Technical Appendix to:
Quantifying the Benefits of Labor Mobility in a Currency Union*

Christopher L. House
University of Michigan and NBER

Christian Proebsting
EPFL | École Polytechnique Fédérale de Lausanne

Linda L. Tesar
University of Michigan and NBER

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*House: chouse@umich.edu; Proebsting: Christian.Probsting@epfl.ch; Tesar: ltesar@umich.edu.
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1 More Details on Calibration

1.1 Bilateral Trade Preference Weights

Since steady-state asset holdings are not uniquely pinned down with incomplete markets, we can choose the initial asset holdings. As is common in the literature, we decide to log-linearize our model around a steady state with zero net foreign asset (i.e. zero net exports) for every region. We choose our bilateral trade matrix $\bar{\omega}$ to satisfy this condition and to look as “similar” as possible to the trade matrix implied by the data, $\tilde{\bar{\omega}}$. In particular, we minimize

$$\min_{\bar{\omega}} \sum_{i} \sum_{j} \frac{1}{2} \left( \frac{\bar{\omega}_{i}^{j} - \tilde{\bar{\omega}}_{i}^{j}}{k + \bar{\omega}_{i}^{j}} \right)^{2}$$

subject to

$$\sum_{i} \bar{\omega}_{i}^{j} \frac{N_{i}Y_{i}}{N_{j}Y_{j}} = 1 + \frac{NX_{j}}{P_{j}Y_{j}} \quad \forall j$$

$$\sum_{j} \bar{\omega}_{i}^{j} = 1 \quad \forall i$$

$$\bar{\omega}_{i}^{j} \geq 0$$

$$1 \geq \bar{\omega}_{i}^{j},$$

with $k > 0$.\(^1\) Our loss function specifies our idea of “similarity” between the two matrices. The first constraint describes the relationship between the trade preference weight and net exports. The second to fourth constraints are purely technical constraints on the parameters. In practice we set $k = 0.1$ and $\frac{NX_{j}}{P_{j}Y_{j}} = 0$. This is a simple problem to solve. Let $\lambda_{j}$ and $\lambda_{i}$ denote the Lagrange multiplier on the first two constraints. We solve for these parameters using the two constraints and setting the preference weights to

$$\bar{\omega}_{i}^{j} = \min \left( 1, \max \left[ 0, \bar{\omega}_{i}^{j} - (k + \tilde{\bar{\omega}}_{i}^{j}) \left( \lambda_{j} + \lambda_{i} \frac{N_{i}Y_{i}}{N_{j}Y_{j}} \right) \right] \right).$$

2 Equilibrium Conditions

In the Technical Appendix, we allow for an intensive margin of labor supply. In particular, the labor supplied by a member of household $i$ living in country $j$ is denoted by $l_{i,j,t}^{t}$. Total labor supply

\(^1\)Notice that we require $k > 0$ because elements in $\tilde{\bar{\omega}}_{i}^{j}$ might be equal to 0.
in country $j$ is then:

$$l_{j,t}N_{j,t} = \sum_i n_{j,t}^i l^i_{j,t} N^i (2.1)$$

where $l_{j,t}$ is labor supply per capita in country $j$. Similarly, total labor supplied by household $i$ is

$$l^i_t = \sum_j n_{j,t}^i l^i_{j,t}.$$ 

The utility function over consumption and labor is described by

$$u(c^i_{j,t}, l^i_{j,t}) = v_i \left( c^i_{j,t} - \kappa_j \frac{l^i_{j,t}}{1 + \frac{1}{\eta}} \right)^{1-\frac{1}{\sigma}},$$

where $\sigma$ is the intertemporal elasticity of substitution, $v_i$ is a household-specific utility weight, $\kappa_j$ is a disutility weight on labor and $\eta$ is the Frisch elasticity of labor supply.

### 2.1 Household’s Budget Constraint

The household’s budget constraint is given by

$$N_i \left( \sum_j S^j_{j,t} P_i n_{j,t}^i c^i_{j,t} \right) + N_{i,t} P_i X_{i,t} + N^i \frac{B^i_t}{(1 + i_t) S_{i,t}},$$

$$= N_i \left( \sum_j S^j_{j,t} n_{j,t}^i \left( W^h_{j,t} n_{j,t}^i + T^j_{i,t} \right) \right) + N_{i,t-1} K_{i,t-1} \left( P^k_i u_{i,t} - P_i a(u_{i,t}) \right) + N_{i,t} (\Pi_{i,t} - T_{i,t}) + N^i \frac{B^i_{t-1}}{S_{i,t}}.$$

Replacing $T_{i,t}$ by the government’s budget constraint

$$P_{i,t} (b_i N_{i,t} U_{i,t} + N_{i,t} G_i) + \sum_j N^j n_{i,t}^j T_{i,t}^j = N_{i,t} T_{i,t}.$$

gives

$$N_i \left( \sum_j S^j_{j,t} P_i n_{j,t}^i c^i_{j,t} \right) + N_{i,t} P_i (X_{i,t} + G_{i,t} + b_i U_{i,t}) + N^i \left( \frac{B^i_t}{(1 + i_t) S_{i,t}} - \frac{B^i_{t-1}}{S_{i,t}} \right),$$

$$= N_i \left( \sum_j S^j_{j,t} n_{j,t}^i \left( W^h_{j,t} n_{j,t}^i + T^j_{i,t} \right) \right) + N_{i,t-1} K_{i,t-1} \left( P^k_i u_{i,t} - P_i a(u_{i,t}) \right) + N_{i,t} \Pi_{i,t} - \sum_j N^j n_{i,t}^j T_{i,t}^j.$$

---

$^2$We ignore the quadratic penalty term on foreign bond holdings and the moving cost because they do not affect the log-linearized equations and are zero in steady state.
Replacing $N_{i,t}P_{i,t}(X_{i,t} + G_{i,t}) + N_{i,t-1}K_{i,t-1}P_{i,t}a(u_{i,t})$ by the market clearing for the final good

$$N_{i,t}Y_{i,t} = N_{i,t}C_{i,t} + N_{i,t}X_{i,t} + N_{i,t}G_{i,t} + a(u_{i,t})N_{i,t-1}K_{i,t-1} + \varsigma N_{i,t}V_{i,t},$$

gives

$$N^i \left( \sum_j S^i_{j,t} P_{j,t} n^i_{j,t} c^i_{j,t} \right) + N_{i,t}P_{i,t}(Y_{i,t} - C_{i,t} - \varsigma V_{i,t} + b_i U_{i,t}) + N^i \left( \frac{B^i_t}{(1 + i_t)S_{i,t}} - \frac{B^i_{i-1}}{S_{i,t}} \right) = N^i \left( \sum_j S^i_{j,t} n^i_{j,t} \left( W^h_{j,t} l^i_{j,t} + T_{j,t}^{i} \right) \right) + R^k_{i,t} u_{i,t} N_{i,t-1} T_{i,t-1} + N_{i,t} \Pi_{i,t} - \sum_j N^j n^j_{i,t} T_{i,t}^{j}. $$

The profit term $N_{i,t} \Pi_{i,t}$ consists of profits by monopolistically competitive producers of varieties, $N_{i,t} \Pi_{i,t}^f$, and profits by labor market firms (employment agencies and HR firms), $N_{i,t} \Pi_{i,t}^f$. Profits by variety producers are given by

$$N_{i,t} \Pi_{i,t}^f = p_{i,t} N_{i,t} Q_{i,t} - W^f_{i,t} N_{i,t} L_{i,t} - R^k_{i,t} u_{i,t} N_{i,t-1} K_{i,t-1}. $$

Employment agencies pay $W^h_{i,t} l_{i,t}$ to households, receive $W^f_{i,t} L_{i,t}$ from HR firms and $P_{i,t} b_i U_{i,t}$ from the government. HR firms pay $W^f_{i,t} L_{i,t}$ to employment agencies, pay a vacancy cost $P_{i,t} \varsigma V_{i,t}$ and receive $W^f_{i,t} L_{i}$ from producing firms. Total profits of employment agencies and HR firms are therefore

$$N_{i,t} \Pi_{i,t}^f = W^f_{i,t} N_{i,t} L_{i,t} - \varsigma P_{i,t} N_{i,t} V_{i,t} + b_i P_{i,t} N_{i,t} U_{i,t} - \frac{W^h_{i,t} L_{i,t}}{1 - ur_{i,t}}. $$

It follows that overall profits, $N_{i,t} \Pi_{i,t} = N_{i,t} \Pi_{i,t}^f + N_{i,t} \Pi_{i,t}^f$, are

$$N_{i,t} \Pi_{i,t} = p_{i,t} N_{i,t} Q_{i,t} - R^k_{i,t} u_{i,t} N_{i,t-1} K_{i,t-1} - \frac{W^h_{i,t} L_{i,t}}{1 - ur_{i,t}} - \varsigma P_{i,t} N_{i,t} V_{i,t} + b_i P_{i,t} N_{i,t} U_{i,t}. $$

Inserting this term into the budget constraint gives

$$N^i \left( \sum_j S^i_{j,t} P_{j,t} n^i_{j,t} c^i_{j,t} \right) + N_{i,t}P_{i,t}(Y_{i,t} - C_{i,t}) + N^i \left( \frac{B^i_t}{(1 + i_t)S_{i,t}} - \frac{B^i_{i-1}}{S_{i,t}} \right) = N^i \left( \sum_j S^i_{j,t} n^i_{j,t} \left( W^h_{j,t} l^i_{j,t} + T_{j,t}^{i} \right) \right) + N_{i,t} \left( p_{i,t} Q_{i,t} - W^h_{i,t} l_{i,t} \right) - \sum_j N^j n^j_{i,t} T_{i,t}^{j}. $$
Re-arranging terms yields

\[
N_i \left( \frac{B_i}{(1 + i_t)S_{i,t}} - \frac{B_{i-1}}{S_{i,t}} \right) = N_{i,t}p_{i,t}Q_{i,t} - N_{i,t}P_{i,t}Y_{i,t} \\
+ N_i \left( \sum_j S_{i,t}^{j}n_{i,j,t}^{j}W_{j,t}^{h,j,i} \right) - N_{i,t}W_{i,t}^{h,i} - \left[ N_j \left( \sum_j S_{i,t}^{j}P_{j,t}n_{i,j,t}^{j}c_{j,t}^{j} \right) \right] - N_{i,t}P_{i,t}C_{i,t} \\
+ N_i \sum_j S_{i,t}^{j}n_{i,j,t}^{j}T_{j,t}^{i} - \sum_j N_{i,t}W_{j,t}^{l,j,i}T_{j,t}^{i}.
\]

