ABSTRACT

We examine the mechanical effect of a multiple vote, proportional representation electoral system on party vote share in n dimensions. In one dimension, Cox (1990) has proven that such a system is centripetal: it drives parties to the center of the political spectrum. However, as populism has swept across Western Europe and the United States, the importance of multiple policy dimensions has grown considerably. We use simulations to examine how a multiple vote system could alter electoral outcomes in all possible parliamentary systems. We find that multiple vote systems act centripetally in multiple dimensions too, though weakly in extreme cases where parties are sorted into ideological clusters at opposite corners of the ideological space. Even in these cases, though, we find that a slight disturbance of the conditions (by introducing an additional party - even if it is very small) strengthens the centripetal properties of the multiple vote system.
INTRODUCTION

Modern electoral competition has become more complicated with the emergence of new issue dimensions (most notably, immigration and economic inequality, but also environment, globalization, institutional efficiency, etc.). It has also become more unpredictable: the successes of Donald Trump in the U.S. and Emmanuel Macron in France were considered hopeless longshots only a year (and sometimes less) before their victories in two major Western democracies. Probably the most telling case in terms of multidimensionality and unpredictability is the United Kingdom, where the “Brexit” dimension was not at all captured by the existing party system. This led to the emergence of new parties, the splitting of old ones, as well as a series of negative votes in Parliament that led to its “prorogation” (suspension) months before the electoral triumph of Johnson. These episodes illustrate that our present understanding of key facets of electoral politics, such as voters’ preference formation, relation between electoral systems and voting, and analyses of voting on the basis of one-dimensional models (Downs 1957) should be reevaluated. Particularly given the modern trend of ideological polarization in many Western democracies, understanding the ramifications of multidimensionality for proposed institutional changes and reforms is imperative.

The goal of this paper is to examine the dynamics of multidimensionality within one particular electoral system: a multiple vote system according to which each voter is endowed with multiple votes and can use as many of them as (s)he wants, to support different parties. In previous work, Cox (1990) has proven that such an electoral system produces centripetal results in a single policy dimension. In this paper, we will demonstrate that the same policy implication is true even in multiple dimensions. We believe this result is of both substantive and methodological importance. Substantively, this result suggests that the multiple vote system described above and analyzed below provides a means for addressing extremism within our increasingly multidimensional political reality. Methodologically, the result helps to overcome the fact that median voter equilibria exist in one dimension but disappear in multiple dimensions, leading us to use computer simulations.

To arrive at these results, we execute a series of simulations that build upon actual electoral results in four different countries: Belgium, Netherlands, Germany, and Romania. Using existing partisan seat distributions from these countries, we calculate the percentage of votes each party would receive under our proposed multiple vote system, and then compare that distribution of seats to seat shares under the prevailing electoral system. Along the way, the procedure incorporates key features of electoral politics such as random non-proximity voting

---

1 The technical term for this system is that it is non-cumulative. In cumulative systems voters can use their votes to support candidates of the same party. For example, in Bremen and Hamburg voters are endowed with five votes each and they can use them to support the same party. We thank an anonymous reviewer for providing this information to us.
and existing national electoral rules, in order to generate hypothetical vote and legislative seat distributions.

From the point of view of the voter, the only difference between our system and the existing ones is that voters may cast more than one vote. However, as we show in the analysis of our results, the resulting differences are significant. First, multiple votes enable voters to express their preferences more completely than in the usual single-vote case. For example, a country with ten political parties, a switch from one to three votes affords voters with 176 unique choice sets—compared to just 11 under a single-vote system (any one particular party or abstention). Second, our system incorporates all these choices into the final outcome. Third, though outside the scope of our paper empirically, we believe this system should lead to a breakdown of “party identification,” since it encourages voters to use multiple criteria when selecting among parties. Consequently, voters would no longer “identify” with any single party in particular. Finally, we show how the forces inherent to this system lead to a party system wherein centrist parties prevail, and wherein the political debate becomes less polarized. We also identify the few conditions under which this kind of result may not prevail, and demonstrate how a small perturbation of these conditions will lead again to a centripetal party system.

