Finding Real Polynomials

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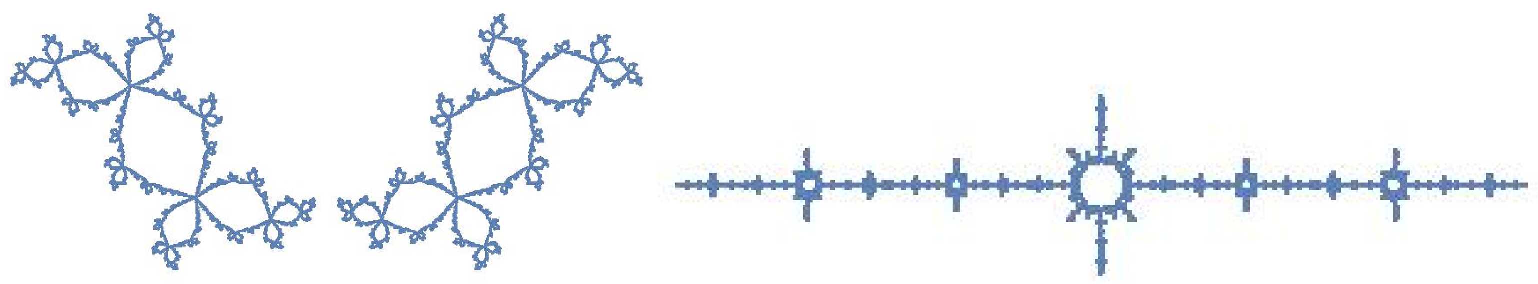
Introduction

Rabbit Polynomial: The rabbit polynomial is P_R(z) such that P_R(z) = z^2 + k for k ≈ -0.12226 + 0.7449i

Rabbit, corabbit, and airplane Special quadratic polynomials whose critical points z = 0 are 3 periodic.



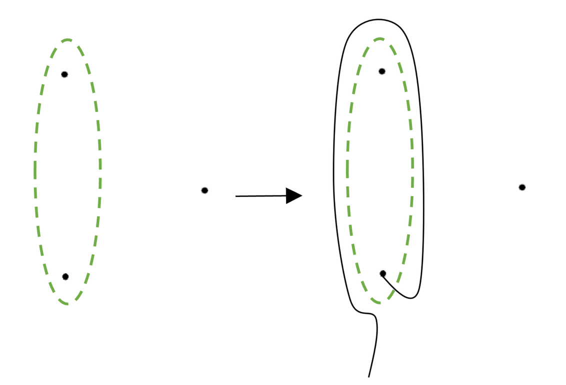
The Julia sets of the rabbit, corabbit, and airplane respectively.

Key Definitions

- **Homeomorphisms** are continuous bijective maps from $\mathbb{C} \rightarrow \mathbb{C}$ with continuous inverses.
- **Mapping classes** are isotopy classes of homeomorphisms.

Theorem 1: (Dehn)

Let ϕ be a mapping class. Then ϕ can be written as a finite product of Dehn twists.

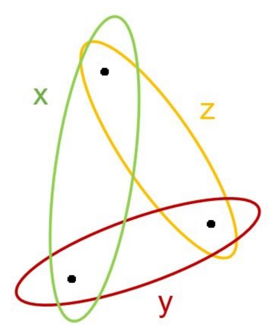


The Dehn twist around the curve x acting on an arc.

Three marked points

Theorem 2

Assume P_R is a polynomial with three marked points. A mapping classϕ can be written uniquely as the product of two finite Dehn twists, T$_x$, T$_y$, and their inverses.

