

Theory Appendix for "Why Do Migrants Return to Poor Countries? Evidence from Philippine Migrants' Responses to Exchange Rate Shocks"

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This Theory Appendix presents the model of migration and household investment described in qualitative terms in Section 2 of the main paper. The model has basic similarities with Stark, et al (1997), Dustmann (2003), and Mesnard (2004). The model is closest to Mesnard (2004), with the primary differences being that I assume a minimum investment level instead of a sunk investment cost, and allow the period of migration to differ from the period of savings prior to enterprise investment.

Let each household have a planning horizon of T discrete periods, and have two working members. Household members supply labor inelastically: one unit is supplied per period by each member. Each household member can supply labor in the domestic labor market (at wage d). One household member has the option of working overseas in each period, at a wage $f > d$; the second household member is restricted to domestic labor. (We can imagine that at least one spouse must stay at home to care for children, or that regulations governing temporary contract work overseas prohibit migration of entire families.)

Households also have the option of investing an amount I in a household enterprise, in which case one (and only one) household member can choose to work in the enterprise and generate profits rI per period. If the household member works in the enterprise, she may not provide wage labor, either domestically or overseas.¹ In addition, there is a minimum investment threshold m ,

¹Because only one person can work in the household enterprise, a household member returning from overseas after the enterprise has been established works for domestic wage labor while the other household member works in the enterprise. In this case, per-period household earnings will be $d + rI$.

below which an investment may not be made (it must be the case that $I \geq m$).² Further, let per period profits from entrepreneurship always exceed the domestic wage, even at the minimum investment level ($rm > d$). To keep the analysis tractable, impose the condition that the capital invested in the enterprise, I , may not be subsequently raised. (Say there are very high capital adjustment costs.)

A crucial assumption is that credit markets for uncollateralized loans do not exist, so households must save the amount of desired enterprise capital before investing. But allow *collateralized* lending, so that households can consume the capital invested in the enterprise. In other words, households are allowed to take out a loan once the enterprise has been established, for the amount I . For simplicity, assume that the interest rate on collateralized loans and the depreciation rate of invested capital are zero. So repayment of the collateralized enterprise loan simply means the enterprise is turned over to the lender at the end of the last period.³ Households may save (transfer income from the current period to future periods) at a zero interest rate. Households start with zero savings, and save from both domestic and foreign income sources.

Households maximize utility over the planning horizon subject to within-period budget constraints and the prohibition against uncollateralized borrowing. Household utility is additively separable across the T discrete time periods. Utility in period t is a strictly concave function of household consumption C_t (utility is $U(C_t)$, with $U' > 0$ and $U'' < 0$). Normalize the price of consumption to unity, and let the household time discount rate be zero.

A simple way to generate a desire for migrants to return to the home country is for consumption overseas to yield less utility than consumption at home (as first proposed by Hill (1987)). I make the simplifying assumption that consumption overseas yields *zero* household utility: overseas work is a pure hardship, and is done exclusively for benefit of future raised consumption in the home country.⁴ In addition to consumption on the part of the overseas worker yielding zero household utility, overseas work by a household member also exacts a cost on the household as a whole. We can imagine this stemming from disutility due to family separation. So let utility in periods when

²This latter condition is reasonable: most investments are likely to be lumpy in this sense. For example, if the household wishes to provide taxi or bus services, there is a minimum cost to purchase a vehicle.

³In other words, a household may not take out a loan *before* starting the enterprise. Simply imagine monitoring problems in the time between provision of the loan and actual establishment of the enterprise, during which the household could abscond with the funds and establish the enterprise in another location (unknown to the lender). But once the enterprise has been established (and physical assets are identifiable), the lender can establish a legal right to the enterprise's assets at the end of the last period.

⁴We can simply think of migrants needing some subsistence level of consumption overseas, that costs a certain amount c . Then think of the foreign wage f as 'disposable foreign income', or total foreign wages net of the amount spent for overseas subsistence, c . This allows me to simply refer to the foreign wage f from now on.

a member is overseas be multiplied by a factor $0 < \gamma < 1$.

1 Describing the household's decision problem

Consider distinct periods a and b , where $b > a$ (b comes after a). Let the non-migrant's earnings be w_a in period a , and w_b in period b . Consider the choice between having the member work overseas in either period a or period b , and domestically in the other period.

Lemma 1 *Let $w_b \geq w_a$ (the income of the domestic household member either stays constant or rises over time). If a household has the choice of supplying labor overseas in either period a or b (but not both), utility is maximized when overseas work occurs in the earlier of the two periods (period a).*

Proof. *When overseas work occurs in period b , utility is $U(d + w_a) + \gamma U(f + w_b)$. When overseas work occurs in period a , utility across the two periods is $\gamma U(f + w_a) + U(d + w_b)$. Proof requires showing that $\gamma U(f + w_a) + U(d + w_b) \geq U(d + w_a) + \gamma U(f + w_b)$. Because of diminishing marginal utility of consumption, because $f > d$, and because $w_b \geq w_a$, it must be true that*

$$U(d + w_b) - U(d + w_a) \geq U(f + w_b) - U(f + w_a).$$

Because $0 < \gamma < 1$, it must also be true that

$$U(d + w_b) - U(d + w_a) \geq \gamma(U(f + w_b) - U(f + w_a)).$$

Expanding and rearranging obtains the required condition:

$$\gamma U(f + w_a) + U(d + w_b) \geq U(d + w_a) + \gamma U(f + w_b).$$

■

It should be clear that because entrepreneurial profits are always larger than domestic wages, a household member will never return to domestic wage labor if investment in the household enterprise has already occurred. So the income of a domestic household member does in fact stay constant or rise over time. This fact, combined with Lemma 1, implies that whenever the household chooses to supply any labor overseas, it must be optimal for every period with migration to precede every period without migration. In other words, there will be a single migration interval

starting at the first time period, and the household migrant either returns home once or not at all.

Let the number of periods of overseas labor supply be denoted t_m , and let the number of periods of household saving prior to investment in the enterprise be denoted t_s . In other words, t_m is the last period overseas, and in period $t_m + 1$ the formerly overseas member works domestically; similarly, t_s is the last period of saving, and in $t_s + 1$ is the first period in which the enterprise generates profits. Let the convention be that if $t_m = 0$ indicates the household never supplies labor overseas, $t_s = 0$ means the household invests at the very beginning and earns profits starting in period 1, $t_m = T$ means the household supplies labor overseas for all periods, and $t_s = T$ means the household never invests in the enterprise.