We believe the political effects of adopting such a system would be significant. First, the increase in parliamentary representation of centrist parties would lead to more centrist governments than the ones that prevailed in certain countries in the recent past (e.g., Netherlands in 2010, Denmark in 2015, Italy in 2018, and Austria in 2017). However, beyond this result, we believe the system would also encourage the creation of more flexible and long-lasting governing coalitions, since the ideological distance of the partners will be smaller and coalitions will be able to respond to unpredicted political events instead of collapsing.\(^2\) Taken together, these advantages represent important improvements over existing arrangements, especially during a volatile time in political history.

**MULTIPLE VOTE SYSTEMS IN THEORY AND PRACTICE**

*Multiple Vote Systems in Theory*

Electoral systems have been shown to affect not only the number of parties (Duverger 1951) but also their positions along the political spectrum (Cox 1990). With respect to the latter, Cox (using a one-dimensional policy space) has demonstrated that granting voters with multiple votes creates centripetal forces inside a political system. More specifically, when the number of candidates is small enough relative to the number of votes per voter, and when cumulation (i.e.,

---

\(^2\) See Tsebelis and Ha (2014) for the theoretical argument and Tsebelis and Crosson (2020) for the application in the case of the Netherlands.
allowing a voter to cast all of her votes for one candidate) is not allowed, centripetal forces will predominate and candidates and parties will be drawn to the center of the political spectrum.

Cox’s results depend on a series of assumptions about voters, candidates, and the policy space, and provide a clear theoretical framework for understanding why a polity might choose to implement a multiple vote system as a means for combatting political polarization. However, Cox’s results are not the only, nor the first, to suggest that multiple vote systems moderate candidates. Indeed, the term “multiple vote” we use in this paper is designed to encompass both approval voting systems and rank-ordering systems, by incorporating their underlying common feature: such systems allow voters to select more than one candidate or party. Under approval voting, voters have an unconditional choice: they use as many of the available votes as they wish. Under rank-order voting, voters must rank their choices, and a subordinate choice is not used unless the higher-ordered option is not operational. Both systems have been proposed for single member districts, while in our paper we combine multiple votes with any district size, as well as adding existing national rules from each corresponding country.

The approval voting literature has suggested that multiple vote systems could, in fact, help moderate candidates— and in the American context, weaken or even destroy the two-party system (Brams and Fishburn 1978). Under approval voting, voters receive m votes that they may or may not choose to use on different candidates in an election. Under this system, cumulation is prohibited, similar to the centripetal case underscored in Cox’s analysis. Its proponents (e.g., Brams and Fishburn 2007 [1980], Kellett and Mott 1977) argue its practical effects, “would probably be to give comparatively more support to moderates” (Brams and Fishburn 1978: 840). Consequently, such proponents have in the past argued that the major parties in America should adopt approval voting as their primary-election voting system, because “most delegates find [moderates] acceptable,” while “extremists […] are only acceptable to ideological factions in their party” (Brams and Fishburn 1978: 840).

A similar logic has evolved in support of ranked choice voting, particularly in state-level elections in the United States. According to one prominent version of ranked-choice voting, voters rank candidates on their ballots according to preference, ranging from most to least favorable. Should a single candidate not receive a majority of first-place votes following the election, the ballots of the last-place candidate flow to the respective remaining candidates—a process that is repeated until the final winner is selected. Proponents of this system argue the procedure generates a winner that is more centrist and/or more widely acceptable to a larger portion of the electorate than does majority rule or (especially) plurality elections (Santucci 2018, Fromuth 2019). These potential advantages have not gone unnoticed outside of political science. For example, a recent study of the American Academy of Arts & Sciences, ReInventing American Democracy for the 21st Century, proposes the adoption of “ranked-choice voting in presidential, congressional, and state elections” as one of the most prominent institutional modifications precisely in order to promote moderation (American Academy of Arts & Sciences 2020,12)
Multiple Vote Systems in Practice: Past and Present

While these systems differ in several important regards, which we discuss below, they nevertheless share a crucial common feature: they grant voters with a larger choice set with which they can express their preferences. The centripetal forces resulting from these systems help to explain why variants of these systems have been adopted in a wide variety of settings over thousands of years. Most of the time, such systems have been adopted in single member districts (with one winner). In ancient Greece, the Spartans’ “acclamation vote” served as an early form of approval voting, as voters were allowed to shout in favor of more than one candidate for the Gerousia (Girard 2010, Tsebelis 2018). Though undoubtedly quieter than the Spartan vote, papal elections from the late thirteenth to the early seventeenth century also took a form that resembled approval voting. According to this voting system, cardinals had the option of voting for more than one papal candidate. The pairing of this system of voting with a 2/3rds qualified majority threshold created long vacancies in the papacy, ultimately leading to the voting system’s demise. However, as Colomer and McClean (1998) argue, the system did encourage the election of largely unobjectionable popes, which helped to address longstanding tension (and even violence) within the Church.

