Flag Triangulations of Low-Dimensional Manifolds

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(Abstract) Simplicial Complex

Def: A *simplicial complex* Δ is a collection of subsets of the vertex set V of Δ, that is closed under taking subsets.

Elements of Δ are called faces.

Example: {$\emptyset$,{1},{2},{3},{1,2},{2,3},{1,3},{1,2,3}}

{$\emptyset$,{1},{2},{3},{1,2},{2,3},{1,3}}

Counter-example: {$\emptyset$,{1},{2},{3},{1,2},{2,3},{1,2,3}}

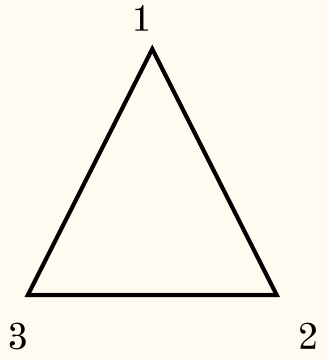


Image from triangle-wiki tionary

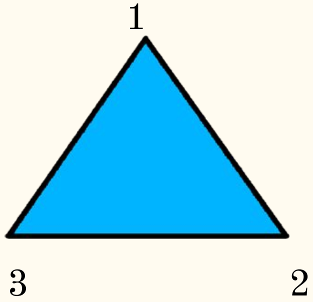
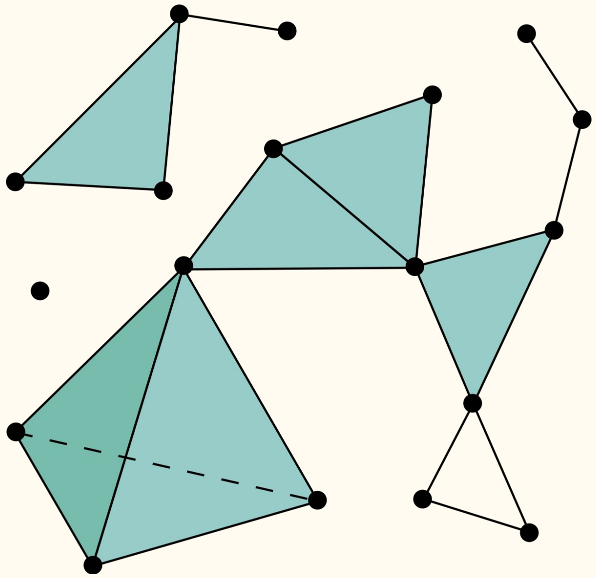
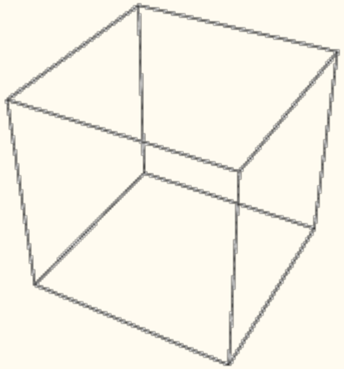


Image from kindersay triangle

Examples



This image is from wikipedia
Simplicial complex page



This image is from
mathworld.wolfram.com/Cube

Motivating example: Triangulation of the Torus

Topologically, a 2-torus is a closed surface defined as the product of two circles: $S^1 \times S^1$.

A *triangulation of a manifold M* is a simplicial complex Δ which is homeomorphic to M.

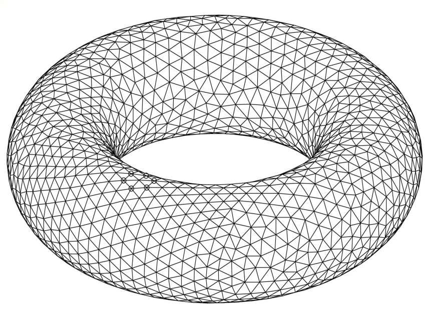
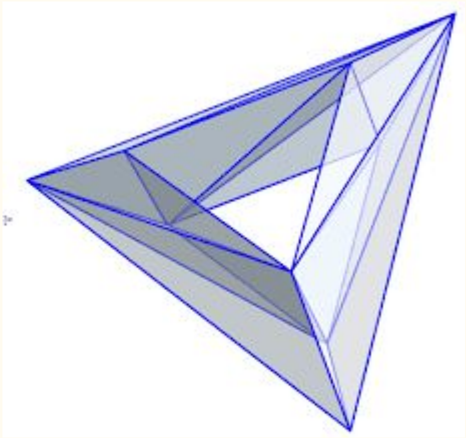
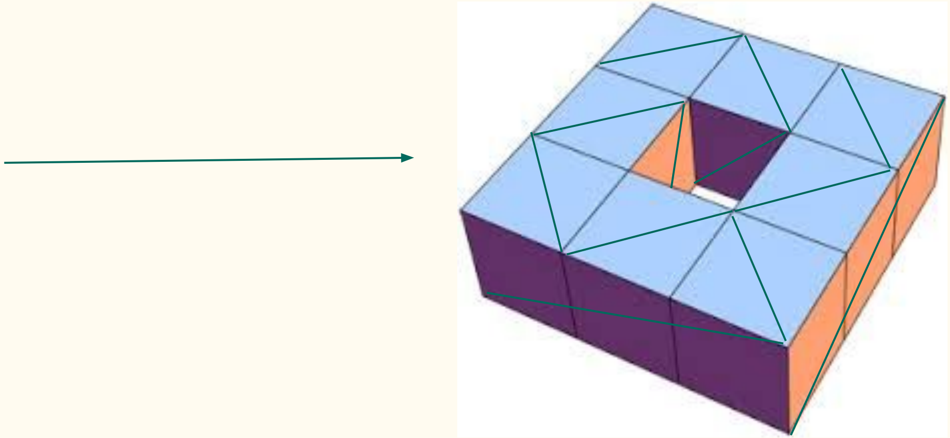