The household's choice of t_m and t_s divides the household's planning horizon into three **intervals** (some of which may collapse to zero length), defined as follows:⁵

1. The *first interval*, from period 1 to $\min[t_m, t_s]$: one household member is overseas and one is at home, and per-period household earnings are $f + d$.
2. The *second interval*, from period $\min[t_m, t_s] + 1$ to $\max[t_m, t_s]$, when there are two possibilities for household earnings:
 - a) If $t_m > t_s$ (return migration follows investment), per-period household earnings are $f + rI$.
 - b) If $t_s > t_m$ (investment follows return migration), per-period household earnings are $2d$.
3. The *third interval*, from period $\max[t_m, t_s] + 1$ to period T : both household members are at home, and per-period household earnings are $d + rI$.

In the second interval, case a), the household also has at its disposal the amount of the collateralized enterprise loan I , which it also can either consume or save. In the third interval, it has at its disposal any savings carried over from the second interval and any remaining amount of the collateralized enterprise loan.

The amount invested in the enterprise is exactly the amount of savings accumulated by the end of period t_s . Because investment profits rise in the investment, and uninvested savings do not earn interest, it is never optimal to invest less than total accumulated assets once the household decides to invest.

In general, households may choose to save from earlier periods to consume or invest in later periods, but cannot transfer resources from later to earlier periods due to the borrowing constraint.

⁵Let the second interval be nonexistent when $t_m = t_s$.

Now consider two periods a and b , that are each within the same defined interval.

Lemma 2 *Utility maximization requires consumption to be the same in any two periods that are within the same ‘interval’.*

Proof. Periods a and b are in the same interval, and so the within-period utility functions are identical in periods a and b . Because of diminishing marginal utility of consumption, maximization of utility therefore requires that consumption in periods a and b also be identical. ■

Lemma 2 suggests that a useful way to express household consumption in a particular period within interval i is as follows. First, ‘spread’ the value of assets accumulated by the end of the previous interval across all periods within the current interval, and add this to each period’s earnings within the interval to create a measure of household per-period ‘resources’ within the interval, R_i . For example, in the second period, case a), we have

$$R_2 = f + rI + \frac{I}{t_m - t_s},$$

where the first two terms on the right-hand-side are household per-period earnings and the third term is the value of the collateralized enterprise loan spread over the number of periods in the interval.

Second, express household consumption C_i in each period within the interval as the interval’s per-period resources R_i multiplied by one minus the savings rate within the interval, s_i :

$$C_i = (1 - s_i) R_i$$

So the household’s optimization problem involves deciding on a savings rate out of each period’s resources that is the same across all periods within the same interval. It should also be clear that savings will be zero in the third interval ($s_3 = 0$), because there are no subsequent intervals after period T .

To summarize, the household’s decision problem simply involves choosing the following to maximize household utility:

1. the number of periods of overseas work, t_m ,
2. the number of periods of saving for investment, t_s ,
3. the savings rate in the first interval, s_1 , and
4. the savings rate in the second interval, s_2 .

Let the utility-maximizing values of the household's choice variables be denoted t_m^* , t_s^* , s_1^* , and s_2^* .

2 Utility functions for given t_m , t_s , s_1 , and s_2

Expressions for household utility when the choice variables take on the (not necessarily optimal) values \widetilde{t}_m , \widetilde{t}_s , \widetilde{s}_1 , and \widetilde{s}_2 are as follows.

If the savings for investment ends before the migrant returns home ($0 \leq \widetilde{t}_s \leq \widetilde{t}_m \leq T$) so that case a) of interval 2 applies, utility is:

$$\begin{aligned} & \widetilde{t}_s \gamma U((1 - \widetilde{s}_1)[f + d]) + (\widetilde{t}_m - \widetilde{t}_s) \gamma U\left((1 - \widetilde{s}_2)\left[f + r\widetilde{I} + \frac{\widetilde{I}}{\widetilde{t}_m - \widetilde{t}_s}\right]\right) \\ & + (T - \widetilde{t}_m) U\left(d + r\widetilde{I} + \frac{\widetilde{s}_2\left[f + r\widetilde{I} + \frac{\widetilde{I}}{\widetilde{t}_m - \widetilde{t}_s}\right](\widetilde{t}_m - \widetilde{t}_s)}{T - \widetilde{t}_m}\right) \end{aligned}$$

where $\widetilde{I} = \widetilde{s}_1[f + d]\widetilde{t}_s$.

If, on the other hand, savings for investment ends after the migrant returns home ($0 \leq \widetilde{t}_m \leq \widetilde{t}_s \leq T$) so that case b) of interval 2 applies, utility is:

$$\begin{aligned} & \widetilde{t}_m \gamma U((1 - \widetilde{s}_1)[f + d]) + (\widetilde{t}_s - \widetilde{t}_m) U\left((1 - \widetilde{s}_2)\left(2d + \frac{\widetilde{s}_1[f + d]\widetilde{t}_m}{\widetilde{t}_s - \widetilde{t}_m}\right)\right) \\ & + (T - \widetilde{t}_s) U\left(d + r\widetilde{I} + \frac{\widetilde{I}}{T - \widetilde{t}_m}\right) \end{aligned}$$

where $\widetilde{I} = \widetilde{s}_2\left[2d + \frac{\widetilde{s}_1[f + d]\widetilde{t}_m}{\widetilde{t}_s - \widetilde{t}_m}\right](\widetilde{t}_s - \widetilde{t}_m)$.

3 A numerical solution

Further results rely on assuming a specific utility function and finding numerical solutions for given parameter values. Let utility in each period j be given by the power function $U(C_j) = C_j^\alpha$ (where $0 < \alpha < 1$). Set the number of periods, T , at 20.

The household chooses among integer values of t_s and t_m in the range $\{0, 1, \dots, 20\}$. In the first and second intervals, the household chooses savings rates s_1 and s_2 from a grid-space of savings rates $\{0, 0.01, 0.02, 0.03, \dots, 0.98, 0.99, 1\}$. 21 possible choices each of t_s and t_m and 101 possible

choices each of s_1 and s_2 yield roughly 4.5 million potential combinations of choice variables for a given set of parameter values. From these possible combinations, the household chooses the combination of t_s , t_m , s_1 , and s_2 values that maximizes household utility.

