

Online Appendix for “Limited Obstruction”

We first present and discuss a revised model of limited obstruction without commitment and compare it with the model with commitment that was emphasized in the paper. We then generalize the original model to cases of any arbitrarily large number to time periods ($T > 2$).

1 Limited obstruction without commitment

A strong assumption in our model is that the obstructer moves first and can credibly commit to how long he will delay each bill if and when the agenda setter places it on the agenda for floor consideration. Alternatively, we can also analyze a variant of the game in which the obstructer cannot make such credible commitments, because the sequence of moves is reversed: the agenda setter brings bills to the floor before the obstructer decides how long to delay the bill. After doing this and isolating the sometimes-mutual benefits from the commitment game, we identify several concrete mechanisms by which credible commitments seem to be made in actual collective choice settings.

1.1 Game form

1. The agenda setter, s , may bring a bill $b_1 \in B$ to the floor. If he does not bring a bill to the floor, the game ends.
2. The obstructer, o , decides whether to delay. If he delays, then b_1 is added to the

agenda, A , and the game ends. If he does not, then b_2 is added to A and the game proceeds to the next step.

3. s may bring a second bill to the floor, $b_2 \in B \setminus b_1$ to the floor. If she does not bring a bill to the floor, the game ends.
4. o decides whether to delay. If he does not delay, then b_2 is added to A . Either way, the game ends.

s gets a payoff of $\sum_{b \in A} u^s(b)$ and o gets a payoff of $\sum_{b \in A} u^o(b)$.

1.2 Subgame-perfect Nash equilibrium

This game can be solved via backward induction.

1. In the second period, if $u^o(b_2) \geq 0$, then o allows it to pass. Otherwise, o kills it by delaying.
2. Let $b_2 = \operatorname{argmax}_{b \in B \setminus \{b_1\} : u^o(b) > 0} u^s(b)$. If $u^s(b_2) \geq 0$, then s brings b_2 to the floor. Otherwise, he does not bring a bill to the floor.
3. In the first period, o delays if $\{b \in B \setminus \{b_1\} : u^s(b) \geq 0 \text{ and } u^o(b) > 0\} = \emptyset$. Otherwise, o does not delay.
4. In the first period, if $u^o(\bar{b}^s) > 0$, then s proposes $\bar{b}^s = \operatorname{argmax}_{b \neq \bar{b}^s} u^s(b)$. Otherwise, s proposes \bar{b}^s .

The equilibrium of the limited obstruction game when the obstruater cannot commit to how long he will obstruct each bill hinges on whether there exists a bipartisan bill that both the agenda-setter and the obstruater value. If no bipartisan bill exists, then the agenda-setter proposes her most preferred bill and the obstruater delays it so that it requires two periods

to pass. If a bipartisan bill exists, the agenda-setter can propose any bill she pleases in the first period. Even if the obstructer abhors the bill, he cannot credibly commit to obstruct it, because he knows that the agenda setter *can* credibly commit to propose a bipartisan bill in the second period. No matter how much he detests the first bill, he would prefer that the first bill pass quickly so that there is enough time to pass a bipartisan bill.

1.3 Comparison of games with and without commitment

The obstructer is worse off when he lacks the ability to commit to how long he will delay each bill. Suppose that there is at least one agenda that both the agenda-setter and the obstructer prefer to the agenda-setter's most preferred bill. That is, in the notation of the main paper, suppose $\mathcal{C} \neq \emptyset$. When the obstructer can commit, the equilibrium outcome is his most preferred pair in \mathcal{C} . When he cannot, the outcome is either the agenda-setter's most preferred bill or the agenda-setter's most preferred bill plus a bipartisan bill of the agenda-setter's choice (note that this pair is an element of \mathcal{C}). Thus, by the construction of \mathcal{C} , the obstructer is weakly worse off without commitment.

Suppose that $\mathcal{C} = \emptyset$, i.e. that there are no pairs that both players prefer to the agenda-setter's most preferred bill. With commitment, the outcome is the agenda-setter's most preferred bill. Without commitment, the outcome is either the agenda-setter's most preferred bill or the agenda-setter's two most preferred bills (which by supposition is even worse for the obstructer). Here, too, the obstructer is weakly worse off when he cannot make credible commitments.

The comparative payoffs for the setter are more surprising. For some choices of B , the agenda setter and the obstructer are *both* better off when the obstructer can make credible commitments. Reconsider the set of bills B in Table 1 of the paper. When the obstructer can make a credible commitment, the equilibrium outcome is $\{y, z\}$. When he cannot make a credible commitment, the equilibrium outcome is $\{x\}$. Therefore, and not surprisingly, the

obstructor does worse if he cannot commit. More surprisingly, the agenda setter, too, is worse off if the obstructor cannot commit, because she also prefers the quick passage of the pair of bills to the slow passage of her most-preferred single bill. This possibility of mutual benefits from commitment has potentially deep implications for institutional design or evolution, which is one focal point of ongoing work on limited obstruction and commitment.

