, $\theta^*(W(s_f), W(s_m), \eta, h)$, decreases with $W(s_f)$, $W(s_m)$, or η. Intuitively, wealthier couples can survive lower quality shocks. Lemma 1 in appendix C.2 proves this statement.

The third part of assumption 4 ensures that η can vary enough to take values in all the meaningful intervals of its range. In other words, we are interested in predicting the efficient investment plan of couples for very low and for very high values of the gains from specialization. But the maximum value that η can take is constrained by the given values of the other exogenous variables. The intuition of this statement is as follows. First, note that the match quality threshold, $\theta^*(W(s_f), W(s_m), \eta, k) \in [\underline{\theta}, \overline{\theta}]$ relates all exogenous variables of the model. In particular, the match quality threshold cannot fall below the lower bound of the distribution of the match quality. But the match quality threshold is decreasing in η (as shown in lemma 1). So, for a given structure of the model, η cannot exceed the value that makes θ equal to its lower bound, $\theta^*(W(s_f), W(s_m), \eta, k) = \underline{\theta}$. The next remark shows how the upper bound $\overline{\eta}$ is uniquely derived given values of the returns to human capital for high skilled individuals, $W(h)$ and $W(H)$, and the lower bound of the match quality, $\underline{\theta}$.

Remark 5 *Recall from section 2 that the match quality, θ follows a uniform distribution with support $[\underline{\theta}, \overline{\theta}]$. Let $\overline{\eta}$ be determined as the value of η such that the following equality is satisfied:*

$$\theta^*(W(h), W(H), \overline{\eta}, k = 1) = \underline{\theta}$$

The range of values of η that are consistent with the model are $\eta \in [0, \overline{\eta}]$. See appendix D for a detailed discussion on how the upper bound $\overline{\eta}$ is determined.

By setting exogenous variables appropriately, hence, we ensure that the model can be solved for high values of η.

B.3 Patterns of efficient specialization

The model so specified generates clear predictions about the optimal marital investment choice, $k = \{0, 1\}$, that each type of couple will make for any given $\eta \in [0, \overline{\eta}]$. Figure 12 illustrates the general patterns of efficient specialization for all four possible types of couples, {(l,L), (h,L), (l,H), (h,H)} as a function of the gains from specialization, η. The vertical axis measures the value of the marital output produced by each couple, while the horizontal axis measures the range of values of $\eta \in [0, \overline{\eta}]$. In the figure, the labels $\varphi(s_f, s_m, k)$ indicate the plot of the marital output generated by each couple $(s_f, s_m) \in \{l, h\} \times \{L, H\}$ that takes action $k \in \{0, 1\}$, as a function of the male gains from specialization, $\eta \in [0, \overline{\eta}]$.

The first feature that figure reveals is that couples with L-type males always find it optimal not to specialize ($k = 0$). The solid and light dotted constant functions, $\varphi(l, L, k = 0)$ and $\varphi(h, L, k = 0)$, represent the marital output of couples with L-type males, (l, L) and (h, L), respectively. Because low skill males have no returns from specialization ($\eta^L = 0$), families with low skill males always produce more by not specializing. Otherwise, under specialization (that is, $k = 1$), these families would loose the first period earnings of wives and gain nothing in the second period. The figure, hence, omits the plots of the marital output for couples (s_f, L) that choose $k = 1$, as those plots would always be below the marital outputs under no specialization.

The second feature the figure reveals is that the choice of $k = \{0, 1\}$ is not trivial, on the contrary, for couples with H-type males. For these couples, there is a threshold value of η below which $k = 0$ is efficient and above which $k = 1$ if efficient. The dashed and solid constant functions, $\varphi(l, H, k = 0)$ and $\varphi(h, H, k = 0)$, represent the marital output of couples with H-type males, (l, H) and (h, H), respectively, under no specialization. The increasing and convex functions $\varphi(l, H, k = 1)$ and $\varphi(h, H, k = 1)$ represent the marital output of these couples when they choose to specialize, for different values of the gains from specialization. For families with high skill males, the male earnings return from specialization $\eta^H = \eta$ is positive. As η increases, the second period earnings of husbands increase, and with it, the expected marital output. But specializing households trade off the first period earnings of wives against the second period boost in earnings of husbands. At low levels of η, the male gains from specialization are not enough to offset loosing the first period earnings of wives. Hence, at low levels of η, the marital output when $k = 0$ exceeds the marital output when $k = 1$. At these values of η, couples with high skill males find it optimal not to specialize. Since the marital output under no specialization is constant in η, but it is increasing under specialization, there is a value of η such that the couple is indifferent between the two investment choices. After that point, the couple finds it optimal to specialize, as the marital output under specialization becomes bigger than the marital output under no specialization. In the graph, the indifference point for (l, H) couples is $\eta = \eta^*_{hH}$. Before that point, the couple produces a higher value by choosing $k = 0$. After that point, the couple produces a higher value by choosing $k = 1$. The indifference point for (h, H) couples is η^*_{hH}, yielding analogous conclusions.

The next proposition states that when the male earnings gains from specialization, η, are low, no couple chooses to specialize. As the gains from specialization increase the marital output under specialization approaches, and eventually offsets, the marital output under no specialization for couples with high skill males. As a result, for high enough values of η, couples with H-type males specialize and couples with L-type males do not specialize.

Proposition 7 **[Efficient specialization patterns]** *Let the model structure satisfy assumption 4. Then, the following results follow:*

1. *For any η, couples (l, L) and (h, L) never choose to specialize.*
2. *There exist a threshold η^*_{lH} for couples (l, H) such that for any $\eta < \eta^*_{lH}$ the couple chooses $k = 0$, and for any $\eta \geq \eta^*_{lH}$ the couple chooses $k = 1$.*
3. *There exist a threshold η^*_{hH} for couples (h, H) such that for any $\eta < \eta^*_{hH}$ the couple chooses $k = 0$, and for any $\eta \geq \eta^*_{hH}$ the couple chooses $k = 1$.*

Proof 7 *See appendix C.2.* ■

In this transferable utility environment, the patterns of efficient specialization determine the value that different types of marriages produce. Hence, the theory presented allows us to predict what is the expected value of marriage for each type of potential couple. In turn, knowing the values associated with each type of potential couple allows us to predict what type of female will be married to which type of male in a stable matching. Hence, the sorting patterns that arise in equilibrium are tied to the investment decision of potential couples. I explore these sorting patterns in the next subsection.

B.4 Sorting patterns

In order to predict the sorting patterns at each value of the gains from specialization $\eta \in [0, \overline{\eta}]$, I study the modularity of the surplus function $\varphi(s_f, s_m)$. In figure 12 underbraces at the bottom indicate the modularity of the surplus function at different values of η. The figure illustrates the statement that is formally proved in proposition 8: at low and high values of η the optimal matching is PAM; whereas at intermediate values of η the optimal matching can be NAM or PAM (the figure illustrates the NAM case).

Proposition 8 **[Characterization of the optimal matching]** *Let assumption 4 be satisfied.*

1. *Suppose the structure of the model is such that* $\varphi(h, H, 1) - \varphi(l, H, 1) = \varphi(h, L, 0) - \varphi(l, L, 0)$ *at* $\eta \in [\eta_{hH}, \overline{\eta}]$*. Then, there exist thresholds* $\eta^*_{P\to N} \geq \eta^*_{lH}$ *and* $\eta^*_{P\leftarrow N} \geq \eta^*_{hH}$ *such that:*
   - *For* $\eta \leq \eta^*_{P\to N}$ *the optimal matching is PAM and no couple specializes.*
   - *For* $\eta \in (\eta^*_{P\to N}, \eta^*_{P\leftarrow N}]$ *the optimal matching is NAM and only low skilled women specialize.*
   - *For* $\eta > \eta^*_{P\leftarrow N}$ *the optimal matching is PAM and only high skilled women specialize.*
2. *Suppose the structure of the model is such that* $\varphi(h, H, 1) - \varphi(l, H, 1) > \varphi(h, L, 0) - \varphi(l, L, 0)$ *at* $\eta \in [\eta_{hH}, \overline{\eta}]$*. Then, the optimal matching is PAM for every* $\eta \in [0, \overline{\eta}]$*. The patterns of specialization for the couples that form in equilibrium are as in proposition 7.*

Sketch of proof. The complete and detailed proof is presented in appendix C.3. Here I outline the main steps of the proof and provide the intuitions. The proof consists of establishing whether the surplus function is supermodular or submodular at different intervals of values for η. There key insight is that the modularity of the marital surplus function changes with η because the patterns of efficient specialization change with η.