Let the model parameters take on the following values: $\alpha = 0.5$, $\gamma = 0.75$, $d = 1$, $r = 0.05$, and $m = 40$. The specific assumptions for the first four parameters are not highly crucial and do not make qualitative differences in the results to follow. What is key for the theoretical results is that the minimum investment level (m) be large enough. $m = 40$ is a large but reasonable minimum investment level. Assume that migrant families have a planning horizon of 5 years, so that $T = 20$ implies that each period is a 0.25 years.⁶ With minimum investment $m = 40$, a household where two members each work for domestic wages of 1 unit per period will take 20 periods to accumulate the minimum investment level. A typical investment is a jeepney (small passenger bus), that costs 400,000 pesos. In the data, median household income among households *without* migrants is 36,000 pesos per half-year, or 18,000 pesos per quarter. For such a household, an investment of 400,000 pesos is equivalent to 22.2 periods' earnings.

The main theoretical analysis examines how household migration and investment decisions depend on the foreign wage, f .

3.1 Three types of households

As the level of the foreign wage (f) varies, it turns out that households can be divided three distinct groups in terms of their periods overseas, their investment decisions, and the number of periods until investment (if investing at all).

The first type of household is one with a high level of the foreign wage, so that earnings are high enough for entrepreneurial investment to occur at a relatively early period. In these households, investment in the household enterprise can occur before the migrant returns from overseas, after which migrants continue to accumulate savings that are simply intended to raise future consumption levels. Such a household's optimal consumption and savings over the planning horizon is illustrated in Appendix Figure 1a (for $f = 6$). The dark solid line depicts the household's consumption level over time, while the light dotted line depicts its savings rate. In the first interval (periods 1 to 11) one household member works overseas while the other works for domestic wages. Consumption and savings are therefore constant during these periods (Lemma

⁶This is reasonable, as stays overseas tend to be fairly short. 84% of migrants away in June 1997 had been overseas for less than 4 years.

2). The first shift in the consumption and savings levels occurs when the household optimally chooses to invest its entire accumulated savings in the household enterprise at the end of period 11 (so that the enterprise first generates profits in period 12, the beginning of the second interval). In the second interval the domestic household member shifts to working in the household enterprise. The household continues to supply labor overseas, accumulating savings for future consumption. Higher resulting domestic earnings, combined with the funds from the collateralized enterprise loan allows the household to raise both its consumption and savings levels. The third interval begins in period 15 (the last period of overseas work was period 14). Savings drops to zero, and in each remaining period the household simply consumes its domestic earnings plus an evenly-distributed portion of its accumulated assets.

Because the investment level ($I = 42.35$) of this first type of household is somewhat above the minimum investment threshold ($m = 40$), it is apparent that these households are not bound by the minimum investment threshold in making their investment decisions. So this first type of household is termed an *unconstrained investor*.

A second group of households has a somewhat lower level of the foreign wage; an example of such a household is depicted in Appendix Figure 1b (for $f = 3.5$). For such households, there is only a first interval (a period of overseas work and savings) and a third interval (where enterprise investment has occurred and the overseas worker has returned home); investment in the household enterprise and the return of the overseas worker are simultaneous (both occurring at the end of period 12), so that the second interval is nonexistent. In this example, household invested capital is 40.5, only slightly higher than the minimum investment threshold.⁷ If there had been no minimum investment threshold (or if it had been somewhat lower), the household would have preferred to invest a lower amount, and would have ceased supplying labor overseas earlier. But the requirement to invest at least m leads the household to supply labor overseas only until it has saved the minimum investment threshold, after which migrants return immediately and the household simultaneously invests. Because these households supply labor overseas only until they have achieved the investment threshold, these households are termed *target-earners* (as in Piore (1979)).

A third group of households has the lowest level of the foreign wage, and an example of such a household is in Appendix Figure 1c (for $f = 2$). These households would take relatively long (and too many periods overseas) to achieve the minimum investment level, allowing too few periods at

⁷Raising the fine-ness of the numerical simulation's grid-spaces can bring the investment level for such a household arbitrarily close to 40.

the end to enjoy the returns from the investment. So they choose not to invest in an enterprise at all. The household simply supplies labor overseas to save for future consumption in the first interval (until the end of period 3 in this example), and each subsequent period it consumes two members' domestic wages plus a portion of accumulated savings from the first interval. I simply term this group of households *non-investors*.

An alternative view of the three groups of households (for a range of values of the foreign wage f) is provided by Appendix Figure 2.⁸ The figure depicts optimal periods overseas t_m^* (the solid line) and optimal periods prior to enterprise investment t_s^* (the dotted line), for a range of values of the foreign wage. Up to a value of f slightly less than 2, households prefer not to supply labor overseas at all (the foreign wage is too low; recall the domestic wage d is 1). At higher foreign wages, optimal periods overseas rise in the foreign wage (until f is slightly above 3). These households reach the last period without having invested ($t_s^* = 20$) indicating they are *non-investors*.

Continuing to higher foreign wages, the solid line dips downward and flattens out for a range (up to between 4 and 5). For these households, return from overseas is simultaneous with enterprise investment (the solid and dotted lines coincide), identifying them as *target-earners*.

At even higher foreign wages, the solid line rises, while the dotted line falls. These households are investing prior to return migration; these households are *unconstrained investors*.

3.2 Impact of exchange rate shock

The empirical analysis examines the impact of exchange rate shocks on return migration and on investment decisions in migrants' source households, so here it is useful to examine the theoretical impact of such shocks. The model predicts that the impact of such shocks varies according to a household's foreign wage, and will contrast starkly with the predictions made by a model with relaxed borrowing constraints.

What exactly is an exchange rate shock in this setting? Denote a household's accumulated savings from foreign earnings at the start of any period j (assumed to be held overseas until

⁸To produce this and all subsequent graphs in the Theory Appendix, utility-maximizing values of the choice variables were found for each discrete value of f in the grid-space [1, 1.125, 1.25, 1.375, ..., 7.75, 7.875, 8]. The range of foreign wages considered is reasonable. For example, domestic servants in Manila typically earn no more than 2,500 per month. By contrast, an anecdotal sampling of typical salaries for domestic servants in foreign countries reported by one Manila recruitment agency range from 10,000 pesos per month in Singapore, Malaysia, and the United Arab Emirates, to 23,400 pesos per month in Hong Kong (a range of 4 to more than 10 times the corresponding Philippine wage.) (Figures acquired in a personal visit by the author in the summer of 2002. At that time, the Philippine peso was trading at roughly 50 pesos to the US dollar.)

the migrant's return) as A_j . Let all monetary variables (f , d , m , I , and A_j) be denominated in households' domestic currency. Now let f and A_j be the exchange rate E (units of domestic currency that can be purchased with every unit of foreign currency) multiplied by these variables denominated in foreign currency (\tilde{f} and \tilde{A}_j respectively):

$$\begin{aligned} f &= E\tilde{f} \\ A_j &= E\tilde{A}_j \end{aligned}$$

An exchange rate shock is simply a change in the exchange rate (ΔE). As such, it changes the domestic currency value of both the foreign wage and any accumulated savings from foreign earnings. Assume exchange rate shocks are *permanent* changes in the exchange rate, and are known to be so by households.