Political entities today have also adopted variations of the multiple vote system. In the most prominent victory for ranked-choice advocates to date, Maine adopted a system of ranked-choice voting for its legislative and gubernatorial elections. As noted above, proponents of the system tout its majority-friendly and centripetal features, though the recency of the reforms have precluded direct empirical tests of these assertions. Nevertheless, reformers ultimately succeeded in Maine due in part to the election and reelection of a widely unpopular Republican governor who never succeeded in securing absolute majority support from voters in the state (Santucci 2018). Similarly, in 2019, New York City residents elected to revise the city’s charter to establish ranked-choice voting for all primary and special elections. New York City is now among more than fifteen cities that use ranked-choice voting (Drutman 2019).³

In multimember districts, STV systems in Ireland and Malta (and the Australian Senate) provide some empirical examples. Generally speaking, analyses of these systems find that “[a]lthough the system provides representation to minor parties, results in single transferable (STV) elections generally have shown that minor centrist parties benefit from the system and minor radical parties are penalized” (Encyclopaedia Britannica). More to the point, researchers have found that voters sometimes transcend party or group barriers under such systems and vote for individual candidates of their liking. For example, Mitchell (2014) compares the electoral results before and after the 1998 Belfast Agreement in Ireland and observes that “prior to the 1998 Agreement

³ Another important reform at the U.S. state level is the “top two” candidate primaries. Here, voters participate in a common “primary” and then select between the prevailing top two candidates in the general elections. Crucially, these general-election candidates may belong to the same party. When they do, the more moderate of the two candidates tends to prevail. Since the adoption of this system in the states of Washington and California, several studies have found evidence of moderation (see Grose 2014, Crosson 2020, and Grose 2020).
inter-ethnic vote-pooling in Northern Ireland was very close to zero.” However, he continues: “[a]fterwards (1998-2007), terminal transfers from the moderate unionist UUP to the moderate nationalist SDLP averaged 32 per cent (and 13 per cent in the opposite direction). Although most transfers clearly remain within ethnic blocs, these inter-ethnic terminal transfers are a change with the past and suggest that SW may be an appropriate electoral system choice for some divided societies.’’

Two Nobel Prize winners in Economics (Maskin and Sen 2016) have argued that a majority requirement in American primaries would have eliminated Trump as the candidate for the Republican party. The same would have happened with an STV system whether at the Republican primaries, or at the general election.

Other modern entities have also either proposed or adopted versions of multiple vote systems. According to a review on approval voting success and failure by Brams and Fishburn (2010), several professional societies have adopted some version of approval voting. These include the Mathematical Association of America, the American Mathematical Society, the Institute for Operations Research and Management Sciences, the American Statistical Association, the Institute of Electrical and Electronics Engineers, the Society for Judgment and Decision Making, the Social Choice and Welfare Society, the International Joint Conference on Artificial Intelligence, and the European Association for Logic, Language and Information, the Econometric Society, and the National Academy of Sciences. While elections in these societies may not be exactly ideological or high stakes, Brams and Fishburn (1978) find that the multiple vote systems appear to advantage candidates who enjoy support from a large cross-section of the societies’ memberships. Similarly, in highly multidimensional contests like gymnastics and diving, Olympians are judged using either multiple rankings or ratings. In doing so, officials hope that the athlete agreed upon as best by the largest group of judges will be selected for a medal. The same voting system is used for the Academy Awards. To our knowledge the only