The sketch of the proof for the case that $\varphi(h, H, 1) - \varphi(l, H, 1) = \varphi(h, L, 0) - \varphi(l, L, 0)$ at $\eta \in [\eta_{hH}, \overline{\eta}]$ is illustrated in figure 12. In the figure, the distances $\Delta^{kk'}_{s_m}$ marked with double-headed arrows ($\updownarrow$) indicate the gain from increasing the female type for male of type $s_m = \{L, H\}$, at a particular value of η, when each type of couple takes the efficient action k and k', respectively (where k can be equal or different to k'). That is,

$$\Delta^{kk'}_{s_m} = \varphi(h, s_m, k) - \varphi(l, s_m, k')$$

For example, $\Delta^{00}_{H} = \varphi(h, H, 0) - \varphi(l, H, 0)$ indicates the increment in marital output that H-type males obtain when they marry an h-type woman and do not specialize ($k = 0$) with respect to marrying an l-type woman and do not specialize ($k' = 0$). As another example, $\Delta^{10}_{H} = \varphi(h, H, 1) - \varphi(l, H, 0)$ indicates the increment in marital output that H-type males obtain when they marry an h-type woman and specialize ($k = 1$) with respect to marrying an l-type woman and do not specialize ($k' = 0$).

Recall that couples with low type males always find it optimal not to specialize ($k = 0$, as shown in proposition 7). To show the optimal matching sorting patterns at different values of

η, I explore the modularity of $\varphi(s_f, s_m)$ at different values of η. That is, I explore whether

$$\Delta_H^{kk'} = \varphi(h, H, k) - \varphi(l, H, k') \lesseqgtr \varphi(h, L) - \varphi(l, L,) = \Delta_L^{00} \tag{21}$$

where k, k', are the efficient investment decisions of each type of couple at a given value of η. When the function is supermodular, the left hand side of equation (21) is bigger. When the function is submodular, the right hand side of equation (21) is bigger. The proof, then, proceeds as follows.

1. I start by showing that for values of η below η^*_{lH} the function $\varphi(s_f, s_m)$ is supermodular (lemma 6). Intuitively, at these values of η no couple specializes. This means that marital economic output is entirely produced with the returns to initial human capital. But in the marital output function, female and male earnings are complements, so marriages of likes produce more than marriages of unlikes. This is illustrated in the figure by the fact that the distance Δ_H^{00} is bigger than the distance Δ_L^{00}

2. At $\eta = \eta^*_{lH}$ couple (l, H) starts specializing. Because the marital output under specialization is strictly increasing in η (lemma 2), the distance $\Delta_H^{01} - \Delta_L^{00}$ decreases as η increases. At point $\eta^*_{P\leftarrow N}$ the two distances equal, $\Delta_H^{01} = \Delta_L^{00}$.

3. This shows that for $\eta \leq \eta^*_{P\leftarrow N}$ the function is supermodular. Hence, the optimal matching is PAM. For these values of η, h-type women marry H-type males and choose not to specialize $(k = 0)$; and l-type women marry L-type males and choose not to specialize $(k = 0)$.

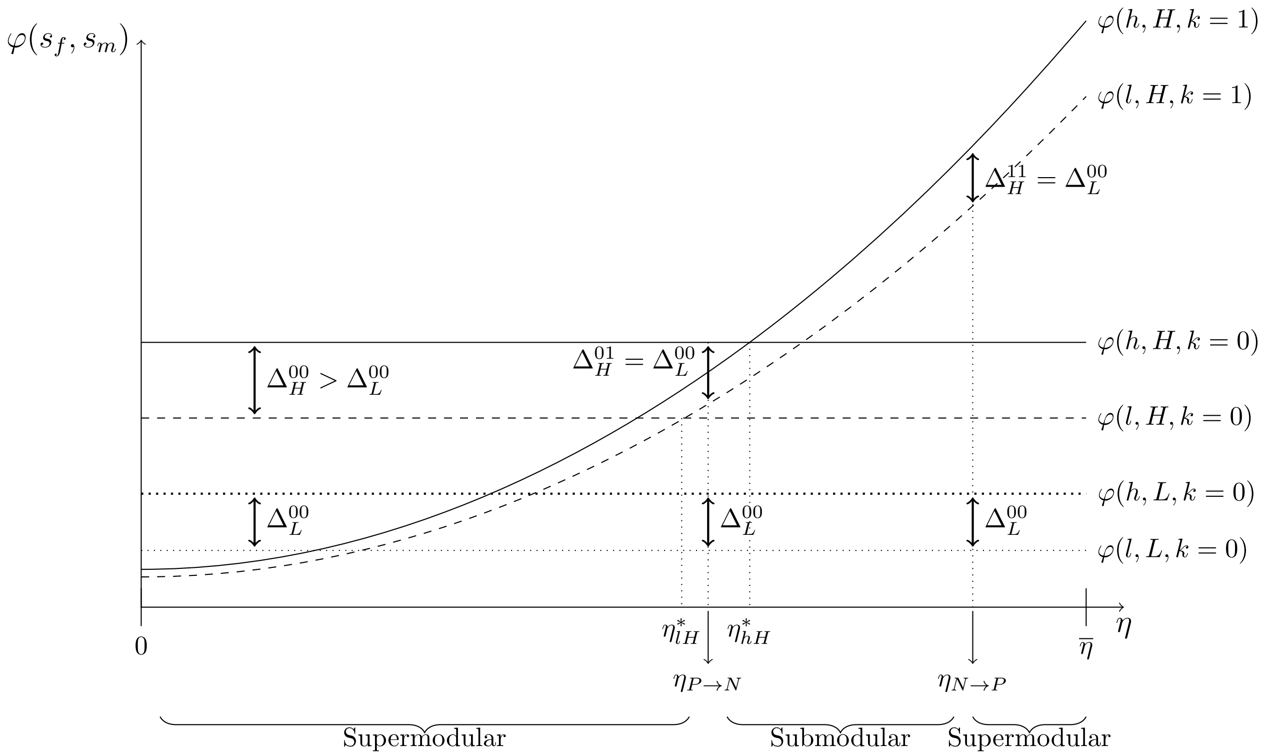
4. The distance Δ_H^{01} continues to decrease below Δ_L^{00} until $\eta = \eta^*_{hH}$. At this point, couples (h, H) start specializing. The gains from marrying a higher female type is increasing for H-type males when both types of couples specialize. That is, the distance Δ_H^{11} is strictly increasing in η (lemma 7).

5. At point $\eta = \eta^*_{N\leftarrow P}$, the two distances equal, $\Delta_H^{11} = \Delta_L^{00}$.

6. This shows that for $\eta \in (\eta^*_{P\leftarrow N}, \eta^*_{N\leftarrow P}]$ the surplus function is submodular. Hence, the optimal matching is NAM. For these values of η, h-type women marry L-type males and choose not to specialize $(k = 0)$; and l-type women marry H-type males and choose to specialize $(k = 1)$.

7. Finally, for $\eta \in (\eta^*_{N\leftarrow P}, \overline{\eta}$ the distance Δ^{11}_H continues to increase above Δ^{00}_L, implying that the surplus function is supermodular. Hence, the optimal matching is PAM. For these values of η, h-type women marry H-type males and choose to specialize ($k = 1$); and l-type women marry L-type males and choose not to specialize ($k = 0$).

Figure 12: Household organization and modularity of the surplus function.



In the next section I solve the model under a different divorce regime and explore whether the marriage market equilibrium remains being the optimal.

Appendix C Proofs[11]

C.1 Proof of proposition 1

Suppose, by contradiction, that k solves problem (7) under MCD but that $k^* \neq k$ maximizes the household surplus as defined in definition 2. Then,

$$V_1^M(k^*) + EV_2^M(k^*) + EV^D(k^*) > V_1^M(k) + EV_2^M(k) + EV^D(k)$$

[11] Some proofs are omitted but available upon request.

Keep the given female utility price, $v(s_f)$ fixed. There will be states and periods where choosing k^* will generate a surplus with respect to choosing k. If those states are such that divorce occurs, allocate all the surplus to the husband. Note that this shift in husband value does not violate participation constraints because it leaves the allocation in marriage unchanged and improves his allocation in divorce. If the surplus states are such that marriages continues, allocate all the surplus in marriage to the husband. Since in those states participation constraints in divorce are not binding, the reallocation does not violate participation constraints. There may be states and periods where choosing k^* will generate a deficit with respect to choosing k. In those states reduce the allocation of the husband in marriage and divorce so that participation constraints in divorce are satisfied. Because the expected surplus of choosing k^* over k is bigger than the expected deficit, $v_m(k^*) > v_m(k)$, a contradiction to k maximizing problem (7).