What assets are held overseas? Assume that when households save, they draw equally across all income sources (foreign wages, domestic wages, and the current period's planned drawdown of the collateralized enterprise loan). Let savings from domestic sources (domestic wages and the collateralized enterprise loan) be held domestically, while foreign savings are held overseas until the migrant returns home.

To examine the impact of an exchange rate shock, the exact timing of events needs to be specified. Consider a given period j , when a household starts with a member working overseas. Let the order of events within period j be as follows:

1. The household observes the exchange rate shock (if any).
2. The household supplies labor in the previously-planned locations (one overseas, one domestic). (Locations of labor supply may not be modified within the same period as an exchange rate shock.)
3. The household saves and consumes. (The savings rate may be modified in response to the exchange rate shock.)
4. The household decides where the overseas member will work (overseas or domestically) for period $j + 1$.
5. If the household has not yet established a household enterprise, the household decides whether or not to establish it (invest), so that profits can be earned in period $j + 1$ and after. If so, all accumulated savings overseas (A_j) are transferred to the home country and invested (in combination with domestic savings).
6. The household takes out the collateralized enterprise loan.

7. Period $j + 1$ begins.

Consider subjecting a subset of households to an exchange rate shock amounting to a 50% increase in the exchange rate ($\frac{\Delta E}{E} = 0.5$). How does this change overseas workers' return decisions? Because the exchange rate shock should affect households differently depending on their elapsed number of periods, some assumption regarding the distribution of households across periods is necessary; simply assume that households are uniformly distributed across periods (within each foreign wage level).

First consider households that experience no change in their exchange rate. The solid line in Appendix Figure 3a represents their 1-period return rate: the fraction of households with a member overseas at the start of a given period j whose migrant returns home at the end of that period. Because households are assumed uniformly distributed across periods, this return rate is simply $\frac{1}{t_m^*}$, the inverse of the optimal number of periods overseas. The return rate is positive for all values of the foreign wage, and naturally is a mirror image of the solid line in Appendix Figure 2: first falling, moving slightly upwards to a temporary plateau, and then falling again in the foreign wage.

The dotted line in the figure is the 1-period return rate for households that *do* experience an exchange rate shock, and it is starkly different from the solid line. Only migrants with intermediate values of the foreign wage return at all at the end of the shock period, and their return rates are substantially higher than those in households without a shock. For all other households, the return rate is zero.

Appendix Figure 3b displays the difference between the return rates of the unshocked and shocked households (the shocked return rate minus the unshocked return rate, for each value of the foreign wage). For households with the lowest and highest values of the foreign wage, the 1-period return rate is lower for shocked vs. unshocked households. By contrast, for households with intermediate values of the foreign wage, the return rate for shocked households is either higher than or the same as the return rate for unshocked households.

The exchange rate shock apparently has opposite effects on return rates for two groups of households: on the one hand, households with intermediate foreign wages, and, on the other, households with either the lowest or highest foreign wages. The explanation becomes clearer when we also examine the impact of the exchange rate shock on household investment.

Define the '1-period investment rate' as the fraction of households with a migrant overseas who make an enterprise investment at the end of the period of the exchange rate shock (so that

an enterprise begins generating profits in the subsequent period).⁹ Appendix Figure 4a depicts the 1-period investment rate for households without (the solid line) and with (the dotted line) an exchange rate shock, and Appendix Figure 4b shows the difference in the investment rate between shocked and unshocked households.¹⁰

In households with the lowest foreign wages, the investment rate is zero for both shocked and unshocked households. These are households who in the unshocked case are ‘non-investors’. When experiencing an exchange rate shock, households in this group either remain non-investors (but are encouraged by the higher foreign wages to extend their overseas stays), or decide to become target-earners (and must stay overseas for longer to save for investment). So no migrants from these households return at the end of the period (the return rate goes to zero). The exchange rate shock also has no effect on investment at the end of the period, either: households are either still non-investors, or, if they have decided to be target-earners, they must accumulate assets for somewhat longer before investing.

Households with intermediate foreign wages have the highest increase in the investment rate. These households are target-earners, who remain overseas only until they have saved at least the minimum investment threshold m . The exchange rate shock, by raising (in domestic currency terms) both the current period’s foreign wage and the accumulated overseas savings, suddenly allows some fraction of these households to exceed the minimum investment threshold in the current period. Thus an exchange rate shock leads to the largest increase in both the return rate and the investment rate for these households.

Households with the highest foreign wages were unconstrained investors prior to the shock. Their return rate falls as they decide on the margin to extend their overseas stays to take advantage of higher foreign wages. The investment rate rises because of the windfall increase in assets, but not by as much as the increase for households with intermediate foreign wages: some fraction of unconstrained investors had *already* invested prior to the shock, and so could not invest again; by contrast, *all* the target-earners were postponing investment until return, and so all had the option to invest sooner in response to the shock.

⁹I restrict attention to households with migrants overseas because the exchange rate shock has no impact on households not supplying labor overseas.

¹⁰The jaggedness of the dotted line derives from the discreteness of the grid-spaces used in the numerical simulation, particularly the restriction that households choose among integer values for t_m and t_s . Substantially finer grid-spaces would eliminate these jags.

3.3 Relaxing the credit constraint

To illustrate the importance of the prohibition on non-collateralized borrowing in generating the theoretical results so far, it is useful to consider the impact of an exchange rate shock in a situation where this borrowing constraint is relaxed somewhat. The non-collateralized borrowing constraint was justified earlier by supposing that lenders could not prevent households from absconding with loans before the funds were invested in the enterprise. Now, instead assume that lenders *are* able to prevent such default.

This allows households to borrow and invest in an enterprise at very beginning, so that one household member earns enterprise profits of rI (instead of the domestic wage d) in all periods. (The other member's options remain overseas work at wage f , or domestic work at wage d .)

Without formally modeling the credit market, the ceiling on how much a given household can borrow is arbitrary. Assume simply that a household's credit ceiling is the amount they would have invested at the original (pre-shock) exchange rate when non-collateralized borrowing was prohibited (analyzed in the previous subsections), with the exception that non-investors (who would have invested zero) are allowed to borrow the minimum investment threshold, m . Retain the assumption that the rate of interest and the rate of depreciation of invested capital are zero, so again repayment of the loan simply means turning the enterprise over to the lender at the end of the last period. Assume that at the minimum credit ceiling per-period enterprise profits exceed the domestic wage, so that all households then borrow and invest their credit ceiling at the very beginning.