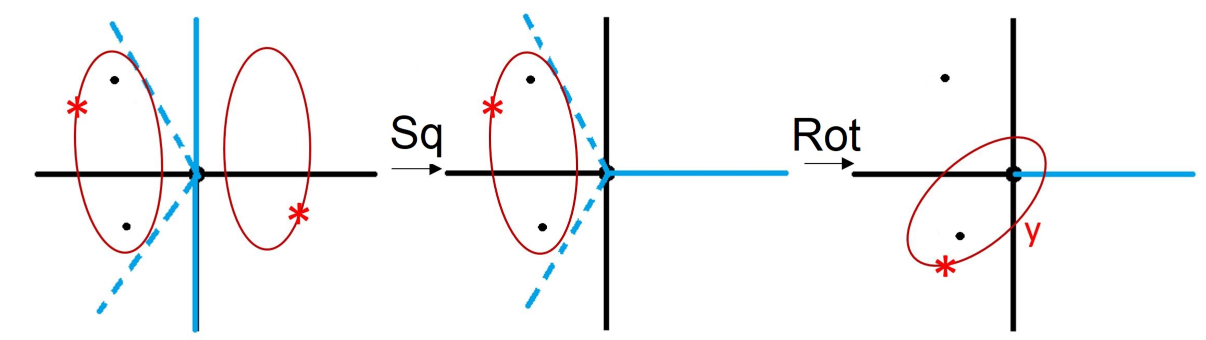


Three marked points and corresponding curves

Lifting

P_R = Sq ∘ Rot

A function f lifts if there exists a function $\tilde{f}$ such that $P_R \circ \tilde{f} = f \circ P_R$

