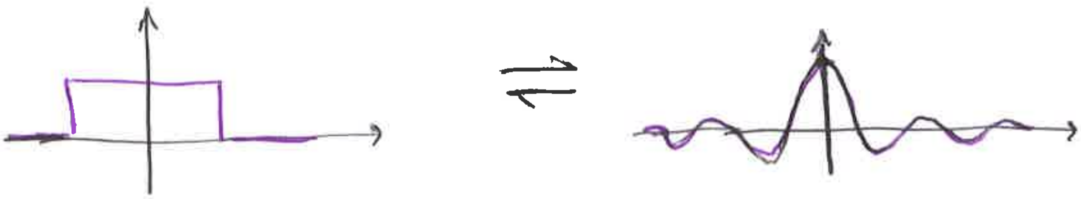
Fourier Optics – 4/23/15 Alexandra Greenbaum

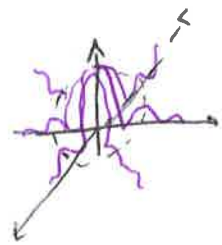
<u>Review</u>: Double slit / Two-hole interferometer (1-D)

2-D image intensity

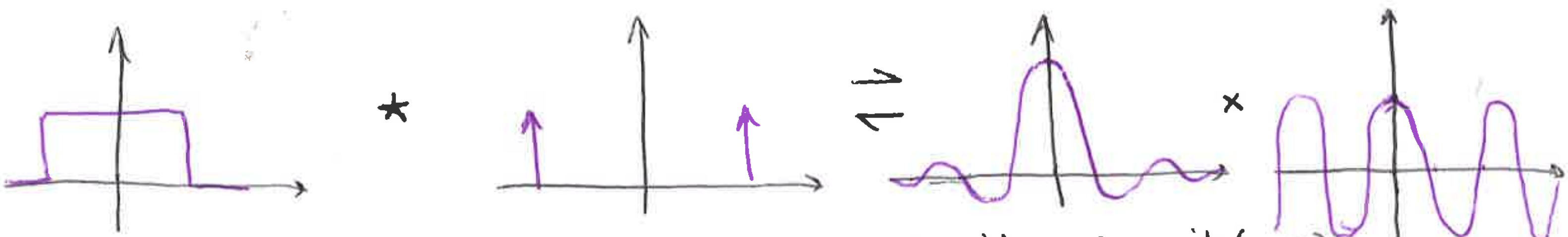
Single:



$P(x) = \Pi(x) \rightleftharpoons p(k) = \text{sinc}\, k$



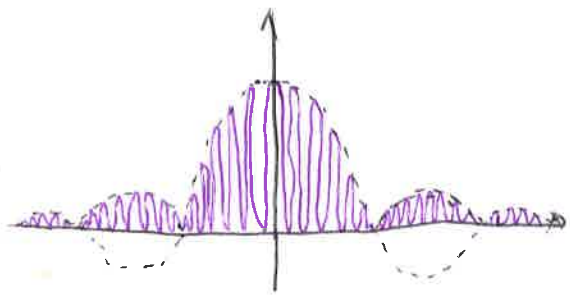
The two-hole is the convolution of the hole shape with delta functions at the hole locations



$$P(x) * [\delta(x-x_1) + \delta(x-x_2)] \rightleftharpoons \underbrace{p(k)\left[e^{-ik(x-x_1)} + e^{-ik(x-x_2)}\right]}$$

For example $x_1 = -x_2$ (symmetric) gives cosine transform

Image $\propto E^2$ ⟶



2-D intensity

$$\| p(k)\left[e^{-ik(x-x_1)} + e^{-ik(x-x_2)}\right] \|^2$$

$$= p^2(k)\left[e^{-ik(x-x_1)}e^{ik(x-x_2)} + e^{-ik(x-x_2)}e^{ik(x-x_2)} + e^{-ik(x-x_1)}e^{ik(x-x_2)} + e^{ik(x-x_1)}e^{-ik(x-x_2)}\right]$$

$$= p^2(k)\left[2 + 2\cos(k(x_2-x_1))\right]$$ All real, non-negative

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Two-hole interferometer: binary source signal

From principles of Long Baseline Stellar Interferometry:

$$V_{binary} = e^{-2\pi i (u\alpha_1 + v\beta_1)} \frac{|V_1| + r|V_2| e^{-2\pi i (u\Delta\alpha + v\Delta\beta)}}{1+r}$$

r = contrast ratio

$$\hookrightarrow \frac{1 + r e^{-2\pi i (u\Delta\alpha + v\Delta\beta)}}{1+r}$$

$$(u, v) = \vec{X}_2 - \vec{X}_1$$

Notice that the signal is baseline-dependent

What can confuse this signal? Unknown phase errors.

$$\text{field:} \quad p(k)\left[e^{-i\phi_1} e^{-ik(x-x_1)} + e^{-i\phi_2} e^{-ik(x-x_2)} \right]$$

$$\text{psf:} \quad p^2(k)\left[2 + \underbrace{e^{i(k(x_2-x_1)+\phi_2-\phi_1)} + e^{-i(k(x_2-x_1)+\phi_2-\phi_1)}}_{2\cos(k(x_2-x_1)+\Delta\phi)} \right]$$

phase delay shifts fringes

BUT φ's don't care about baseline length!

↳ NRM can be useful for measuring hole-dependent errors!

