Bioeconomics: Cyclical Equilibrium in a Sequential-Game Ecosystem

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December 11, 2022

1 Abstract

This paper is one of applied Game Theory. It aims to use a Game-Theoretic framework to provide a mathematical, recursive model of natural selection and establish an equilibrium for an ecosystem. By our definition of natural selection, we can then compute from a notion of fitness which animals will be selected and which will not.

To do this, we consider an ecosystem to be a sequential game, with nature itself as each player. Nature can then "select" strategies, which are the animals of the ecosystem, and it will eventually settle on some equilibrium based on the fitness of the animals and how they interact with each other. Using our definition of fitness and this sequential game, we can iterate through the "stages" of an ecosystem, represented as individual games that play out consecutively, giving us a way to represent and understand the progression of the ecosystem through time.

Ultimately, we will be able to form a complete profile of the ecosystem, including how it evolves over time and the distribution of animals after a selection. We show that every ecosystem, given infinite time to develop, will always reach some cyclical equilibrium. This will include some set of animals (represented as strategies) that deterministically occur in the same order repeatedly as the game progresses.

2 Introduction

The prisoner's dilemma is the quintessential example of modern game theory. The problem is as follows: two accomplices of a crime are interrogated individually for the same crime. Each player faces two choices: cooperate (stay silent) or defect (betray the other). If both cooperate, they'll get two jail years each. If one cooperates and defects, the one who defects gets off free, but the one who cooperates gets 10 years. If both defect, they both get 7 years. As it turns out, the equilibrium outcome is for both to defect, even though that's the worst case scenario (most jail time combined). The prisoner's dilemma is supposed to always end this way in the long run, because both players will always try to maximize their utility. But it doesn't always turn out this way. Other factors like loyalty, emotion, or fear may add or detract from their utility, which can cause difficulty when creating a real-life model. Many academics think this model isn't actually the best way to use game theory, which is best for modeling clear notions of utility and predictable economic agents.

There exists a different but clear use for classical game theory: Darwinian Evolution. In the fight for survival, organisms make strategic decisions regarding food, competition, habitat, and reproduction, and these strategies fit well into a game theoretic model. An ambiguous notion of utility is replaced by a linear and measurable notion of fitness. Additionally, the payoffs of different strategies are on a single scale as payoffs are only measured in terms of evolutionary fitness. Utility can be measured on different scales, depending on whether it contributes to livelihood, happiness, posterity, etc., all of which are hard to define and measure.

3 Overview

This paper aims to create a mathematical model for the prediction of an ecosystem's evolution. A philosophical choice of this paper is that evolution is a *sequential game*, not a simultaneous one. This means that nature does not select all animals at the same time. Instead, nature applies selective pressures, which result in one selection. That selection then becomes a selective pressure for the second selection, which influences the third selection, so on and so on. Evolution is iterative, and it occurs in discrete steps. The truth of this assumption is debatable, but it will be taken axiomatically for this paper.

Using this assumption, it is possible to develop an algorithm to map out each stage of evolution, and the evolution of the ecosystem as a whole. By determining how much *fitness* (the evolutionary advantage to reproduce) each animal gains from each interaction, it is possible to distinguish some interactions as better than others, because they contribute more to evolutionary fitness. Similarly, we can distinguish some animals as more fit than others, because they have more fitness. The animals with the most fitness will be positively selected by the environment. We can then iterate through the ecosystem to see which animals are most fit at each stage, forming a complete picture of how natural selection operates in the system. Finally, we can find equilibrium after some indefinite number of rounds.

The model does not account for a simultaneous ecosystem, which may be possible. By nature of the assumption that evolution occurs sequentially, I omit possibilities of a simultaneous evolution, where multiple different animals are all selected at the same time. In a simultaneous case, a Nash Equilibrium is a more appropriate measure of optimum than the Equilibrium Selection Profile (ESP) that I describe. Furthermore, the model is oversimplified. It assumes that the frequency of each animal is constant, even as the ecosystem evolves. This cannot be true, because as some animals are selected positively and negatively, their frequencies increase and decrease respectively, according to the definition of natural selection.

