Homework 8

Please solve the problems and show all your work. Hand in your homework on Nov 15 class.

Problem 1: (a) Show that $\frac{1}{z} = \frac{1}{z}$. (b) Evaluate $\text{Im} \frac{a+ib}{c+id}$. (c) Evaluate $|(1-2i)^2 + (1+i)^2|$. (d) Show whether or not $|e^z| = e^{|z|}$.

Problem 2: Recall that we define

$$
\cos z = \frac{e^{iz} + e^{-iz}}{2}, \quad \sin z = \frac{e^{iz} - e^{-iz}}{2i}, \quad \cosh z = \frac{e^z + e^{-z}}{2}, \quad \sinh z = \frac{e^z - e^{-z}}{2}.
$$

(a) Show that $\sin(-z) = -\sin z$. (b) Show that $\cos(z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2$. (c) Show that $\cos(x + iy) = \cos x \cosh y - i \sin x \sinh y$, where $x$ and $y$ are real.

Problem 3: (a) Recall that the argument of $z = x + iy$ can be determined by $\text{arg}(z) = \text{arctan} \frac{y}{x}$ with the help of showing the complex number in the complex plane. Find all values of $z^{1/2}$ and $z^{1/5}$ for $z = 1$ and $z = 3 - 2i$, express those values in polar form. (b) Obtain all values of $\log z$ for $z = 1$ and $z = 2 - i$.

Problem 4: One can define

$$
e^z = e^{z(\ln |c|+ic)}.
$$

Use this to evaluate $(1 + \sqrt{3}i)^{(2-5i)}$.

Problem 5: Given $f(z)$, determine $f'(z)$, where it exists, and state where $f$ is differentiable and where it is not.

(a) $\frac{1}{z^3 + 1}$   (b) $z \sin z$