C.2 Proof of proposition 7

For all $(s_f, s_m) \in \{l, h\} \times \{L, H\}$, let the household gain from specialization be denoted $\Delta\varphi(s_f, s_m, k)$:

$$\Delta\varphi(s_f, s_m, k) = \varphi(s_f, s_m, k = 1) - \varphi(s_f, s_m, k = 0)$$

And let the household gain from specialization at a particular value of η, $\eta = \tilde{\eta}$ be denoted $\Delta\varphi(s_f, s_m, k)|_{\eta=\tilde{\eta}}$:

$$\Delta\varphi(s_f, s_m, k)|_{\eta=\tilde{\eta}} = \varphi(s_f, s_m, k = 1)|_{\eta=\tilde{\eta}} - \varphi(s_f, s_m, k = 0)$$

To ease notation, let $y = W(s_f) + W(s_m)$.

The proof makes use of the following lemmas.

Lemma 1 *The match quality threshold $\theta^*(W(s_f), W(s_m), \eta, k)$ is strictly decreasing in $W(s_f)$, $W(s_m)$, and η.*

Proof 8 *To show that θ^* is strictly decreasing in $W(s_f)$, $W(s_m)$, and η, recall that*

$$\theta^* = \frac{1}{2}\Big[W(s_f) + W(s_m) + \eta - \Big(\frac{W(s_f) + W(s_m) + \eta}{2}\Big)^2\Big] \tag{22}$$

Note that $W(s_f)$, $W(s_m)$, and η play the same role in the expression of θ^*. Hence, it suffices to show that the derivative of θ^* with respect to one of the exogenous variables is negative. Consider the derivative with respect to η:

$$\frac{\partial\theta^*}{\partial\eta} = \frac{1}{2}\Big[1 - \Big(\frac{W(s_f)+W(s_m)+\eta}{2}\Big)\Big] < 0 \tag{23}$$

by assumption 4. ■

Lemma 2 *For all $s_f = \{l,h\}$, $\Delta\varphi(s_f,H,k)$ is strictly increasing in η. In particular,*

$$\frac{\partial\Delta\varphi(s_f,H,k)}{\partial\eta} > 1 \quad \forall s_f = \{l,h\}$$

Proof 9 *First, note that $\varphi(s_f,H,k=0)$ does not vary with η. Hence, it suffices to prove that $\frac{\partial\varphi(s_f,H,k=1)}{\partial\eta} > 1$. Recall that*

$$\varphi(s_f,H,k=1) = \Big(\frac{W(h)}{2}\Big)^2 + \frac{\overline{\theta}-\theta^*}{\overline{\theta}-\underline{\theta}}\Big[\Big(\frac{y+\eta}{2}\Big)^2 + \overline{\theta}+\theta^*\Big] + \Big(\frac{\theta^*-\underline{\theta}}{\overline{\theta}-\underline{\theta}}\Big)(y+\eta)$$

The derivative with respect to η is:

$$\begin{aligned}
\frac{\partial\varphi(s_f,H,k=1)}{\partial\eta} &= \frac{\theta^{*\prime}_\eta}{\overline{\theta}-\underline{\theta}}\Big(y+\eta-\Big(\frac{y+\eta}{2}\Big)^2 - \overline{\theta}-\theta^*\Big) + \frac{\overline{\theta}-\theta^*}{\overline{\theta}-\underline{\theta}}\Big(\frac{y+\eta}{2}+\theta^{*\prime}_\eta\Big) + \frac{\theta^*-\underline{\theta}}{\overline{\theta}-\underline{\theta}} \\
&= \underbrace{\frac{\theta^{*\prime}_\eta}{\overline{\theta}-\underline{\theta}}}_{<0,\text{ by lemma 1}} \underbrace{\Big[\frac{1}{2}\Big(y+\eta-\Big(\frac{y+\eta}{2}\Big)^2\Big)-\overline{\theta}\Big]}_{<0,\text{ by assumption 4}} + \\
&\quad \frac{\overline{\theta}-\theta^*}{\overline{\theta}-\underline{\theta}} \underbrace{\frac{1}{2}\Big(\frac{y+\eta}{2}+1\Big)}_{>1,\text{ by assumption 4}} + \frac{\theta^*-\underline{\theta}}{\overline{\theta}-\underline{\theta}} > 1
\end{aligned} \tag{24}$$

where the second equality follows from replacing θ^* and $\theta^{*\prime}_\eta$ by their expressions (22) and (23), respectively. □

Lemma 3 *For all $(s_f,s_m) \in \{l,h\}\times\{L,H\}$, $\Delta\varphi(s_f,s_m,k)|_{\eta=0} < 0$.*

Proof 10 *Note that since $\eta = \eta(L) = 0$, the expected value of marriage and divorce in the second period is the same under both choices of $k = \{0,1\}$ for all types of couples. Hence, the* *difference between choosing to specialize ($k=1$) or not ($k=0$) is,*

$$\Delta\varphi(s_f, s_m, k)|_{\eta=0} = \Big(\frac{W(s_m)}{2}\Big)^2 - \Big(\frac{W(s_m)+W(s_f)}{2}\Big)^2 < 0 \tag{25}$$

by assumption 4. □

Lemma 4 *For all $s_f = \{l, h\}$, $\Delta\varphi(s_f, H, k)$ is a strictly convex function of η.*

Proof 11 *Again, note that $\varphi(s_f, H, k=0)$ does not vary with η. Hence, it suffices to prove that $\dfrac{\partial^2\varphi(s_f, H, k=1)}{(\partial\eta)^2} > 0$.*

Differentiating expression (24) with respect to η:[12]

$$\begin{aligned}\frac{\partial^2\varphi(s_f, H, k=1)}{(\partial\eta)^2} &= \underbrace{\frac{\theta^{*\prime\prime}_{\eta\eta}}{\overline{\theta}-\underline{\theta}}}_{<0}\underbrace{\Big[\frac{1}{2}\Big(y+\eta-\Big(\frac{y+\eta}{2}\Big)^2\Big)-\overline{\theta}\Big]}_{<0,\ by\ assumption\ 4}+ \\ &+ \underbrace{\frac{\theta^{*\prime}_{\eta}}{\overline{\theta}-\underline{\theta}}}_{<0,\ by\ lemma\ 1}\underbrace{\Big(1-\frac{y+\eta}{2}\Big)}_{<0,\ by\ assumption\ 4}+\frac{1}{4}\frac{\overline{\theta}-\theta^*}{\overline{\theta}-\underline{\theta}} > 0\end{aligned}$$

□

[**Couples** (l, L) **and** (h, L)] I show that couples (l, L) and (h, L) never choose to specialize, for any value of η. First, note that by construction $\eta(L) = 0$ and the marital surplus of couples (l, L) and (h, L) do not vary with $\eta = \eta(H)$. Hence, lemma 3 implies that for any value of $\eta \in [0, \overline{\eta}]$, couples with L-type males produce a higher marital output by choosing $k = 0$.

[**Couples** (h, H)] Next, I show that for (h, H) couples there exists a threshold $\eta^*_{hH} < \overline{\eta}$ such that the optimal choice is $k = 0$ for $\eta < \eta^*_{hH}$ and $k = 1$ for $\eta \geq \eta^*_{hH}$.

First, by lemma 2 the marital gains from specialization, $\varphi(h, H, h=1) - \varphi(h, H, h=0)$, is a strictly increasing and continuous function of η. Second, by lemma 3 the function takes a strictly negative value at the lower bound $\eta = 0$.