4 Literature Review

Game theory as an academic subject is a product of the 20th century. It started off famously with mathematician John Nash, who invented the field in his 1950 paper "Equilibrium points in N-person games." There, the concept of the Nash Equilibrium was born, founding the basis of behavioral analysis as we know it today in mathematics.

.Soon after, it was applied outside of its original context for human decision-making . In the 1960s and 1970s, several authors began to write preliminary pieces on animal game theory, specifically on strategic interactions and natural selection, but the most important of all of them was John Maynard Smith, a Professor of Biology at the University of Sussex.

Smith had several important works. In 1972, he published "Game Theory and the Evolution of Fighting," where he introduced the concept of an evolutionary stable strategy (the details are not important for this paper), which was the first rigorous introduction of Game Theory into ecology. In 1982, ten years later, he published the foundation of evolutionary game theory: *Evolution and the Theory of Games*. In this book, Smith models several evolutionary games, including the Hawk-Dove game, the war of attrition, and several asymmetric games. Smith's Hawk-Dove game has become the quintessential example of evolutionary game theory. This article will use the Hawk Dove game as a basis for an evolutionary game, and then extend this game to model an entire ecosystem. Some important terms and conventions for this paper come from this book.

One particularly important point for this paper is to make a paradigm shift introduced by Smith. In traditional game theory, individuals play strategies involving specific decisions. There, the individuals are the decision-makers . In modern literature about evolutionary game theory, however, the animals are not the players, but they are the strategies themselves. They are the ones being selected, not doing the selecting, so it is more helpful to think of them this way. In this case, the selecting agent (player) is nature itself. This book will think of animals the same way, which will be reflected in the later analysis. This philosophy, outlined by Smith, will provide useful when thinking about the process of natural selection. Eventually, with Smith's framework, it will be possible to map the selection chronology of an entire ecosystem.

5 Methodology

The hypothesis of this paper is as follows: Natural selection is a predictable phenomenon that can be accurately modeled with a Game Theoretic Model. Here, though, there is no conception of data from real ecosystems. This paper only aims to outline a theoretical model for predicting an arbitrary ecosystem.

This paper aims to evaluate the ecosystem at some notion of *optimum*, which would be some stable state of selection. For this paper, I will define the *Selection Profile* (SP) simply as a list of strategies selected by nature.

It is important to distinguish some assumptions and terminology that the model is based on.

First, I assume that interactions between animals happen according to their frequency in the Ecosystem, and I treat each animal's frequency as a discrete random variable, each independent of every other. (I ignore the effects of predatory relationships. For this paper, the frequency of one animal in a population is not dependent on any other). Because the intersection of two animals is the product of their independent events, the intersection itself is independent. (The intersection of event L with frequency $P(L)$ and event A with frequency $P(A)$ is event LA with frequency $P(L)P(A)$). Because intersections of animals happens with random frequency, I suggest that players cannot predict what animals they will interact with, so their most effective valuation of a strategy's payoff is it's *expected fitness* of a strategy. Then, a player's best strategy is the one with the highest *expected fitness*. This corresponds to a mechanism of natural selection; the strategy (animal) with the highest expected fitness is most likely to be selected by the player (nature). The intersection of 2 players' best strategies is a *Selection Profile*.

Second, I assume that the game of natural selection is *sequential*, meaning animals are selected in *sequence, not simultaneously*. This means that nature will select one animal, and that animal will select another, which will select another, (etc) until all animals have been accounted for.

Third, I will assume that each Player has a *unique* best response to each strategy. This means that the fitness gained from each strategy (in response to another player's strategy) is different from all others. The reason for this is to be able to calculate an equilibrium, which is discussed later.

Finally, the Ecosystems I discuss for a Multi-Player, Multi-Step Model, will be considered infinite when considering equilibrium. That is, to calculate an equilibrium, it is assumed that there are infinite time periods where a player could choose a strategy. This is because the discussed equilibrium is cyclical, and a finite time horizon could truncate the game in the middle of an equilibrium cycle. An infinite time horizon would allow the complete equilibrium to play out.

6 The Lion-Antelope Game

For this article's preliminary example, consider two strategies in an evolutionary game: A (cannibalistic) Lion (L) and an Antelope (A). The relationship between a predator and prey involves strategic interaction and planning for both parties to maximize utility, which in this case is Darwinian Fitness: the capacity to survive and reproduce. The strategies are as follows:

Lion: Fight, regardless of opponent, until incapacitated or it wins. If it wins, eat opponent to gain evolutionary fitness F. Assume fighting is always associated with cost Q. Assume $F > Q$.

