Marriage Market and Labor Market Sorting: Online Appendix

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Throughout this Online Appendix, we indicate figures and tables within this appendix by O# ('O' for Online). In turn, figures and tables from the main paper are denoted by just 1,2,...

OA Empirical Evidence

OA.1 Marriage Market and Labor Market Sorting

We provide additional evidence on the relationship between marriage market and labor market sorting presented in Figure 1 (right), using regressions. We run the following regression at the individual level:

$$\text{Occup.Rank}_{its} = \beta_0 + \beta_1 \text{Educ}_i + \beta_2 \text{Educ}_i \times \text{PAM}_{it} + \beta_3 \text{PAM}_{it} + X_{it}\mathbf{\Gamma} + \delta_t + \delta_s + \epsilon_{its}$$

where $Occup.Rank_{its}$ is the percentile rank of individual i's occupation at time t in state s, as defined in Appendix C.3 of the paper. $Educ_i$ is defined as the highest education bin an individual attains during the time we observe them in the sample, where the education bins are defined as in Section 3.2 of the paper. PAM_{it} is an indicator variable that takes value 1 if individual i and their spouse are both in the same education bin at time t. We also control for the age of the individual, the income of the household and the presence of children (all summarized in vector X_{it}). Finally, δ_t and δ_s capture year and state fixed effects, while ϵ_{its} is a mean-zero error term. Coefficients β_1 and β_2 are of most interest here.

We pool data from the period 2010-2016 and restrict our attention to individuals in couples (married or cohabiting). We run the regression separately for women, and men, and also for the pooled sample, displayed in column (1), (2) and (3) of Table O1. The results in Table O1 suggest a positive correlation between own education and productivity attribute of the job (as indicated by a positive and significant β_1), indicative of positive labor market sorting. This correlation becomes *larger* when spouses are well matched on education, as suggested by a positive and significant coefficient on the interaction term, β_2. This in line with the evidence in the paper's Figure 1, right panel, on the inverse u-shaped relation between labor market sorting and marriage market sorting.

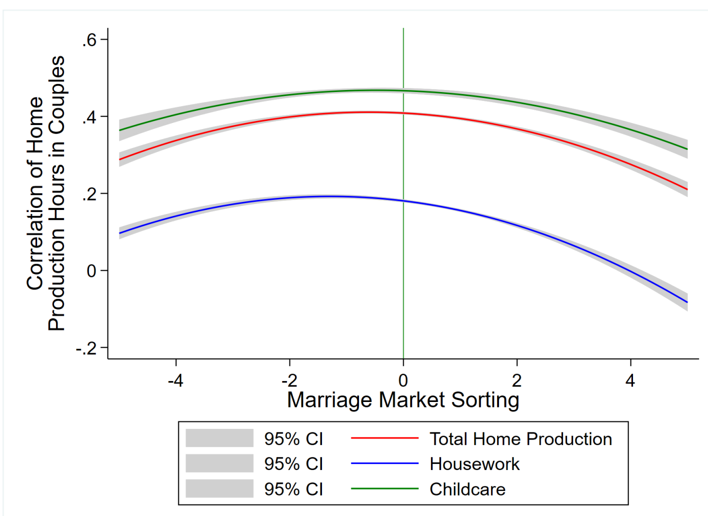
Table O1: Labor Market Sorting and Marriage Market Sorting

	(1) Occupation Rank	(2) Occupation Rank	(3) Occupation Rank
Educ	0.162***	0.179***	0.174***
	(0.008)	(0.005)	(0.004)
Educ × PAM	0.081***	0.044***	0.060***
	(0.010)	(0.007)	(0.006)
PAM	-0.144***	-0.107***	-0.121***
	(0.021)	(0.015)	(0.012)
Observations	5,079	5,948	11,027
R-squared	0.380	0.450	0.403
Demographic Controls	Yes	Yes	Yes
State and Year FE	Yes	Yes	Yes
Sample	Women	Men	All
Period	2010-2016	2010-2016	2010-2016

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Occupation Rank is the job attribute percentile of the individual's occupation. Educ is measured as the individual's highest education bin we observe during the sample period. PAM is an indicator variable that takes value 1 when spouses are in the same education bin. We restrict the sample to individuals in couples, both married and cohabiting. Demographic controls: age, household income and presence of a child in the household.

OA.2 Home Production Complementarities

Figure O1: Hours Correlation and Marriage Sorting by Home Production Components



Note: We split total home production between time allocated to childcare-related activities and housework (including household chores, such as cooking and cleaning, errands, and repairs around the house and the garden).

OA.3 Home Production and Labor Hours Complementarities

In Tables O2 and O3 we further explore the correlation between male and female hours, and how it is related to marriage market sorting. This is to rule out some of the concerns discussed in Section 3.2 of the paper.

We run the following regression at the couple level:

$$hours^{m}_{cts} = \alpha_0 + \alpha_1 hours^{f}_{cts} + X_{ct}\mathbf{\Gamma} + \delta_t + \delta_s + \epsilon_{cts} \tag{1}$$

where $hours^{f}_{cts}$ and $hours^{m}_{cts}$ are either the home or the market hours (depending on the regression) of the female and the male partner in couple c at time t and state s, respectively. In all regressions, we control for the years of education of the male and female partners, the age of both partners, the income of the household and the presence of children (captured by vector X_{ct}). Finally, δ_t and δ_s are year and state fixed effects, and ϵ_{cts} is a mean-zero error term. Our main coefficient of interest is α_1.

We first run (1) for *home production hours*. Our results in Table O2 suggest strong complementarities in home production hours between spouses (column 1). In line with our findings in Figure 2, left, complementarities are stronger when spouses are better sorted in the marriage market (column 2).[1]

We also provide additional evidence on the source of complementarities. In columns 3 and 4, Table O2, we restrict attention to couples with children younger than 18 years old in the household, and run the regression for childcare time and (child-unrelated) housework time separately. We find that complementarities in childcare are larger than complementarities in housework (which includes household chores, errands, and repairs around the house), in line with Figure O1.