Maintaining all other assumptions from the previous subsections, Appendix Figures 5a, 5b and 5c illustrate optimal choices for households allowed such non-collateralized borrowing. Appendix Figure 5a shows that the optimal number of periods overseas rises continuously in the foreign wage. There are no target-earners to generate kinks in this curve, unlike in the case depicted in Appendix Figure 2. (Because all households invest at the very beginning, Appendix Figure 5a shows no curve for optimal periods prior to investment. For the same reason, the investment rate is not meaningful.)

Appendix Figure 5b depicts the 1-period return rate for households that do (dotted line) and do not (solid line) experience an exchange rate shock. The return rate of unshocked households declines continuously in the foreign wage. For shocked households, the return rate in the period of the shock is zero for households below a certain foreign wage (around 4.5); such households have reoptimized and extended their desired periods of overseas work, and so none return right after

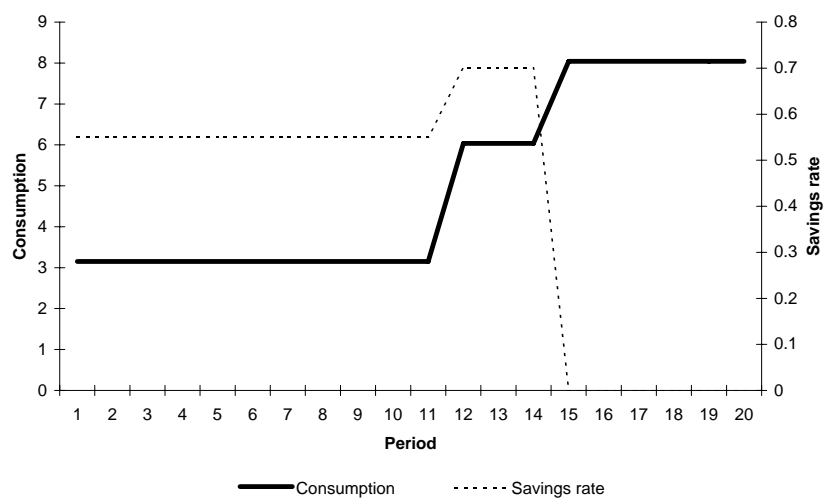
the shock. For shocked households with higher foreign wages, the 1-period return rate coincides with the rate for unshocked households. The fact that periods overseas is unchanged for these households reflects the fact that households with higher wages have on average accumulated more overseas savings at any point in time, and so experience a larger increase in wealth when the exchange rate shock occurs. An increase in wealth raises the desirability of return migration (an income effect), which for these households is large enough to offset the substitution effect of the increase in foreign wages. On net, then, the return rate is unchanged for these households.¹¹

All told, then, the change in the 1-period return rate due to the shock declines in the foreign wage (Appendix Figure 5c) in a model with a relaxed borrowing constraint.

¹¹In the model with the non-collateralized borrowing constraint, the wealth increase is in general not large enough to offset the substitution effect of the increase in foreign earnings because high-foreign-wage households hold less in overseas savings on average (many have already invested, and so a large fraction of their assets have been transferred to home country and are not affected by the exchange rate shock).

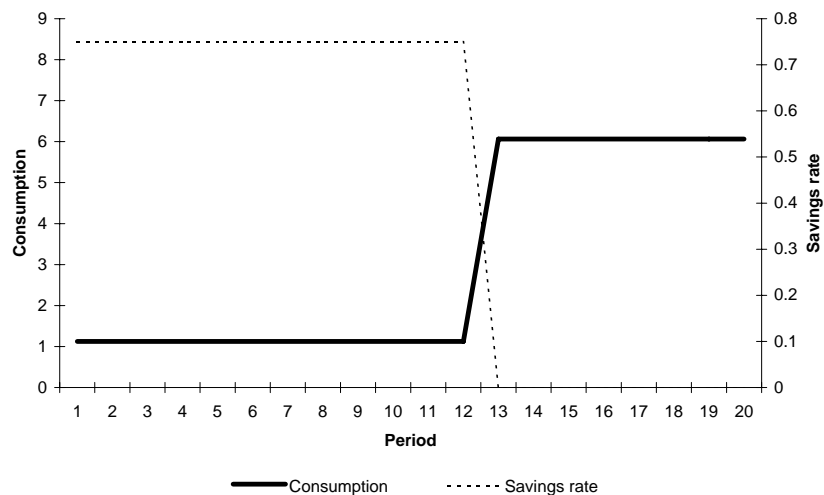
Appendix Figures 1a, 1b, and 1c: Optimal consumption and savings over the life-cycle

1a: Unconstrained investor: $f=6$



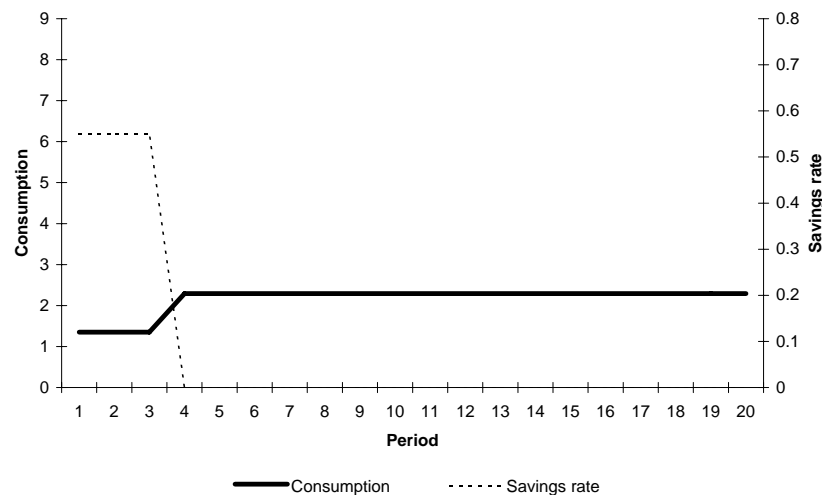
Optimal number of periods of saving before investment (t_s^*): 11
 Optimal number of periods working overseas (t_m^*): 14
 Investment in enterprise: 42.35

1b: Target-earner: $f=3.5$



Optimal number of periods of saving before investment (t_s^*): 12
 Optimal number of periods working overseas (t_m^*): 12
 Investment in enterprise: 40.5