The definition of consumption and the labor force are

\[
C_{i,t}N_{i,t} = \sum_j n_{i,j,t}^{j}c_{i,t}^{j}N_{j}^{j} \\
l_{i,t}N_{i,t} = \sum_j n_{i,j,t}^{j}l_{i,t}^{j}N_{j}^{j}.
\]

Substituting this in and re-arranging yields:

\[
N_i \left( \frac{B_i}{(1 + i_t)S_{i,t}} - \frac{B_{i-1}}{S_{i,t}} \right) = N_{i,t}p_{i,t}Q_{i,t} - N_{i,t}P_{i,t}Y_{i,t} \\
+ \sum_{j \neq i} \left( S_{i,t}^{j}n_{i,j,t}^{j}N_{j}^{j}W_{j,t}^{h,j,i} - n_{i,j,t}^{j}N_{j}^{j}W_{j,t}^{h,i} \right) - \left( \sum_j S_{i,t}^{j}n_{i,j,t}^{j}N_{i,t}P_{j,t}c_{j,t}^{j} - n_{i,t}^{j}N_{j}^{j}P_{j,t}C_{i,t} \right) \\
+ \sum_{j \neq i} \left( S_{i,t}^{j}n_{i,j,t}^{j}N_{j}^{j}T_{j,t}^{i} - n_{i,j,t}^{j}N_{j}^{j}T_{j,t}^{j} \right).
\]

The current account (LHS) equals net exports (1st row), net primary income from abroad (2nd row) and current transfers (3rd row). Net exports is the value of total production less the value of total domestic absorption. Net primary income is labor income earned abroad less labor income earned at home by foreigners, minus consumption expenditure abroad less consumption expenditure at home by foreigners. Current transfers consist of total government transfers received abroad less government transfers paid out at home to foreigners.

In steady state, this equation becomes

\[
N_i(\beta - 1) \frac{B_i}{S_i} = N_i(p_iQ_i - P_iY_i) + \sum_{j \neq i} \left( S_{i}^{j}n_{i,j}^{j}N_{j}^{j}T_{j,t}^{j} - n_{i}^{j}N_{j}^{j}T_{j,t}^{j} \right) \\
+ \sum_{j \neq i} \left( S_{i}^{j}n_{i,j}^{j}N_{j}^{j}W_{j,t}^{h,j,i} - n_{i}^{j}N_{j}^{j}W_{j,t}^{h,i} \right) - \left( \sum_j S_{i}^{j}n_{i,j}^{j}N_{j}^{j}P_{j,t}c_{j,t}^{j} - n_{i}^{j}N_{j}^{j}P_{j,t}C_{i} \right)
\]
Re-arranging yields:
\[ \mathbb{N}^i(\beta - 1) \frac{B^i}{S_i} = \mathbb{N}_i(p_i Q_i - P_i Y_i) + \mathbb{N}^i \sum_{j \neq i} S^i_j n^i_j \left( T r^i_j - W^h j^i_j P_j c^i_j \right) - \sum_{j \neq i} \mathbb{N}^j n^j_i \left( T r^j_i - W^h i^j_i - P_i c^j_i \right). \]

Since governments set transfers in steady state such that
\[ T r^i_j = P_i c^j_i - W^h i^j_i, \]
we are left with
\[ \mathbb{N}^i(\beta - 1) \frac{B^i}{S_i} = \mathbb{N}_i(p_i Q_i - P_i Y_i). \]
That is, in steady state net primary income and current transfers cancel each other out and the net foreign asset position is proportional to net exports. We start from a steady state with zero net exports, i.e. the net foreign asset position is zero as well.

Log-linearizing the household’s budget constraint around a steady state with a zero net foreign asset position (\( B^i = 0 \) for all \( i \)) yields
\[
\begin{align*}
\mathbb{N}^i \left( \frac{1}{\beta} \Delta B^i_{i-1} - \Delta B^i_i \right) &= \mathbb{N}_i \left( Y_{i,t} \bar{Y}_{i,t} - Q_i \left( \bar{P}_i, \bar{P}_{i,t} + \bar{Q}_{i,t} \right) \right) \\
&= \left[ \sum_{j \neq i} n^i_j n^i_j w^h j^i_j \left( \bar{w}^h j^i_j + \bar{\bar{h}}_{j,t} \right) - n^i_j n^i_j w^h i^j_i \left( \bar{w}^h i^j_i + \bar{\bar{h}}_{i,t} \right) \right] \\
&\quad - \left[ \sum_{j \neq i} n^i_j n^j_i \bar{c}^j_{i,t} - n^j_i n^j_i \bar{c}^j_{i,t} \right].
\end{align*}
\]

### 2.2 Steady State

To solve for the steady state, we proceed in three steps:

1. We first solve for real prices (rental price of capital, \( r^k_i \), real price of the intermediate good, \( p_i^P_i \)) and shares in GDP (share of consumption expenditure in GDP, \( C_i^Q_i \), share of investment in GDP, \( X_i^Q_i \), share of net exports in GDP, \( \frac{NX_i^Q_i}{Q_i} \)). This requires data on relative country size in terms of countries’ domestic absorption, \( \frac{N_i Y_i}{N_i Y_i} \), but it does not require separate information on population, \( N_i \) or domestic absorption per capita, \( Y_i \). In a standard international DSGE model, only total country size matters, but not GDP per capita. Importantly, at this stage we cannot and do not need to solve for the real wage \( w^f_i \). The model only pins down wage payments as a share of GDP, i.e. \( \frac{w^f_i L_i}{Q_i} \), but not the real wage \( w^f_i \).

2. We next solve for the steady-state values related to migration (labor income of nationals, \( w^h j^i_j \), and consumption of nationals, \( c^j_{i,j} \), for all locations \( j \)). Here, we require information
on population measured in terms of persons, $N_i$, and the share of the labor force in total population.

3. We then solve for the steady-state values related to the search and matching block. Here, we require data on unemployment rates, $ur_i$, to solve for employment and the real wage $w^f_i$. Given this real wage and unemployment benefits (as percent of GDP), $b_i$, we can solve for all remaining variables pertaining to the search and matching block.

2.2.1 DSGE block

We solve the model in a neighborhood of a non-stochastic steady state with zero inflation. Because inflation is zero, the Euler equations associated with the noncontingent nominal bonds imply that the nominal interest rate is $1 + i_i = \frac{1}{\beta}$ for all $i$. Next, we use the capital Euler equation

$$\frac{\mu_i}{P_i} = \beta \left[ u_i r^k_i + \frac{\mu_i}{P_i} (1 - \delta) - a(u_i) \right].$$

Note that the households’ first-order condition for investment implies that $\mu_i = P_i$ because $\Lambda = \Lambda' = 0$ in steady state. Inserting this back into the entrepreneurs’ first-order condition for capital and noting that $a(u_i) = 0$ and $u_i = 1$ gives

$$r^k_i = \frac{1}{\beta} - 1 + \delta.$$ (2.2)

This equation determines the real rental price of capital $r^k_i$ in each country.

With zero inflation, the steady state price of intermediates is a constant markup over the nominal marginal cost,

$$p_i = \frac{\psi_q}{\psi_q - 1} MC_i.$$

This can be seen from the reset equation and the law of motion for the nominal price of the intermediate good.

Next, cost minimization of the first-stage producers implies

$$R^k_i = MC_i \alpha Z_i \left[ \frac{K_i}{L_i} \right]^{\alpha - 1}$$

$$r^k_i = \frac{\psi_q - 1}{\psi_q} \frac{p_i}{P_i} \alpha Z_i \left[ \frac{K_i}{L_i} \right]^{\alpha - 1}$$

$$\frac{p_i}{P_i} = r^k_i \frac{\psi_q}{\psi_q - 1} \alpha Z_i \left[ \frac{K_i}{L_i} \right]^{1 - \alpha}$$

We adjust the technology levels $Z_i$ so that all intermediate goods prices equal the price of the
respective final good: $p_i = P_i$. Notice that the real wage paid by the producing firms is given by

$$w_i^f = \frac{\psi_q - 1}{\psi_q} \frac{p_i}{P_i} (1 - \alpha) \frac{Q_i}{L_i}.$$ 

The price index formula for the final good states

$$P_i = \left( \sum_{j=1}^{N} \bar{\omega}_i^j \left[ \frac{S_j}{S_i} p_j \frac{P_j}{P_i} \right]^{1-\psi_y} \right)^{\frac{1}{1-\psi_y}}$$

$$P_i S_i = \left( \sum_{j=1}^{N} \bar{\omega}_i^j \left[ \frac{P_j S_j}{P_j P_i} \right]^{1-\psi_y} \right)^{\frac{1}{1-\psi_y}}$$

One can easily verify that $P_i S_i = 1$ solves this equation, that is the real exchange rate $s_i = P_i S_i$ is unity.

We directly calibrate some steady-state variables to match their empirical counterparts. Those are the shares of government purchases, $G_i$, the relative country sizes, $\frac{N_i Y_i}{N_i Y}$, and the bilateral import shares $\frac{y_i^n}{Y_i}$. We now derive the shares of the remaining variables, $NX_i$, $C_i$ and $X_i$, and later show that these non-targeted shares implied by our model match their empirical counterparts quite closely.