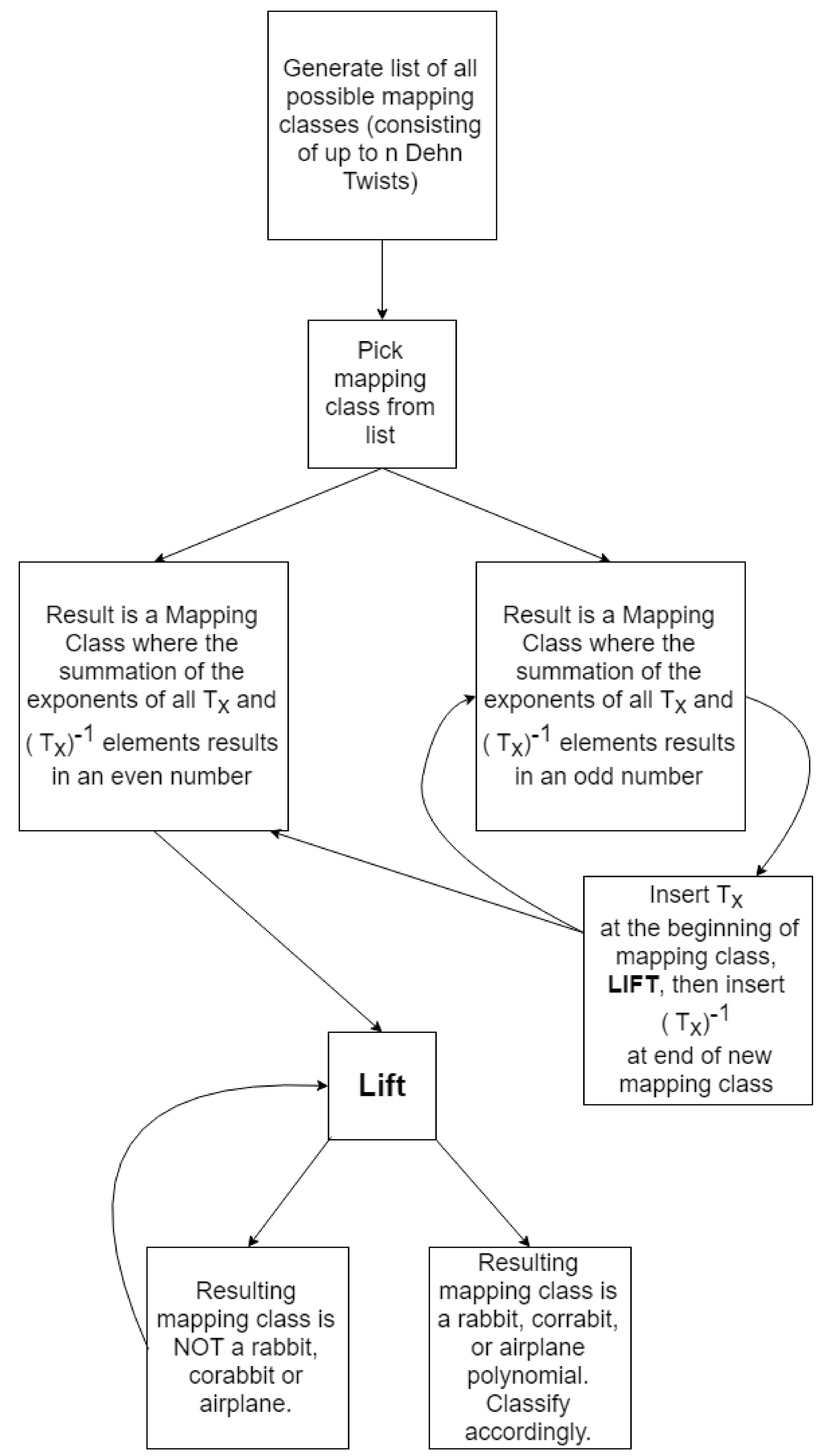


T$_y$ lifts to T$_x$.

Goal

Make a program that finds proportion of mapping classes f such that f ∘ P_R is equivalent to the rabbit, corabbit, or airplane

Program Methodology



Program Results

Mapping Classes of Length 10

- Total Mapping Classes: 118,096
- Number of Rabbits: 25,095 / 21.25%
- Number of Corabbits: 25,134 / 21.28%
- Number of Airplanes: 67,867 / 57.47%

Rules for Lifting

Given a mapping class written as the product of T$_x$, T$_y$, and their inverses,

- If the sum of exponents on T$_x$ are even, the mapping class lifts.
- T$_x^2$ lifts to T$_x^{-1}$T$_y^{-1}$
- T$_y$ lifts to T$_x$
- T$_x$T$_y$T$_x^{-1}$ lifts to the identity
- End conditions:
  - Id: rabbit
  - T$_x$: airplane
  - T$_x^{-1}$: corabbit

n > 3 marked points

Problems

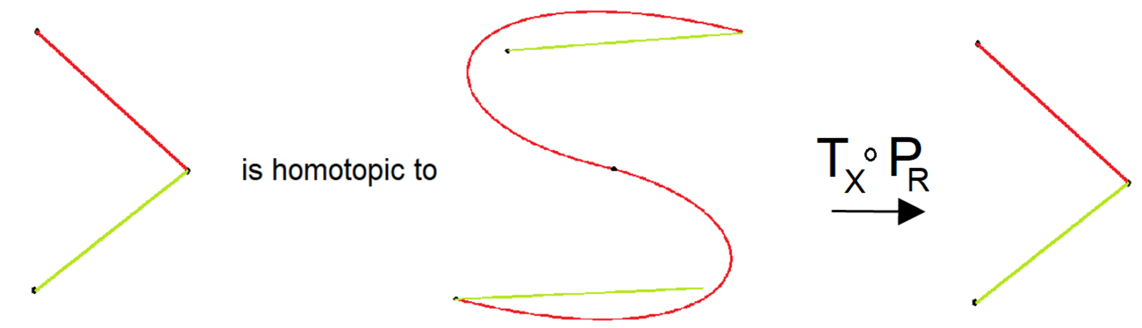
- Any mapping class can be written as a product of (n-1) Dehn twists, but not uniquely.
- How do we determine our end condition polynomials?
- What are all of the lifting rules?

Promising Directions

Using Trees to Classify Mapping Classes

Theorem 3: (Poirier)

Every quadratic polynomial has a unique tree in $\mathbb{C}$ such that every marked point is a vertex, all leaves are marked points, unmarked vertices have valence ≥ 3, and the preimage of the tree under the rabbit is homotopic to itself.



T$_x$∘P$_R$ is equivalent to the Airplane polynomial.

- Trees can similarly be used to identify the polynomial in question. We've learned that each branched cover that is equivalent to a polynomial (any of our Dehn-twisted rabbits) has a unique tree fixed when we take the pre-image.
- With this information, we can then determine that the trees associated with our twisting can also determine which polynomial will be lifted to.
- Why Trees? Trees give us a combinatorial method of understanding mapping classes. In other words, trees are easier to understand.

Acknowledgements

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