One concern is that there is unobserved heterogeneity driving both male and female hours. To control for time-invariant unobserved characteristics, we exploit the panel structure of the data and include individual fixed effects in the regressions of Table O2. Our main results are robust (available on request).

We show our results for *labor market hours* in Table O3.[2] We first replicate our results from Column 1 and 2 of Table O2 but using market hours instead of home production hours. Our coefficients are close to zero and not significant. Several factors could be driving this result: First, market hours are more rigid than home hours, which makes it more difficult to adjust them in the short run. Second, gender norms in Germany are such that men are less likely to adjust their labor market hours in response to life-events, such as the arrival of a child, since they are less involved in childcare. Third, confounding factors could affect both male and female hours. For instance, as in our model in Section 6, unobserved

[1] For consistency with our regressions in Table O3, we condition on the sample of couples in which both partners work in the labor market. Our results are qualitatively unchanged (but smaller) when we do not impose this restriction.

[2] As mentioned above, we restrict our attention to couples in which both partners participate in the labor market. We winsorize market hours to 10 hours at the bottom and 60 hours at the top. For the IV regressions (see below) we extend our sample period from 2010-2016 to 2006-2016, in line with the timing of our instrument. We also include in all regressions a control for the presence of children between 1 and 3 years old in the household.

ability could impact both own hours and partner's hours; also, as in our model, there could be household labor supply shocks affecting both partners' market hours.

In order to address these concerns, we take advantage of an exogenous change in childcare availability in Germany to instrument for female hours in regression (1). In particular, we exploit the variation across time and across states in the share of the childcare slots available for children between 1 and 3 years old, in response to a law passed in December 2008. This law ("Das Kinderförderungsgesetz") aimed to guarantee universal and subsidized childcare for children aged 1 to 3 years by August 1, 2013.

Specifically, we use as instrument for female market hours the share of children between 1 and 3 years old enrolled in childcare in a state and year, and its interaction with the presence of a child in that age range in the household.[3] Our identification assumption is that the exogenous increase in childcare availability affects female labor force participation and hours, but it only affects male hours through changes in his partner's hours. We present different pieces of evidence to support our identifying assumption: first, gender norms in Germany are such that women take most of the burden of childcare (almost 80% of childcare is done by women) independently of the education levels of the partners. Second, women with small children are likely to drop out of the labor force or to work part-time, while most men (even with small children) in Germany tend to work fulltime. In particular, we find that the presence of a small child in the household has a strong negative impact on female market hours, but a non-significant impact on male market hours. Therefore, we would expect that policies that allow households to outsource childcare would have a direct impact on the allocation of maternal time, but no direct effect on paternal time. Finally, previous literature looking at the impact of childcare expansions in Germany on labor market outcomes focus on women's outcomes, with positive impacts on maternal employment (Boll and Lagemann, 2017; Müller and Wrohlich, 2020; Bauernschuster and Schlotter, 2015).[4]

We report the results of the IV regressions in columns 3 and 4 in Table O3. Once we instrument for female market hours we find a sizable positive impact of female market hours on male market hours (column 3). The effect is larger for well sorted couples (column 4)—in line with our descriptive evidence from Figure 2, right.[5]

In column 5 of Table O3, we report the first stage corresponding to the IV regression in column 3. The results suggest that our instruments are indeed relevant in the regression of female hours. The F-statistic of the first stage is 1130.

[3]Both the number of children enrolled in childcare and the total population by age group, state and year were obtained from the Federal Statistical Office (Statistisches Bundesamt). We combined them to construct the share of children enrolled in childcare by state and year. Under the assumption of excess demand for childcare slots and full take-up of available slots, this corresponds to the share of slots offered at each year and state (Müller and Wrohlich, 2020).

[4]In particular, Müller and Wrohlich, 2020 use the same policy as we do to estimate the impact of childcare availability on maternal time allocation, in line with our first stage. However, different from us, they use data from the German Micro-census and their empirical strategy is different from ours.

[5]Unfortunately, we cannot use the same instrument for home hours in Table O2, since the presence of a small child has a significant direct impact on the hours allocated to home production by *both* parents, violating the exclusion restriction.

Table O2: Complementarity in Home Production Hours

	(1) Home Hours Male	(2) Home Hours Male	(3) Childcare Hours Male	(4) Housework Hours Male
Home Hours Female	0.174***	0.184***		
	(0.017)	(0.021)		
Childcare Hours Female			0.176***	
			(0.032)	
Housework Hours Female				0.136***
				(0.027)
Demographic Controls	Yes	Yes	Yes	Yes
State and Year FE	Yes	Yes	Yes	Yes
Sample	All	Same Education	Children Present	Children Present
Period	2010-2016	2010-2016	2010-2016	2010-2016
Observations	9,422	5,475	3,791	3,649
R-squared	0.173	0.186	0.195	0.095

Notes: Standard errors clustered at the state level in parentheses. *** p<0.01, ** p<0.05, * p<0.1. All regressions include demographic controls (years of education and age of the spouses, household income, and presence of children). Missing data on home hours are imputed using market hours and considering 70 hours of available time to work (home + market) per week. We pooled observations from West and East Germany for consistency with regression on market hours in Table O3.