To derive the share of net exports, we first use the demand equation for intermediate goods,

$$y_i^j = Y_i \bar{\omega}_i^j \left[ \frac{S_j}{S_i} p_j \frac{P_j}{P_i} \right]^{-\psi_y} = Y_i \bar{\omega}_i^j \left[ \frac{s_j}{s_i} \frac{P_j}{P_i} \right]^{-\psi_y}.$$ 

It follows that $\bar{\omega}_i^j$ is country $i$’s import share of country $j$’s good, measured in terms of the privately-produced good $Y_i$:

$$\bar{\omega}_i^j = \frac{y_i^j}{Y_i}.$$ 

The implied net export share can then be expressed in terms of country sizes and the import preference parameters. Inserting the market clearing condition for $Q_i$ into the definition of net exports, $NX_i = p_i Q_i - P_i Y_i$, we have

$$\frac{NX_i}{P_i Y_i} = \left( \sum_{j=1}^{N} \frac{N_j p_i y_i^j}{N_i P_i Y_i} \right) - 1$$

$$= \left( \sum_{j=1}^{N} \frac{N_j Y_j}{N_i Y_i} \bar{\omega}_i^j \right) - 1.$$ (2.3)
Notice that this also gives us
\[
\frac{p_i Q_i}{P_i Y_i} = 1 + \frac{N X_i}{P_i Y_i}.
\]  
(2.4)

To derive the share of investment, we insert the marginal product of capital equation, \( p_i Q_i = \frac{\psi_q}{\psi_q - 1} \alpha_i K_i \), into the definition of net exports, \( N X_i = p_i Q_i - P_i Y_i \):

\[
\frac{\psi_q}{\psi_q - 1} \alpha_i X_i = P_i Y_i + N X_i,
\]

\[
\frac{X_i}{Y_i} = \frac{\alpha \delta}{\psi_q - 1} r_i \left(1 + \frac{N X_i}{P_i Y_i}\right),
\]

where \( X_i = \delta K_i \).

Finally, the consumption share is the residual of the market clearing condition \( Y_i = C_i + X_i + G_i + \varsigma V_i \):

\[
\frac{C_i}{Y_i} = 1 - \frac{X_i}{Y_i} - \frac{G_i}{Y_i} - \frac{\varsigma V_i}{Y_i}.
\]  
(2.6)

To summarize, we solve for the steady state values as follows:

1. Calibrate the government expenditure shares \( \frac{G_i}{Y_i} \) to the counterpart in the data.

2. Solve for the real rental price \( r_i \) using equation (2.2).

3. Calibrate the import preference parameters \( \omega^j_i \) using data on country \( j \)'s share of country \( i \)'s imports, and calibrate the relative size of countries in terms of their domestic absorption, \( \frac{N_j Y_j}{N_i Y_i} \).

4. Solve for the net export share \( \frac{N X_i}{P_i Y_i} \) using equation (2.3), the GDP share using equation (2.4), the investment share \( \frac{X_i}{Y_i} \) using equation (2.5) and the consumption share \( \frac{C_i}{Y_i} \) using equation (2.6)

2.2.2 Migration

For the DSGE block, we only require data on a country’s total domestic absorption, \( N_i Y_i \), as a measure of an economy’s size. Given \( N_i Y_i \), the values for population, \( N_i \), and domestic absorption per capita, \( Y_i \), are irrelevant. This is no longer true if we allow for migration. For the migration block, we require data on countries’ population, \( N_i \). Given values for \( N_i \), we immediately have domestic absorption per capita, \( Y_i \) from

\[
Y_i = \frac{N_i Y_i}{N_i}.
\]  
(2.7)
This allows us to write all shares previously expressed in terms of domestic absorption, \((2.6)\), \((2.5)\), \((2.3)\) and \((2.4)\) in absolute values. For instance, \(Q_i\), is then calculated from equation \((2.4)\) as

\[
Q_i = \frac{Q_i Y_i}{Y_i}.
\]

Next, we solve for steady-state shares of migrants, \(n_{ij}\), population size of nationals, \(N^j\), the bilateral labor supply matrix, \(l^j_i\), and the bilateral consumption matrix, \(c^i_j\).

The steady-state shares of migrants, \(n_{ij}\), are determined by the equation governing the location choice:

\[
u(c^j_i, l^j_i) - u(c^i_i, l^i_i) + A^j_i - \frac{1}{\gamma} (\ln(n^j_i) + 1) = \left(c^j_i - w^h_j l^j_i - tr^i_j\right) u^i_{1,j} - \left(c^i_i - w^h_i l^i_i - tr^i_i\right) u^i_{1,i}.
\]

Notice that the terms related to migration costs are zero in steady state. Given our assumption on how transfers \(tr^j_i\) are set in steady state, we have

\[
u(c^j_i, l^j_i) - u(c^i_i, l^i_i) + A^j_i = \frac{1}{\gamma} (\ln(n^j_i) + 1).
\]

Since steady-state values for \(c^j_i\) and \(l^j_i\) are independent of \(A^j_i\), migration shares \(n^j_i\) are pinned down by \(A^j_i\). In practice, we adjust \(A^j_i\) to match bilateral stocks of migrants observed in the data.

Given the matrix of migration shares \(n^j_i\), we solve for the population size of nationals (household size), \(N^j\), from the linear equation system

\[
N_i = \sum_j n^j_i N^j.
\]

**Consumption and labor supply.** Given GHH preferences, labor supply in steady state satisfies

\[
\kappa_j \left(\frac{l^j_i}{l^j_j}\right)^{\frac{1}{\eta}} = \frac{w^h_j}{\kappa_j} \nonumber
\]

\[
l^i_j = \left(\frac{w^h_j}{\kappa_j}\right)^{\frac{1}{\eta}}.
\]

which implies that labor supply only depends on a household member’s current location, i.e. \(l^j_i = l^j_j = l^i_j\). We can therefore write

\[
l^i_j = \left(\frac{w^h_j}{\kappa_j}\right)^{\frac{1}{\eta}}.
\]
Given values for the household wage, \( w_i^h \), and the disutility weight \( \kappa_j \), we could solve for \( l_j \). Here, we use a slightly different strategy and calibrate \( l_j \) directly to the data. We then adjust \( \kappa_j \) accordingly.\(^3\)

To solve for the consumption matrix, we make use of the Backus-Smith risk-sharing condition:

\[
C_j^i - \kappa_j (l_j)^{1 + \frac{1}{\eta}} = C_i^i - \kappa_i (l_i)^{1 + \frac{1}{\eta}}
\]

\[
C_j^i - \frac{w_j^h l_j}{1 + \frac{1}{\eta}} = C_i^i - \frac{w_i^h l_i}{1 + \frac{1}{\eta}}
\]

Then, \( C_j^i \) can be solved from the linear equation system

\[
C_i N_i = \sum_j n_i^j c_i^j N_j^j
\]

\[
C_i N_i = \sum_j n_i^j \left( C_j^i - \frac{w_j^h l_j - w_i^h l_i}{1 + \frac{1}{\eta}} \right) N_j^j. \tag{2.9}
\]

Notice that we have solved for \( C_i \) in equation (2.6) together with equation (2.7), \( N_i \) is country \( i \)'s population, given by the data, \( n_i^j \) are migration shares given by the data, \( N_j^j \) is the size of household \( j \), given by (2.8), labor supply is calibrated to the data and the household wage is solved below (see equation (2.13)).\(^4\)

To summarize, we solve for the steady state values as follows:

1. Calibrate population sizes \( N_i \) to rewrite quantities in per capita terms \( (Y_i, Q_i, ...) \)

\(^3\)We choose this strategy because there is little guidance on the actual value of \( \kappa_j \). If the disutility on labor \( \kappa_j \) was the same across countries, our calibration would imply large differences in the labor force, \( l_j \), because countries substantially differ in their real wage rates.

\(^4\)With separable preferences, the risk-sharing condition

\[
(c_i^{\frac{1}{\sigma}})^{-\frac{1}{\sigma}} = (c_j^{\frac{1}{\sigma}})^{-\frac{1}{\sigma}},
\]

ensures that \( c_i^i = c_j^i = c^i \); independent of \( \kappa_j \). We can solve for \( c^i \) using

\[
C_i N_i = \sum_j n_i^j c^i N_j^j.
\]

From labor supply in steady state, we have

\[
w_i^h l_i^i = \kappa_i \left( l_i^i \right)^{1 + \frac{1}{\eta}} \left( c^i \right)^{\frac{1}{\sigma}}
\]

and

\[
w_j^h l_j^i = \kappa_i \left( l_j^i \right)^{1 + \frac{1}{\eta}} \left( c^i \right)^{\frac{1}{\sigma}}.
\]

Combining, we obtain

\[
l_j^i = l_i^i \left( \frac{c_j^i}{c_i^i} \right)^{\frac{2}{\sigma}}.
\]
2. Calibrate migrant shares $n^j_i$ and obtain the household size from (2.8).  
3. Calibrate the size of the labor force (per capita), $l_i$, and use $l^j_i = l_i$.  
4. Solve for the consumption matrix $c^j_i$ from (2.9).  

2.2.3 Search block  

In this section, we solve for the real wage paid by the HR firm to the employment agency, $w_i$, the wage received by the household, $w_i^h$, and the values of a filled vacancy and of being employed, $J_i$ and $E_i$. We solve for these values as functions of the real wage paid by the producing firm, $w_i^f$ and the parameters of the model. The real wage paid by firms equals the marginal product of labor:

$$w_i^f = (1 - \alpha) \frac{\psi_q - 1 \frac{Q_i}{L_i}}{\psi_q}.$$  \hfill (2.10)  

Employment, $L_i$, is calculated as

$$L_i = l_i (1 - ur_i), \hfill (2.11)$$

where $l_i$ is the labor force and $ur_i = \frac{U_i}{L_i}$ is the unemployment rate. We calibrate both the labor force and the unemployment rate to the data.