Antelope: Retreat at threat, gain evolutionary fitness $G > 2F$. Assume the probability of success of retreating is $\frac{1}{4}$. This choice is arbitrary. There is a gain in fitness here because compared to the alternative of dying, running away and surviving is a net gain. Also, it is assumed that the fitness gained from running away is much greater than the fitness gained by the lion by eating after victory, because it preserves more fitness (survival) than eating. This is not necessarily true in the wild, but will stand for this article.

Otherwise, the antelope will do nothing and gain evolutionary fitness 0. No fitness is gained from this situation.

For each payoff, consider the following equations, where $P(X)$ describes the expected payoff of an action for player X. Let $p(X)$ describe the probability of winning for player X. Let $U(S)$ describe the utility gained from an outcome, with outcomes W and L representing a win and loss respectively. Then the payoffs can be described by the weighted average equation

$$P(X) = p(X)(U(W)) + (1 - p(X))(U(L)) \tag{1}$$

And the payoffs of Lions and Antelopes respectively are described by

$$P(L) = p(L)(F - Q) + (1 - p(L))(0) \tag{2}$$
$$P(A) = p(A)(G) + (1 - p(A))(0) \tag{3}$$

Then the payoff matrix for the situation (first payoff is for the left player, the second is for the right player) is as following:

$$\begin{bmatrix} & L & A \\ L & \frac{1}{2}(F - Q), \frac{1}{2}(F - Q) & \frac{3}{4}(F - Q), \frac{1}{4}G \\ A & \frac{1}{4}G, \frac{3}{4}(F - Q) & 0, 0 \end{bmatrix}$$

To explain the payoffs, consider each strategy combination. For L/L, each lion has a $\frac{1}{2}$ probability of winning, which comes with gain F and cost Q.

For L/A and A/L, the Lion had a $\frac{3}{4}$ probability of winning, with gain F and cost Q. The antelope has a $\frac{1}{4}$ probability of succeeding in running, with payoff G.

For A/A, neither antelope will fight, and neither will gain any payoff.

7 Extension to an n-Strategy game

Now consider a model with more than two strategies. Instead of just lions and antelopes, consider a full ecosystem E with n animals (strategies). Suppose for now that there are only two players, and that each animal represents a strategy.

Then we extend the payoff matrix from a 2 x 2 matrix to an n x n matrix. For simplicity, instead of one matrix representing all payoffs for all players, I will distinguish 2 separate n x n matrices, one with payoffs for the player on the left and another one with payoffs for the player on top.

Consider matrices E_λ E_α and $\in (\mathbb{R})^{nxn}$, which are extended versions of the matrix shows above.

$$\begin{bmatrix} & S_1 \cdots S_n \\ \\ S_1 & \lambda_{11} \cdots \lambda_{1n} \\ \vdots & \\ S_n & \lambda_{n1} \cdots \lambda_{nn} \end{bmatrix}, \begin{bmatrix} & S_1 \cdots S_n \\ \\ S_1 & \alpha_{11} \cdots \alpha_{1n} \\ \vdots & \\ S_n & \alpha_{n1} \cdots \alpha_{nn} \end{bmatrix} \tag{4}$$

where E_λ is the first matrix and E_α is the second. In these cases λ_{ij} denotes payoff for the player on the left and α_{ij} denotes payoff for the player on top. Payoffs are calculated according to (1). For both, $1 \leq i, j \leq n$. Also, for notation purposes, let $\vec{\lambda_i}$ denote the rows of E_λ and $\vec{\alpha_j}$ denote the columns of E_α.

8 The Two-Player, Two-Step Model

The goal here is to develop an algorithm that will rigorously track the evolution of an ecosystem. Given the players and circumstances, the algorithm should be able to develop the evolution of an ecosystem, and portray a profile of that ecosystem (and its evolution) concisely. It is helpful to distinguish the players as *Player on Left (PL)* and *Player on Top (PT)*. PL has payoffs λ_{ij} and PT has payoffs α_{ij}.