Note that since $\varphi(h, H, k=0)$ is a constant finite value and $\varphi(h, H, k=1)$ is strictly increasing and convex (lemmas 2 and 4), an intersection between the two occurs at a finite value of η, η^*_{hH}. It remains to be shown that $\eta^*_{hH} < \overline{\eta}$. Hence, the proof for this couple is complete by showing that at the upper bound $\eta = \overline{\eta}$,

[12] Differentiating expression (23) with respect to η $\dfrac{\partial^2\theta^*}{(\partial\eta)^2} = \dfrac{\partial}{\partial\eta}\Big\{\dfrac{1}{2}\Big[1-\Big(\dfrac{W(s_f)+W(s_m)+\eta}{2}\Big)\Big]\Big\} = -\dfrac{1}{2} < 0$

$$\Delta\varphi(h,H,k)\big|_{\eta=\overline{\eta}} > 0$$

First, note that in remark 5, $\overline{\eta}$ was derived as the value of η at which $\theta^* = \underline{\theta}$. Hence, when $\eta = \overline{\eta}$ the ex ante probability of divorce is zero. Hence, when the couple specializes,

$$\varphi(h,H,k=1)\big|_{\eta=\overline{\eta}} = \Big(\frac{W(H)}{2}\Big)^2 + \Big(\frac{W(h)+W(H)+\overline{\eta}}{2}\Big)^2 + \frac{\overline{\theta}+\underline{\theta}}{2}$$

When the couples does not specialize, they get gains from specialization $\eta = 0 < \overline{\eta}$, so that the ex ante probability of divorce, $\Pi = \dfrac{\theta^*(\eta=0)-\underline{\theta}}{\overline{\theta}-\underline{\theta}}$, is positive. The marital output when the couples chooses $h = 0$ is then,

$$\begin{aligned}\varphi(h,H,k=0) &= \Big(\frac{W(h)+W(H)}{2}\Big)^2 + \\ &+(1-\Pi)\Big(\frac{W(h)+W(H)}{2}\Big)^2 + \frac{\overline{\theta}^2-\theta^{*2}}{2(\overline{\theta}-\underline{\theta})} + \Pi(W(h)+W(H))\end{aligned}$$

The difference between the two is

$$\begin{aligned}\Delta\varphi(h,H,k)\big|_{\eta=\overline{\eta}} &= \Big(\frac{W(H)}{2}\Big)^2 + \Big(\frac{W(h)+W(H)}{2}\Big)^2 + \Big(\frac{\overline{\eta}}{2}\Big)^2 + \frac{W(h)\overline{\eta}}{2} + \frac{W(H)\overline{\eta}}{2} - \\ &-\Big(\frac{W(h)+W(H)}{2}\Big)^2 - (1-\Pi)\Big(\frac{W(h)+W(H)}{2}\Big)^2 - \Pi(W(h)+W(H)) + \Delta\theta\end{aligned}$$

where $\Delta\theta = \dfrac{\overline{\theta}+\underline{\theta}}{2} - \dfrac{\overline{\theta}^2-\theta^{*2}}{2(\overline{\theta}-\underline{\theta})}$.

Rearranging the terms that survive after cancellation,

$$\begin{aligned}\Delta\varphi(h,H,k)\big|_{\eta=\overline{\eta}} &= (1-\Pi)\Big[\Big(\frac{W(H)+\overline{\eta}}{2}\Big)^2 - \Big(\frac{W(h)+W(H)}{2}\Big)^2\Big] + \\ &\Pi\Big[\Big(\frac{W(H)+\overline{\eta}}{2}\Big)^2 - (W(h)+W(H))\Big] + \Delta\theta + \frac{W(h)\overline{\eta}}{2} > 0 \qquad (26)\end{aligned}$$

by assumption 4.

By conditions (24) (lemma 2), (25) (lemma 3), and (26), there exist a threshold η^*_{hH} at which

$$\varphi(h,H,k=1)\big|_{\eta=\eta^*_{hH}} - \varphi(h,H,k=0)\big|_{\eta=\eta^*_{hH}} = 0$$

For values $\eta < \eta^*_{hH}$ the difference is negative implying that the marital output maximizing choice is $k = 0$. For values $\eta > \eta^*_{hH}$ the difference is positive implying that the marital output maximizing choice is $k = 1$.

[**Couples** (l, H)] Finally, I show that for (l, H) couples there exists a threshold η^*_{lH} such that the optimal choice is $k = 0$ for $\eta < \eta^*_{lH}$ and $k = 1$ for $\eta \geq \eta^*_{lH}$.

Note again that $\varphi(l, H, k = 0)$ is a constant finite value. By lemmas 2 and 4, $\varphi(l, H, k = 1)$ is a strictly increasing, continuous, and strictly convex function of η. By lemma 3 the difference $\Delta\varphi(l, H, k)$ takes a strictly negative value at the lower bound $\eta = 0$. Hence, an intersection between the two occurs at a finite value of η, η^*_{lH}. It remains to be shown that $\eta^*_{lH} < \overline{\eta}$. I first show the following lemma:

Lemma 5 *For all* $s_f = \{l, h\}$, $\eta^* : \Delta\varphi(s_f, H, k)\big|_{\eta=\eta^*} = 0$ *is increasing in* $W(s_f)$.

Proof 12 *Let* $y = W(s_f) + W(H)$. *Let* $\theta^* = \theta^*(s_f, H, k = 1)$ *and* $\theta^\dagger = \theta^*(s_f, H, k = 0)$. *Recall that*

$$\begin{aligned}
\Delta\varphi(s_f, H, k)\big|_{\eta=\eta^*} \quad &= \quad \Big(\frac{W(H)}{2}\Big)^2 + \underbrace{\frac{\overline{\theta} - \theta^*}{\overline{\theta} - \underline{\theta}}\Big[\Big(\frac{y + \eta}{2}\Big)^2 + \overline{\theta} + \theta^*\Big] + \Big(\frac{\theta^* - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\Big)(y + \eta)}_{V_2(s_f,H,k=1)} - \\
&\quad -\Big[\Big(\frac{y}{2}\Big)^2 + \underbrace{\frac{\overline{\theta} - \theta^\dagger}{\overline{\theta} - \underline{\theta}}\Big[\Big(\frac{y}{2}\Big)^2 + \overline{\theta} + \theta^\dagger\Big] + \Big(\frac{\theta^\dagger - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\Big)y}_{V_2(s_f,H,k=0)}\Big]
\end{aligned}$$

From lemma 2,

$$\frac{\partial\Delta\varphi(s_f, H, k)}{\partial\eta}\Big|_{\eta=\eta^*} \quad = \quad \frac{\partial V_2(s_f, H, k = 1)}{\partial\eta} > 0$$

Hence, $Sign\Big(\dfrac{\partial\eta^*}{\partial W(s_f)}\Big) = -Sign\Big(\dfrac{\partial[V_2(s_f, H, k = 1) - V_2(s_f, H, k = 0)]}{\partial W(s_f)}\Big|_{\eta=\eta^*} - \dfrac{\partial\Big(\frac{y}{2}\Big)^2}{\partial W(s_f)}\Big)$.

Note that both terms are positive. The first term captures the complementarity between household specialization and female wages in the second period: the higher the female wage, the higher the gains from specialization in the second period. The cost is the loss in the complementarity between female and male earnings in the first period. By the assumptions of the *model,*

$$\frac{\partial[V_2(s_f,H,k=1)-V_2(s_f,H,k=0)]}{\partial W(s_f)}\Big|_{\eta=\eta^*}<\frac{y}{2}=\frac{\partial\left(\frac{y}{2}\right)^2}{\partial W(s_f)}$$

implying the result: $\dfrac{\partial\eta^*}{\partial W(s_f)}>0.$ □

Lemma 5 implies that the household gains from specialization for couple (l,H) is strictly positive at the upper bound of η: $\Delta\varphi(l,H,k)\big|_{\eta=\overline{\eta}}>0$. For suppose it is not. Then, the root η^*_{lH} at which $\Delta\varphi(l,H,k)\big|_{\eta=\eta^*_{lH}}=0$ is a finite value higher than $\overline{\eta}$. But then, by condition (26), $\eta^*_{hH}<\overline{\eta}<\eta^*_{lH}$, a contradiction to lemma 5.

Hence, by lemmas 2, 3, and 5, there exist a threshold $\eta^*_{lH}<\eta^*_{hH}<\overline{\eta}$ at which

$$\varphi(h,H,k=1)\big|_{\eta=\eta^*_{lH}}-\varphi(h,H,k=0)\big|_{\eta=\eta^*_{lH}}=0$$

For values $\eta<\eta^*_{lH}$ the difference is negative implying that the marital output maximizing choice is $k=0$. For values $\eta>\eta^*_{lH}$ the difference is positive implying that the marital output maximizing choice is $k=1$.