---

4 One may reasonably object that the results of the 2020 Irish election cut against the trends Mitchell underscores. That is, because Sinn Fein made significant gains during the 2020 election, one may argue that STV permitted seemingly centrifugal results. However, as several accounts persuasively underscore, Sinn Fein’s 2020 gains did not occur in a vacuum: rather, the results occurred after not only a great deal of “learning” on the part of Sinn Fein strategists, but also amid a rapidly changing status quo (Aodha 2020). As a result, Sinn Fein’s appeal evolved over time, particularly as the party worked to expand its message and render it more positive (O’Leary 2020). Indeed, dating back as far as the 2008 financial crisis, Sinn Fein chose to emphasize economic issues “to move away from its image as a single-issue party on Irish unity.” In doing so, it successfully “positioned itself as an anti-austerity party akin to European left-wing parties like Spain’s Podemos and Greece’s Syriza” (Aodha 2020). These appeals later extended to social issues such as the 2015 referenda on same-sex marriage and abortion, which “broadened [Sinn Fein’s] appeal beyond an identity as a single-issue party for unification, helping it gain popularity among a younger generation that overwhelmingly backed the reforms” (Aodha 2020). Taken together, then, Sinn Fein’s historic gains in 2020 do not represent a centrifugal result. To the contrary, they reflect long-term learning and refinement by a political party that found itself within a centripetal (as we argue in this paper) electoral system, that is, an electoral system that rewards broader political appeal.
case of approval voting application at a national level election was the Greek electoral system from 1864-1920 (Tsebelis 2014: 172).5

Theoretical and Practical Challenges for Understanding Multiple Vote Systems

While previous implementations of multiple vote systems have encountered some success in terms of electing broadly supported, moderate candidates, they nevertheless face limitations in both theory and practice. First, as we noted at the outset of the paper, although studies like Cox’s prove in one dimension that multiple vote systems can draw candidates to the center of the political spectrum, he (nor anyone else to date) does not offer a proof in more than a single dimension. This stands as a challenge to the multiple vote system, as a second or third dimension can change the definition of “moderate” in a political system and create possibilities for candidates to be close to one another in one dimension while remaining dispersed in another.

Perhaps the most relevant example in contemporary politics lies in the current populist movements across the United States and Western Europe. While populists lie to the far right of the political spectrum on cultural issues, they nevertheless often support interventionist policies in the economy. In the United States, for example, top advisers to President Donald Trump have expressed strong support for spending on infrastructure, while maintaining culturally rightist positions on religion and immigration. Similarly, in Great Britain, populists have maintained an anti-Muslim posture while simultaneously driving the Brexit campaign and a swell of anti-free-trade rhetoric. In short, the rise in popularity of populist ideas has muddied the neat left-right distinction implied by unidimensional models. As we find, this can present challenges for the effectiveness of multiple votes systems.

In addition, though the STV system asks voters to rank candidates, the multiple votes it provides are conditional choices only. That is, the voters’ second or third choices influence the election only after the first vote is invalidated. This feature makes each additional choice less important than the prior, and complicates the system (although the logic of each successive choice is the same as in the system we analyze below).

Given these challenges to the current theory and practice of multiple vote systems, we focus in this paper on the mechanical consequences of adding m issue dimensions and a proportional voting mechanism to a multiple vote system. In doing so, we find that multidimensionality and proportional representation create centripetal party systems—just as current literature predicts. We also find that under certain extreme centrifugal conditions that the electoral system cannot overcome in the first election, but is likely to modify in the medium and long run. We conclude by discussing some scope conditions of this effect.

---

5 The election of Doges of Venice was also done through approval voting but in multiple rounds, though a deliberately complicated system so that the influence of organized clan interests would be minimized.
A MULTIDIMENSIONAL, MULTIPLE VOTE MODEL

Voting System Design

Tsebelis (2014) has proposed a multiple vote electoral system, which permits a certain number of votes, and is combined with the national distribution requirements of different countries. For example, countries like the Netherlands and Israel have absolutely proportional electoral systems; others, like Germany have national quotas (5%), while still others like Greece may give seat bonuses to the first party (a fixed amount or, a number proportional to its size). In this study, we simulate such a system by calculating the number of votes that each party receives under the multiple vote system and distribute the seats on the basis of the national features of the electoral system.

The fact that we use multiple member constituencies and distribute the seats on the basis of national (more or less proportional) rules is the major difference between the system we analyse in this paper and approval voting, which has been applied in single member constituencies (see previous section). Under the system we examine, voters may cast up to $n$ votes in total, with a maximum of one vote per party. In other words, even if a voter strongly prefers one party to the next best option, she may not cast a second or third vote for that preferred party (no cumulation of votes). However, if she strongly dislikes all other options besides her most preferred party, she can opt against casting more than one ballot at all. Thus, voters may cast any number of votes they desire, with a maximum of $m$ and a minimum of 1 (all abstainers are assumed to have already been removed). In our implementation of this system, the ballot entities are conceived of as parties (although one could imagine implementing a similar system with actual candidates, instead of parties).