For reference, the expected value of event X with probability function $p(x)$ and outcomes $x_i, 1 \leq i \leq n$ takes form

$$EV = \sum_{i=1}^{n} p(x_i)(x_i) \tag{5}$$

To define our expected values most efficiently, I define a vector of probabilities

$$\vec{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix} \tag{6}$$

where $p_i, 1 \leq i \leq n$ describes the frequency of animal i in ecosystem E. ($\vec{p} \in (\mathbb{R})^{nx1}$). To find expected values using $\vec{p}$, define the transformation

$$EV(\vec{p}) = E\vec{p} : \mathbb{R}^n \to \mathbb{R}^n \tag{7}$$

where EV is a vector of expected values, E is the payoff matrix for some Ecosystem E and vector $\vec{p}$ defined as before.

Recall that evolution here is considered a sequential game, where the first selection is done by nature. Nature will select the strategy that maximizes *expected fitness*, because that strategy will have the highest selective advantage. To compute this, consider the matrix product

$$E_\lambda \vec{p} = \begin{bmatrix} \vec{\lambda_1} \cdot \vec{p_1} \\ \vdots \\ \vec{\lambda_n} \cdot \vec{p_n} \end{bmatrix} \tag{8}$$

is a vector where each $\vec{\lambda_i} \cdot \vec{p_i}$ computes the expected value for the left Player when playing strategy S_i. (in expanded form, each $\vec{\lambda_i} \cdot \vec{p} = \lambda_{i1} * p_1 + \ldots + \lambda_{in} * p_n$, which is the expected payoff for event S_i with outcomes λ_{ij} and associated frequencies p_j). Then the maximal component of vector $E_\lambda \vec{p}$, denoted Ŝ, with row σ, is the best strategy for PL. This strategy, which maximizes expected fitness, will be called PL's *efficient selection* (ES).

This is the first step in our two-step sequential game. Recall that there are two "players" in this game. Nature will select the first strategy, and then that strategy will contribute to the selection of the second. Ŝ denotes the first selection, done by nature.

To compute PT's Efficient Selection (denoted S̃), it is helpful to create some notation. If Ŝ is the first selection, then the second selection will be denoted S̃|Ŝ: S̃ given Ŝ. This makes it clear that S̃ is chosen conditionally based on Ŝ.

The computation of S̃|Ŝ is relatively simple. Using E_α, the matrix with payoffs according to the top player, find the row σ that corresponds to strategy

Ŝ. This row is the only relevant row of payoffs to choose from, because we assume that PL has already chosen its optimal strategy Ŝ. Now, PT must find its optimal choice *given* Ŝ. To do this, choose the $\alpha_{\sigma j}$ with the highest payoff. This is PT's best choice out of the options they have after player 1 chose Ŝ; it is PT's efficient selection..

Now an SP can be evaluated at the strategy profile (Ŝ,S̃|Ŝ). As an interpretation, this is the profile that will be selected in order – given the circumstances. Nature will select Ŝ, and Ŝ will help select S̃|Ŝ. The SP provides an evolutionary profile of an ecosystem after the game has been played. In classical game theory, this algorithm is called *best response iteration*, where Player 2 chooses their *best response* given a response by Player 1.

9 The Mutli-Player, Mutli-Step Model

This kind of iteration can be used in a game with more than two steps. The algorithm uses the process used in the Two-Player, Two-Step model iteratively to calculate the optimal strategy for each subsequent player, conditioned on the strategy of the previous player.

To follow this algorithm, use $E_\lambda \vec{p}$ to calculate Player 1's best choice, and then use the process above to determine Player 2's best response. To calculate Player 3's best response, use the same process used when calculating Player 2's best response. Now, however, Player 2 becomes PL, and their best response (calculated in the previous round) becomes the conditioning strategy to choose Player 3's best response.

For notation purposes the optimal strategy of, $P_i, 1 \leq i \leq n$ will be denoted S'_i. Furthermore, denote the E_α matrix with PL= P_i and PT=P_j as $E_{\alpha_{ij}}$. Lastly, the SP for round i, denoted SP_i, will be $(S'_1, S'_2, \ldots, S'_i, S'_{i+1})$.