The matching function, $f_i = \frac{M_i}{H_i}$, combined with the law of motion for employed workers, $L_i = (1 - d)L_i + M_i$ and the equation describing the number of job hunters, $H_i = U_i + dL_i$ gives

$$f_i = \frac{dL_i}{U_i + dL_i} = \frac{d}{\frac{ur_i}{1 - ur_i} + d}, \hfill (2.12)$$

where we used that $\frac{U_i}{L_i} = ur_i \frac{l_i}{L_i} = \frac{ur_i}{1 - ur_i}$. The equation describing the value of being employed is given by

$$w_i^h = w_i - [1 - (1 - d)\beta] E_i \hfill (2.13)$$

Then, we can solve for $l_i$ using

$$l_i N_{i,t} = \sum_{j} n^j_i l_i^j N^j$$

$$l_i N_{i,t} = l_i \sum_{j} n^j_i \left( \frac{c^j_i}{c_i} \right)^{\frac{\sigma}{2}} N^j$$

$$l_i = l_i N_{i,t} \left( \sum_{j} n^j_i \left( \frac{c^j_i}{c_i} \right)^{\frac{\sigma}{2}} N^j \right)^{-1}.$$
where we can replace \( w^h_i \) by the free entry condition for employment agencies

\[
\mathcal{E}_i = \frac{1 - f_i}{f_i} \left( w^h_i - b_i \right),
\]

to get

\[
\left[ 1 - (1 - d)\beta + \frac{f_i}{1 - f_i} \right] \mathcal{E}_i = w_i - b_i.
\]

To solve for \( w_i \) as a function of parameters, we combine the wage bargaining equation,

\[
\varrho \mathcal{J}_i = (1 - \varrho) \left( \mathcal{E}_i - b_i + w^h_i \right),
\]

with the free entry condition for employment agencies and the equation describing the value of a filled vacancy

\[
\mathcal{J}_i = \frac{w^f_i - w_i}{1 - (1 - d)\beta}
\]

to obtain

\[
\mathcal{E}_i = (1 - f_i) \frac{\varrho}{1 - \varrho} \frac{w^f_i - w_i}{1 - (1 - d)\beta}.
\]  

(2.15)

Using this expression to replace \( \mathcal{E}_i \), we obtain

\[
\frac{1 - (1 - f_i)(1 - d)\beta}{1 - (1 - d)\beta} \frac{\varrho}{1 - \varrho} (w^f_i - w_i) = w_i - b_i
\]

\[
\Xi_i w^f_i = (1 + \Xi) w_i - b_i
\]

\[
w_i = \frac{\Xi_i}{1 + \Xi_i} w^f_i + \frac{1}{1 + \Xi_i} b_i,
\]

where \( \Xi_i = \frac{1 - (1 - f_i)(1 - d)\beta}{1 - (1 - d)\beta} \frac{\varrho}{1 - \varrho} \). We calibrate \( b_i \) using data on replacement values, \( rv_i = \frac{b_i}{w_i} \). Given \( rv_i \), the wage \( w_i \) is given by

\[
w_i = \frac{\Xi_i}{1 + \Xi_i - rv_i} w^f_i.
\]  

(2.16)

We can then directly solve for the value of a filled vacancy, \( \mathcal{J}_i \), the value of being employed, \( \mathcal{E}_i \) and the wage rate paid to the household, \( w^h_i \).

Notice that if the bargaining power of the worker, \( \varrho \) goes to 1, we have \( \Xi_i \to \infty \) and the steady state wage, \( w_i \), equals the marginal product of labor, \( w_i^f \). If, in addition, the steady-state unemployment rate goes to zero, \( ur_i \to 0 \), then, the probability to find a job goes to 1, \( f_i \to 1 \), the value of being employed goes to 0, \( \mathcal{E}_i \to 0 \), so that \( w^h_i = w_i = w_i^f \).
To summarize, we solve for the steady state values as follows:

1. Solve for employment per capita, $L_i$, from (2.11), using data on unemployment rates, the real wage paid by producing firms, $w^f_i$, from (2.10) and the job finding rate, $f_i$, from (2.12).

2. Solve for the real wage, $w_i$, from (2.16), using data on replacement values, $rv_i$. Solve for real unemployment benefits, $b_i = \frac{b_i}{w^f_i}$.

3. Solve for the value of a filled vacancy, $J_i$, from (2.14), the value of being employed, $E_i$, from (2.15), and the real wage received by households, $w^h_i$, from (2.13).
2.3 Log-linearized Equilibrium Conditions

1. Marginal utility of consumption\(^5\)
   
   With separable preferences:
   
   $$\tilde{u}_{1,j,t} = -\frac{1}{\sigma} \tilde{c}_{j,t}$$
   
   with GHH preferences:
   
   $$-\sigma \left( u_{1,j} \right)^{-\sigma} \tilde{u}_{1,j,t} = c_{j,t} - w_j \tilde{p}_j$$
   
2. Marginal rate of substitution
   
   With separable preferences
   
   $$\tilde{mrs}_{j,t} = \frac{1}{\eta} \tilde{s}_{j,t} - \tilde{u}_{1,j,t}$$
   
   With GHH:
   
   $$\tilde{mrs}_{j,t} = \frac{1}{\eta} \tilde{s}_{j,t}$$
   
3. Risk sharing among household members (FOC \(c_{j,t}\))
   
   $$u_{i,t} = u_{i,j,t}, \quad \text{for} \quad j \neq i$$
   $$\tilde{u}_{1,t} + \tilde{s}_{j,t} = \tilde{u}_{1,j,t}, \quad \text{for} \quad j \neq i$$
   
4. Labor supply
   
   $$mrs_{i,t} = w_{j,t}$$
   $$\tilde{mrs}_{j,t} = \tilde{w}_{j,t}$$

\(^5\) GHH preferences:

$$u_{i,t} = \frac{1}{1 - \frac{1}{\sigma}} \left( c_{i,t} - \kappa \left( l_{i,t}^{1+\frac{1}{\eta}} \right) \right)^{1-\frac{1}{\sigma}}$$

$$\left( u_{i,j,t} \right)^{-\sigma} \tilde{a}_{i,j,t} = c_{j,t} l_{j,t} \left( l_{i,t}^{1+\frac{1}{\eta}} \right) \frac{4}{\eta} \tilde{l}_{j,t},$$

where we can replace \(\kappa \left( l_{i,t}^{\frac{1}{\eta}} \right) = w_{j,t}^{h} \).
5. Definition of consumption

\[ C_{i,t}N_{i,t} = \sum_j n_{i,t,j}^j c_{i,t,j}^j N_{j} \]

\[ (\tilde{C}_{i,t} + \tilde{N}_{i,t}) = \sum_j \frac{N_{j}}{N_{i}} \frac{c_{i,t,j}^j}{c_{i,t}^j} \left( \tilde{n}_{i,t,j}^j + \tilde{c}_{i,t,j}^j \right) \]

6. Definition of the labor force

\[ l_{i,t}N_{i,t} = \sum_j n_{i,t,j}^j l_{i,t,j}^j N_{j} \]

\[ (\tilde{l}_{i,t} + \tilde{N}_{i,t}) = \sum_j \frac{N_{j}}{N_{i}} \frac{l_{i,t,j}^j}{l_{i,t}^j} \left( \tilde{n}_{i,t,j}^j + \tilde{l}_{i,t,j}^j \right) \]

7. Capital Euler equation

\[ \frac{\mu_{i,t}}{P_{i,t}} = \beta E_t \left\{ \frac{u_{i,t+1}^{i}}{u_{i,t}^{i}} \left[ u_{i,t+1}^{i} r_{i,t+1}^{k} + \frac{\mu_{i,t+1}}{P_{i,t+1}} (1 - \delta) - a (u_{i,t+1}) \right] \right\} \]

\[ \beta r_{i,t}^{k} r_{i,t+1}^{k} = \beta \Delta i_{i,t} - \tilde{\pi}_{i,t+1} + \frac{\mu_{i,t}}{P_{i,t}} - \beta (1 - \delta) \left( \frac{\mu_{i,t+1}}{P_{i,t+1}} \right) \]

8. Price of capital

\[ 1 = \frac{\mu_{i,t}}{P_{i,t}} \left( 1 - \Lambda_{i,t} - \frac{N_{i,t}X_{i,t}}{N_{i,t-1}X_{i,t-1}} \Lambda_{i,t} \right) + \beta E_t \left\{ \frac{u_{i,t+1}^{i}}{u_{i,t}^{i}} \left[ u_{i,t+1}^{i} \frac{\mu_{i,t+1}}{P_{i,t+1}} \frac{N_{i,t+1}X_{i,t+1}}{N_{i,t}X_{i,t}} \right] \Lambda_{i,t+1}^{i} \right\} , \]

\[ \left( \frac{\mu_{i,t}}{P_{i,t}} \right) \frac{1}{N^i} = (1 + \beta) \left( \tilde{N}_{i,t} + \tilde{X}_{i,t} \right) - \left( \tilde{N}_{i,t-1} + \tilde{X}_{i,t-1} \right) - \beta \left( \tilde{N}_{i,t+1} + \tilde{X}_{i,t+1} \right) \]

9. Law of motion for the capital stock

\[ N_{i,t}K_{i,t} = N_{i,t-1}K_{i,t-1} (1 - \delta) + \left[ 1 - \Lambda \left( \frac{N_{i,t}X_{i,t}}{N_{i,t-1}X_{i,t-1}} \right) \right] N_{i,t}X_{i,t} \]

\[ \tilde{K}_{i,t} = (1 - \delta) \left( \tilde{N}_{i,t-1} + \tilde{K}_{i,t-1} - \tilde{N}_{i,t} \right) + \delta \tilde{X}_{i,t} \]

10. Optimal capital utilization

\[ r_{i,t}^{k} = a' (u_{i,t}) \]

\[ r_{i,t}^{k} \tilde{r}_{i,t}^{k} = a'' u_{i,t} \tilde{u}_{i,t} \]
11. Optimal factor employment

\[
\frac{\alpha}{1 - \alpha} \frac{W_{i,t}^f}{R_{i,t}^k} = \frac{u_{i,t}N_{i,t-1}K_{i,t-1}}{\bar{N}_{i,t}L_i}
\]

\[
\tilde{r}_{i,t}^k - \tilde{w}_{i,t}^f = \tilde{N}_{i,t} + \tilde{L}_{i,t} - \bar{u}_{i,t} - \tilde{K}_{i,t-1} - \bar{N}_{i,t-1}
\]

12. Real marginal costs

\[
MC_{i,t} = \left(\frac{W_{i,t}^f}{Z_{i,t}}\right)^{1-\alpha} \left(\frac{R_{i,t}^k}{Z_{i,t}}\right)^\alpha \left(\frac{1}{1 - \alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^\alpha
\]

\[
\tilde{mc}_{i,t} = -\tilde{Z}_{i,t} + \alpha \tilde{x}_{i,t}^k + (1 - \alpha) \bar{w}_{i,t}^f
\]

