Finding Real Polynomials
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Introduction
Rabbit Polynomial: The rabbit polynomial is $P_R(z)$ such that $P_R(z) = z^2 + k$ for $k \approx -0.12226 + 0.7449i$

Rabbit, corabbit, and airplane Special quadratic polynomials whose critical points $z = 0$ are 3 periodic.

The Julia sets of the rabbit, corabbit, and airplane respectively.

Key Definitions
- **Homeomorphisms** are continuous bijective maps from $\mathbb{C} \to \mathbb{C}$ with continuous inverses.
- **Mapping classes** are isotopy classes of homeomorphisms.

Theorem 1: (Dehn) Let $\phi$ be a mapping class. Then $\phi$ can be written as a finite product of Dehn twists.

Three marked points

Theorem 2
Assume $P_R$ is a polynomial with three marked points. A mapping class can be written uniquely as the product of two finite Dehn twists, $T_x, T_y$, and their inverses.

Lifting

$P_R = \text{Sq} \circ \text{Rot}$
A function $f$ lifts if there exists a function $\tilde{f}$ such that $P_R \circ \tilde{f} = f \circ P_R$

Goal
Make a program that finds proportion of mapping classes $f$ such that $f \circ P_R$ is equivalent to the rabbit, corabbit, or airplane

Program Methodology

Rules for Lifting
Given a mapping class written as the product of $T_x, T_y$, and their inverses,
- If the sum of exponents on $T_x$ are even, the mapping class lifts.
- $T_x^2$ lifts to $T_x^{-1}T_y^{-1}$
- $T_x$ lifts to $T_x^{-1}$
- $T_yT_xT_y^{-1}$ lifts to the identity

End conditions:
- $\text{Id}$: rabbit
- $T_x$: airplane
- $T_x^{-1}$: corabbit

$n > 3$ marked points

Program Results

Mapping Classes of Length 10
- Total Mapping Classes: 118,096
- Number of Rabbits: 25,095 / 21.25% 
- Number of Corabbits: 25,134 / 21.28%
- Number of Airplanes: 67,867 / 57.47%

Promising Directions
Using Trees to Classify Mapping Classes

Theorem 3: (Poirier) Every quadratic polynomial has a unique tree in $\mathbb{C}$ such that every marked point is a vertex, all leaves are marked points, unmarked vertices have valence $\geq 3$, and the preimage of the tree under the rabbit is homotopic to itself.

- Trees can similarly be used to identify the polynomial in question. We’ve learned that each branched cover that is equivalent to a polynomial (any of our Dehn-twisted rabbits) has a unique tree fixed when we take the pre-image.
- With this information, we can then determine that the trees associated with our twisting can also determine which polynomial will be lifted to.
- Why Trees? Trees give us a combinatorial method of understanding mapping classes. In other words, trees are easier to understand.

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