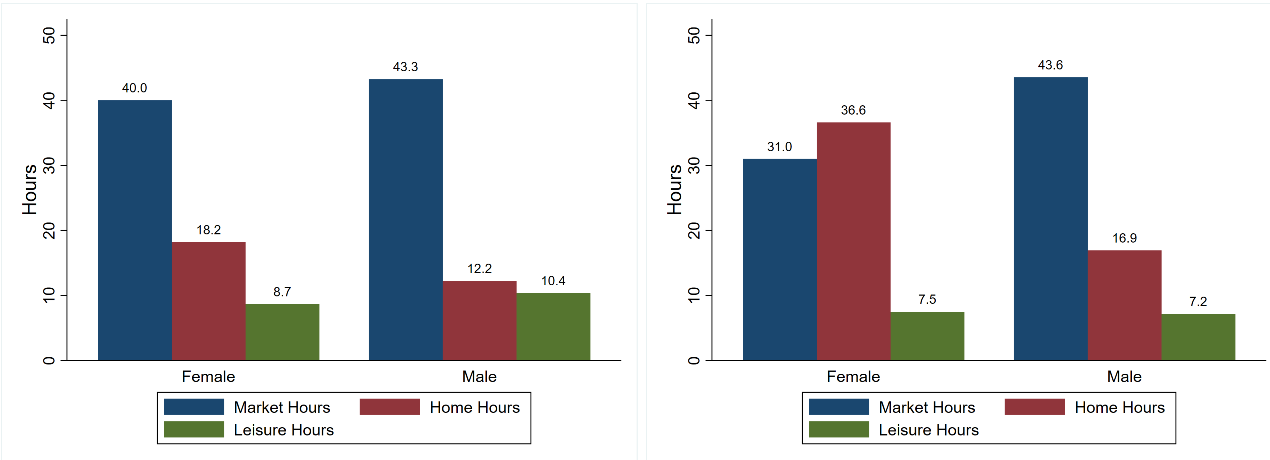
Table O3: Complementarity in Labor Market Hours

	OLS Market Hours Male	OLS Market Hours Male	IV Market Hours Male	IV Market Hours Male	First Stage Market Hours Female
Market Hours Female	-0.004	0.008	0.334*	0.444*	
	(0.013)	(0.020)	(0.176)	(0.230)	
Share of Childcare Slots (1-3 yo)					7.893*
					(4.037)
Share of Childcare Slots (1-3 yo) ×					6.870***
Presence of a Child in the HH (1-3 yo)					(1.378)
Demographic Controls	Yes	Yes	Yes	Yes	Yes
State and Year FE	Yes	Yes	Yes	Yes	Yes
Sample	All	Same Education	All	Same Education	All
Period	2010-2016	2010-2016	2006-2016	2006-2016	2006-2016
Observations	9,316	5,414	15,047	8,746	17,816
R-squared	0.035	0.038			0.237

Notes: Standard errors clustered at the state level in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$. All regressions include demographic controls (years of education and age of the spouses, household income, and presence of children). Columns (3) and (4) instrument female market hours with the share of available slots for daycare for children 1 to 3 years old, and its interaction with the presence of a child of that age group in the household. Columns (2) and (4) restrict the sample to couples in which both spouses have the same level of education. We pooled observations from West and East Germany to capture more of the regional variation in childcare availability.

OA.4 Time Allocation and Marital Status

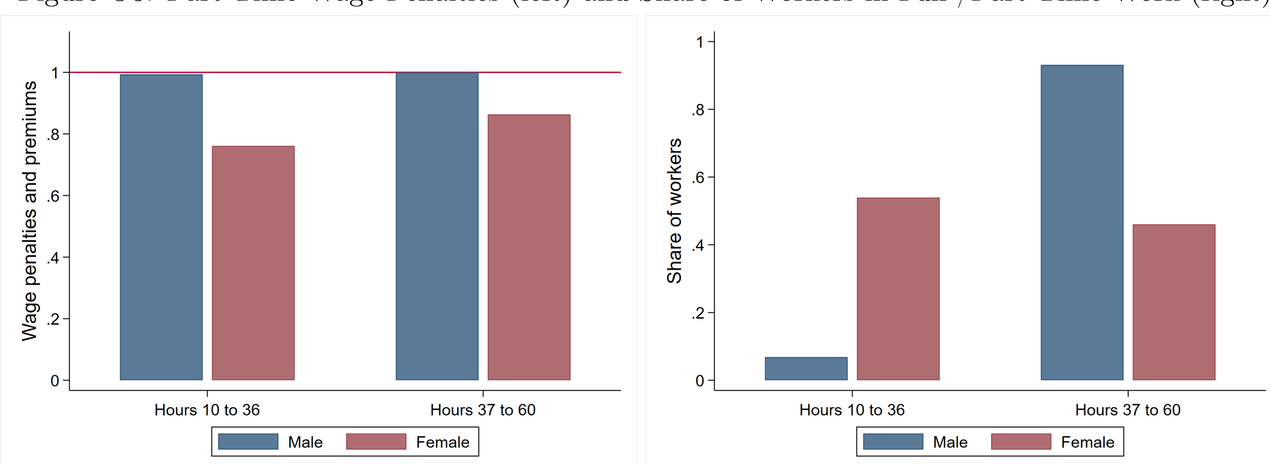
Figure O2: Time allocation for Singles (left) and Couples (right)



OA.5 Time Allocation and Wages

The left panel of Figure O3 shows the wage penalties of various groups *relative* to men working at least fulltime (blue bar, which height equals one), where 'fulltime' is defined as working between 37.5 and 60 hours per week. The height of the bars in Figure O3 (left) reflect the estimates of the regression of the logarithm of hourly wages on an indicator for part-time work, a gender indicator and the interaction between both. We control for age, marital status, birth place and education. While fulltime women have a wage penalty of 14.7 percentage points relative to fulltime men, when they work part-time the wage penalty increases to 26.6 percentage points. Figure O3 (right) shows that while few men work less than fulltime (less than 10% of employed men), more than 50% of employed women do so.

Figure O3: Part-Time Wage Penalties (left) and Share of Workers in Full-/Part-Time Work (right)



OB Theory

OB.1 TU Representation: Examples

We give two additional examples of utility functions under which our problem features TU. First, consider a utility function that is multiplicatively separable in private and public consumption, $u(c,p) = m(c)k(p)$, where, as standard, m and k are assumed to be strictly increasing. Then, the wife's constraint in (3) reads $m(c)k(p) = \overline{v} \Leftrightarrow c = m^{-1}(\overline{v}/k(p))$, and the constrained household maximization problem (3) can be simplified as follows, when substituting in both the budget constraint and the constraint on the wife's utility:

$$\begin{aligned}
&\max_{h_m,h_f} m(w(\tilde{x}_m) + w(\tilde{x}_f) - m^{-1}(\overline{v}/k(p)))k(p) \\
\Leftrightarrow \quad &\max_{h_m,h_f} m([w(\tilde{x}_m) + w(\tilde{x}_f)][m^{-1}(k(p))] - m^{-1}(\overline{v})) \\
\Leftrightarrow \quad &\max_{h_m,h_f} [w(\tilde{x}_m) + w(\tilde{x}_f)][m^{-1}(k(p))]
\end{aligned}$$