13. FOC wrt \(y_{i,t}^j\)

\[
y_{i,t}^j = Y_{i,t}^{\psi_y} \left[\frac{S_{j,t}}{S_{i,t}^{\psi_y}}\right]^{\psi_y} \sum_{j=1}^{\mathcal{N}} \bar{\omega}_{i,t} \left(\frac{p_{j,t}}{P_{i,t}}\right)
\]

\[
\left(\frac{p_{j,t}}{P_{i,t}}\right) + \bar{s}_{j,t} - \bar{r}_{i,t} = \frac{1}{\psi_y} \left(Y_{i,t}^{\psi_y} + \left(\frac{\psi_y}{\psi_y - 1}\left(\bar{y}_{i,t}^j - \sum_k \bar{\omega}_{i,t}^k \varepsilon_{i,t}^k\right)\right)\right)
\]

14. Production of intermediate good

\[
\bar{N}_{i,t}Q_{i,t} = Z_{i,t} \left(u_{i,t}N_{i,t-1}K_{i,t-1}\right)^\alpha \left(N_{i,t}L_{i,t}\right)^{1-\alpha}
\]

\[
Q_{i,t} = \tilde{Z}_{i,t} + \alpha \left(\bar{u}_{i,t} + \tilde{N}_{i,t-1} + \tilde{K}_{i,t-1} - \bar{N}_{i,t}\right) + (1 - \alpha) \tilde{L}_{i,t}
\]

15. Production of final good\(^6\)

\[
Y_{i,t}^{\psi_y} \tilde{Y}_{i,t} = \left(\sum_{j=1}^{\mathcal{N}} \left(\frac{\bar{\omega}_{i,t}^{\psi_y}}{\psi_y}\left(\bar{y}_{i,t}^j - \sum_k \bar{\omega}_{i,t}^k \varepsilon_{i,t}^k\right)\right)\right)^{\psi_y}
\]

\[
\tilde{Y}_{i,t} = \sum_{j=1}^{\mathcal{N}} \bar{\omega}_{i,t}^j \left(\bar{y}_{i,t}^j + \frac{1}{\psi_y - 1}\left(\varepsilon_{i,t}^j - \sum_k \bar{\omega}_{i,t}^k \varepsilon_{i,t}^k\right)\right)
\]

\(^6\)Our calibration of the shares \(\bar{\omega}_{i,t}^j\) is \(\bar{\omega}_{i,t}^j = \frac{\psi_y}{\psi_y - 1}\), so that

\[
Y^{\psi_y - 1} \tilde{Y}_{i,t} = \sum_{j=1}^{\mathcal{N}} \left[\left(\frac{\bar{\omega}_{i,t}^j}{\psi_y}\right)^{\psi_y - 1} \left(\bar{y}_{i,t}^j + \frac{1}{\psi_y - 1}\left(\varepsilon_{i,t}^j - \sum_k \bar{\omega}_{i,t}^k \varepsilon_{i,t}^k\right)\right)\right]
\]

can be simplified.
16. Market clearing for intermediate goods

\[ Q_i \left( \tilde{N}_{i,t} + \tilde{Q}_{i,t} \right) = \sum_{j=1}^{N} \frac{N_j Y_j^i}{N_i Y_i^j} \left( \tilde{N}_{j,t} + \tilde{y}_{j,t} \right) \]

17. Market clearing for final goods

\[ Y_{i,t} = C_{i,t} + X_{i,t} + G_{i,t} + a \left( u_{i,t} \right) \frac{N_{i,t-1} K_{i,t-1}}{N_{i,t}} + \varsigma V_{i,t} \]

\[ \tilde{Y}_{i,t} = \frac{C_i}{Y_i} \tilde{\tilde{C}}_{i,t} + \frac{X_i}{Y_i} \tilde{X}_{i,t} + \frac{G_i}{Y_i} \tilde{G}_{i,t} + r_i^k (1 - r_i^K) K_i Y_{i,t} + \frac{\varsigma V_i}{Y_i} (\tilde{f}_{i,t} - \tilde{g}_{i,t} + \tilde{H}_{i,t}) \]

18. Domestic Euler equation

\[ \frac{u_{1,i,t}^j}{P_{i,t}} = (1 + i_{i,t}) \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \beta \frac{u_{1,i,t+1}^j}{P_{i,t+1}} \]

\[ \beta \Delta i_{i,t} - \pi_{i,t+1} = \tilde{u}_{1,i,t} - \tilde{u}_{i,t+1} \]

19. Phillips curve

\[ \theta_p \left( \tilde{\pi}_{i,t} + \tilde{ToT}_{i,t} \right) = (1 - \theta_p) \left( 1 - \theta_p \beta \right) \left[ \tilde{mc}_{i,t} - \left( \frac{\tilde{p}_{i,t}}{P_{i,t}} \right) \right] + \theta_p \beta \left( \tilde{\pi}_{i,t+1} + \tilde{ToT}_{i,t+1} \right) \]

20. Definition of Terms of Trade

\[ \tilde{ToT}_{i,t} = \left( \frac{\tilde{p}_{i,t}}{P_{i,t}} \right) - \left( \frac{\tilde{p}_{i,t-1}}{P_{i,t-1}} \right) \]

21. Monetary Policy

\[ a(u_i) = u_i r_i^k + 1 - \frac{F_i}{\beta} - \delta \]

and is zero if \( u_i = 1 \). Also: \( V_{i,t} = \frac{f_{i,t}}{\hat{u}_{i,t}} \) and \( a'(u_i) = r_i^k (1 - r_i^K) \)
• Floating exchange rate:

\[ \Delta i_{i,t} = \phi_i \Delta i_{i,t-1} + (1 - \phi_i) \left( \phi Q \tilde{Q}_{i,t} + \phi \tilde{\pi}_{i,t} \right) \]

• Fixed exchange rate:
  
  - Leader \( n \):
    
    \[ \Delta i_{i,t} = \phi_i \Delta i_{i,t-1} + (1 - \phi_i) \sum_{j \in CU} \text{weight}_j \left( \phi Q \tilde{Q}_{j,t} + \phi \tilde{\pi}_{j,t} \right), \]
    
    where \( \text{weight}_j \) is the share of \( Q_j \) in the gdp of the currency union.
  
  - Follower \( j \):
    
    \[ \Delta S_{j,t} = \Delta S_{i,t} \]

22. Definition of change in nominal exchange rate

\[ \Delta S_{i,t} = (\tilde{s}_{i,t} - \tilde{s}_{i,t-1} - \tilde{\pi}_{i,t} \]

23. International Euler equation\(^9\)

- With complete markets

  \[ \tilde{u}_{1,i,t} = \tilde{s}_{i,t} \]

- With incomplete markets (Uncovered interest rate parity)

  \[ 0 = \tilde{s}_{1,t} \]

  \[ \beta \Delta i_{i,t} - \tilde{\pi}_{i,t+1} + \tilde{s}_{i,t+1} - \tilde{s}_{i,t} = \beta \Delta i_{1,t} - \tilde{\pi}_{1,t+1} + \tilde{s}_{1,t+1} - \tilde{s}_{1,t} + t \frac{B_1^*}{Y_1} B_{1,t}^* \]

  for \( n > 1 \)

\(^9\)Our baseline model features incomplete markets. As an alternative, we could assume complete markets. Here, we present the “international Euler equation” for both cases for the sake of completeness.
24. Budget constraint (for incomplete market case)

\[ \frac{\Delta B^i_t}{Y_{1,t}} = 0 \]

\[ N^i \left( \frac{1}{\beta} \Delta B^i_{t-1} - \Delta B^i_t \right) = N^i \left( Y^i_{1,t} - Q^i \left( \frac{\bar{p}^i_{t}}{P^i_{t-1}} + \bar{Q}^i_{t} \right) \right) \]

\[ - \sum_{j \neq i} n^i_j N^i w^h_{j,t} \left( \bar{w}^h_{j,t} + \bar{\rho}^i_{j,t} \right) - n^i_j N^i u^h_{i,t} \left( \bar{w}^h_{i,t} + \bar{\rho}^i_{i,t} \right) \]

\[ + \sum_{j \neq i} n^i_j N^i \gamma^i_{j,t} - n^i_j N^i c^i_{j,t} \] for \( i > 1 \)

25. Location choice\(^\text{10}\)

\[ \frac{1}{\gamma} \left( \ln(n^i_{j,t}) + 1 \right) = u(c^i_{j,t}, l^i_{j,t}) - u(c^i_{i,t}, l^i_{i,t}) + A^i_j - \left( c^i_{j,t} - w^h_{j,t} l^i_{j,t} + \Phi^i_{j,t} \right) n^i_{j,t} \]

\[ - \left( \frac{n^i_{j,t}}{n^i_{j,t-1}} \Phi^i_{j,t} \right)^{n^i_{j,t-1}} \left( \Phi^i_{j,t} \right)^{n^i_{j,t}} \right) u_{1,j,t} \]

\[ + \beta \left( \frac{n^i_{j,t} + 1}{n^i_{j,t}} \right)^2 \left( \Phi^i_{j,t+1} \right)^{n^i_{j,t+1}} \] for \( i \neq j \)

\[ \frac{1}{\gamma} \tilde{n}^i_{j,t} = w^h_{j,t} l^i_{j,t} n^i_{j,t} - w^h_{i,t} l^i_{i,t} - \Phi^i \left( (1 + \beta) \tilde{n}^i_{j,t} - \tilde{n}^i_{j,t-1} - \beta \tilde{n}^i_{j,t+1} \right) \] for \( i \neq j \)

\[ \sum_j n^i_{j,t} = 1 \]

\[ \sum_j n^i_j \tilde{n}^i_{j,t} = 0 \]

\(^{10}\)Log-linearizing yields

\[ \frac{1}{\gamma} \tilde{n}^i_{j,t} = \left( u^i_{1,j} - u^i_{1,i} \right) c^i_{j,t} + \left( u^i_{2,j} - w^h_{j,i} u^i_{1,j} \right) l^i_{j,t} \tilde{c}^i_{j,t} - \left( c^i_{j,t} - w^h_{j,t} l^i_{j,t} - \Phi^i_{j,t} \right) u^i_{1,j} \tilde{n}^i_{j,t} + w^h_{j,t} l^i_{1,j} \tilde{w}^h_{j,t} \]

\[ - \left( u^i_{1,i} - u^i_{1,i} \right) c^i_{j,t} + \left( u^i_{2,i} - w^h_{i,i} u^i_{1,i} \right) l^i_{i,t} \tilde{c}^i_{i,t} - \left( c^i_{i,t} - w^h_{i,t} l^i_{i,t} - \Phi^i_{i,t} \right) u^i_{1,i} \tilde{n}^i_{i,t} + w^h_{i,t} l^i_{1,i} \tilde{w}^h_{i,t} \]

\[ - u^i_{1,i} \Phi^i \left( (1 + \beta) \tilde{n}^i_{j,t} - \tilde{n}^i_{j,t-1} - \beta \tilde{n}^i_{j,t+1} \right) \]