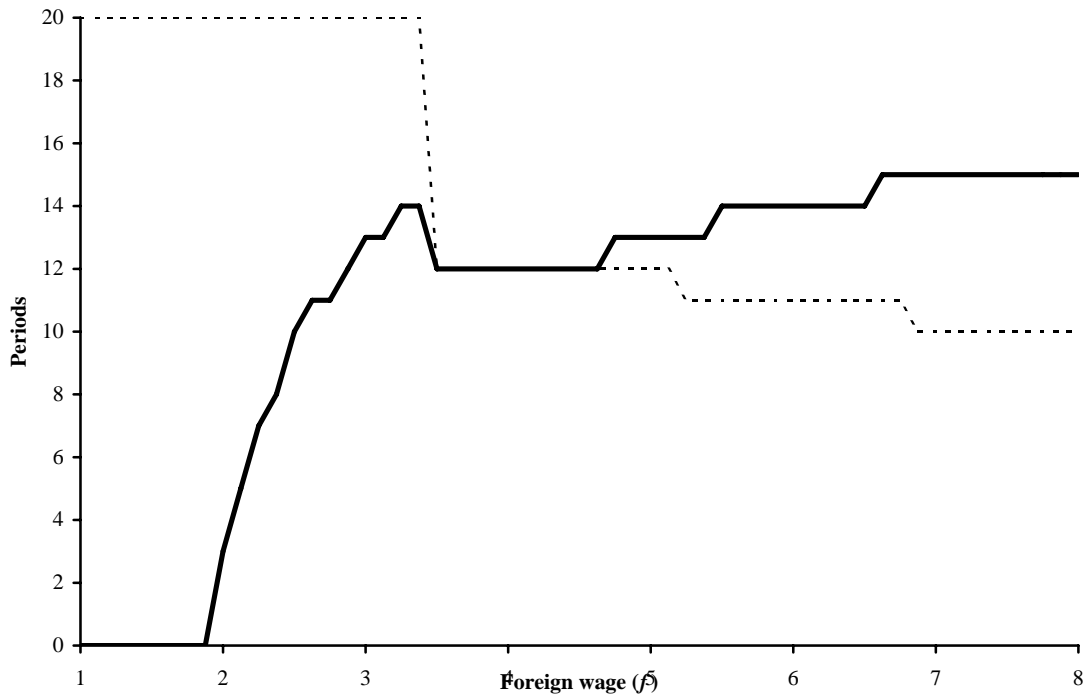
1c: Non-investor: $f=2$



Optimal number of periods of saving before investment (t_s^*): (No investment)
 Optimal number of periods working overseas (t_m^*): 3
 Investment in enterprise: (No investment)

NOTES: Optimal values of choice variables chosen from the following sets: t_s, t_m from integers in the range $\{0, 1, \dots, 20\}$; s_1, s_2 from discrete values in the range $\{0, 0.01, 0.02, \dots, 0.98, 0.99, 1\}$. Assumes within-period utility function is $U(C) = C^\alpha$. Parameter values assumed are: $\alpha=0.5, \gamma=0.75, d=1, T=20, m=40, r=0.05$.

Appendix Figure 2: Optimal periods before return migration and household investment by foreign wage level

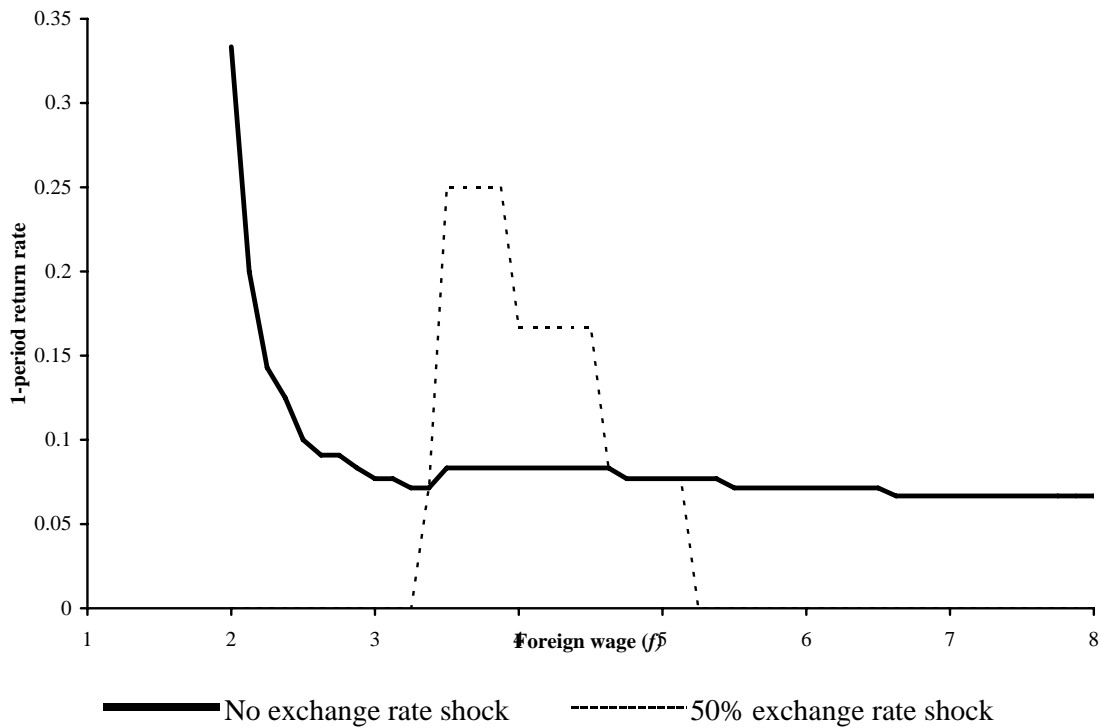


Legend:
 ————— Optimal periods overseas (t_m^*)
 - - - - - Optimal periods before investment (t_s^*)