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Suppose: three-hole interferometer

$$\text{pupil} = P(x) * [\delta(x-x_1) + \delta(x-x_2) + \delta(x-x_3)] \quad \text{in general}$$

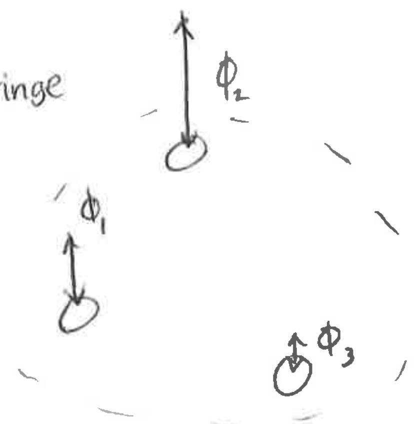
$$\rightleftharpoons p(k)\left[e^{-ik\cdot(x-x_1)} + e^{-ik\cdot(x-x_2)} + e^{-ik(x-x_3)}\right]$$

$$\text{psf} = p^2(k)\left[3 + e^{ik(x_2-x_1)} + e^{-ik(x_2-x_1)} + e^{ik(x_3-x_2)} + e^{-ik(x_3-x_2)} + e^{ik(x_1-x_3)} + e^{-ik(x_1-x_3)}\right]$$

Again, cosine fringes

$\frac{N(N-1)}{2}$ unique fringe phases

Attach constant phases to each hole



For a point source, measured phases:

$\phi_2 - \phi_1$

$\phi_3 - \phi_2$

$\phi_1 - \phi_3$

in general how do we go from pupil phases to fringe phases?

Call this matrix "A"

$$\begin{bmatrix} & & \\ & ? & \\ & & \end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\end{bmatrix} = \begin{bmatrix}\phi_2-\phi_1\\ \phi_3-\phi_2\\ \phi_1-\phi_3\end{bmatrix}$$

If these are phase "errors" how can we get rid of them?

In other words, is there a matrix K such that:

$K \cdot A = 0$ (leaving only physical signal?)

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3-hole interferometer

Closure phase $= \phi_{12} + \phi_{23} + \phi_{31} = \phi_2 - \phi_1 + \phi_3 - \phi_2 + \phi_1 - \phi_3$

(hole phase errors only) $= \phi_2 - \phi_2 + \phi_1 - \phi_1 + \phi_3 - \phi_3 = 0$

Only 1 closure phase for three holes

$$\underbrace{\begin{bmatrix} 1 & 1 & -1 \end{bmatrix}}_{K} \begin{bmatrix} \phi_{12} \\ \phi_{23} \\ \phi_{13} \end{bmatrix} = 0 \qquad \binom{N}{3} \text{ closure phases}$$

in observations:

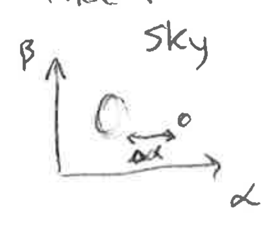
$$\Theta_{measured} = A \cdot \Phi_{holes} + \Theta_{physical}$$

$$K \cdot \Theta_{measured} = \underbrace{\cancel{K \cdot A\,\Phi_{holes}}}_{0} + K \cdot \Theta_{physical}$$

Closure phases ------ measure physical structure

what does the physical signal look like?

$$V_{binary} = \frac{1 + r e^{-2\pi i (u\Delta\alpha + v\Delta\beta)}}{1+r}$$



1-D in general, $r = 0.5$ example:

$$V_1 = \frac{1 + \frac{1}{2} e^{-2\pi i \Delta\alpha u_{12}}}{1+\frac{1}{2}}$$

$$V_2 = \frac{1 + \frac{1}{2} e^{-2\pi i \Delta\alpha u_{23}}}{1+\frac{1}{2}}$$

$$V_3 = \frac{1 + \frac{1}{2} e^{-2\pi i \Delta\alpha u_{31}}}{1+\frac{1}{2}}$$

$$V_1 V_2 V_3 = \Big[1 + \tfrac{1}{2} e^{-2\pi i \Delta\alpha u_{12}} + \tfrac{1}{2} e^{-2\pi i \Delta\alpha u_{23}} + \tfrac{1}{2} e^{-2\pi i \Delta\alpha u_{31}} + \tfrac{1}{4} e^{-2\pi i \Delta\alpha u_{32}} + \tfrac{1}{4} e^{-2\pi i \Delta\alpha u_{21}} + \tfrac{1}{4} e^{-2\pi i \Delta\alpha u_{13}} + \tfrac{1}{8}\Big] \Big/ \left(1 + \tfrac{1}{2}\right)^3$$

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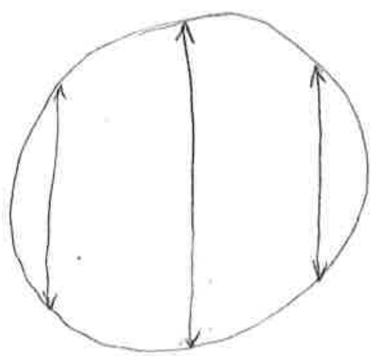
→ Redundant pupils?

?

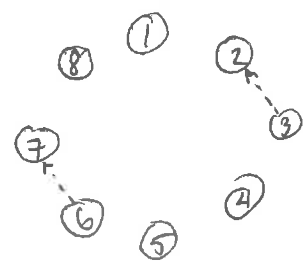
Goal: $\theta_{measured} = A \cdot \phi_{\text{"holes"}} + \theta_{physical}$

Example: partially redundant pupil – annulus



Doubly redundant in all except longest baselines

Break it into 8 supapertures



One row in "A": to define baseline

$[0\ \ -0.5\ \ 0.5\ \ 0\ \ 0\ \ 0.5\ \ -0.5\ \ 0]$

$$\theta_{measured} = A\,\phi_{subapertures} + \theta_{physical}$$

Choose K so $K \cdot A = 0$

$$K\theta_{measured} = \theta_{kernel} = K\,\theta_{physical}$$

For arbitrary pupil shapes Kernel phase is a self-calibrating quantity.