The terms related to \( \tilde{c}^i_{j,t} \) and \( \tilde{c}^i_{i,t} \) drop out. Similarly, the terms related to \( \tilde{p}^i_{j,t} \) and \( \tilde{p}^i_{j,t} \) drop out because in steady state, \( w^h_0 = \frac{w^h}{u^i_{1,j}} \) for all \( j \). Given our assumption that governments set transfers so as to balance out labor income and consumption expenditures in steady state, the terms related to \( \tilde{u}^i_{1,j,t} \) and \( \tilde{u}^i_{1,i,t} \) drop out. Finally, notice that the Backus-Smith condition among household members in steady state is simply \( u^i_{1,i} = u^i_{1,j} \) (since all real exchange rates are set to 1 in steady state).
26. Definition of population

\[ N_{i,t} = \sum_j n_{i,t}^j \]

\[ \tilde{N}_{i,t} = \sum_j \frac{\bar{N}_j}{\bar{N}_i} n_{i,t}^j \tilde{n}_{i,t}^j \]

27. Number of job hunters

\[ N_{i,t}H_{i,t} = N_{i,t-1}U_{i,t-1} + dN_{i,t-1}L_{i,t-1} + N_{i,t}u_{i,t} - N_{i,t-1}l_{i,t-1} \]

\[ [(1 - d)ur_i + d] \tilde{H}_{i,t} = ur_i\tilde{U}_{i,t-1} + d(1 - ur_i)\tilde{L}_{i,t-1} + (\bar{l}_{i,t} - \bar{l}_{i,t-1}) + (1 - d)(1 - ur_i) \left( \tilde{N}_{i,t} - \tilde{N}_{i,t-1} \right) \]

28. Labor market clearing

\[ U_{i,t} = l_{i,t} - L_{i,t} \]

\[ ur_i\tilde{U}_{i,t} = \bar{l}_{i,t} - (1 - ur_i)\bar{L}_{i,t} \]

29. Law of motion for employed workers

\[ N_{i,t}L_{i,t} = N_{i,t-1}L_{i,t-1} (1 - d) + N_{i,t}M_{i,t} \]

\[ \tilde{L}_{i,t} = (1 - d)\tilde{L}_{i,t-1} + d\tilde{M}_{i,t} - (1 - d) \left( \tilde{N}_{i,t} - \tilde{N}_{i,t-1} \right) \]

30. Matching function

\[ M_{i,t} = H_{i,t}\bar{m}\lambda_{i,t}^{1 - \zeta} \]

\[ \tilde{M}_{i,t} = \tilde{H}_{i,t} + (1 - \zeta)\tilde{\lambda}_{i,t} \]

31. Job finding rate

\[ f_{i,t} = \bar{m}\lambda_{i,t}^{1 - \zeta} \]

\[ \tilde{f}_{i,t} = (1 - \zeta)\tilde{\lambda}_{i,t} \]

32. Job filling rate

\[ g_{i,t} = \bar{m}\lambda_{i,t}^{-\zeta} \]

\[ \tilde{g}_{i,t} = -\zeta\tilde{\lambda}_{i,t} \]
33. Value of a posted vacancy

\[ V_{i,t} = \nabla + g_{i,t} \bar{J}_{i,t} + (1 - g_{i,t}) \beta \bar{V}_{i,t+1} \]

\[ V_{i,t} = \nabla (J_{i} - \nabla) \bar{J}_{i,t} + g_{i,t} \bar{J}_{i,t} + (1 - g_{i,t}) \beta V_{i,t+1} \]

\[ \Delta V_{i,t} = g_{i,t} \bar{J}_{i,t} + g_{i,t} \bar{J}_{i,t} + (1 - g_{i,t}) \beta \Delta V_{i,t+1} \]

34. Value of a filled vacancy

\[ J_{i,t} = w_{i,t} - w_{i,t} + \beta E_{t} \{ \Psi_{i,t+1} [(1 - d) J_{i,t+1} + dV_{i,t+1}] \} \]

\[ J_{i,t} = w_{i,t} - w_{i,t} + \beta J_{i,t+1} + d \beta \Delta V_{i,t+1} \]

35. Vacancy adjustment cost

\[ \nu_{i,t} = \Psi_{i,t} + V_{i,t} - 1 \nu'_{i,t} - \beta \nu'_{i,t+1} \]

\[ \frac{1}{\nu_{i,t}} \Delta \nu_{i,t} = (1 + \beta) \left( \nu_{i,t+1} + \nu_{i,t} \right) - \beta \left( \nu_{i,t+1} + \nu_{i,t} \right) \]

36. Value of being employed

\[ \tau_{i,t} = w_{i,t} - w_{i,t} + (1 - d) \beta \tau_{i,t+1} \]

\[ \tau_{i,t} = w_{i,t} - w_{i,t} + (1 - d) \beta \tau_{i,t+1} \]

37. Free entry of employment agencies

\[ f_{i,t} \tau_{i,t} = (1 - f_{i,t}) \left( w_{i,t} - b_{i} \right) \]

\[ f_{i,t} \tau_{i,t} = (1 - f_{i,t}) \left( w_{i,t} - b_{i} + \tau_{i,t} \right) \]

38. Wage determination

\[ \theta^{w} w_{i,t} = \theta^{w} w_{i,t-1} + (1 - \theta^{w}) \left[ g \bar{J}_{i,t} - (1 - \theta^{w}) (\tau_{i,t} - U_{i,t}) \right] \]

\[ \theta^{w} w_{i,t} = \theta^{w} w_{i,t-1} + (1 - \theta^{w}) \left[ g \bar{J}_{i,t} - (1 - \theta^{w}) (\tau_{i,t} + \bar{J}_{i,t}) \right] \]

39. Percentage point change in the unemployment rate

\[ \Delta ur_{i,t} = ur_{i} \left( \bar{U}_{i,t} - \bar{U}_{i,t} \right) . \]
2.4 Combined Log-Linearized Equilibrium Conditions

We combine a few equations to reduce the number of equations and variables in the system.

2.4.1 DSGE block

We remove the following equations

- Marginal utility of consumption (1) \( c_i^j,t \)
- Marginal rate of substitution (2) \( l^i_j,t \)
- Risk sharing among household members (3) \( u^i_j,t, j \neq i \)
- Labor supply (4) \( mrs^i_{j,t} \)
- Optimal capital utilization (10) \( u^i_{i,t} \)
- FOC wrt \( y^i_{i,t} \) (13) \( y^i_{i,t} \)
- Definition of Terms of Trade (20) \( ToT_{i,t} \)
- Definition of change in nominal exchange rate (22) \( \Delta S_{i,t} \)

**Definition of consumption (5)** Combining the Risk sharing among household members (3)

\[
\tilde{u}_{1,j,t}^i = \tilde{u}_{1,i,t}^i + \tilde{s}_{j,t} - \tilde{s}_{i,t}
\]

with the Marginal utility of consumption (1)

\[
\tilde{u}_{1,j,t}^i = -\frac{1}{\sigma} \tilde{c}_{j,t} \quad \text{separable}
\]

\[
-\sigma (u^i_{1,j})^{-\sigma} \tilde{u}_{1,j,t}^i = c^i_j \tilde{c}_{j,t}^i - \eta l^i_j w^h \tilde{w}_{j,t}^h \quad \text{GHH}
\]

and (for GHH preferences) with the Marginal rate of substitution (2)

\[
\tilde{mrs}_{j,t}^i = \frac{1}{\eta} \tilde{l}_{j,t}^i
\]

plus the labor supply condition, (4) to replace \( \tilde{mrs}_{j,t}^i = \tilde{w}_{j,t}^h \) gives

\[
\tilde{u}_{1,i,t}^i + \tilde{s}_{j,t} - \tilde{s}_{i,t} = -\frac{1}{\sigma} \tilde{c}_{j,t}^i \quad \text{separable}
\]

\[
-\sigma (u^i_{1,j})^{-\sigma} (\tilde{u}_{1,i,t}^i + \tilde{s}_{j,t} - \tilde{s}_{i,t}) = c^i_j \tilde{c}_{j,t}^i - \eta l^i_j w^h \tilde{w}_{j,t}^h \quad \text{GHH}
\]
Inserting this into the Definition of consumption (5):

\[
\left( \tilde{C}_{i,t} + \tilde{N}_{i,t} \right) = \sum_j \frac{N_j^i}{N_i^i} c_j^i \left( \tilde{n}_j^i + \tilde{c}_j^i \right)
\]

gives

\[
\left( \tilde{C}_{i,t} + \tilde{N}_{i,t} \right) = \sum_j \frac{N_j^i}{N_i^i} \frac{c_j^i}{C_i^i} n_j^i \left( \tilde{n}_j^i - \sigma \left( \tilde{u}_{1,j,t} - \tilde{s}_{j,t} \right) \right) \quad \text{separable}
\]

\[
\left( \tilde{C}_{i,t} + \tilde{N}_{i,t} \right) = \sum_j \frac{N_j^i}{N_i^i} \frac{c_j^i}{C_i^i} n_j^i \left( \tilde{n}_j^i - \sigma \left( \frac{\left( u_{1,j}^j \right)^{\sigma}}{c_j^i} \left( \tilde{u}_{1,j,t} + \tilde{s}_{j,t} - \tilde{s}_{i,t} \right) + \eta l_i \frac{w_h^j}{c_j^i} \tilde{w}_h^j \right) \right) \quad \text{GHH}
\]