In order to demonstrate the significant growth in voters’ choice sets under this arrangement, consider a country with 10 parties (like the Netherlands in our simulations below). The current single-vote proportional representation system provides the voters with 11 choices (abstention, plus selection of any one of 10 parties) in such a party system. In a two-vote system, however, the choices increase to 56 (abstention, 10 single party choices plus 45 two-party vote combinations); in a three-vote system the number of choice profiles increases to 176 (abstention, plus the 10 single party votes, 45 two party votes, and 120 three party votes). Even more impressively, a four-vote system affords the voter 386 unique choices. The maximum number of available choices lies at 5 total ballots, wherein Dutch voters would enjoy 638 total choice profiles. One may object that the number of choices is overwhelming for the voter; but in

---

6 The total number of available choice sets decreases after 5 votes, since the voters now face the decision of who to exclude in their ballot, rather than who to include.
reality, it is a simple, repetitive task, since the voter only has to determine whether she likes enough each one of the parties to vote for them (as long as (s)he has available votes).\footnote{The very fact that this system is the “acclamation vote” of ancient Sparta indicates that the logic is simple and straightforward! Nevertheless, we do not mean to suggest that, in practice, voters would not take some time to acclimate to the system. Certainly, voters would learn how to use their new choice set to express their preferences. However, we aim primarily to underscore the potential effects of the system after these transitional factors have run their course.}

According to this model, parties receive the same proportion of representation in the legislature as a proportion all votes cast. In our multiple vote case, this proportion is not as straightforward as the single-vote case. In our system, representation is allotted by

\[ P_i = \frac{V_i}{m \times N - A} \]

where \( P_i \) is the legislative proportion earned by party i, \( V_i \) is the total number of votes cast for party i, \( m \) is the number of votes allotted to each voter in the system, and \( N \) is the total number of voters. \( A \) is an important term in this fraction, as it signifies the total number of abstentions present in an election. As noted above, voters can choose against casting all their multiple votes if they deem some unacceptable. Thus, the inclusion of this term is necessary for calculating the actual proportion of total votes cast.

Our proportion \( P_i \) differs from ranked choice voting in that it all votes count equally in the final tabulation of \( V_i \). This is a significant difference from the STV system, where the multiple votes count only conditionally (i.e., only after incapacitation of the previous vote). This difference has the double effect of: 1) Simplifying the system for both voters and authorities alike (two of the main criticisms of STV) and 2) Providing more motivation for voters to use many of their votes, since such a behavior increases their contribution to the electoral result. We believe that casting multiple votes is less cognitively demanding for voters as it only requires them to inquire whether a given candidate is sufficiently acceptable to warrant one of their votes. Given that some literature has demonstrated that voter exhaustion leads ranked-choice voting to rely on only a fraction of total ballots in the final vote distribution used to select a winner (Burnett and Kogan 2015), we believe the equally weighted and singularly tabulated votes in our system improve upon this particular weakness of ranked-choice voting.

**Modeling Assumptions and Mode of Analysis**

To examine how multidimensionality and proportional representation impact the centripetal nature of multi-vote systems, we create a voting simulation in R, using the electoral system defined above. A simulation is necessary in this context, because of our interest in multidimensional issue spaces (analytic proofs in n dimensions are impossible). Instead, we run simulations and examine the results that obtain for various parameter specifications.
To proceed with the simulation, we created a customizable function that implements the aforementioned multiple-vote PR system. The function proceeds as follows. First, the user specifies several system-wide parameters of interest. These include both the number of votes \( m \) allotted per voter and the total number of voters \( N \) in the political system. The user must also define the platform location points of each party \( i \) in each issue dimension \( d \). The function generalizes to any number of parties and dimensions, on the condition that the user provides a platform location estimate for every party in each dimension. In addition, parties may decide not to take position in some dimension.

Beyond these parameters, there are several other user-defined parameters of note, including one related to abstentions (which are incorporated directly to the voting decision rules programmed into the model). In the model, voters have single-peaked preferences. Therefore, they vote on the basis of ideological proximity: voter \( n \in N \) casts each vote on the basis of the following decision rule:

\[
\arg\min_i (||n - i||^d)
\]

where \( ||*||^d \) represents the Euclidean distance in \( d \) dimensions between voter \( n \)'s ideal point and party \( i \)'s platform location. As noted earlier, voters may vote for each party only once (like in approval or transferable voting).