For each subsequent player $P_j, 2 \leq j \leq n$, use the previous player's (P_{j-1}) strategy as the conditioning event, and use that to choose the optimal strategy for P_j. To do this, use $E_{\alpha_{j-1,j}}$, and use PL=$P_j - 1$'s optimal strategy to condition and find PT=P_j's best strategy. This algorithm is iterative, because it recursively uses the previous player's strategy to calculate the next player's strategy.

10 Equilibrium

An interesting outcome of the multi-player, multi-step model is the ability to form an equilibrium. The claim of this paper is that, the multi-step, multi-player model will *always* reach an equilibrium, specifically a *cyclical equilibrium.* This will be proved recursively.

To prove, this, consider an ecosystem E with n strategies $S_1, S_2, \dots, S_n$. Let S'_1 be Player 1's unique best strategy, determined as before above, according to their highest expected utility (Use $E\lambda\vec{p}$). Let $S'_2, \dots, S'_k$ be the best response strategies for players $P_2, \dots, P_k$, conditioned on the previous player's response (Use $E_{\alpha_{ij}}$as described above). Note that is assumed that $S'_1, \dots, S'_k$ are all uniquely determined, so for every computation of optimal strategies, the $S'_1, \dots, S'_k$ will always be the same.

To prove this recursively, start with $S'_1 = S_{p_1}$, with $1 \leq p_1 \leq n$. This simply means that Player 1's optimal strategies is one of the available strategies. Let $S'_2 = S_{p_2}$, with $1 \leq p_2 \leq n$. Continue through this process, until some $S'_q = S_{p_w}$, where $1 \leq w \leq q-1$. This only means that strategy S_{p_w} is a repeat of a strategy seen somewhere before. Notice that before, strategy S_{p_w} deterministically led to strategy $S_{p_{w+1}}$, which deterministically led to strategy $S_{p_{w+2}}$, until the process eventually returns to strategy S_{p_w}. Then the cycle repeats. Consider the following diagram illustratively.

$$
\begin{aligned}
S'_1 = S_{p_1} \to S'_2 = S_{p_2} \to \dots \to S'_w = S_{p_w} \to \dots \to S'_{q-1} = S_{p_{q-1}} \to S'_q = S_{p_w} \\
S'_q = S_{p_w} \to \dots \to S'_{2q-w-1} = S_{p_{q-1}} \to S'_{2q-w} = S_{p_w} \\
S'_{2q-w} = S_{p_w} \to \dots \to S'_{3q-2w-1} = S_{p_{q-1}} \to S'_{3q-2w} = S_{p_w} \\
\vdots
\end{aligned}
$$

Here, the Strategies S'_w through $S'_{q-1} = S_{p_w}$ through $S_{p_{q-1}}$form a cycle, because they repeat forever in a loop as the game continues infinitely. From now, the SP $(S_{p_w}, \dots, S_{p_{q-1}})$ will be called the Equilibrium Strategy Profile (ESP), because it describes an equilibrium; a stable state that the ecosystem settles on as the game continues.

To prove that this game *always* has a cyclical ESP like the one shown above, consider a game where $S'_1 = S_{p_1}$, (with p_1 described above), $S'_2 = S_{p_2}$, ... $S'_n = S_{p_n}$, where each of these strategies is different from all others. In this case, the game has exhausted all n strategies that are possible. Notice, however, that the game is infinite, so there will be a player P_{n+1} who must also have an optimal strategy. In this case, $S'_{n+1} = S_{p_{n+1}}$, but $S_{p_{n+1}}$ must be a strategy that has already been chosen before, because it must be one of the available n strategies, all of which have already been played. In that case, $S_{p_{n+1}} = S_{p_w}$, where $1 \leq w \leq n$, and strategies $(S'_w, \dots, S'_n)$ form the ESP. In this case, *any* game must have an ESP, because if it does not reach an ESP before exhausting all possible strategies, it must reach one after.

