Please solve the problems and show all your work. Hand in your homework on Dec 6 class.

**Problem 1:** (a) Evaluate the line integral $\int_{\gamma_1} z \, dz$ where $\gamma_1$ is the straight line from $z = 0$ to $z = 1 + i$. (b) After that evaluate $\int_{\gamma_2} e^z \, dz$ where $\gamma_2$ is a union of two straight lines, one from $z = i$ to $z = 1 + i$, the second from $z = 1 + i$ to $z = 1 - 2i$. (c) Consider the line integral $\int_{\gamma} z \, dz$ for any path $\gamma$ starting at $z = 0$ and terminating at $z = 1 + i$. Show that this integral is path dependent by choosing two different paths from 0 to $1 + i$ and obtaining different values for the line integral.

**Problem 2:** Evaluate each of the following integrals.

(a) $\int_{|z|=1} 3z \, dz$  
(b) $\int_{|z|=1} \frac{dz}{z(z + 2)}$  
(c) $\int_{|z|=3} \frac{z}{z^3 - 1} \, dz$

**Problem 3:** Evaluate each integral using the (generalized) Cauchy integral formula

(a) $\int_{|z|=3} \frac{\sin z}{z} \, dz$  
(b) $\int_{|z|=3} \frac{z + 1}{(z - 1)(z + 2)^3} \, dz$

(c) $\int_{|z|=3} \frac{z^2 - 1}{z^2 + 1} e^z \, dz$  
(d) $\int_{|z|=3} \frac{z^3}{z^2 + 1} \, dz$, where all the contours are oriented counterclockwise.

**Problem 4:** Evaluate the given integral by means of the residue theorem.

(a) $\int_{|z|=1} \frac{dz}{z^2 e^z}$  
(b) $\int_{|z|=1} \frac{z^2 \, dz}{\sinh(2z)}$  
(c) $\int_{|z-1|=2} \frac{dz}{z^2 - 2iz - 2}$, where all the contours are oriented counterclockwise.
Problem 5: Evaluate the given integral by means of the residue theorem.

(a) $\int_0^{2\pi} \frac{dt}{(a + b \cos t)^2}, \quad a > b > 0 \quad$ (b) $\int_0^{\pi} \frac{dt}{(a + \cos t)^2}, \quad a > 1.$

Problem 6: Evaluate the given integral by means of the residue theorem.

(a) $\int_{-\infty}^{\infty} \frac{dx}{x^4 + 1}$ \quad (b) $\int_{0}^{\infty} \frac{x \, dx}{x^4 + 1}$ \quad (c) $\int_{0}^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}, \quad a > 0, b > 0.$

Problem 7: Evaluate the given integral by means of the residue theorem.

(a) $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + 1)^2} \, dx$ \quad (b) $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + a^2)(x^2 + b^2)} \, dx, \quad a > 0, b > 0, a \neq b.$