Because voters are prohibited from casting multiple voters for their top choice, they are not obligated to make use of all their vote choices \( m \). Instead, voters will only cast a vote for a party if and only if the following condition obtains:

\[
(||n - i^-||) < a
\]

where \( i^- \) refers to the nearest available party and \( a \) refers to a user-defined range of acceptability. In other words, once the distance between voter \( n \)'s ideal point and the remaining parties’ platform locations exceeds the user-defined range of acceptability \( a \), voter \( n \) will stop casting votes. If the user is not interested in restricting voter behavior in this way, \( a \) can be easily set to a very large number.\(^8\)

Finally, to render our model more “realistic,” we incorporate an error term in the voter’s calculations. Voters make their choices on the basis of distance between their preferences and the parties’ programs; however, with a probability \( 1 - r \) they may not select the party closer to them. In this case, the voter casts her vote randomly to one of the available parties. This behavior can also be generated if a voter attributes higher significance to a particular issue (quality of leadership) or is willing to vote for a party that is closer to her in a particular dimension, despite the fact that the overall distance (taking into account all dimensions) is large--which is

---

\(^8\) Practically speaking, this means that the voter stops casting votes entirely (rationally or randomly) once the \( a \) threshold is reached. For precision, one may define \( v_j \) as the set of parties that lie within voter \( j \)'s range of acceptability \( a \). By definition, \( i^- \) for voter \( j \) must be in the set \( v_j \) in order for the voter to actually cast their vote.
sometimes called “valence” in the literature (Green and Hobolt 2008, Bittner 2011, Green and Jennings 2017). While this parameter adds noise to our results, we believe it is a useful way to relax the strictly single peaked preference account of voting inherent to the model’s implementation. Like $a, m, N, d,$ and $i$, $r$ is a user-specified parameter that represents the probability that voter $n$ selects the party closest to her.

**Simulation Procedure**

The simulation proceeds by first transforming a matrix of party shares into a society of voters. Because multidimensional ideological estimates do not exist for entire citizenries, we begin first with a user-specified list of proportions of the legislature held by each party. From these proportions, the simulation creates a vector of length $N$ with voter identities and ideal points equal the proportions and ideological locations of the legislative parties. In other words, if Parties X, Y, and Z occupied 20, 30, and 50 percent of the legislature, respectively, then a 10-person society would include 2 citizens who identify with X, 3 who identify with Y, and 5 who identify with Z. The first vote vector is always equal to the actual electoral outcomes from the year in question—20 percent X, 30 percent Y, and 50 percent Z in the example above.

After generating this initial vector of voters, the algorithm then calculates the Euclidean distance between all voters and parties and determines which party lies second-closest to each party’s voters. If this distance is greater than the acceptability parameter $a$, the voter refrains from casting any more ballots. If the distance is less than $a$, the voter (with probability $r$) casts a vote for the most proximate party. With probability $1-r$, however, she casts her vote randomly. Once this process occurs for all voters, votes are tabulated for each party and representation is allotted accordingly.

For multiple vote systems that feature more than two ballots, the algorithm then proceeds as follows. For the third ballot (and beyond), rather than assuming that all voters in a given party share exactly the same preferences, the algorithm instead assumes that each voter $n$ is likely between her first-choice and second-choice parties. Consequently, when the voters cast their third ballots, their choices are based on this assumed position—a position that evolves as the algorithm continues from one ballot to the next. As a result, by the end of the simulation, the estimated distribution of voters is considerably different than the distribution of parties. This is far more realistic, of course, than voters sharing the ideal points of their chosen party. An example of the resulting estimated voter locations is depicted in Figure 1.

[INSERT FIGURE 1 HERE]

In this paper, we base our simulations on actual countries, using the classification of party systems generated by Laver and Benoit (2015). They present a mutually exclusive and collectively exhaustive classification of party systems into 5 basic categories. According to this system of classification, Category “A” countries exhibit a single “winning” party that controls all legislative decisions; Category “B” countries are led by a single, dominant party that governs in
coalition with a smaller party; in Category “C”, the legislature is led primarily by three parties—any two of which are large enough to form a coalition government; Category “D” countries, on the other hand, are dominated by two “top” parties; Category “E” countries exhibit a party system that is truly “open,” in that no winning two-party coalition is possible (based on the sizes of the parties in the system).