These manipulations yield an objective function, which is linear in money, complying with the Gorman form. A second example is the log-utility function $u(c,p) = \log(c) + \log(p)$. We then obtain from (3):

$$\begin{aligned}
&\max_{h_m,h_f} \log\left(w(\tilde{x}_m) + w(\tilde{x}_f) - \frac{\exp(\overline{v})}{p}\right) + \log(p) \\
\Leftrightarrow \quad &\max_{h_m,h_f} \log\left((w(\tilde{x}_m) + w(\tilde{x}_f))p - \exp(\overline{v})\right) \\
\Leftrightarrow \quad &\max_{h_m,h_f} (w(\tilde{x}_m) + w(\tilde{x}_f))p
\end{aligned}$$

which is again linear in money and independent of $\overline{v}$. Here the arguments of the log operator are linear but they can be specified in a flexible way, e.g. CRRA utility for each component, as long as the overall utility function is log-additive in consumption of the private and the public good.

OB.2 Monotone Equilibrium in the Quantitative Model

We now show that the key properties of our analytical model (Propositions 1 and 2 of the paper) are preserved in our quantitative model, when adjusted for the stochastic nature of hours and matching. Since Propositions 1 and 2 are flip sides of each other, we here focus on generalizing Proposition 1. We will show that under similar conditions as in the baseline model, the properties of monotone equilibrium hold *on average*.

Proposition O1. *If p is strictly supermodular in (ℓ_m, ℓ_f) and z are strictly supermodular in $(\tilde{s}, y)$ and convex $\tilde{s}$, then the properties of monotone equilibrium are satisfied on average, i.e.*

1. *labor market: there is positive sorting between effective human capital $\tilde{s}$ and firm types y;*

2. *households: on average, labor hours h_i are increasing in own type s_i and in partner's type s_j, $i \neq j, i, j \in \{f, m\}$;*
3. *marriage market: on average, there is positive sorting in the sense that higher s_f match with higher s_m.*

Proof.

Part 1. Recall the firm's problem

$$\max_{\tilde{s}} \quad z(\tilde{s}, y) - w(\tilde{s}).$$

Based on well-known arguments, given that z is assumed to be strictly supermodular in $(\tilde{s}, y)$, the optimal matching satisfies positive sorting in $(\tilde{s}, y)$.

Part 2. We want to show that $\widehat{\Pi}_m(h_m|s_m)$ and $\widehat{\Pi}_f(h_f|s_m)$ are decreasing in s_m, where

$$\widehat{\Pi}_m(h_m|s_m) \equiv \sum_{s_f} \Pi_m(h_m|s_m, s_f)\eta(s_f|s_m)$$
$$\widehat{\Pi}_f(h_f|s_m) \equiv \sum_{s_f} \Pi_f(h_f|s_m, s_f)\eta(s_f|s_m)$$

are marginal cdfs of male and female hours conditional on male (own and partner) type, so that higher s_m types are associated with stochastically more labor market hours both for them and their partner (the argument for hours being increasing in s_f is analogous and omitted). Note that $\eta(s_f|s_m)$ is the conditional marriage matching probability mass function (pmf), and $\Pi_m(h_m|s_m, s_f)$ and $\Pi_f(h_f|s_m, s_f)$ are the marginal cdfs of male and female hours obtained from the joint pmf of spouses' hours, $\pi_{(s_f,s_m)}(h_f, h_m)$ introduced in the paper:

$$\Pi_m(h_m|s_m, s_f) \equiv \sum_{\tilde{h}_m < h_m} \sum_{\tilde{h}_f} \pi_{(s_f,s_m)}(\tilde{h}_f, \tilde{h}_m)$$
$$\Pi_f(h_f|s_m, s_f) \equiv \sum_{\tilde{h}_f < h_f} \sum_{\tilde{h}_m} \pi_{(s_f,s_m)}(\tilde{h}_f, \tilde{h}_m),$$

where we recall that the probability of a couple with human capital $\mathbf{s} = (s_f, s_m)$ of choosing hours alternative $\mathbf{h} = (h_f, h_m)$ is given by:

$$\pi_{(s_f,s_m)}(h_f, h_m) \quad = \quad \frac{\exp(\overline{u}_{\mathbf{s}}(\mathbf{h})/\sigma_\delta)}{\sum_{\tilde{\mathbf{h}} \in \{\mathcal{H} \cup \emptyset\}^2} \exp(\overline{u}_{\mathbf{s}}(\tilde{\mathbf{h}})/\sigma_\delta)}$$

where we dropped the household type superscript t to reduce notation (we exclusively focus on couples here). We will derive step-by-step the conditions under which $\widehat{\Pi}_m(h_m|s_m)$ is decreasing in s_m; the steps and conditions for showing that $\widehat{\Pi}_f(h_f|s_m)$ is decreasing are analogous.

As we are interested in the impact of s_m on $\widehat{\Pi}_m(h_m|s_m)$, we apply the discrete chain rule to obtain:

$$\Delta_{s_m}\widehat{\Pi}_m(h_m|s_m) = \sum_{s_f}\Pi_m(h_m|s_m,s_f)\Delta_{s_m}\eta(s_f|s_m) + \sum_{s_f}\Delta_{s_m}\Pi_m(h_m|s_m,s_f)\eta(s_f|s_m) \tag{2}$$

where we denote the discrete derivative of a function $f(n)$ by $\Delta_n f(n) = f(n+1) - f(n)$. We want to establish conditions under which $\Delta_{s_m}\widehat{\Pi}_f(h_f|s_m) \leq 0$ in (2).