This completes the proof of proposition 7. ■

C.3 Proof of proposition 8

Lemma 6 *If, for any $\eta\in[0,\overline{\eta}]$, all couples choose $k=0$, the surplus maximizing matching is positive assortative.*

Proof 13 *It is sufficient to show that when the wife works ($k=0$), the marital surplus generated is supermodular, that is,* $\dfrac{\partial^2\varphi(s_f,s_m,k=0)}{\partial W(s_f)\partial W(s_m)}=\dfrac{\partial^2\varphi(s_f,s_m,k=0)}{(\partial y)^2}>0$

Given the derivations in the previous proofs and the interchangeable role of η and y, it is easy to derive the expression for the second derivative of $\varphi(s_f,s_m,k=0)$ with respect to y

twice:[13]

$$\frac{\partial^2\varphi(s_f,s_m,k=0)}{(\partial y)^2} = \frac{1}{2} + \underbrace{\frac{\theta^{*\prime\prime}_{yy}}{\overline{\theta}-\underline{\theta}}}_{<0}\underbrace{\left[\frac{1}{2}\left(y-\left(\frac{y}{2}\right)^2\right)-\overline{\theta}\right]}_{<0,\ by\ assumption\ 4} + \underbrace{\frac{\theta^{*\prime}_{y}}{\overline{\theta}-\underline{\theta}}}_{<0,\ by\ lemma\ 1}\underbrace{\left(1-\frac{y}{2}\right)}_{<0,\ by\ assumption\ 4} + \frac{1}{4}\frac{\overline{\theta}-\theta^*}{\overline{\theta}-\underline{\theta}} > 0$$

□

Lemma 7 *The difference $\varphi(h,H,1)-\varphi(l,H,1)$ is strictly increasing in η.*

Proof 14 *It is sufficient to show that $\frac{\partial^2\varphi(s_f,H,k=1)}{\partial W(s_f)\partial\eta} > 0$. The proof is analogue to the proof of strict convexity of $\varphi(s_f,H,k=1)$ on η (lemma 4), given that η and $W(s_f)$ have interchangeable roles in the marital output function. Differentiating expression (24) with respect to $W(s_f)$:*[14]

$$\frac{\partial^2\varphi(s_f,H,k=1)}{\partial W(s_f)\partial\eta} = \underbrace{\frac{\theta^{*\prime\prime}_{W(s_f)\eta}}{\overline{\theta}-\underline{\theta}}}_{<0}\underbrace{\left[\frac{1}{2}\left(y+\eta-\left(\frac{y+\eta}{2}\right)^2\right)-\overline{\theta}\right]}_{<0,\ by\ assumption\ 4} + \underbrace{\frac{\theta^{*\prime}_{W(s_f)}}{\overline{\theta}-\underline{\theta}}}_{<0,\ by\ lemma\ 1}\underbrace{\left(1-\frac{y+\eta}{2}\right)}_{<0,\ by\ assumption\ 4} + \frac{1}{4}\frac{\overline{\theta}-\theta^*}{\overline{\theta}-\underline{\theta}} > 0$$

□

C.4 Proof of proposition 3

Consider relationship (9). Suppose, first, that $\varphi(s_f,s_m,k=0) > \varphi(s_f,s_m,k=1)$. Consider female utility prices $v_f(s_f) \in [W(s_f),\varphi(s_f,s_m,k)-W(s_m)-\eta k]$. Note first that

$$[W(s_f),\varphi(s_f,s_m,k=1)-W(s_m)-\eta] \subset [W(s_f),\varphi(s_f,s_m,k=0)-W(s_m)]$$

and that $W(s_m)+\eta(s_m) \geq W(s_m)$, because $\eta(s_m) \geq 0$. These two features imply that all female prices that are feasible and satisfy the participation constraints for both spouses at choice $k=1$

[13] Differentiating expression (23) with respect to η $\frac{\partial^2\theta^*}{(\partial\eta)^2} = \frac{\partial}{\partial\eta}\left\{\frac{1}{2}\left[1-\left(\frac{W(s_f)+W(s_m)+\eta}{2}\right)\right]\right\} = -\frac{1}{2} < 0$

[14] Differentiating expression (23) with respect to $W(s_f)$:

$$\frac{\partial^2\theta^*}{\partial\eta\partial W(s_f)} = \frac{\partial}{\partial W(s_f)}\left\{\frac{1}{2}\left[1-\left(\frac{W(s_f)+W(s_m)+\eta}{2}\right)\right]\right\} = -\frac{1}{2} < 0$$

are also feasible and satisfy the participation constraints for both spouses at choice $k = 0$. Because the lifetime surplus is higher at $k = 0$, the same argument used in proof C.1 proves that for $v_f(s_f) \in [W(s_f), \varphi(s_f, s_m, k = 1) - W(s_m) - \eta(s_m)]$ the choice of k that maximizes (9) is $k = 0$. For values of $v_f(s_f) > \varphi(s_f, s_m, k = 1) - W(s_m) - \eta(s_m)]$, choosing $k = 1$ would violate the participation constraint of the husband, so that the constrained efficient optimal choice is $k = 0$ for $v_f(s_f) \in [\varphi(s_f, s_m, k = 1) - W(s_m) - \eta(s_m), \varphi(s_f, s_m, k = 0) - W(s_m)]$.

Suppose, next, that $\varphi(s_f, s_m, k = 1) > \varphi(s_f, s_m, k = 0)$. Now again note that all female prices that are feasible and satisfy the participation constraints for both spouses at choice $k = 1$ (that is, $v_f \in [W(s_f), \varphi(s_f, s_m, k = 1) - W(H) - \eta)$ are also feasible and satisfy the participation constraints for both spouses at choice $k = 0$. Hence, for $v_f \in [W(s_f), \varphi(s_f, s_m, k = 1) - W(H) - \eta$ the optimal choice for the couple is $k = 1$. However, female prices $v_f \in [\varphi(s_f, s_m, k = 1) - W(H) - \eta, \varphi(s_f, s_m, k = 0) - W(H)]$ are feasible and satisfy the participation constraints only if the couple chooses the unconstrained inefficient level $k = 0$. ■

C.5 Proof of proposition 5

Suppose, contrary to PAM under assumption 3, that the female proposing mechanism delivers an equilibrium, μ, where both types of mixed couples are observed: $(l, H) \in \mu \wedge (h, L) \in \mu$. Stability implies that, at the equilibrium prices $(v_f(l), v_f(h), v_m(L), v_m(H))$, l and h do not want to switch partners

$$v_f(l) = \max_{\substack{k\in\{0,1\} \\ v_m = v_m(H)}} \{\varphi(l, H, k)\} - v_m(H) > \varphi(l, L, 0) - v_m(L)$$

$$v_f(h) = \varphi(h, L, 0) - v_m(L) > \max_{\substack{k\in\{0,1\} \\ v_m = v_m(H)}} \{\varphi(h, H, k)\} - v_m(H)$$

Rearranging the two inequalities, adding the positive term $W(L)$ to both sides of both inequalities, and using the definition of thresholds $\overline{v}_{H \leftarrow l}$ and $\overline{v}_{H \leftarrow h}$ from the definition of PTC (definition 4):

$$\underbrace{\max_{\substack{k\in\{0,1\}\\ v_m=v_m(H)}} \{\varphi(l,H,k)\} - \varphi(l,L,0) + W(L)}_{\overline{v}_{H\leftarrow l}} > v_m(H) - v_m(L) + W(L)$$

$$\underbrace{\max_{\substack{k\in\{0,1\}\\ v_m=v_m(H)}} \{\varphi(h,H,k)\} - \varphi(h,L,0) + W(L)}_{\overline{v}_{H\leftarrow h}} < v_m(H) - v_m(L) + W(L)$$

Hence, $\overline{v}_{H\leftarrow h} < \overline{v}_{H\leftarrow l}$, a contradiction to PTC. ■

C.6 Proof of proposition 6

The proof makes use of the following lemma.