11 A Worked Example

Consider an Ecosystem E with four animals: an Alligator (A), Bear (B), Cat (C) and Dog (D). Let this be a 3 round game with 4 players. There are 30 organisms who will play strategy 1, 50 who play strategy 2, 20 who play strategy 3, and 70 who play strategy 4. For simplicity, assume each animal always has a 0.5 chance to win and a 0.5 chance to lose each interaction. Moreover, assume that payoff for losing is always 0. assume payoffs are as follows:

1. Alligator: Gains 2 for winning against an Alligator, 4 for Bear, 8 for Cat, and 8 for Dog.
2. Bear: Gains 4 for winning against an Alligator, 6 for Bear, 10 for Cat, and 10 for Dog.
3. Cat: Gains 8 for winning against an Alligator, 10 for Bear, 6 for Cat, and 12 for Dog.
4. Dog: Gains 12 for winning against an Alligator, 12 for Bear, 6 for Cat, and 6 for Dog.

Then, using equation (1),

$$E_{\lambda_{12}} = \begin{bmatrix} & A & B & C & D \\ A & 1 & 2 & 4 & 4 \\ B & 2 & 3 & 5 & 5 \\ C & 4 & 5 & 3 & 6 \\ D & 6 & 6 & 3 & 3 \end{bmatrix}, E_{\alpha_{12}} = \begin{bmatrix} & A & B & C & D \\ A & 1 & 2 & 4 & 6 \\ B & 2 & 3 & 5 & 6 \\ C & 4 & 5 & 3 & 3 \\ D & 4 & 5 & 6 & 3 \end{bmatrix}$$

Also, arbitrarily, assume frequencies are as following: $p(A) = 0.2, p(B) = 0.4, p(C) = 0.3, p(D) = 0.1$. Then,

$$\vec{p} = \begin{bmatrix} 0.2 \\ 0.4 \\ 0.3 \\ 0.1 \end{bmatrix}$$

The matrix product

$$E_\lambda \vec{p} = \begin{bmatrix} \vec{\lambda_1} \cdot \vec{p}_1 \\ \vdots \\ \vec{\lambda_n} \cdot \vec{p}_n \end{bmatrix} = \begin{bmatrix} 2.6 \\ 3.6 \\ 4.3 \\ 4.8 \end{bmatrix}$$

Therefore, strategy D, which maximizes expected fitness at $EV = 4.8$ is S_1'. Using $E_{\alpha_{12}}$, we look at the 4th row, corresponding to $S_1' = D$, and we pick the best option for Player 2. This is option C, which has a payoff (fitness) of 6. Now $S_2' = C$ Also, our SP for$SP_1 = (S_1', S_2') = (D, C)$.

In the next round, Player 2 becomes the left player, and Player 3 becomes the top player. The relevant matrix now is

$$E_{\alpha_{23}} = \begin{bmatrix} & A & B & C & D \\ A & 1 & 2 & 4 & 6 \\ B & 2 & 3 & 5 & 6 \\ C & 4 & 5 & 3 & 3 \\ D & 4 & 5 & 6 & 3 \end{bmatrix}$$

Now, we try to maximize Player 3's outcome given Player 2's choice of $S_2' = C$. We look at row 3 of the matrix, corresponding to Player 2's choice C, and maximize Player 3's payoff accordingly. This comes when player 3 chooses B, which corresponds to the highest value in the row, 5. Now $S_3' = B$. Also, $SP_2 = (S_1', S_2', S_3') = (D, C, B)$.

In the last round, Player 3 becomes the left player, and Player 4 becomes the top player. The relevant matrix now is

$$E_{\alpha_{12}} = \begin{bmatrix} & A & B & C & D \\ A & 1 & 2 & 4 & 6 \\ B & 2 & 3 & 5 & 6 \\ C & 4 & 5 & 3 & 3 \\ D & 4 & 5 & 6 & 3 \end{bmatrix}$$

Now, we try to maximize Player 4's outcome given Player 3's choice of $S_3' = B$. We look at row 2 of the matrix, corresponding to Player 3's choice B, and maximize Player 4's payoff accordingly. This comes when player 4 chooses D, which corresponds to the highest value in the row, 6. Now $S_4' = D$. Also, $SP_3 = (S_1', S_2', S_3', S_4') = (D, C, B, D)$.

This indicates that the ecosystem evolved so that 30 Dogs were selected first, 50 Cats were selected after, 20 Bears were selected after that, and 70 more Dogs were selected last. It gives us a complete evolutionary picture of the dynamics of the ecosystem according to natural selection. In this case, alligators were never the most fit given their circumstances, so they were never selected.