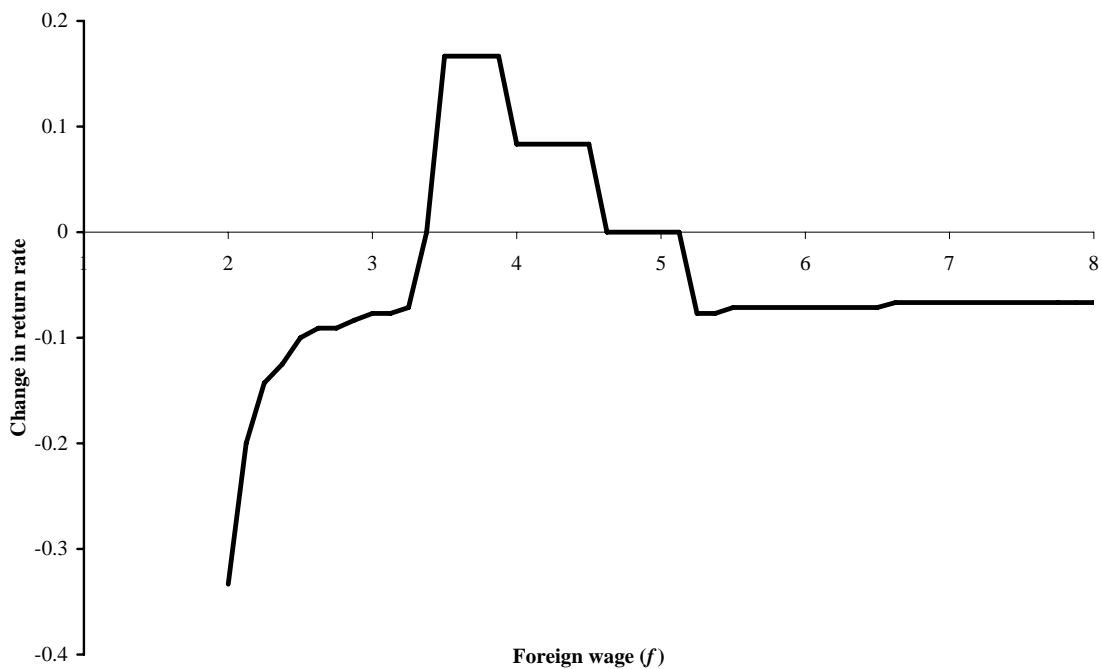
NOTES: Results from numerical simulation; see notes to previous appendix figure for details.

**Appendix Figures 3a and 3b: Theoretical impact of exchange rate shock
on return migration**

3a: 1-period return rate with and without exchange rate shock



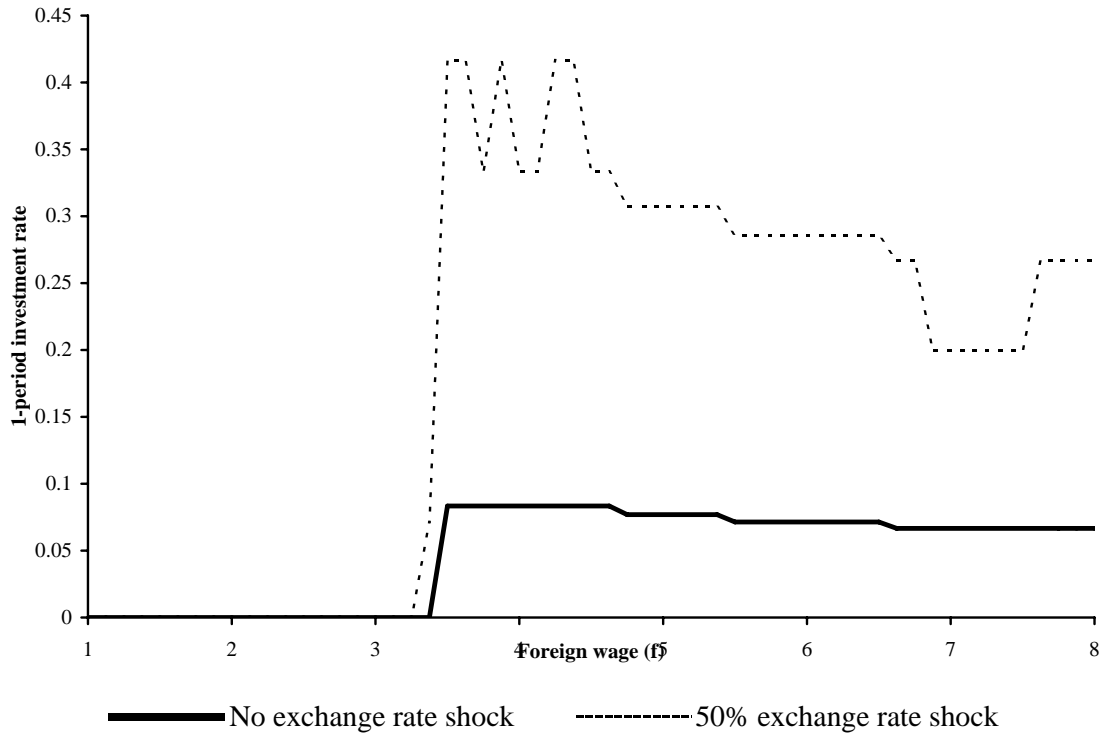
3b: Difference in return rate between households without and with exchange rate shock



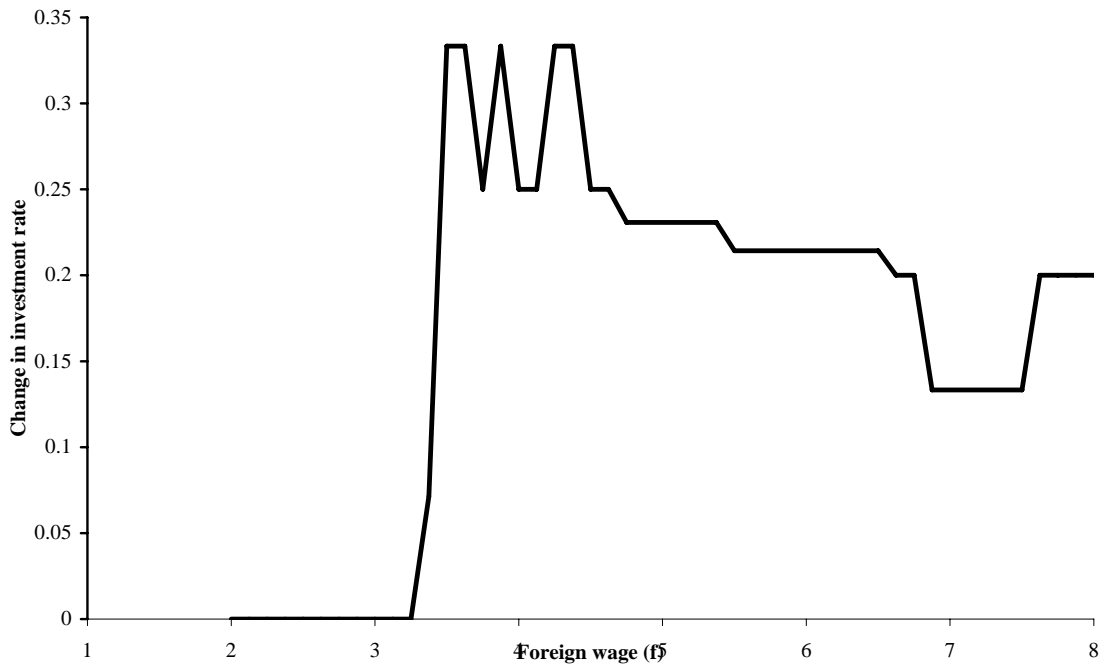
NOTES: Exchange rate shock raises both the foreign wage and accumulated assets held overseas by 50%. Shock assumed to occur at very beginning of a period (period j), with overseas work and enterprise investment decision assumed fixed in period j . "1-period return rate" is fraction of migrants overseas who return immediately after the period of the exchange rate shock (period j), so as to be working domestically in period $j+1$. For each level of the foreign wage, households assumed uniformly distributed across elapsed periods of life. See first appendix figure for other notes.