Lemma 8 *For any given couple of initial human capital composition (s_f, H), the three following statements are true:*

1. *There exist a value $\eta^{**}_{s_f,H}$ such that $\varphi(s_f,H,k=1)\big|_{\eta=\eta^{**}_{s_f,H}} - \varphi(s_f,H,k=0) = \eta^{**}_{s_f,H}$.*
2. *For all $0 \leq \eta < \eta^{**}_{s_f,H}$: $\varphi(s_f,H,k=1)\big|_{\eta} - \varphi(s_f,H,k=0) < \eta$.*
3. *$\eta^{**}_{l,H} < \eta^{**}_{h,H}$.*

Proof 15 *The statement is analogous to parts 2 and 3 of proposition 7, except in this lemma we look for the intersection of $\Delta^{10}_{s_f,H} - \eta = \varphi(s_f,H,k=1)\big|_{\eta} - \varphi(s_f,H,k=0) - \eta$, with 0. First, note that lemmas 2 and 4 imply that $\Delta^{10}_{s_f,H} - \eta$ is a strictly increasing and strictly concave function of η. Moreover, note that at $\eta = 0$, $\Delta^{10}_{s_f,H} - \eta < 0$ (lemma 3). So, an intersection of $\Delta^{10}_{s_f,H} - \eta$ with 0 occurs at a finite value of η, η^{**}_{hH}. It remains to be shown that $\eta^{**}_{s_f,H} < \overline{\eta}$. This follows from condition (26) in the proof of of proposition 7 and assumption 4. Finally, the proof of lemma 5 guarantees that $\eta^{**}_{l,H} < \eta^{**}_{h,H}$.* □

Lemma 8 and lemma 5 imply that there are two possible structures of the model, depicted in figures 13 and 14:

- $\eta^{*}_{lH} < \eta^{*}_{hH} < \eta^{**}_{l,H} < \eta^{**}_{h,H}$ (figure 13)
- $\eta^{*}_{lH} < \eta^{**}_{l,H} < \eta^{*}_{hH} < \eta^{**}_{h,H}$ (figure 14)

Lemma 9 *In the female proposing mechanism, females always start proposing to $H-type$ males.*

Proof 16 *For all women of type $s_f = \{l, h\}$, the minimum acceptable offer to L-males is $W(L)$. The associated female gain is*

$$v^0_{s_f \to L} = \varphi(s_f, L, 0) - W(L)$$

Minimum acceptable offers to H-males depend on females' decision to specialize in home production or not. If female s_f chooses $k = 0$, the minimum acceptable offer to H-males is $W(H)$, while if chooses $k = 1$, the minimum acceptable offer is $W(H) + \eta$. Hence, the female gain after offering the minimum acceptable price to males of type H is

$$v^0_{s_f \to H} = max\{\varphi(s_f, H, 0) - W(H); \varphi(s_f, H, 1) - W(H) - \eta\}$$

I next show that

$$v^0_{s_f \to H} > v^0_{s_f \to L}$$

Suppose, first, that for any $s_f = \{l, h\}$, $\varphi(s_f, H, 0) - W(H) > \varphi(s_f, H, 1) - W(H) - \eta$. The difference between the values associated to the initial offers to H and L males is:

$$\begin{aligned}
v^0_{s_f \to H} - v^0_{s_f \to L} &= \varphi(s_f, H, 0) - \varphi(s_f, L, 0) - (W(H) - W(L)) \\
&= \frac{W(H)^2 - W(L)^2}{4} + \frac{W(s_f)}{2}(W(H) - W(L)) + \Delta V_2 - (W(H) - W(L)) \\
&= \Big(\underbrace{\frac{1}{4}(W(H) + W(L)) - 1}_{>0 \text{ by assumption } 4} + \frac{W(s_f)}{2}\Big)(W(H) - W(L)) + \underbrace{\Delta V_2}_{>0 \text{ by lemma } 2} \\
&> 0
\end{aligned}$$

Next, suppose that $\varphi(s_f, H, 0) - W(H) < \varphi(s_f, H, 1) - W(H) - \eta$. Since the inequality for the first case holds for any value of η, the proof for the second case follows immediately:

$$\begin{aligned}
v^0_{s_f \to H} - v^0_{s_f \to L} &= \varphi(s_f, H, 1) - W(H) - \eta - (\varphi(s_f, L, 0) - W(L)) \\
&> \varphi(s_f, H, 0) - W(H) - (\varphi(s_f, L, 0) - W(L)) > 0 \\
&> 0
\end{aligned}$$

$\square$

Lemma 10 *Supermodularity of the surplus function is sufficient for PTC on the female side.*

Proof 17 *Let $k^*(s_f) = \{0, 1\}$ be the optimal female choice of k in potential couples $(s_f, H) \in \{l, h\} \times H$ when the divorce regime is unilateral. Let $k^{**}(s_f) = \{0, 1\}$ be the efficient household choice in potential couples $(s_f, H) \in \{l, h\} \times H$. Recall that PTC means that*

$$\overline{v}_{H\leftarrow h} - \overline{v}_{H\leftarrow l} = \varphi(h, H, k^*(h)) + \varphi(l, L, 0) - \varphi(l, H, k^*(l)) - \varphi(h, L, 0) > 0$$

Suppose, contrary to PTC, that

$$\overline{v}_{H\leftarrow h} - \overline{v}_{H\leftarrow l} < 0 \tag{27}$$

Depending on the structure of the model, there are many cases to analyze.

Structure 1 (figure 13). *Suppose the structure of the model is such that $\eta^*_{lH} < \eta^*_{hH} < \eta^{**}_{l,H} < \eta^{**}_{h,H}$ (figure 13) and that the surplus is supermodular in each case.*

[Case i] *When $\eta \leq \eta^*_{lH}$, $k^{**}(l) = k^{**}(h) = 0$ (proposition 7) and for all $v_m \in \mathcal{F}^{s_f,H}_H$, $k^*(l) = k^*(h) = 0$ (proposition 3). Condition (27) implies*

$$\varphi(h, H, 0) + \varphi(l, L, 0) < \varphi(l, H, 0) + \varphi(h, L, 0)$$

a contradiction to supermodularity of the surplus.

[Case ii] *When $\eta^*_{lH} < \eta \leq \eta^*_{hH}$, $k^{**}(l) = 1$ and $k^{**}(h) = 0$ (proposition 7); $k^*(l) = \{0, 1\}$ depending on feasible price $v_m \in \mathcal{F}^{l,H}_H$ and $k^*(h) = 0$ for all $v_m \in \mathcal{F}^{h,H}_H$ (proposition 3). If $\overline{v}_{H\leftarrow l}$ occurs when $k^*(l) = 0$, condition (27) implies*

$$\varphi(h, H, 0) + \varphi(l, L, 0) < \varphi(l, H, 0) + \varphi(h, L, 0)$$

a contradiction to supermodularity of the marital output function when all types of couples choose $k = 0$. If $\overline{v}_{H\leftarrow l}$ occurs when $k^*(l) = 1$, condition (27) implies

$$\varphi(h, H, 0) + \varphi(l, L, 0) < \varphi(l, H, 1) + \varphi(h, L, 0)$$

a contradiction to supermodularity of the surplus.

[Case iii] *When $\eta^*_{hH} < \eta \leq \eta^{**}_{lH}$, $k^{**}(l) = 1$ and $k^{**}(h) = 1$ (proposition 7); $k^*(l) = \{0, 1\}$ and $k^*(h) = \{0, 1\}$ depending on feasible price $v_m \in \mathcal{F}^{l,H}_H$ and $v_m \in \mathcal{F}^{h,H}_H$, respectively (proposition 3). If $\overline{v}_{H\leftarrow l}$ occurs when $k^*(l) = 0$ and $\overline{v}_{H\leftarrow h}$ occurs when $k^*(h) = 0$, condition (27) implies*

$$\varphi(h,H,0)+\varphi(l,L,0)<\varphi(l,H,0)+\varphi(h,L,0)$$

a contradiction to supermodularity of the marital output function when all types of couples choose $k=0$*. If* $\overline{v}_{H\leftarrow l}$ *occurs when* $k^*(l)=1$ *and* $\overline{v}_{H\leftarrow h}$ *occurs when* $k^*(h)=1$*, condition (27) implies*

$$\varphi(h,H,1)+\varphi(l,L,1)<\varphi(l,H,1)+\varphi(h,L,1)$$

a contradiction to supermodularity of the surplus. If $\overline{v}_{H\leftarrow l}$ *occurs when* $k^*(l)=0$ *and* $\overline{v}_{H\leftarrow h}$ *occurs when* $k^*(h)=1$*, condition (27) implies*

$$\varphi(h,H,1)+\varphi(l,L,0)<\varphi(l,H,0)+\varphi(h,L,0)$$

a contradiction to supermodularity of the surplus. Finally, suppose $\overline{v}_{H\leftarrow l}$ *occurs when* $k^*(l)=1$ *and* $\overline{v}_{H\leftarrow h}$ *occurs when* $k^*(h)=0$*. This means that the maximum that* l *can obtain by choosing* $k=1$ *is higher than the maximum* l *can obtain by marrying an* L*-type male:*

$$\varphi(l,H,1)-W(H)-\eta>\varphi(l,L,0)-W(L) \tag{28}$$

Similarly, because $\overline{v}_{H\leftarrow h}$ *occurs when* $k^*(h)=0$*,*

$$\varphi(h,H,1)-W(H)-\eta<\varphi(h,L,0)-W(L) \tag{29}$$

Hence,

$$\varphi(h,H,1)-\varphi(h,L,0)-W(H)+W(L)\underbrace{<}_{by(29)}\eta\underbrace{<}_{by(28)}\varphi(l,H,1)-\varphi(l,L,0)-W(H)+W(L)$$

a contradiction to supermodularity of the surplus.