1.4 Mechanisms that facilitate or resemble commitment

The preceding analysis shows the obstructor has an incentive to find and deploy commitment devices, and the setter sometimes shares that incentive. Two facets of this commitment problem should be emphasized. First, while the obstructor wants to make credible promises to refrain from imposing excessive delay ($t(b) = 1$), he sometimes has an incentive to renege on his promise and delay the bill once it comes to the floor ($t(b) = 2$). In the second period, doing so may allow him to kill a bill that he dislikes. In the first period, doing so might ensure that there is not enough time for the agenda setter to pass a second harmful bill. Second, the obstructor wants to make credible threats to impose delay ($t(b) = 2$), but he sometimes has an incentive to renege on his promise and allow the bill to pass quickly ($t(b) = 1$). In the second period, if the agenda setter brings a bipartisan bill to the floor, the obstructor wants to allow it to pass even if he prefers a different bill. In the first period, no matter what bill the agenda setter brings to the floor, the obstructor may want to allow it to pass quickly so that a bipartisan bill can be passed in the second period.

Do real-world institutions have ways to address these maladies of limited obstruction caused specifically by the difficulty of potential obstructers to make credible commitments? We suggest they do. However, commitment devices in reality are much more nuanced and informal than the explicit and rigorous ways in which they are characterized in game theory. Consider, therefore, three concrete examples of real-world commitment.

The first commitment problem discussed above may be solved in part by institutional

devices such as the U.S. Senate’s regularized use of *unanimous consent agreements* (UCAs). In a smoothly functioning UCA process, the Senate majority leader brings a bill to the floor only after having negotiated with the minority leader various terms of floor debate. UC requests vary in their scope of activities covered and the severity of the restraints imposed on waivers, points of order, and permissible amendments. They are often explicitly used to set a specific time limit on floor debate thereby automatically precluding a filibuster and the cloture process; indeed, UCAs were originally called “time limitation agreements.” As its name suggests, a unanimous consent agreement can be agreed to only by the consent of every Senator, including the senatorial counterpart to our game’s obstructer. Furthermore and crucially in the present context, once the agreement has been made, it cannot be repealed unless by unanimous consent. In this way, by consenting to UCA, the obstructer *can*, in reality, credibly commit to a pre-specified level of obstruction.

The second commitment problem—that of making good on threats to selectively obstruct bills—is more challenging. A second plausible commitment device is obstructers’ ability to make *salient public statements* about their intentions to delay or hasten the passage of specified bills. When intentions are highly publicized, renegeing on commitments is costly to the obstructer inasmuch as his constituents are led to believe he is weak, untrustworthy, or ineffective. Similarly, our third example of a commitment device is *repeated interaction* of legislators. It is easy to envision a version of the initial, limited obstruction game that is played repeatedly and in which the credibility of an obstructer’s commitments depended on whether or not he had followed through on his previous commitments. Then, the obstructer may find it to his advantage to build and maintain his *reputation* even when doing so is costly in the short run. The fact that his payoff is higher when he can credibly commit ensures that the short-term gains from renegeing can be more than offset by long-term gains in establishing a reputation of credibility and trustworthiness.

In summary, the theoretical commitment problem has several viable (but, as yet, hypo-

thetical) empirical solutions.

2 Obstruction with general lengths of sessions

The game proceeds as follows:

- t announces $t : B \rightarrow \{1, 2\}$, which specifies whether each bill in B will take one or two periods to pass.
- o selects $A \subseteq B$ subject to $\sum_{b \in A} t(b) = B$.¹
- Payoffs are awarded.

2.1 Notation

It is convenient to define the equilibrium strategies algorithmically. In doing so, we use the variable assignment operator common in computer science. For instance, $A \leftarrow A \cup \{b\}$ can be informally read as “add bill b to agenda A .” Formally, the value of set A is being updated to the union of whatever A was before and the singleton set $\{b\}$. With this notation, it is possible to specify a set of rules for constructing A even though it is difficult to write down A in closed form.

¹The equality constraint is imposed to simplify the proofs.

2.2 Assumption: No indifference

Throughout the arguments below, we assume that neither player is indifferent between two bills, between two pairs of bills, or between one bill and some pair of bills:

$$u^s(b) \neq u^s(b') \quad \forall b, b' \in B$$

$$u^o(b) \neq u^o(b') \quad \forall b, b' \in B$$

$$u^s(b') + u^s(b') \neq u^s(b'') + u^s(b''') \quad \forall b, b', b'', b''' \in B$$

$$u^o(b') + u^o(b') \neq u^o(b'') + u^o(b''') \quad \forall b, b', b'', b''' \in B$$

$$u^s(b') + u^s(b') \neq u^s(b'') \quad \forall b, b', b'' \in B$$

$$u^o(b') + u^o(b') \neq u^o(b'') \quad \forall b, b', b'' \in B$$

Although the characterization of optimal solutions would be identical if ties were allowed, this set of assumptions implies a unique optimal agenda and thereby greatly simplifies the proofs.