We can further simplify this expression by applying summation by parts to the first term. To do so, let's index the different female types by k, so $s_{f_k}, k \in \{0, ..., n\}$. Then:

$$\begin{aligned}\sum_{s_f}\Pi_m(h_m|s_m,s_f)\Delta_{s_m}\eta(s_f|s_m) &= \sum_{k=0}^{n}\Pi_m(h_m|s_m,s_{f_k})\Delta_{s_m}\eta(s_{f_k}|s_m) \\ &= \Pi_m(h_m|s_m,s_{f_n})\sum_{k=0}^{n}\Delta_{s_m}\eta(s_{f_k}|s_m) - \sum_{j=0}^{n-1}\Delta_{s_{f_j}}\Pi_m(h_m|s_m,s_{f_j})\sum_{k=0}^{j}\Delta_{s_m}\eta(s_{f_k}|s_m)\end{aligned}$$

where in the first term, we have

$$\sum_{k=0}^{n}\Delta_{s_m}\eta(s_{f_k}|s_m) = \sum_{k=0}^{n}\eta(s_{f_k}|s_{m+1}) - \sum_{k=0}^{n}\eta(s_{f_k}|s_m) = 0$$

and thus the first term vanishes due to the standard property of cdfs. Therefore,

$$\sum_{s_f}\Pi_m(h_m|s_m,s_f)\Delta_{s_m}\eta(s_f|s_m) = -\sum_{j=0}^{n-1}\Delta_{s_{f_j}}\Pi_m(h_m|s_m,s_{f_j})\sum_{k=0}^{j}\Delta_{s_m}\eta(s_{f_k}|s_m)$$

where

$$\sum_{k=0}^{j}\Delta_{s_m}\eta(s_{f_k}|s_m) = \sum_{k=0}^{j}\eta(s_{f_k}|s_{m+1}) - \sum_{k=0}^{j}\eta(s_{f_k}|s_m) \leq 0$$

if the cdf $H(s_f|s_m) \equiv \sum_{k=0}^{j}\eta(s_{f_k}|s_m)$ is decreasing in s_m—something we will show below—so that higher s_m types are matched with higher s_f in the FOSD sense.

Thus, the first term in (2) is negative if $\Delta_{s_f}\Pi_m(h_m|s_m,s_f) \leq 0$ and the second term is negative if $\Delta_{s_m}\Pi_m(h_m|s_m,s_f) \leq 0$. To derive conditions under which this holds, i.e. under which $\Pi_m(h_m|s_m,s_f)$ is decreasing in s_m and s_f, it suffices to show conditions under which the *marginal* pmf of male hours $\sum_{\tilde{h}_f}\pi_{(s_f,s_m)}(\tilde{h}_f,h_m)$ satisfies the monotone likelihood property (or, equivalently, is log-supermodular) in (h_m,s_m) (for fixed s_f), and in (h_m,s_f) (for fixed s_m), as these properties imply FOSD of the marginal cdf $\Pi_m(h_m|s_m,s_f)$ in male and female types, respectively.

The denominator of the joint pmf, $\pi_{(s_f,s_m)}(h_f,h_m)$, does not depend on the specific (h_f,h_m), nor does the denominator of the marginal pmf, $\sum_{\tilde{h}_f}\pi_{(s_f,s_m)}(\tilde{h}_f,h_m)$, so we can focus on the numerator when establishing log-supermodularity. We thus aim to show under which conditions

$$\sum_{\tilde{h}_f} \exp(\overline{u}_{(s_f,s_m)}(\tilde{h}_f, h_m)/\sigma_\delta)$$

is log-supermodular in (h_m, s_m) (for fixed s_f), and in (h_m, s_f) (for fixed s_m). For this is suffices that the log-transformed summand is supermodular pairwise[6], meaning in (h_m, s_m), (h_m, h_f), (h_f, s_m) (for fixed s_f) and in (h_m, s_f), (h_m, h_f), (h_f, s_f) (for fixed s_m), where

$$\log(\exp(\overline{u}_{(s_f,s_m)}(h_f, h_m)/\sigma_\delta)) = (w\left(\psi s_f h_f\right) + w(s_m h_m) + 2p^M(1-h_m, 1-h_f))/\sigma_\delta.$$

Supermodularity in (h_f, s_m) and in (h_m, s_f) is trivially satisfied (at equality). Supermodularity in (h_m, h_f) holds under the premise of strictly supermodular p. Finally, supermodularity in (h_m, s_m) and (h_f, s_f), holds if the wage function is supermodular in these pairs, which—as in the baseline model—is true if z is strictly supermodular and convex, as assumed.

A similar argument and analogous conditions establish that $\widehat{\Pi}_f(h_f|s_m)$ is decreasing in s_m. We have thus shown that under the premise of the proposition (and if marriage sorting is positive in a stochastic sense, to which we will turn next), hours are stochastically increasing in own and partner's type.

Part 3. Recall the probability that a man s_m chooses woman s_f on the marriage market, conditional on marrying, is given by

$$\eta_{(s_f,s_m)} = \frac{\exp(\Phi(s_m, s_f, v(s_f)))/\sigma_\beta^M)}{\sum_{\tilde{s}_f} \exp(\Phi(s_m, \tilde{s}_f, v(\tilde{s}_f)))/\sigma_\beta^M)}.$$

We are interested in conditions under which:

$$\frac{\eta_{(s''_f,s''_m)}}{\eta_{(s'_f,s''_m)}} \geq \frac{\eta_{(s''_f,s'_m)}}{\eta_{(s'_f,s'_m)}}$$

for all $s''_f > s'_f$ and $s''_m > s'_m$. In words, we are seeking conditions under which probability $\eta_{(s_f,s_m)}$ is log-supermodular, or equivalently, conditions that ensure that the monotone likelihood ratio property of $\eta_{(s_f,s_m)}$ holds. The important implication is that higher s_m-type men are matched to higher s_f-type women in a FOSD sense or, equivalently, using the notation for $H(s_f|s_m) \equiv \sum_{k=0}^{j} \eta(s_{f_k}|s_m)$ from above, that $H(s_f|s_m)$ is decreasing in s_m.

[6]This step uses the fact that log-supermodularity is preserved under discrete sums, as shown by de Clippel, Eliaz, and Rozen (2014), Proof of Theorem 4, Step 3.

