1) (25 pts) This question tests your understanding of the concept of capital and wealth in a macroeconomic model.
   a) (8 pts) State minimal assumptions or conditions that make the market value of businesses equal to the current resale value of their capital stock.
   b) (8 pts) State minimal assumptions or conditions that make household wealth equal to the market value of (domestic) businesses.
   c) (9 pts) Naturally, the input in the production function should be the quantity of capital rather than its market value. Conceptually, the two are distinct. State minimal assumptions or conditions that justify using the value of capital instead of its quantity as an input in the production function.

2) (25 pt) Consider the Solow model with labor-augmenting technological change. The production function is $Y_t = K_t^\alpha(A_tL_t)^{1-\alpha}$, where $L_t$ grows at a constant rate $n > 0$, and $A_t$ grows at a constant rate $g_A > 0$. The capital accumulation equation is $\dot{K}_t = sY_t - dK_t$. At date $T$, the economy is in its long-run equilibrium. The law of motion for the capital per effective worker, $k_t = K_t/A_tL_t$, is

$$\dot{k}_t = sk_t^\alpha - (n + d + g_A)k_t.$$

At date $T$, an oil price shock makes $A_t$ jump down permanently. The growth rate $g_A$ is left unchanged.

   a) (7 pt) Plot the trajectory for the capital per effective worker, $k_t$ (with time on the horizontal axis), before and after date $T$. Indicate whether the long-run value of $k_t$ is above, below or equal to what it was prior the shock. (Hint: Because $A_t$ jumps at date $T$, the capital per effective worker should jump also).
   b) (6 pt) Plot the trajectory for the interest rate (with time on the horizontal axis), before and after date $T$. Indicate whether the long-run value of the interest rate is above, below or equal to what it was prior to the shock. Indicate any kinks or jumps in your trajectory.
   c) (12 pt) Plot the trajectory for the wage (with time on the horizontal axis), before and after date $T$. Does the wage have a balanced growth path, and what is its slope? Does this balanced growth path shift up, down or does not change as a result of the shock? Does the wage jump at time $T$ and by how much relative to its eventual balanced growth path?
3) (35 pts) Consider a version of the canonical overlapping generations model where agents have a concern for status, as measured by their wealth, $a_{t+1}$, relative to the average wealth, $\bar{a}_{t+1}$, of the other members of their birth cohort. Time is discrete. Agents live for 2 periods. The size of generation $t$ is $L_t = (1 + n)^t$. Agents receive labor income $w_t$ in youth. The utility maximization problem of generation $t$ is

$$\max_{c_{1,t},c_{2,t+1},a_{t+1}} \left[ \ln c_{1,t} + \ln c_{2,t+1} + \varphi \ln \left( \frac{a_{t+1}}{\bar{a}_{t+1}} \right) \right]$$

s.t. $a_{t+1} = w_t - c_{1,t}$, $c_{2,t+1} = (1 + r_{t+1})a_{t+1}$.

Output is produced by competitive firms with the production function

$$Y_t = K_t^\alpha L_t^{1-\alpha}, \quad \alpha \in (0, 1)$$

where $K_t$ is physical capital and $L_t$ is labor. The law of motion for capital is

$$K_{t+1} = I_t + K_t.$$ (3)

There is an initial group of old agents (generation 0) alive at $t = 1$ who own the initial capital stock $K_1$ and die at the end of period 1.

a) (10 pts) Solve the utility maximization problem (1). Derive the expression for the demand for assets by generation $t$ (i.e. $a_{t+1} L_t$) as a function of $K_t$, $L_t$ and the model’s parameters.

b) (10 pts) Treat $\alpha$ as an unknown parameter. What is the range of $\alpha$ for which the long-run equilibrium of this model is Pareto inefficient? (Hint: Your range should depend on $\varphi$. Note that (3) implies that capital fully depreciates in one period. If you did not get part a), don’t worry. Proceed as if $\varphi = 0$ to get partial credit).

c) (15 pts) Suppose that the economy’s long-run equilibrium is Pareto-inefficient, and that the government comes to the rescue. It does so by issuing an initial amount of debt, $D_1 > 0$, that it borrows from the initial young (generation 1), and transferring the proceeds from borrowing as a gift to the initial old (generation 0). The government has zero taxes and zero spending at every date, so it’s law of motion for debt is

$$D_{t+1} = (1 + r_{t+1})D_t, \quad t \geq 1.$$ (4)

Derive the expression for the maximum debt-to-GDP ratio, $D_t/Y_t$, that is sustainable in a long run equilibrium, as a function of $n$, $\alpha$, $\varphi$. (Hint: Look at the law of motion for $d_t = D_t/L_t$ implied by (4). Will a growing per-capita government debt be sustainable in a long-run equilibrium?)
4) **(35 points) Ramsey meets Macron.** Consider an otherwise standard Ramsey model where households are subject to an ongoing wealth tax at rate $\tau > 0$. Specifically, the representative household optimization problem (with the appropriate transversality condition) is

$$\max_{\{c_t\}} \int_0^\infty e^{-\rho t} L_0 e^{nt} \ln c_t dt$$

s.t. $\dot{A}_t = (r_t - \tau) A_t + w_t L_t - c_t L_t$, $A_0 > 0$ – given,

where $A_t$ is the dynasty’s current financial wealth, $L_t = L_0 e^{nt}$ is the dynasty size (equal to its labor supply), $c_t$ is per capita consumption, $r_t$ is the market rate of return on household wealth, $w_t$ is the wage. The government has a balanced budget in every period, so the economy’s resource constraint with tax is

$$\dot{k}_t = f (k_t) - c_t - (n + d) k_t - \tau k_t$$

where $k_t$ is the market value of capital per worker and $f (k_t)$ is a neoclassical production function in intensive form.

**a) (15 pts)** Derive the household Euler equation and plot the phase diagram ($k_t$ on the horizontal axis and $c_t$ on the vertical axis) for the economy’s competitive equilibrium for two scenarios: positive wealth tax, $\tau > 0$ combined with balanced government budget, and zero wealth tax, $\tau = 0$, combined with zero government expenditure.

**b) (20 pts)** Assume that initially the economy is in its long-run equilibrium with $\tau > 0$. At date $t_A$, Macron is elected by surprise. Immediately upon election, he announces that the tax on wealth (and the government spending associated with it) will be eliminated forever at date $t_C > t_A$. Plot the resulting trajectory $(c_t, k_t)$ on the phase diagram for $t \geq t_A$. Explain the economic intuition for the behavior of consumption. **(Hint: The pre-tax wealth does not jump at date $t_C$. Your phase diagram analysis may depend on whether the stable arm of the eventual phase diagram, the one after $t \geq t_C$, is relatively steep or relatively flat).**