For our purposes, the most interesting countries are of Types C and E, because the mechanics of the proposed system are straightforward in the other three cases: the dominant party or one of the two competing coalitions will have a majority. Countries of Type “C” or “E” however, provide unpredictability of the results which provides a fertile ground for voters to use their preferences to influence the electoral results in a more significant way. If new parties emerge in the other systems, the policy space dimensions will increase and the party system will move to one of the two categories we examine. Interested readers may use our appendix to apply our model to analyze any system or particular country they want. In this application, we select Germany and Romania to serve as examples of Type C, and the Netherlands and Belgium as examples of Type E. We consider these four cases as sufficient to investigate the properties of the multiple vote system we propose.

To generate ideological positions for each party (as well as initial party sizes), we rely upon data from the Manifesto Project (Krause et al. 2018). More specifically, we use the 15-dimension refinement of the Manifesto Project scores generated by Lowe et al. (2011). Lowe et al. (2011) generate these 15 dimensions from a much larger number of topical categories found within the Manifesto Project data. The authors reduce the Project’s dimensionality in a principled way, by pairing opposing positions within the Project’s data into individual dimensions--rather than incorporating some positions that lack a clear “opposite” position within the data.

Once the parties of the selected countries were matched to the Lowe et al. Manifesto scores, our algorithm measured $n$ dimensional Euclidean distances between our generated voter populations and the locations of each of the parties. Given the high dimensionality of the data, providing visual representation of the parties’ locations is impossible. However, as we present our results, we ultimately present ideological centrism as each party’s distance from the “center of gravity” of the ideological distribution of voters. We define these measures more precisely in the results section.

Taken together, our expectations are as follows:

*Proposition 1 (centripetal effect):* Multiple votes will increase the shares held by centrist parties (and reduce extremist ones).

The logic underlying this proposition is simply that centrist parties will receive votes from all directions, while extremist ones only from their own area (if there are neighbor parties).
Proposition 2 (redistributive effect): Multiple votes will have a negative effect on the initial size of parties.

Indeed, smaller parties will get a higher number of “transfer” votes than larger ones and vice versa. However, beyond these two propositions, we do not anticipate that the other variables will have a systematic effect on party shares, but will depend on the distribution of parties in space.

RESULTS

In order to show the centripetal effects of the multiple voting system we present our results as a comparison between the \( m \)-vote cases and the classic, one-vote system of proportional representation, asking which parties gain (and lose) as a result of the \( m \)-vote system? In general, we present our results using the multidimensional center of gravity as our measure of moderation.

As noted above, our simulation features parameters that may affect our findings. These include the probability \( 1-r \) that voters will fail to vote on the basis of ideological proximity, the range of “acceptability” (ideological distances within which an individual is willing to actually cast a vote), and the total number of ballots. Thus, in presenting our results, we regress the gains from the \( m \)-vote system (relative to the one-vote) on each of these parameters: the probability of voting based on proximity (1-\( r \)), acceptability (\( a \)), and the number of ballots (\( b \)). Inclusion of each of these covariates ensures that we hold factors besides ideological centrism constant when examining the centripetal forces present in the \( m \)-vote system.

[FIGURE 2 HERE]

Figure 2 presents a coefficient plot of the pooled fixed-effect regression described above. Taken together, the results depicted in the figure provide support for the centripetal (Proposition 1) and redistributive (Proposition 2) effects of the multiple vote system. Indeed, as a party’s distance from the ideological center increases, vote share decreases. Also as predicted, initial party size is an important variable to control for, as smaller parties clearly benefit most from \( m \)-vote alterations. The other variables are overall close to zero.

[TABLE 1 HERE]

Although these results are suggestive, they tell only part of the story. As demonstrated by the country-specific results displayed in Table 1, the centripetal and redistributive properties of the system exist in all countries when pooled together. While the coefficients of the parameters used in the model are overall as expected (higher acceptability, higher number of votes, and lower error term in the single peaked preferences lead to more vote gains for the average party), they also show variability across different countries. This finding indicates that the significance of these parameters depends on the party distribution in each country. Similarly, in Germany, while
the signs of the coefficients of centripetal and redistributive effects are the “correct” ones and statistical significance is high, the relative size of the coefficients indicates that the redistributive effect is much more significant than the centripetal one.