Note that

$$\frac{\eta_{(s''_f,s''_m)}}{\eta_{(s'_f,s''_m)}} = \frac{\dfrac{\exp(\Phi(s''_m,s''_f,v(s''_f)))/\sigma_\beta^M)}{\sum_{\tilde{s}_f}\exp(\Phi(s''_m,\tilde{s}_f,v(\tilde{s}_f)))/\sigma_\beta^M)}}{\dfrac{\exp(\Phi(s''_m,s'_f,v(s'_f)))/\sigma_\beta^M)}{\sum_{\tilde{s}_f}\exp(\Phi(s''_m,\tilde{s}_f,v(\tilde{s}_f)))/\sigma_\beta^M)}}$$
$$= \frac{\exp(\Phi(s''_m,s''_f,v(s''_f)))/\sigma_\beta^M)}{\exp(\Phi(s''_m,s'_f,v(s'_f)))/\sigma_\beta^M)}$$

Therefore, it suffices to show under which conditions $\exp(\Phi(s_m,s_f,v(s_f)))/\sigma_\beta^M$ is log-supermodular, in (s_f,s_m) or, equivalently, under which conditions $\Phi(s_m,s_f,v(s_f))/\sigma_\beta^M$ is supermodular in (s_f,s_m). Recall that:

$$\frac{\Phi(s_m,s_f,v(s_f))}{\sigma_\beta^M} = \frac{\sigma_\delta}{\sigma_\beta^M}\Big[\kappa + \log\Big(\sum_{h_f,h_m}\exp\Big\{\frac{w(s_m h_m) + w(\psi s_f h_f) + 2p^M(1-h_m,1-h_f)}{\sigma_\delta}\Big\}\Big)\Big] - v(s_f)$$

And so $\Phi(s_m,s_f,v(s_f))/\sigma_\beta^M$ is supermodular if

$$\log\Big(\sum_{h_f,h_m}\exp\Big\{\frac{w(s_m h_m) + w(\psi s_f h_f) + 2p^M(1-h_m,1-h_f)}{\sigma_\delta}\Big\}\Big)$$

is supermodular, or if

$$\sum_{h_f,h_m}\exp\Big\{\frac{w(s_m h_m) + w(\psi s_f h_f) + 2p^M(1-h_m,1-h_f)}{\sigma_\delta}\Big\}$$

is log-supermodular in (s_f,s_m), which is the case if

$$\exp\Big\{\frac{w(s_m h_m) + w(\psi s_f h_f) + 2p^M(1-h_m,1-h_f)}{\sigma_\delta}\Big\}$$

is log-supermodular *pairwise* or if $w(s_m h_m) + w(\psi s_f h_f) + 2p^M(1-h_m,1-h_f)$ is supermodular pairwise in $(s_f,s_m),(s_f,h_f),(s_f,h_m),(s_m,h_f),(s_m,h_m),(h_f,h_m)$.[7] Thus, $\eta_{(s_f,s_m)}$ is log-supermodular—and therefore higher s_f women match with higher s_m in the FOSD sense and thus on average—if the wage function is supermodular in (s_i,h_i) (which again is satisfied if z is supermodular and weakly convex) and if the home production function p is supermodular in (ℓ_m,ℓ_f). □

[7] This step again uses the fact that log-supermodularity is preserved under discrete sums, as shown by de Clippel, Eliaz, and Rozen (2014), Proof of Theorem 4, Step 3.

OC Data and Sample Construction

The main dataset used for our empirical analysis in Section 3.2 and estimation in Section 7.3 is the GSOEP, a household survey conducted by the German Institute of Economic Research (in German: DIW Berlin) starting in 1984. In 1990, it was extended to include states from the former German Democratic Republic. The core study of the GSOEP surveys about 25,000 individuals living in 15,000 households each year. All individuals aged 16 and older respond to the individual questionnaire, while the head of household additionally answers a household questionnaire. This survey is longitudinal in nature, and collects very rich information on demographics (such as marital status, education level, nationality, and family background), labor market variables (including actual and contractual hours worked, wages, and occupation), and detailed time use in home production. Importantly for us, it contains the same information for the head of household and their partner (whether married or cohabiting). Additionally, we link this information to marital and birth histories, and hence we observe presence, number, and ages of children in the household.

OC.1 Sample for Empirical Facts

We now describe the sample restrictions and the variables used for our empirical facts in Section 3.2.

OC.1.1 Sample restrictions

To construct our *Main sample*, we pool observations from the period 2010-2016, for the original GSOEP samples and their refreshments.[8] We restrict our attention to West Germany. Following other papers in the literature, such as Heise and Porzio (2019), we drop Berlin from the sample since it cannot be unambiguously assigned to East or West Germany.

We impose the following demographic restrictions: We keep all individuals in private households, either single or in heterosexual couples (married or cohabiting). We restrict our analysis to the first marital spell of the life of an individual, which could be either never married or the individual's first marriage.[9] We restrict our sample to individuals in their prime working age, 22-55 years old. We apply these restrictions at the individual level, but not at the couple level, which implies that in some cases, one of the spouses could be part of the sample, even if the other spouse is not.

Regarding the labor market, we exclude from our sample those individuals who are self-employed or those who are still in school, those working in odd occupations (identified with the occupational code `kldb92`$\geq$ 9711) and the employed individuals with missing occupational code.

[8] We exclude from our analysis the migrants and refugees samples, the oversampling of low income individuals and single parents, and the oversampling of high income earners.

[9] Since cohabitants are defined as never married, we cannot rule out that they are in a cohabiting relationship that is not the first one.

OC.1.2 Variable description

We now describe the variables we use in the empirical analysis.