Image from Wikipedia (triangulation)

Flag! Triangulation Of A Torus

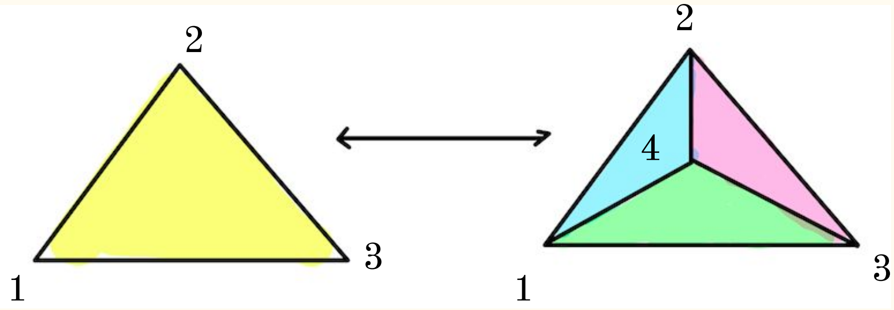
A simplicial complex is said to be flag if it has no empty simplices (for instance, triangles and tetrahedrons).



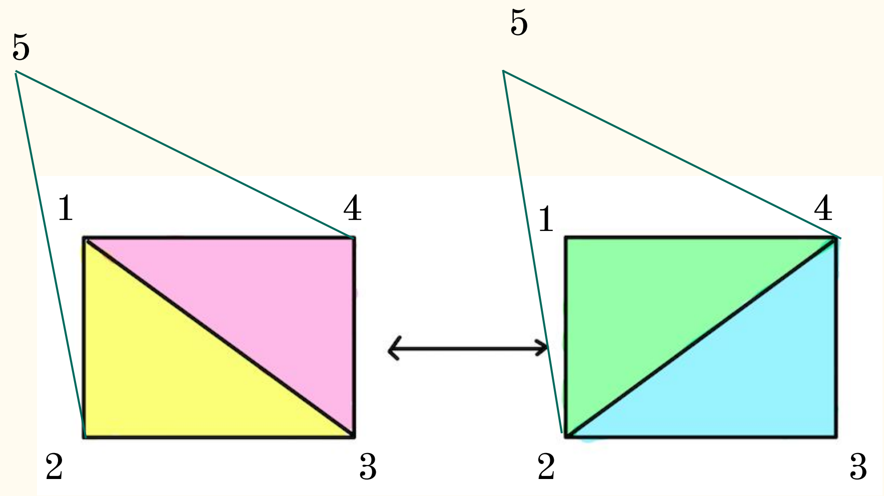


Bistellar flip

Pachner's theorem: Two closed combinatorial manifold are piecewise linear homeomorphic if and only if they are bistellar equivalent.

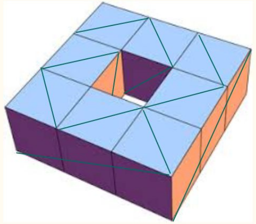


Flip 13 (31)



Flip 22

Final comment



Edge subdivision: For a simplicial complex Δ and its edge $\{u,v\}$, the edge subdivision deletes $\{u,v\}$ and add two edges $\{u,w\}$ and $\{w,v\}$ along with the new vertex w.

Nevo & Lutz theorem:

Two flag simplicial complexes are piecewise linearly homeomorphic if and only if they can be connected by a sequence of flag complexes, each obtained from the previous one by either an edge subdivision or its inverse.

Reference

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