Notice that the SP (D,C,B) forms an ESP, because it will continue to play out for the rest of the game cyclically. (Notice that if calculations were continued, $S_5' = C$, $S_6' = B$, $S_7' = D$, $\ldots$). This is the cyclical equilibrium for the game.

12 Discussion and Conclusion

Our results show that we are able to form an algorithm to mathematically predict natural selection and find equilibrium. It takes theoretical data about selective pressures, and iterates through an ecosystem by applying natural selection. At the end, it provides a comprehensive profile of the chronology and number of selection by using best response iteration. Eventually, after enough rounds, the ecosystem will fall into a cyclical equilibrium, where natural selection selects the same strategies (animals) repeatedly.

As discussed before, this algorithm's major shortfall is the assumption that evolution is a sequential game as opposed to a simultaneous one. In an ecosystem where different animals get selected at the same time, the4 algorithm would require higher dimensions and more conditions. A useful extension of this paper would be one that models a simultaneous game, making time an independent variable instead of mechanically iterating through periods.

This algorithm is particularly useful in evolutionary biology and ecology. Natural Selection, as a doctrine, is easy to imagine for a small number of in interactions, but it becomes hard to generalize for an ecosystem of arbitrary size. Now, with best response iteration, we have a useful, recursive, and rigorous way to predict the evolution of a closed system. The algorithm provides an insight into the process by which natural selection operates.

One exciting feature of this algorithm is its ability to track an ecosystem through time. For example, this process allows us to predict the future of an ecosystem, which could help us concentrate conservation efforts on species that will struggle. Some animals may no longer be selectively adequate, like bees, but they're still important to the ecosystem. Using this algorithm would help us predict which animals will need conservationist help in the future. Also, it would allow us to track how an ecosystem was in the past. If we know the current state of the ecosystem, this algorithm can be applied backwards to trace the history of an ecosystem. Lastly, If we think of ourselves as one of the strategies, we can evaluate the selective pressures we face and create. It allows us to understand our impact on our own ecosystems, and may provide us with the understanding to change it for the benefit of other species.

Also, a notion of equilibrium could explain the state of ecosystems that we know today. Some ecosystems, which have been around for millennia, have already reached an ESP, which explains why the animals present today have persisted for so long. It also explains why some animals went extinct; they weren't part of the equilibrium selection profile and were selected out. It gives us a way to trace the history of evolution and persistence, and possibly predict how extinction will play out in unbalanced ecosystems (ones that have not reached an ESP). This again would allow us to properly allocate resources for conservation efforts. Most excitingly, however, this model shows us that some principles of social science, like Game Theory, are applicable to other, non-human pursuits, like animal evolution. It extends some principles of rationality and utility to other forms of life, which provides us with a useful framework to model them. In the pursuit of human knowledge and action, the model provides us with the rigor, generality, and methodology to understand and interact with our environment.

13 References

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14 Appendix

1. Expected Value - A predicted value of a variable that can take different values with different probabilities.
2. Efficient Selection (ES)- The strategy for a player that maximizes expected payoff/fitness.
3. Ecosystem - A system containing natural selective pressures and a countable number of animals that all interact with each other.
4. Equilibrium Selection Profile (ESP) - an SP that describes a finite, cyclical equilibrium that an ecosystem repeatedly selects over time.
5. Fitness - the evolutionary advantage to reproduce
6. Best Response Iteration - an algorithm developed to determine the recursive outcomes of a sequential game
7. Payoff - A quantitative measure of utility associated with one strategy
8. Player on Left (PL) - The player on the left of a payoff matrix, with payoffs described by λ_{ij} as described above.
9. Player on Top (PT) - The player on the top of a payoff matrix, with payoffs described by α_{ij} as described above.
10. Select - To gain an evolutionary favor as a result of having higher fitness (compared to some other strategy)
11. Selection Profile (SP) - A set containing the Efficient Selection of multiple players. It is useful in determining how the efficient selection of one player will select the efficient selection of other players. It is also useful in mapping out how natural selection works chronologically through an ecosystem, because items in a selection profile are in chronological order.
12. Sequential Game - A game where strategies are played in a countable amount of discreet steps

13. Simultaneous Game - A game where all strategies are played at the same time
14. Utility - An arbitrary measure of happiness, considered to be the useful measure of value when comparing decisions