1. **Education Variables:** We classify individuals into three education bins: low education includes those with a high school degree or with a vocational degree, with less than 11 years of schooling. Medium education includes those individuals with a vocational degree and more than 11 years of schooling. High education includes those with a college degree or higher. Education levels are defined based on the ISCED-97 classification. Alternatively, we use years of education as our education measure, and we left-truncate this variable at 10 years of education.
2. **Marriage Market Sorting:** For the figures, we define marriage market sorting as the difference between years of education of the partners in couples (own years of education minus partner's years of education). When the analysis is at the household level, we compute marriage market sorting as the difference between the years of education of the male partner and the years of education of the female partner, for consistency across couples.[10]
3. **Matching Function:** Our matching function is given by the task complexity of the occupation in which the individual is employed, defined at the 4-digit level of the `kldb92` classification, as a function of individuals' education. For a detailed description of how the task complexity measure is constructed, refer to Appendix C.3.
4. **Labor Market Sorting:** We define labor market sorting as the correlation between the individual's years of education and their matched job characteristic (task complexity).
5. **Hours:**
   (a) **Market hours**: We define market hours as the number of self-reported hours of an individual in the labor market in a given week (including overtime). We winsorize market hours to 10 and 60 hours, at the bottom and the top, respectively.
   (b) **Home Hours:** We measure home hours as the weekly time an individual allocates to the following activities: childcare, housework (which includes household chores such as cooking, cleaning, etc.), running errands, repairs around the house or the car, and garden work. Since home hours are measured on a typical week day, we multiply these hours by 5, for consistency with market hours. When home hours are not available, we use information on market hours to impute them, assuming a total of 70 hours available per week to allocate to home production and market work.
   (c) **Leisure Hours:** We measure leisure hours as the weekly hours allocated to hobbies and other leisure activities.

[10] We exclude couples for which the difference in years of education between the partners is 6.5 years or more. We pool together couples in which the difference in years of education between the partners is either 5 or 6 years, in order to increase the sample size in the extreme bins.

OC.2 Estimation Sample

In this section we describe the sample restrictions and variable definitions of our estimation sample.

OC.2.1 Sample Restrictions

In order to construct our *Baseline Estimation Sample*, we use data from West Germany for the period 2010-2016 of the original GSOEP panel and its refreshments (as discussed above).[11] For our *Past Estimation Sample* we use data for the period 1990-1996. We drop those individuals that appear in both periods. We then apply the following restrictions, similar to our *Main Sample* above:

1. **Age restrictions**: We restrict our attention to individuals between 22 and 55 years. In the case of couples, we keep them in our sample if both partners are within this age range.[12]
2. **Marital Status Restrictions**: We focus on individuals who are either single or in heterosexual couples (married or cohabiting). We restrict our analysis to the first marital spell of the life of an individual, as discussed above. We drop observations corresponding to periods after the first marriage ended, or for which the end date of the first marriage cannot be identified. We drop observations for which we can identify more than one spouse/partner during the sample period.
3. **Labor Market Restrictions:**
   (a) We exclude from our sample observations corresponding to individuals working in odd occupations (`kldb92`$\geq$ 9711) or employed but with missing occupation code.
   (b) We exclude observations of employed individuals for whom information on hourly wages is missing.
4. **Additional restrictions:**
   (a) We exclude observations from individuals to whom we cannot assign a 'type', based on our estimation of human capital types, described in Appendix C.2.
   (b) We exclude observations of individuals in couples for whom information on the spouse/partner is not available in any of the years, in which they appear in the sample.

OC.2.2 Definition of Typical Occupation, Hours, Wages and Marital Status

In this section, we explain our methodology to create summary variables for the individuals in the GSOEP panel (to which we refer to as 'typical outcomes'), to align the static nature of our model with dynamic features of the data. We define the concepts of typical occupation, typical work and home production hours, and typical wages. For each individual we also define the typical marital status and the corresponding variables for their partner.

[11] We drop from our sample individuals who are observed both in the West and in the East during the time they are in their typical occupation, as defined below.

[12] Even if we impose the age restriction at the individual level, we would drop observations for married individuals for which information of their spouses is not available in any of the years they appear in the sample.

Typical occupation: we apply the following rules to determine the typical occupation of an individual:

1. If the individual appears in the sample only once, we assign them the occupation corresponding to that particular year.
2. If the individual appears more than once in the sample, but only reports one occupation, we assign them the unique occupation that we observe over the sample period.
3. If an individual appears more than once in the sample, and at some point was 'self-employed', 'studying' or 'not-employed', we check the duration of these states.
   (a) If the individual was either 'not-employed', 'self-employed' or 'studying' for strictly more than half of the time they appear in the sample, we consider that status as the typical occupation.
   (b) If an individual was in one of these states exactly half of the time they appear in the sample, but they spent the other half in the other two status, we still classify them to that status.
   (c) If the conditions above do not hold but the individual spent more than 75% of their time between 'not-employed', 'self-employed' and 'studying', even if there is another occupation, we assign them to 'not-employed', 'self-employed' and 'studying' depending on which one has the longest duration.
4. If we observe an individual more than once, and only in one occupation other than 'not employed', 'self-employment' and 'studying', and they spent less than 75% of the time in these three states, we assign the individual to that unique occupation.
5. If we observe an individual having more than one occupation during the period they appear in the sample, we follow these rules:
   (a) We construct the difference in percentiles between the highest ranked occupation and the lowest ranked occupation the individual held during the sample period, where we rank occupations as described in Appendix C.3.
   (b) When the difference is larger than 0.1, we assign the individual to the highest ranked occupation held.
   (c) If this difference is lower than 0.1, we assign the individual to the occupation with the longest tenure.
   (d) If the difference is lower than 0.1, but we observe the individual in more than one occupation with the same tenure, we assign the individual to the better ranked occupation (between those with the longest tenure).
6. After applying all these rules, we exclude from our sample those individuals whose typical occupation is 'self-employment' or 'studying'.

Typical market hours: For each individual, we define typical market hours as the mean of the self-reported total work hours (including overtime) during the years when they were working in their typical occupation, defined as above. We winsorize market hours to 10 and 60 hours, at the bottom and the top, respectively. For those individuals whose typical occupation is 'not employed', typical market hours take value zero.

Typical wages: For each individual, we define their typical wage as the mean of hourly wages earned during the years the individual worked in their typical occupation.[13]

Typical home hours: For each individual, we construct typical home hours (defined as in Appendix OC.1) as the mean of hours the individual works at home while employed in their typical occupation. We impute missing home hours using data on labor market hours, as discussed in Appendix OC.1.

