Please solve the problems and show all your work. Hand in your homework on Nov 1 class.

**Problem 1:** Solve the following 2d wave equation using separation of variables

\[
c^2 (w_{xx} + w_{yy}) = w_{tt}, \quad 0 < x < a, 0 < y < b, \quad t > 0,
\]
\[
w(0, y, t) = w(a, y, t) = w(x, 0, t) = w(x, b, t) = 0, \quad t > 0,
\]
\[
w(x, y, 0) = 0, \quad w_t(x, y, 0) = g(x, y), \quad 0 < x < a, 0 < y < b.
\]

**Problem 2:** Solve the Laplace equation using separation of variables

\[
u_{xx} + u_{yy} = 0, \quad 0 < x < 3, 0 < y < 2,
\]
\[
u(0, y) = u(x, 2) = u_x(3, y) = 0, u(x, 0) = 50H(x - 2),
\]

where \(H(x)\) is the Heaviside function

\[
H(x) = \begin{cases} 
0, & x \leq 0 \\
1, & x > 0.
\end{cases}
\]

**Problem 3:** Consider a thin flat plate of radius \(b\), that is thermally insulated on its two flat faces. With a hacksaw we make a radial cut along \(\theta = 0\), say, from \(r = b\) to \(r = 0\). The small gap, due to the cut, may be approximated as a thermal insulator, so that \(u_{\phi} = 0\) on the edges \(\theta = 0\) and \(\theta = 2\pi\). If the circumference of the plate is held at the temperature \(50(1 + \sin \theta)\) for a long time, the steady-state temperature field \(u(r, \theta)\) is governed by the boundary-value problem

\[
\nabla^2 u = 0, \quad 0 < r < b, 0 < \theta < 2\pi,
\]
\[
u_{\phi}(r, 0) = u_{\phi}(r, 2\pi) = 0,
\]
\[
u(b, \theta) = 50(1 + \sin \theta),
\]
\[
u \quad \text{bounded}.
\]

Solve for \(u(r, \theta)\).

**Problem * (extra 1/5 of this assignment set):** Consider the Poisson equation

\[
\nabla^2 u = f(x, y, z)
\]
in some three-dimensional domain $D$ with surface $S$. Integrating the Poisson equation over $D$, show that the following solvability condition holds

$$\int_S \frac{\partial u}{\partial n} dA = \int_D f dV,$$

where $n$ is the unit outward normal on $\partial D$. You may use Green’s identity.