**Definition of the labor force (6)** Combining the marginal rate of substitution (2)

\[
\tilde{mrs}_s^i = \frac{1}{\eta} \tilde{l}_{j,t} - \tilde{w}_h^j \quad \text{separable}
\]

\[
\tilde{mrs}_s^i = \frac{1}{\eta} \tilde{l}_{j,t} \quad \text{GHH}
\]

with the labor supply condition, (4) to replace \( \tilde{mrs}_s^i = \tilde{w}_h^j \) and (for separable preferences) the Risk sharing condition among household members (3) gives

\[
\tilde{w}_h^j = \frac{1}{\eta} \tilde{l}_{j,t} - \tilde{u}_{1,i,t} - \tilde{s}_{j,t} + \tilde{s}_{i,t} \quad \text{separable}
\]

\[
\tilde{w}_h^j = \frac{1}{\eta} \tilde{l}_{j,t} \quad \text{GHH}
\]

Inserting this into the Definition of the labor force (6)

\[
\left( \tilde{l}_{i,t} + \tilde{N}_{i,t} \right) = \sum_j \frac{N_j^i}{N_i^i} \frac{l_j^i}{l_i^i} n_j^i \left( \tilde{n}_j^i + \tilde{p}_j^i \right)
\]

gives

\[
\left( \tilde{l}_{i,t} + \tilde{N}_{i,t} \right) = \sum_j \frac{N_j^i}{N_i^i} \frac{l_j^i}{l_i^i} n_j^i \left( \tilde{n}_j^i + \eta \left( \tilde{w}_h^j + \tilde{w}_{1,j,t} - \tilde{s}_{j,t} + \tilde{s}_{i,t} \right) \right) \quad \text{separable}
\]

\[
\left( \tilde{l}_{i,t} + \tilde{N}_{i,t} \right) = \sum_j \frac{N_j^i}{N_i^i} \frac{l_j^i}{l_i^i} n_j^i \left( \tilde{n}_j^i + \eta \tilde{w}_h^j \right) \quad \text{GHH}
\]
Optimal factor employment (11) Inserting the Optimal capital utilization (10)

\[ \bar{u}_{i,t} = \frac{r^k_i}{a'' u_i} \bar{r}_{i,t} \]

into the Optimal factor employment (11)

\[ \bar{r}^k_{i,t} - \bar{w}^f_{i,t} = \tilde{N}_{i,t} + \bar{L}_{i,t} - \bar{u}_{i,t} - \tilde{K}_{i,t-1} - \tilde{N}_{i,t-1} \]

gives

\[ \bar{r}^k_{i,t} - \bar{w}^f_{i,t} = \tilde{N}_{i,t} + \bar{L}_{i,t} - \frac{r^k_i}{a'' u_i} \bar{r}_{i,t} - \tilde{K}_{i,t-1} - \tilde{N}_{i,t-1}. \]

Production of intermediate good (14) Inserting the Optimal capital utilization (10)

\[ \bar{u}_{i,t} = \frac{r^k_i}{a'' u_i} \bar{r}_{i,t} \]

into the Production of intermediate good (14)

\[ \tilde{Q}_{i,t} = \tilde{Z}_{i,t} + \alpha \left( \bar{u}_{i,t} + \tilde{N}_{i,t-1} + \tilde{K}_{i,t-1} - \tilde{N}_{i,t-1} \right) + (1 - \alpha) \tilde{L}_{i,t} \]

gives

\[ \tilde{Q}_{i,t} = \tilde{Z}_{i,t} + \alpha \left( \frac{r^k_i}{a'' u_i} \bar{r}_{i,t} + \tilde{N}_{i,t-1} + \tilde{K}_{i,t-1} - \tilde{N}_{i,t-1} \right) + (1 - \alpha) \tilde{L}_{i,t}. \]

Production of final good (15) Inserting the FOC wrt \( y^j_{i,t} \) (13)

\[ \tilde{y}^j_{i,t} = \tilde{Y}_{i,t} + \left( \varepsilon^j_t - \sum_k \tilde{\omega}_i^k \varepsilon^k_t \right) - \psi_y \left( \frac{\tilde{p}_{j,t}}{\tilde{P}_{j,t}} \tilde{s}_{j,t} - \tilde{s}_{i,t} \right) \]

into the Production of final good (15)

\[ \tilde{Y}_{i,t} = \sum_{j=1}^N \tilde{\omega}_i^j \left( \tilde{y}^j_{i,t} + \frac{1}{\psi_y - 1} \left( \varepsilon^j_t - \sum_k \tilde{\omega}_i^k \varepsilon^k_t \right) \right) \]

gives

\[ \psi_y \sum_{j=1}^N \tilde{\omega}_i^j \left( \frac{\tilde{p}_{j,t}}{\tilde{P}_{j,t}} + \tilde{s}_{j,t} - \tilde{s}_{i,t} \right) = \sum_{j=1}^N \tilde{\omega}_i^j \left( 1 + \frac{1}{\psi_y - 1} \right) \left( \varepsilon^j_t - \sum_k \tilde{\omega}_i^k \varepsilon^k_t \right) \]

\[ (\psi_y - 1) \sum_{j=1}^N \tilde{\omega}_i^j \left( \frac{\tilde{p}_{j,t}}{\tilde{P}_{j,t}} + \tilde{s}_{j,t} - \tilde{s}_{i,t} \right) = \sum_{j=1}^N \tilde{\omega}_i^j \left( \varepsilon^j_t - \sum_k \tilde{\omega}_i^k \varepsilon^k_t \right) \]
Market clearing for intermediate good (16) Inserting the FOC wrt $y_{j,t}^i$ (13)

$$\tilde{y}_{j,t}^i = \tilde{Y}_{j,t} + \left( \varepsilon_t^j - \sum_k \tilde{w}_{j,k}^i \varepsilon_t^k \right) - \psi_y \left( \frac{\tilde{p}_{i,t}}{\tilde{P}_{i,t}} + \tilde{s}_{i,t} - \tilde{s}_{j,t} \right)$$

into the Market clearing for intermediate good (16)

$$\frac{Q_i}{Y_i} \left( \tilde{N}_{i,t} + \tilde{Q}_{i,t} \right) = \sum_{j=1}^{N} \frac{N_j Y_j}{N_i Y_i} \tilde{\omega}_{i,j} \left( \tilde{N}_{j,t} + \tilde{y}_{j,t}^i \right)$$

gives

$$\frac{Q_i}{Y_i} \left( \tilde{N}_{i,t} + \tilde{Q}_{i,t} \right) = \sum_{j=1}^{N} \frac{N_j Y_j}{N_i Y_i} \tilde{\omega}_{i,j} \left( \tilde{N}_{j,t} + \tilde{Y}_{j,t} + \left( \varepsilon_t^j - \sum_k \tilde{w}_{j,k}^i \varepsilon_t^k \right) - \psi_y \left( \frac{\tilde{p}_{i,t}}{\tilde{P}_{i,t}} + \tilde{s}_{i,t} - \tilde{s}_{j,t} \right) \right)$$

Market clearing for final good (17) Inserting the relationship between $\tilde{f}_{i,t}$ and $\tilde{g}_{i,t}$,

$$\tilde{g}_{i,t} = -\frac{\zeta}{1 - \zeta} \tilde{f}_{i,t},$$

and the Optimal capital utilization (10)

$$\tilde{u}_{i,t} = \frac{r_{k}^i}{a'' u_{i}} \tilde{r}_{i,t}$$

into the Market clearing condition for final goods (17):

$$\tilde{Y}_{i,t} = \frac{C_i}{Y_i} \tilde{C}_{i,t} + \frac{X_i}{Y_i} \tilde{X}_{i,t} + \frac{G_i}{Y_i} \tilde{G}_{i,t} + r_{k}^i \frac{K_i}{Y_i} \tilde{u}_{i,t} + \frac{\varsigma V_i}{Y_i} \left( \tilde{f}_{i,t} - \tilde{g}_{i,t} + \tilde{H}_{i,t} \right)$$

gives

$$\tilde{Y}_{i,t} = \frac{C_i}{Y_i} \tilde{C}_{i,t} + \frac{X_i}{Y_i} \tilde{X}_{i,t} + \frac{G_i}{Y_i} \tilde{G}_{i,t} + r_{k}^i \frac{K_i}{Y_i} \tilde{u}_{i,t} + \varsigma V_i \left( \frac{1}{1 - \zeta} \tilde{f}_{i,t} + \tilde{H}_{i,t} \right)$$

Phillips curve (19) Inserting the Real marginal costs (12)

$$\tilde{mc}_{i,t} = -\tilde{Z}_{i,t} + \alpha \tilde{r}_{i,t} + (1 - \alpha) \tilde{w}_{i,t}^f$$

and the Definition of Terms of Trade (20)

$$\tilde{ToT}_{i,t} = \left( \frac{\tilde{p}_{i,t}}{\tilde{P}_{i,t}} \right) - \left( \frac{\tilde{p}_{i,\text{t-1}}}{\tilde{P}_{i,\text{t-1}}} \right)$$

27
into the Phillips curve (19)

\[
\theta_p \left( \tilde{\pi}_{i,t} + \tilde{T}_o \tilde{T}_{i,t} \right) = (1 - \theta_p)(1 - \theta_p \beta) \left[ \tilde{m} \tilde{c}_{i,t} - \left( \frac{\tilde{p}_{i,t}}{\tilde{P}_{i,t}} \right) \right] + \theta_p \beta \left( \tilde{\pi}_{i,t+1} + \tilde{T}_o \tilde{T}_{i,t+1} \right)
\]

gives

\[
\theta_p \left( \tilde{\pi}_{i,t} - \left( \frac{\tilde{p}_{i,t-1}}{\tilde{P}_{i,t-1}} \right) \right) = (1 - \theta_p)(1 - \theta_p \beta) \left( -\tilde{Z}_{i,t} + \alpha \tilde{r}_{i,t} + (1 - \alpha) \tilde{w}_{i,t} \right) - (1 - \theta_p \beta) \left( \frac{\tilde{p}_{i,t}}{\tilde{P}_{i,t}} \right) + \theta_p \beta \left( \tilde{\pi}_{i,t+1} + \left( \frac{\tilde{p}_{i,t+1}}{\tilde{P}_{i,t+1}} \right) \right)
\]

