1) (25 pts) *Production with non-competitive labor market.* The economy’s labor force, \( L_t = L_0 (1 + n)^t \), consists of \( N_t \) labor unions. Each union has a given and fixed size \( l = L_t / N_t \) that never changes. The number of firms equals the number of unions. Each firm has to hire all of its union’s members, that is, firm-level employment is \( l \) workers at all times. As the labor force grows, new unions (of size \( l \) each) are formed and new firms can enter the market and join production. The production function for firm \( i \) is
\[
y_i = k_i^\alpha l_i^{1-\alpha}, \quad \alpha \in (0, 1),
\]
where \( y_i \) is firm-level output, \( k_i \) is capital input and \( l_i \) is labor input. The union contract requires that the firm pay each worker an equal share of its output, so the wage at firm \( i \) is given by
\[
w_i = \lambda \frac{y_i}{l}, \quad \lambda \in (0, 1). \tag{1}
\]

a) (12 pts) Assume that capital leasing firms and capital goods producers are competitive sectors, and that the law of motion for capital is
\[
K_{t+1} - K_t = I_t - dK_t, \quad d > 0
\]
where \( K_t \) is the market value of capital stock and \( I_t \) is investment expenditure in units of final output. Define the production-side equilibrium for this economy and derive the expression for the interest rate as a function of \( K_t, L_t \) and given parameters. (**Hint:** think carefully about the firms’ maximization problems when labor is unionized. You do not need to know the aggregate production function to define the production side equilibrium)

b) (8 pts) Suppose that aggregate supply of capital, \( K_t \), and of labor, \( L_t \), are given. Define and derive the aggregate production function \( Y_t = F (K_t, L_t) \) where \( Y_t \) is the economy-wide output.

c) (5 pts) Compare the equilibrium rental fee on capital from part a) to \( MPK_t = \frac{\partial F}{\partial K} (K_t, L_t) \). Which is smaller or are they equal to one another? Explain the economic intuition for your result.
2) (30 pts) Consider a canonical overlapping generations model with the non-competitive labor market and production as described in Question 1. Time is discrete. Agents live for 2 periods. The size of generation $t$ is $L_t = (1 + n)^t$. Agents receive labor income $w_t$ in youth. The law of motion for capital is

$$K_{t+1} = I_t, \text{ (i.e. } d = 1).$$

There is an initial group of old agents (generation 0) alive at $t = 1$ who own the initial capital stock $K_1$ and die at the end of period 1.

Old agents own all capital (and, therefore, all firms) at the beginning of every period. Thus they receive both capital income on their financial assets and a fraction of the firms’ economic profits in proportion to the assets invested in firms.

The utility maximization problem of generation $t$ is

$$\max_{c_{1,t}, c_{2,t+1}, a_{t+1}} \left( \ln c_{1,t} + \ln c_{2,t+1} \right)$$

subject to:

$$a_{t+1} = w_t - c_{1,t},$$

$$c_{2,t+1} = (1 + r_{t+1}) a_{t+1} + (1 + r_{t+1}) a_{t+1} \cdot \varphi,$$

where $(1 + r_{t+1}) a_{t+1} \varphi$ captures the additional old-age income stemming from firm ownership.

a) (8 pts) Derive the expression for $\varphi$ as function of exogenous parameters. (Hint: If you did not get question 1, assume that $\varphi = \text{const} > 0$ and proceed to part b. Otherwise, note that $Y_t = w_t L_t + R_t K_t + \Pi_t$ where $\Pi_t$ is aggregate economic profit. Express $\Pi_t$ through $RK$. One old generation-$t$ agent owns fraction $a_{t+1}/K_{t+1}$ of total assets, and she gets the same fraction of total economic profits).

b) (12 pts) Derive the expression for the long-run interest rate, $\bar{r}$, as function of exogenous parameters. (Hint: What is the equilibrium marginal product of capital in this economy?)

c) (10 pts) Suppose that you are a policy maker in the context of the above model. You do not know what $\alpha$ is. Your objective is to avoid a long-run equilibrium that is not Pareto optimal. The policy choice is whether to strengthen the unions (i.e. increase $\lambda$) or to weaken them (decrease $\lambda$). Which policy choice is more likely to meet your objective? Explain the economic intuition for your policy recommendation. (Hint: What is the marginal product of capital in this economy?)
3) **(25 pts)** Consider the standard Ramsey model. Let $C_t$ denote aggregate consumption and let $L_t$ denote the number of workers. The lifetime utility of a representative household is

$$U = \int_0^\infty e^{-\rho t} L_t u \left( \frac{C_t}{L_t} \right) dt, \quad 0 \leq \sigma \leq 1$$  (3)

Population grows at a constant rate $n = \frac{\dot{L}_t}{L_t}$, and there is no technological change. The production function is

$$Y_t = K_t^\alpha L_t^{1-\alpha}, \quad \alpha \in (0, 1),$$

and the aggregate resource constraint is

$$\dot{K} = Y_t - C_t - dK_t.$$  (4)

The status quo is the per-person utility function $u(x) = \ln x$. Assume that before time 0, the economy is in the status quo long-run equilibrium. At time 0, a new, “hedonistic”, generation is born, and the household’s utility function temporarily changes to $u(x) = 2 \ln x$. At this time, agents know that preferences will revert back to status quo at date $T > 0$, and that no further parameter changes will ever occur.

Depict the economy’s trajectory $(k_t, c_t)$ on the phase diagram. Label the initial equilibrium ”A”, the point where the economy is at time $+0$ ”B”, the point where the economy is at time $T - 0$ ”C”, the point where the economy is at time $T + 0$ ”D”. Do some of these points coincide with one another? Why or why not? Use the first-order conditions for the planner’s problem to explain your answer.