2.3 Agenda setter's strategy

Let A be the agenda, τ be the number of periods in the session that will remain after passing all bills in A , B_1 be the set of bills that could be passed in one period that are not already on the agenda, and B_2 be the set of all bills that could be passed in two periods that are not already on the agenda.

Initialize $B_1 = \{b \in B : t(b) = 1\}$, $B_2 = \{b \in B : t(b) = 2\}$, $A = \emptyset$, and $\tau = T$.

1. If $\tau = 1$, $A \leftarrow A \cup \{\bar{b}_1^s\}$, then terminate. Otherwise, proceed to Step 2.
2. If $u^s(\bar{b}_1^s) + u^s(\bar{b}_1^s) > u^s(\bar{b}_2^s)$, then $A \leftarrow A \cup \{\bar{b}_1^s\}$, $B_1 \leftarrow B_1 \setminus \{\bar{b}_1^s\}$, and $\tau \leftarrow \tau - 1$.
Otherwise, $A \leftarrow A \cup \{\bar{b}_2^s\}$, $B_2 \leftarrow B_2 \setminus \{\bar{b}_2^s\}$, and $\tau \leftarrow \tau - 2$.

At termination, A will be the optimal agenda.

2.3.1 Lemma: Decomposition

Claim: Suppose A^* is the optimal agenda when $T = \tau$ and A^{**} is the optimal agenda when $T = \tau + 2$. Then $A^* \subset A^{**}$.

Proof: Suppose $A^* \not\subset A^{**}$. There are two cases to consider:

1. Suppose $t(b) = 2$. If $\exists b' \in A^{**} \setminus A^*$ such that $t(b') = 2$, then by Step 2 of the equilibrium strategy, $u^s(b) > u^s(b')$. If not, then $\exists b', b'' \in A^{**} \setminus A^*$ such that $t(b') = t(b'') = 1$ (because A^{**} must consume two more periods than A^*). By Step 2, $u^s(b) > u^s(b') + u^s(b'')$.
2. Suppose $t(b) = 1$. If $\exists b' \in A^{**} \setminus A^*$ such that $t(b') = 1$, then by Steps 1 and 2 of the equilibrium strategy, $u^s(b) > u^s(b')$. Otherwise, $\exists b' \in A^{**} \setminus A^*$ such that $t(b') = 2$ (because A^{**} must consume two more periods than A^*) and $\exists \hat{b} \in A^* \setminus A^{**}$ such that $t(\hat{b}) = 1$ (because the sizes of A^* and A^{**} are either both odd or both even). By construction of Step 2 and transitivity, $u^s(b) + u^s(\hat{b}) > u^s(b')$.

In both cases, there is a profitable improvement available to A^{**} . This contradicts optimality, so $A^* \subset A^{**}$.

This lemma shows that any problem of session length $T = \tau + 2$ can be decomposed into two subproblems: the problem of constructing an optimal agenda for a session of length τ and the problem of constructing an optimal agenda for a session of length 2 after removing the bills used in the first agenda. From the analysis in the main body of the paper, we already know how to solve the second subproblem.

2.3.2 Proof of optimality

The algorithm obviously obtains an optimal solution for $T = 1$ and $T = 2$. Assume the algorithm obtains an optimal solution for $T = \tau$. By construction, running the algorithm for a session of length $\tau + 2$ is equivalent to running the algorithm for a session of length τ and then constructing an optimal agenda for a session of length 2 from the remaining bills. If the algorithm obtains the optimal agenda for a session of length τ , then the decomposition lemma implies that it will also obtain the optimal agenda for a session of length $\tau + 2$. By induction, the algorithm always obtains the optimal agenda.

2.4 Obstructor's strategy

For each $i = 0, 1, \dots, \frac{T}{2}$, let $\mathcal{C}(i)$ be the set of partitions that satisfy the following properties:

1. Each $C \in \mathcal{C}(i)$ partitions B into two subsets, $B_1(C)$ and $B_2(C)$, such that $|B_1(C)| = 2i$ if T is even and $|B_1(C)| = 2i + 1$ if T is odd.
2. Let $b_{x,(y)}$ be the y th largest element of set B_x , where the elements are ordered by $u^s(b)$. Then for each $C \in \mathcal{C}(i)$, $u^s(b_{1,(|B_1(C)|-1)}) + u^s(b_{1,(|B_1|)}) > u^s\left(b_{2,(\frac{T-|B_1(C)|}{2}+1)}\right)$.