Typical marital status: We define the typical marital status following these rules:

1. If the individual had only one marital status during the sample period, we consider that marital status as the typical one.
2. If the individual switched from being single to being married during the time they appear in the sample, we assign the marital status observed when employed in the typical occupation. If they were observed in both marital status while working in their typical occupation, we assign them to marriage.
3. We exclude from the sample those individuals that, even if assigned to marriage based on the previous rules, report more than one spouse during the time they appear in the sample. We also exclude their corresponding partners.

Corresponding variables for spouses: For every variable defined above (typical occupation, typical market and home hours, and typical wages) we define the analogous variable for the spouse.

Following the rules above, our *Baseline Estimation Sample* (West Germany, 2010-2016) has 3,857 individuals living in 2,326 households. Of these households, 1,531 are couples and 795 are single individuals (418 are single women and 377 are single men).

Our *Past Estimation Sample* (West Germany, 1990-1996) consists of 2,336 individuals, living in 1,294 households, of which 1,042 are couples and 252 are singles (117 single women and 135 single men).

OD Estimation

OD.1 Internal Estimation

OD.1.1 Construction of Moments

Our main estimation targets the 17 moments defined in Table O4. We now provide details on how these moments are constructed both in the data and in the model.

For moment M1, we compute the female to male ratio in labor force participation in the full sample (married individuals and single). We define labor force participation in the data and in the model as a dichotomous variable that takes value 1 when the individual work positive hours, and zero otherwise.

[13] Hourly wages are constructed based on inflation adjusted monthly earnings, divided by monthly hours (constructed as weekly hours times 4.3). Hourly wages are trimmed at the bottom and top 1% percentile. The data for inflation adjustment comes from the OECD: `https://data.oecd.org/price/inflation-cpi.htm`

For moment M2, we compute the ratio of fulltime work by women to fulltime work by male, where fulltime in the data is defined as working more than 37.5 hours per week, which corresponds to working more than 46% of the available time.[14] In the model, we define fulltime as working more than 46% of total time in the labor market, corresponding to the 5th entry in our hours grid. To compute this moment, we consider both individuals that are single and those in couples.

For M3 and M4, we compute the married to single ratio in labor force participation, for men and women separately.

In M14-M15, we compute the female labor force participation rate for women in couples where both partners are of similar human capital type. For M14, we pool couples where both partners are either of human capital type 3 or type 4 (see columns 3 and 4 in Table 7, main document), while for M15 we pool couples in which both partners are either of type 5 or type 6 (see columns 5 and 6 in Table 7).

For M16 and M17, we focus on the sample of single women. M16 computes the female labor force participation of single women of types 3 and 4, while M17 pools together single women of types 5 and 6.

In M5, to compute the correlation of spousal home production time, we first construct home hours $(1-h)$ as the share of home hours in total time (defined as hours worked at home plus hours worked in the market), in both data and model. To deal with the fact that different individuals report different number of total hours allocated to home production and market work, we first constructed a common denominator for all individuals, given by the 95th percentile of the sum of home and market hours (81.6 hours per week in our estimation sample). Then, we use the hours allocated to home production by each individual to construct their share of home hours in total time, using this common denominator.[15] M5 is computed based on the sample of individuals in couples.

Wage moments (moments M6-M9) are based on data of hourly wages for all individuals in our estimation sample (singles and in couples), conditional on employment. The model counterpart of these moments is constructed in the same way.

Moment M10 is constructed as the correlation of partners' s-types in both data and model.

Moment M11 is the share of single men in the sample, both in data and model.

Finally, moments M12 and M13 measure the gender wage gap by (s, h)-types. To construct these moments in the data, we focus on two (s, h)-type combinations: all individuals (singles or in couples) of s-type 2 and s-type 4 that work fulltime in the labor market. We define fulltime work in the data and the model as described above.

[14] To determine the total available time, we take the 95th percentile of the distribution of total time spent in the labor market and in home production and add them, which is 81.6 hours in our data.

[15] For those individuals that report more than 81.6 hours of home production, we assign them the value of the 95th percentile, to avoid that the share of time spent in home production is larger than 1.

Table O4: Moments

Moment Description	**Definition**
Labor Force Participation Female to Male Ratio (M1)	$\frac{Pr(h_f>0)}{Pr(h_m>0)}$
Full Time Work Female to Male Ratio (M2)	$\frac{Pr(h_f=\hat{h})}{Pr(h_m=\hat{h})}, \hat{h} \geq 37.5$
Labor Force Participation Married to Single Ratio by Gender (M3-M4)	$\frac{Pr(h_i>0\|Married)}{Pr(h_i>0\|Single)}, i \in \{f, m\}$
Correlation of Spouses' Home Hours (M5)	$corr(1-h_f, 1-h_m)$
Mean Hourly Wage (M6)	$\mathbb{E}[w]$
Variance Hourly Wage (M7)	$Var[w]$
Upper Tail Wage Inequality (M8)	$\frac{w^{90}}{w^{50}}$
Overall Wage Inequality (M9)	$\frac{w^{90}}{w^{10}}$
Correlation between Spouses' Human Capital Types (M10)	$corr(s_m, s_f)$
Fraction of Single Men (M11)	$\frac{\#SingleM}{\#M}$
Gender Wage Gap by Effective Type (M12-M13)	$\frac{\mathbb{E}[w(h_i s_i)\|i=m,h_i=\hat{h},s_i=\hat{s}]-\mathbb{E}[w(s_i h_i)\|i=f,h_i=\hat{h},s_i=\hat{s}]}{\mathbb{E}[w(h_i s_i)\|i=m,h_i=\hat{h},s_i=\hat{s}]}$
Female Labor Force Participation by Couple Type (M14-M15)	$Pr(h_f > 0\|s_f = s_m = \hat{s})$
Female Labor Force Participation of Single Women by Type (M16-M17)	$Pr(h_f > 0\|Single, s_f = \hat{s})$

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