[Case iv] *When* $\eta^{**}_{lH}<\eta\leq\eta^{**}_{hH}$*,* $k^{**}(l)=1$ *and* $k^{**}(h)=1$ *(proposition 7);* $k^*(l)=1$ *for all* $v_m\in\mathcal{F}^{l,H}_H$ *and* $k^*(h)=\{0,1\}$ *for all* $v_m\in\mathcal{F}^{h,H}_H$ *(proposition 3). If* $\overline{v}_{H\leftarrow h}$ *occurs when* $k^*(h)=1$*, condition (27) implies*

$$\varphi(h,H,1)+\varphi(l,L,0)<\varphi(l,H,1)+\varphi(h,L,0)$$

a contradiction to supermodularity. Suppose, now, that $\overline{v}_{H\leftarrow h}$ *occurs when* $k^*(h)=0$*. This means that the maximum that* h *can obtain by choosing* $k=1$ *in a marriage with an*

H-type male is lower than the maximum h can obtain by marrying an L-type male:

$$\varphi(h,H,1) - W(H) - \eta < \varphi(h,L,0) - W(L)$$
$$\varphi(h,H,1) - \varphi(h,L,0) < \eta + W(H) - W(L) \tag{30}$$

The fact that the utility possibility sets for (l,H) couples do not overlap implies that

$$\varphi(l,H,1) - \varphi(l,H,0) - W(H) > \eta - W(H) \tag{31}$$

Hence,

$$\varphi(l,H,1) - \eta - W(H) \underbrace{>}_{\text{by (31)}} \varphi(l,H,0) - W(H) \underbrace{>}_{\text{by lemma 9}} \varphi(l,L,0) - W(L) \tag{32}$$

Finally, condition (32) together with (30) imply that

$$\varphi(l,H,1) - \varphi(l,L,0) \underbrace{>}_{\text{by (32)}} \eta + W(H) - W(L) \underbrace{>}_{\text{by (30)}} \varphi(h,H,1) - \varphi(h,L,0)$$

a contradiction to supermodularity of the surplus.

[Case v] *When $\eta > \eta^{**}_{hH}$, $k^{**}(l) = k^{**}(h) = 1$ (proposition 7) and for all $v_m \in \mathcal{F}_H^{s_f,H}$, $k^*(l) = k^*(h) = 1$ (proposition 3). Condition (27) implies*

$$\varphi(h,H,1) + \varphi(l,L,0) < \varphi(l,H,1) + \varphi(h,L,0)$$

a contradiction to supermodularity of the surplus.

Structure 2 (figure 14). *Suppose the structure of the model is such that $\eta^*_{lH} < \eta^{**}_{l,H} < \eta^*_{hH} < \eta^{**}_{h,H}$ (figure 14) and that the surplus is supermodular in each case. The proofs for the five corresponding cases is similar.* □

Lemma 11 *There exist values of η such that the surplus is submodular and PTC is satisfied. In particular, this occurs for η:*

$$\eta_{P\to N} < \eta < \overline{\eta^{nn}},$$

where $\overline{\eta^{nn}}$:

$$\varphi(l,H,1)-\varphi(l,L,0)-(W(H)-W(L))-\eta=0 \tag{33}$$

Proof 18 *Note that condition (33) is strictly increasing in* η *(by lemma 2) and strictly negative at the lower bound of* $\eta=0$*. Hence, for values of* $\eta<\overline{\eta^{nn}}$*, we have that* $\varphi(l,H,1)-\varphi(l,L,0)<W(H)-W(L)+\eta$*, which implies that females of type* l *become indifferent between* L*-type males and* H*-type males when they are optimally choosing* $k^*(l)=0$*. Because when both types of females choose* $k=0$ *the surplus is supermodular,* $\overline{v}_{H\leftarrow l}<\overline{v}_{H\leftarrow h}$*, meaning that PTC is satisfied. Because this also occurs when* $\eta>\eta_{P\rightarrow N}$*, the surplus is submodular and yet, PTC holds. An example of this situation is provided in Case I in table 2 which corresponds to the model with* $\eta=\eta^i$ *in figure 7.* □

By lemma 9, in all cases the female proposing mechanism starts with l*-type and* h*-type females competing for* H*-type males. In all rounds while both types find it optimal to propose to* H*-type males,* L*-type males receive no offer. By lemma 10, whenever the surplus is supermodular, there exists a round* n *at which* l*-type females become indifferent between* L*-type males and* H*-type males,* h*-type females still prefer to make an offer to the* H*-type males, and before which both types of females prefer to make offers to* H*-type males. Because at this round* l*-type males have no offers on hold, all* l*-type males that receive an offer from an* L*-type woman accept. Because the number of females is lower than the number of males, the female proposing mechanism stops at round* n *and the resulting matching is positive assortative. Because the surplus is supermodular, the optimal matching is also positive assortative. This proves that a change in divorce law cannot decrease assortativeness. Lemma 11 shows that there exists values of* η *such that the surplus is submodular and PTC is satisfied. Both types of females start making offers to* H*-type males at* $k=0$ *(lemma 9). PTC being satisfied in the example implies that there exists a round* n *at which* l*-type females become indifferent between* L*-type males and* H*-type males,* h*-type females still prefer to make an offer to the* H*-type males, and before which both types of females prefer to make offers to* H*-type males. Since* L*-type males have no offers on hold, no rejections are issued in that round. The resulting matching is positive assortative. However, the optimal matching, where couples with* H*-type males efficiently choose* $k=1$*, is negative assortative. This shows that assortativness can strictly increase. One can find examples where* *the surplus is submodular and PTC is not satisfied, implying that the female proposing mechanism delivers a negative assortative matching, just as the optimal matching. This proves that assortativeness can only increase, which concludes the proof.* ■

Figure 13: Structure 1: $\eta^*_{lH} < \eta^*_{hH} < \eta^{**}_{l,H} < \eta^{**}_{h,H}$