**Monetary policy (21)**

Inserting the Definition of change in nominal exchange rate (22)

\[
\tilde{\Delta} S_{i,t} = (\tilde{s}_{i,t} - \tilde{s}_{i,t-1}) - \tilde{\pi}_{i,t}
\]

into the monetary policy rule for followers under fixed exchange rates (21)

\[
\tilde{\Delta} S_{j,t} = \tilde{\Delta} S_{i,t}
\]

gives

\[
(\tilde{s}_{j,t} - \tilde{s}_{j,t-1}) - \tilde{\pi}_{j,t} = (\tilde{s}_{i,t} - \tilde{s}_{i,t-1}) - \tilde{\pi}_{i,t}
\]

**Budget constraint (24)**

Combining the marginal rate of substitution (2) with the labor supply condition, (4) to replace \( \tilde{m} \tilde{r} s_{j,t} = \tilde{w}_{j,t} \) and (for separable preferences) the Risk sharing condition among household members (3) gives

\[
\tilde{l}_{j,t} = \eta \left( \tilde{w}_{j,t} + \tilde{u}_{i,t} + \tilde{s}_{j,t} - \tilde{s}_{i,t} \right) \quad \text{separable}
\]
\[
\tilde{l}_{j,t} = \eta \tilde{w}_{j,t} \quad \text{GHH.}
\]

Similarly, combining the Risk sharing among household members (3) with the Marginal utility of consumption (1) and (for GHH preferences) with the Marginal rate of substitution (2) plus the labor supply condition, (4) to replace \( \tilde{m} \tilde{r} s_{j,t} = \tilde{w}_{j,t} \) gives

\[
\tilde{c}_{j,t} = \sigma c_{j,t} \left( -\tilde{u}_{i,t} + \tilde{s}_{j,t} - \tilde{s}_{i,t} \right) \quad \text{separable}
\]
\[
\tilde{c}_{j,t} = -\sigma (\tilde{u}_{i,j,t})^{-\sigma} \left( \tilde{u}_{i,t} + \tilde{s}_{j,t} - \tilde{s}_{i,t} \right) + \eta l_{j} w_{j} \tilde{w}_{j,t} \quad \text{GHH.}
\]
Inserting this into the Budget constraint (24) (for \( i > 1 \))

\[
N_i \left( \frac{1}{\beta} \Delta B_{i,t-1} - \Delta B_i^t \right) = N_i \left( Y_i \bar{Y}_{i,t} - Q_t \left( \frac{\bar{p}_{t,i}}{\bar{p}_{i,t}} + \bar{Q}_{i,t} \right) \right)
- \sum_{j \neq i} n_j^i N_i w_{j,t}^h l_{i,t}^j \left( \bar{w}_{j,t}^h + \bar{l}_{i,t}^j \right) - n_i^j p_{j,t}^h l_{i,t}^j \left( \bar{w}_{i,t}^h + \bar{l}_{i,t}^j \right)
+ \sum_{j \neq i} n_j^i N_i c_{j,t}^i - n_i^j p_{j,t}^h c_{i,t}^j
\]

gives for separable preferences

\[
N_i \left( \frac{1}{\beta} \Delta B_{i,t-1} - \Delta B_i^t \right) = N_i \left( Y_i \bar{Y}_{i,t} - Q_t \left( \frac{\bar{p}_{t,i}}{\bar{p}_{i,t}} + \bar{Q}_{i,t} \right) \right)
- \sum_{j \neq i} n_j^i N_i w_{j,t}^h l_{i,t}^j \left( \bar{w}_{j,t}^h (1 + \eta) + \eta \left( \bar{u}_{1,i,t}^j + \bar{s}_{j,t} - \bar{s}_{i,t} \right) \right)
+ \sum_{j \neq i} n_j^i N_i w_{j,t}^h l_{i,t}^j \left( \bar{w}_{j,t}^h (1 + \eta) + \eta \left( \bar{u}_{1,j,t}^i + \bar{s}_{i,t} - \bar{s}_{j,t} \right) \right)
+ \sum_{j \neq i} n_j^i N_i \sigma c_{j,t}^i \left( \bar{u}_{1,i,t}^j - \bar{s}_{j,t} + \bar{s}_{i,t} \right) - n_i^j p_{j,t}^h \sigma c_{j,t}^i \left( -\bar{u}_{1,j,t}^i - \bar{s}_{j,t} + \bar{s}_{i,t} \right)
\]

and for GHH preferences

\[
N_i \left( \frac{1}{\beta} \Delta B_{i,t-1} - \Delta B_i^t \right) = N_i \left( Y_i \bar{Y}_{i,t} - Q_t \left( \frac{\bar{p}_{t,i}}{\bar{p}_{i,t}} + \bar{Q}_{i,t} \right) \right)
- \sum_{j \neq i} n_j^i N_i w_{j,t}^h l_{i,t}^j \left( (1 + \eta) \bar{w}_{j,t}^h - n_j^i N_i w_{j,t}^h l_{i,t}^j (1 + \eta) \bar{w}_{j,t}^h \right)
+ \sum_{j \neq i} n_j^i N_i \left( \sigma (u_{1,j}^i)^{-\sigma} \left( -\bar{u}_{1,i,t}^j - \bar{s}_{j,t} + \bar{s}_{i,t} \right) + \eta l_{j}^i w_{j,t}^h \bar{w}_{j,t}^h \right)
+ \sum_{j \neq i} n_j^i N_i \left( \sigma (u_{1,i}^j)^{-\sigma} \left( -\bar{u}_{1,j,t}^i - \bar{s}_{j,t} + \bar{s}_{i,t} \right) + \eta l_{i}^j w_{i,t}^h \bar{w}_{i,t}^h \right)
\]

### 2.4.2 Search block

We remove the following equations

- Labor market clearing (28) \((U_{i,t})\)
• Matching function (30) \((M_{i,t})\)
• Job finding rate (31) \((\lambda_{i,t})\)
• Job filling rate (32) \((g_{i,t})\)

**Number of job hunters (27)** Inserting the Labor market clearing condition (28)

\[
ur_i \tilde{U}_{i,t} = \tilde{l}_{i,t} - (1 - ur_i)\tilde{L}_{i,t}
\]

into the Number of job hunters (27):

\[
[(1 - d)ur_i + d] \tilde{H}_{i,t} = ur_i \tilde{U}_{i,t-1} + d(1 - ur_i) \tilde{L}_{i,t-1} + (\tilde{l}_{i,t} - \tilde{l}_{i,t-1}) + (1 - d)(1 - ur_i)(\tilde{N}_{i,t} - \tilde{N}_{i,t-1})
\]
gives

\[
[(1 - d)ur_i + d] \tilde{H}_{i,t} = (d - 1)(1 - ur_i) \tilde{L}_{i,t-1} + \tilde{l}_{i,t} + (1 - d)(1 - ur_i)(\tilde{N}_{i,t} - \tilde{N}_{i,t-1}).
\]

**Law of motion for employed workers (29)** Inserting the Matching function (30)

\[
\tilde{M}_{i,t} = \tilde{H}_{i,t} + (1 - \zeta)\tilde{\lambda}_{i,t}
\]

and the Job finding rate (31)

\[
\tilde{f}_{i,t} = (1 - \zeta)\tilde{\lambda}_{i,t}
\]

into the Law of motion for employed workers (29)

\[
\tilde{L}_{i,t} = (1 - d)\tilde{L}_{i,t-1} + d\tilde{M}_{i,t} - (1 - d)\left(\tilde{N}_{i,t} - \tilde{N}_{i,t-1}\right)
\]
gives

\[
\tilde{L}_{i,t} = (1 - d)\tilde{L}_{i,t-1} + d\left(\tilde{H}_{i,t} + \tilde{f}_{i,t}\right) - (1 - d)\left(\tilde{N}_{i,t} - \tilde{N}_{i,t-1}\right).
\]

**Value of a posted vacancy (33)** Combining the Job filling rate (32)

\[
\tilde{g}_{i,t} = -\zeta\tilde{\lambda}_{i,t}
\]

with the Job finding rate (31)

\[
\tilde{f}_{i,t} = (1 - \zeta)\tilde{\lambda}_{i,t}
\]

gives

\[
\tilde{g}_{i,t} = -\frac{\zeta}{1 - \zeta}\tilde{f}_{i,t}
\]
Inserting this into the Value of a posted vacancy (33)

\[ \Delta V_{i,t} = g_i J_i \tilde{g}_{i,t} + g_i J_i \tilde{J}_{i,t} + (1 - g_i) \beta \Delta V_{i,t+1} \]

yields

\[ \Delta V_{i,t} = -g_i J_i \frac{\zeta}{1 - \zeta} \tilde{f}_{i,t} + g_i J_i \tilde{J}_{i,t} + (1 - g_i) \beta \Delta V_{i,t+1}. \]

Vacancy adjustment cost (35) Inserting the Job finding rate (31)

\[ \tilde{f}_{i,t} = (1 - \zeta) \tilde{\lambda}_{i,t} \]

into the Vacancy adjustment cost (35)

\[ \frac{1}{\Upsilon''} \Delta V_t = (1 + \beta) \left( \tilde{\lambda}_t + \tilde{H}_t \right) - \left( \tilde{\lambda}_{t-1} + \tilde{H}_{t-1} \right) - \beta \left( \tilde{\lambda}_{t+1} + \tilde{H}_{t+1} \right) \]

gives

\[ \frac{1}{\Upsilon''} \Delta V_t = (1 + \beta) \left( \frac{1}{1 - \zeta} \tilde{f}_t + \tilde{H}_t \right) - \left( \frac{1}{1 - \zeta} \tilde{f}_{t-1} + \tilde{H}_{t-1} \right) - \beta \left( \frac{1}{1 - \zeta} \tilde{f}_{t+1} + \tilde{H}_{t+1} \right) \]

Definition of unemployment rate (39) Inserting the Labor market clearing condition (28)

\[ ur_i \tilde{U}_{i,t} = \tilde{l}_{i,t} - (1 - ur_i) \tilde{L}_{i,t} \]

into the Percentage point change in the unemployment rate (39)

\[ \Delta ur_{i,t} = ur_i \left( \tilde{U}_{i,t} - \tilde{l}_{i,t} \right) \]

gives

\[ \Delta ur_{i,t} = (1 - ur_i) \left( \tilde{l}_{i,t} - \tilde{L}_{i,t} \right) \]