Appendix Figures 4a and 4b: Theoretical impact of exchange rate shock on household investment

4a: 1-period investment rate, with and without exchange rate shock



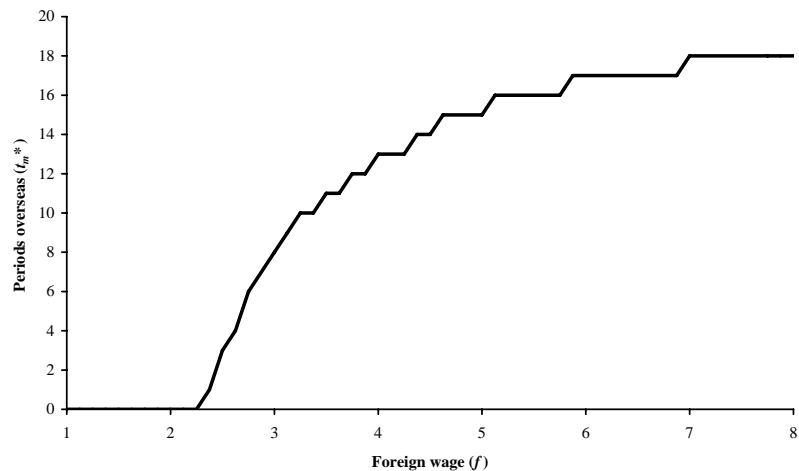
4b: Change in 1-period investment rate



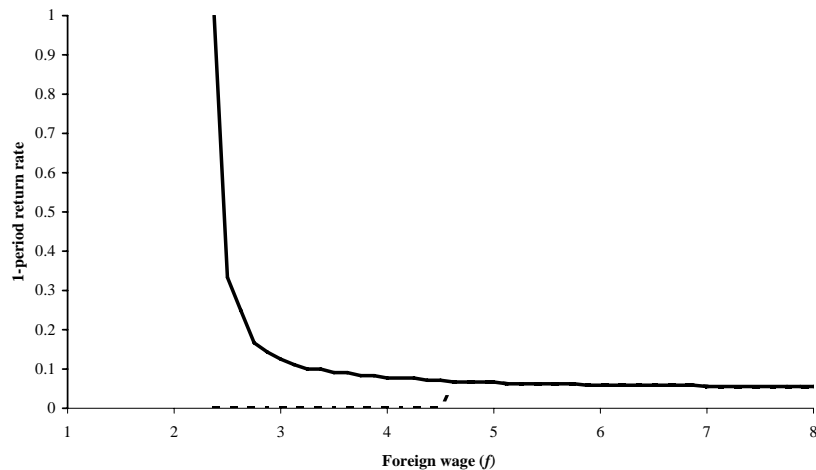
NOTES: Exchange rate shock raises both the foreign wage and accumulated assets held overseas by 50%. Shock assumed to occur at very beginning of a period (period j), with overseas work and enterprise investment decision assumed fixed in period j . "1-period investment rate" is fraction of households making enterprise investment immediately after the period of the exchange rate shock (period j), so enterprise is operating in period $j+1$. For each level of the foreign wage, households assumed uniformly distributed across elapsed periods of life. See first appendix figure for other notes.

Appendix Figures 5a, 5b, and 5c: Theoretical impact of exchange rate shock on return migration (borrowing constraint relaxed)

5a: Optimal periods overseas by foreign wage level (average across households)

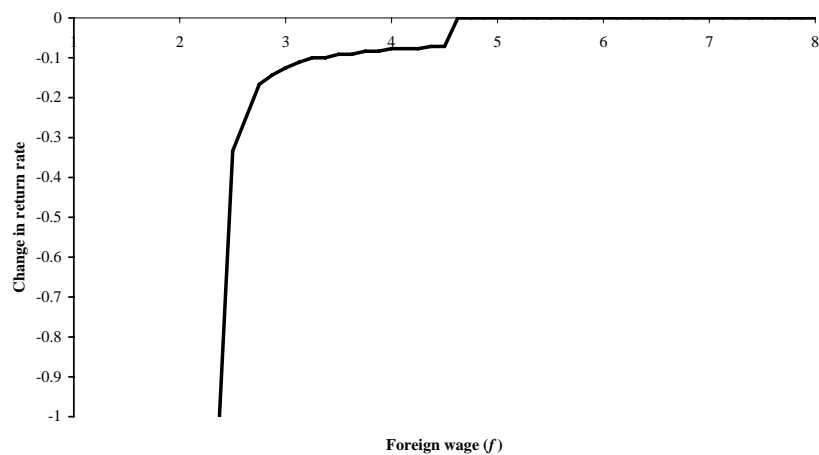


5b: 1-period return rate with and without exchange rate shock



— No exchange rate shock - - - 50% exchange rate shock

5c: Change in 1-period return rate



NOTES: Prior to period 1, households assumed able to borrow amount they would have invested in borrowing constraint case (Figure 4), or the minimum investment threshold (m) if a non-investor. Exchange rate shock raises both the foreign wage and accumulated assets held overseas by 50%. Shock assumed to occur at very beginning of a period (period j), with overseas work and enterprise investment decision assumed fixed in period j . "1-period return rate" is fraction of migrants overseas who return immediately after the period of the exchange rate shock (period j), so as to be working domestically in period $j+1$. For each level of the foreign wage, households assumed uniformly distributed across elapsed periods of life. See first appendix figure for other notes.