Any partition satisfying these properties can be interpreted as follows. If the obstructor sets $t(b) = 1$ for all $b \in B_1(C)$ and $t(b) = 2$ for all $b \in B_2(C)$, then the agenda setter passes all of the bills from $B_1(C)$ and uses whatever time she has leftover to pass bills from $B_2(C)$. This is because the worst pair of bills in $B_1(C)$ (from the perspective of the agenda setter) is still preferable to the best bill in $B_2(C)$ that does not make it onto the agenda.

Let $U^o(C) = \sum_{j=1}^{|B_1(C)|} u^0(b_{1,(j)}) + \sum_{j=1}^{\frac{T-|B_1(C)|}{2}} u^o(b_{2,(j)})$. By Step 2 of the agenda setter's best response algorithm, the agenda setter's best response to $t(b) = 1$ for $b \in B_1(C)$ and $t(b) = 2$ for $b \in B_2(C)$ is $A = B_1(C) \cup \{b_{2,(j)}\}_{j=1}^{\frac{T-|B_1(C)|}{2}}$, so $U^o(C)$ gives the utility to o for the agenda

induced by C .

Define $C(i) = \operatorname{argmax}_{C \in \mathcal{C}(i)} U^o(C)$, and $C^* = \operatorname{argmax}_{i=0, \dots, \frac{T}{2}} U^o(C(i))$. Then setting $t(b) = 1$ for $b \in B_1(C^*)$ and $t(b) = 2$ for $b \in B_2(C^*)$ yields the optimal t .

2.4.1 Definitions

An agenda A is politically feasible if A is the agenda setter's best response to some t .

Suppose $C \in \mathcal{C}(i)$ for some i . The agenda, A , is induced by C if A is the agenda setter's best response to $t(b) = 1$ for $b \in B_1(C)$ and $t(b) = 2$ for $b \in B_2(C)$.

2.4.2 Lemma: Minimal acquiescence

Claim: Suppose there exists a t such that A is the agenda setter's best response to t . Then there exists an obstruction schedule t' such that A is the agenda setter's best response to t' and $t'(b) = 2 \forall b \in B \setminus A$.

Proof: Suppose $t(b') = 1$ for some $b' \in B \setminus A$. Step 2 of the agenda setter's strategy implies that $u^s(b') < u^s(b)$ for all $b \in A$. Thus, a t' such that $t'(b) = t(b)$ for all $b \neq b'$ and $t'(b') = 2$ would yield the same agenda. This logic can be iterated until $t(b) = 2 \forall b \in B \setminus A$.

2.4.3 Lemma: Exhaustiveness

Claim: If A is politically feasible, then there is some element of $\bigcup_{i=0}^{\frac{T}{2}} \mathcal{C}(i)$ that induces A .

Proof: Suppose A is politically feasible. Let t be a schedule of obstruction that induces A as the agenda setter's best response with the property that $t(b) = 2 \forall b \in B \setminus A$. The fact

that A is politically feasible coupled with the minimal acquiescence lemma guarantees the existence of t .

Let C be a partition with $B_1(C) = \{b \in A : t(b) = 1\}$ and $B_2(C) = B \setminus B_1(C)$. Then $\sum_{b \in A} t(b) = T$ and $t(b) \in \{1, 2\} \forall b \in B \implies |B_1(C)|$ is odd if T is odd and even if T is even. Additionally, $|B_1(C)| \leq T$. Thus, C satisfies the first condition of $\mathcal{C}(i)$ for $i = \frac{|B_1(C)|}{2}$ if T is even and for $i = \frac{|B_1(C)|-1}{2}$ if T is odd.

Additionally, by Steps 1 and 2 of the agenda setter's best response function, $u^s(b') + u^s(b'') > u^s(b_{2, (\frac{T-|B_1(C)|}{2})})$ for b', b'' such that $t(b') = t(b'') = 1$, because the last two bills from $B_1(C)$ to enter A must have a payoff for s that exceeds all remaining bills in $B_2(C) \setminus A$. Thus, C satisfies the second condition, which implies $C \in \mathcal{C}(i)$, and A is induced by some element of $\cup_{i=0}^{\frac{T}{2}} \mathcal{C}(i)$.

2.4.4 Proof of optimality

Every politically feasible agenda is induced by some $C \in \cup_{i=0}^{\frac{T}{2}} \mathcal{C}(i)$. C^* yields the largest payoff for the obstructor in $\cup_{i=0}^{\frac{T}{2}} \mathcal{C}(i)$, so it is the obstructor's most preferred politically feasible agenda. By the definition of a politically feasible agenda, this implies that setting $t(b) = 1$ for $b \in B_1(C)$ and $t(b) = 2$ otherwise is the obstructor's optimal strategy.