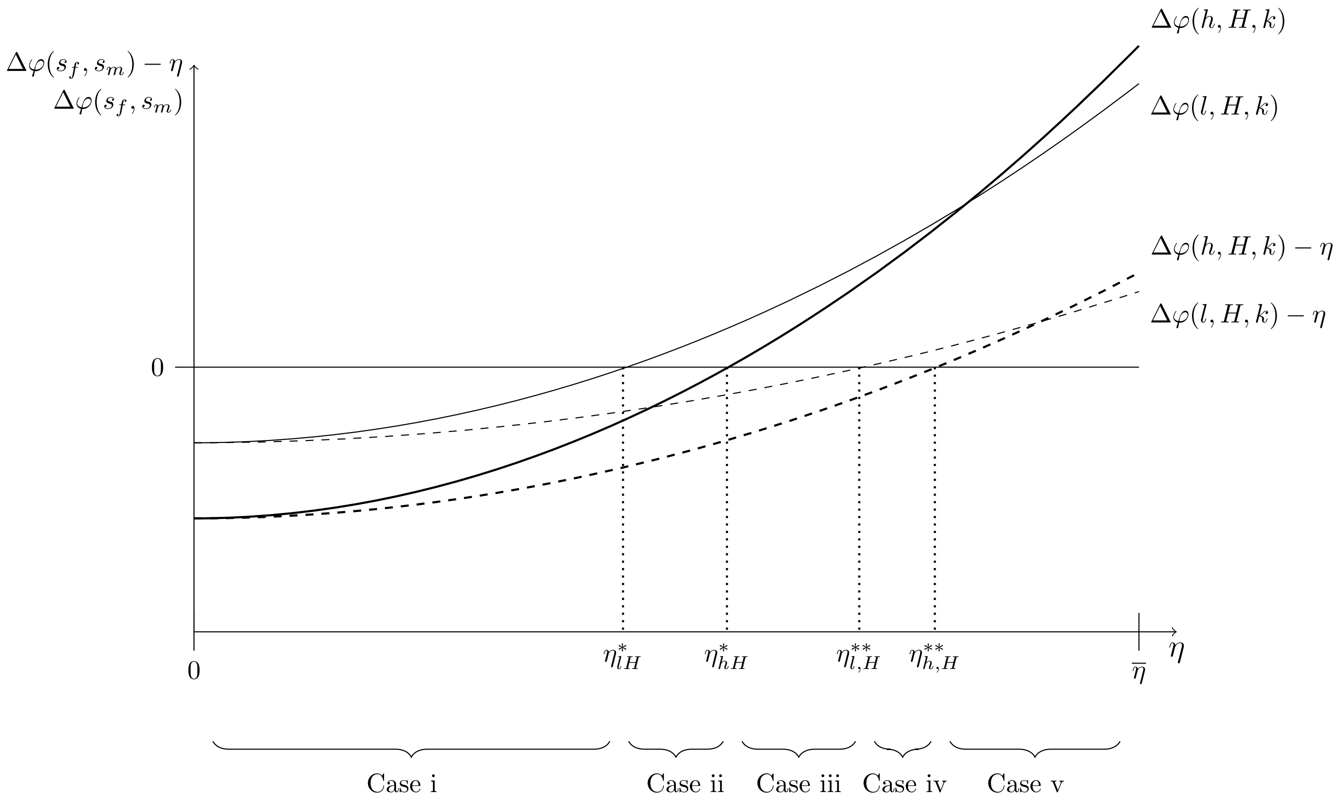
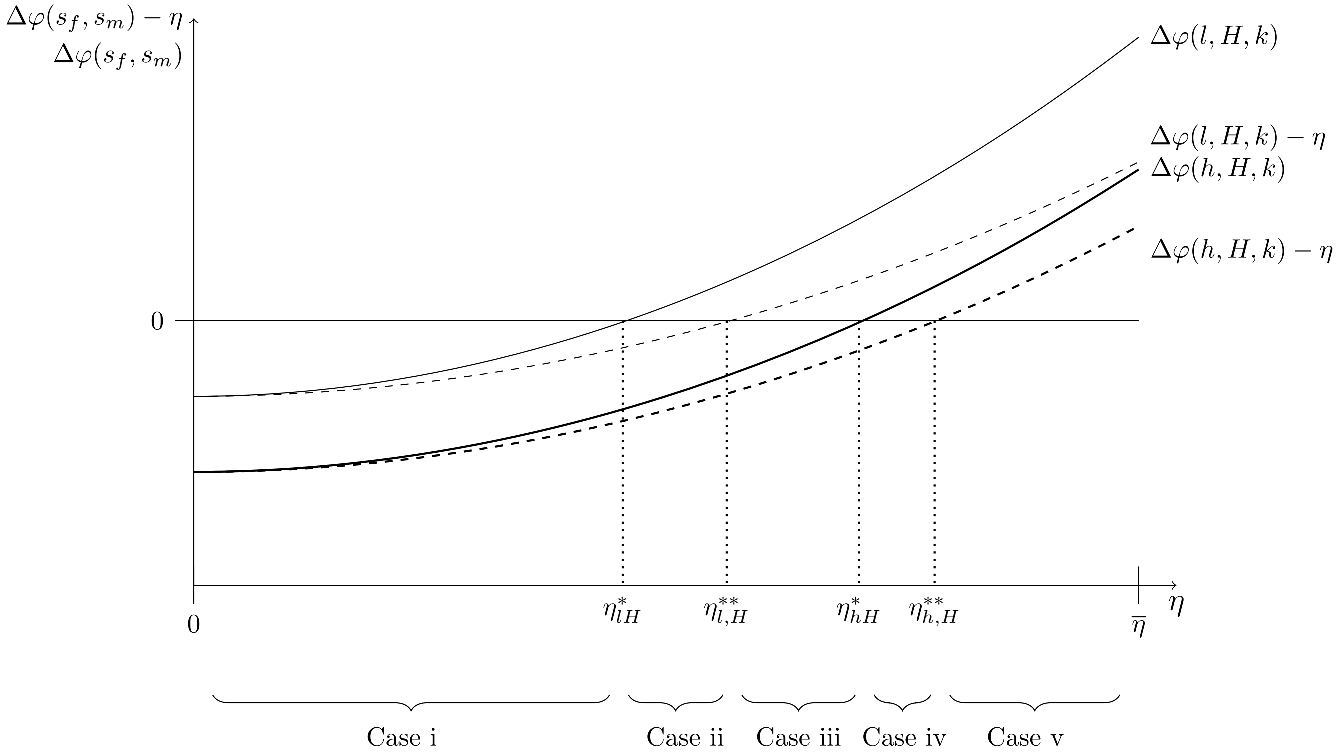


Figure 14: Structure 2: $\eta^*_{lH} < \eta^{**}_{l,H} < \eta^*_{hH} < \eta^{**}_{h,H}$



Appendix D Derivation of the bounds to the gains from specialization, η

The lower bound of η is zero by construction. The upper bound of η, $\overline{\eta}$, is derived as an implicit function of $W(h)$, $W(H)$, and $\underline{\theta}$:

$$\overline{\eta} : \underbrace{\frac{1}{2}\Big[W(h) + W(H) + \overline{\eta} - \Big(\frac{W(h) + W(H) + \overline{\eta}}{2}\Big)^2\Big]}_{\theta^*(W(h), W(H), \overline{\eta}, h=1)} = \underline{\theta}$$

That is, the maximum value that η can take is the value such that the match quality divorce threshold when the couple with highest human capital composition, (h, H), specializes equals the lower bound of the θ distribution. This condition follows from two aspects of the model.

First, lemma 1 establishes that θ^* is decreasing in η. This implies that as η increases θ^* approaches the lower limit of its distribution, $\underline{\theta}$. Therefore, η must not take values so high that θ^* falls below $\underline{\theta}$.

Second, lemma 1 also establishes that θ^* is decreasing in in $W(s_f)$. This implies that it is sufficient to consider only the returns to skill of spouses (h, H), $W(h)$ and $W(H)$, to bound η from below. In effect, the corresponding value of η such that the match quality threshold reaches the lower bound $\underline{\theta}$ for couples with less skilled women, (l, H), is higher than $\overline{\eta}$:

$$\overline{\eta}^{lH} : \theta^*(W(l), W(H), \overline{\eta}^{lH}, k = 1) = \underline{\theta} > \overline{\eta}$$

To see this, note that for a given η, a lower $W(s_f)$ increases θ^*, so to make it equal to the lower bound, $\underline{\theta}$, θ^* must decrease. This is achieved by increasing $\overline{\eta}$.

Appendix E Numerical examples: model values

Table 2 displays the values associated with the solution of the model in the three cases described in section 4.5. For each case, the first column portrays marital output when the marital investment decision is to not specialize $(k = 0)$ and the second column when it is to specialize $(k = 1)$. For each marital investment choice, the table displays the value produced by the marriage for each type of couple (shown in the four rows labeld *expected household output*) and the ex ante probability of divorce (shown in the subsequent four rows). The last three rows show the optimal matching that results from maximizing the total surplus, and the equilibria under Unilateral divorce regime that result from simulating the female and male utility proposing mechanisms.

Table 2: Examples: a change from MC to UD weakly increases assortativeness.

Case:	*Case I*		*Case II*		*Case III*	
Household specialization (k):	0	1	0	1	0	1
Parameters:						
$\mathcal{W} = (W(l), W(h), W(L), W(H))$	(2, 2.2, 2.1, 2.4)		(2, 2.2, 2.1, 2.4)		(2, 2.2, 2.1, 2.4)	
$\eta^s = (\eta^L, \eta^H)$	(0, 2)		(0, 1.6)		(0, 2.8)	
Expected household output, φ:						
(l,L)	9.68	6.58	**9.68**	6.58	9.68	6.58
(l,H)	10.78	**11.60**	10.78	10.56	10.78	**14.07**
(h,L)	**10.40**	6.88	10.40	6.88	**10.40**	6.88
(h,H)	11.55	12.16	**11.55**	11.07	11.55	14.79
Ex ante probability of divorce:						
(l,L)	0.51	0.51	0.51	0.51	0.51	0.51
(l,H)	0.48	0.17	0.48	0.24	0.48	0.02
(h,L)	0.50	0.50	0.50	0.50	0.50	0.50
(h,H)	0.45	0.15	0.45	0.21	0.45	0
Optimal matching:	NAM		PAM		NAM	
Female proposing under UD	PAM		PAM		NAM	

Notes: **Optimal matching**. Female proposing equilibrium.