Given these significant across country differences, we examine the results more closely by country. For visualization purposes, we plot the parties’ locations in Figure 3 according to Lowe et al.’s scaling of Laver and Benoit’s (2007; p.98, Table 2) two-dimensional party scores. These scores place parties along “social conservatism” and “state involvement in the economy” dimensions. Given this procedure, we remind the reader that the graphs may exhibit some differences from a figure that plots any pair of the “raw” dimensions found in the Manifesto Project.

[FIGURE 3 HERE]

Among the countries that do experience centripetal results, this visualization provides some context as to which parties tend to drive the result. In Belgium, for example, sizeable gains by Centre Democrat Humaniste and ECOLO (as well as losses by Mouvement Reformateur and Libertair, Direct, Democratisch) generate rather strong centripetal results.

Nevertheless, as is plainly depicted for Germany, a different set of dynamics is at play. Here, we observe a party system that is ideologically scattered, with parties located in loose clusters that lie far apart from one another. Under these conditions, the multiple vote system lacks a party to attract votes from the extremes. Instead, parties exchange votes within ideological clusters, limiting the effectiveness of a multiple vote system at alleviating preference polarization in the legislature. In fact, such preference configurations reward a different sort of centrality: cluster-specific centrality.

To be clear, as Table 1 indicates, our multiple vote system generates a significantly more centripetal result in Germany than does the single-vote alternative. However, compared to the other countries in our simulations, this result is smaller in magnitude. Figure 3 depicts why this is the case. In particular, non-centrist parties such as The Left and FDP make gains due to the fact that they receive votes in multiple directions: FDP receives votes from CDU/CSU and SPD, while The Left draws votes from the Greens and SPD. In a perfectly centripetal case, another centrist party would serve as a vote-trading partner with SPD, allowing SPD to make gains instead of The Left and/or FDP (depending on the exact location of the hypothetical party).

In order to corroborate this argument in all 15 dimensions, we perform an additional test below, wherein we perturb the Germany’s party distribution. That is, we show that when a small party is added in the hypothesized neighborhood, centripetal forces become significantly more pronounced.

[FIGURE 4 HERE]
**Reinforcing Moderation in Polarized Systems**

To test our claim that a small spatial deviation can strengthen centripetal results, we introduce a small centrist party (though bordering on “populist” given its social conservativism) into Germany’s party system. This party occupies four percent of the total vote, representing the smallest vote share out of any party in our simulation.9

The results of this test are depicted in Figure 4, where we have again collapsed the parties into Laver and Benoit’s two dimensions. When we introduce this small-but-centrist party, we find that even a weak centripetal configuration, such as Germany’s, may exhibit stronger centripetal properties. In Figure 4, our added party, labeled as “Centrist Party,” makes sizable gains as a result of the \( m \)-vote system. Indeed, because the party draws votes from both parties in the rightward bloc (CSU/CDP and FDP) and parties in the center-left bloc (SPD and, in the three-vote case, the Greens) this new party experiences significant gains that improve the centripetal nature of the system.

![TABLE 2 HERE]

This trend does not necessarily apply only to perfectly centrist parties. Indeed, the “centrist” party itself appears as fairly conservative in the overall distribution. However, so long as any “new” party lies within the acceptability range of the innermost parties from each cluster, we observe a significant coefficient on Distance to Center variable in the above models. In this case, the coefficient on Distance to Center is nearly twice as large in the perturbed case than in the original simulation in Table 1. This result is supported by the fact that, because more “centrist” parties performed no worse (and, in fact, marginally better) under this configuration than in the original, the introduction of a centrist party benefited other more centrally located party blocs. For example, while SPD still loses votes relative to the single-vote case, it nevertheless performs better than in the original simulation, having received votes from Centrist Party (in addition to retaining votes from The Greens and FDP).

**CONCLUSIONS**

This paper examines the mechanical effect of a multiple vote system, using actual countries’ party distributions as a means for examining how and when our system should encourage the election of centrist parties or candidates. In doing so, we demonstrate that moderation effects in one dimension, demonstrated by Cox (1991), obtain in multiple dimensions. As we caution throughout, our analysis does not examine the strategic effects of such a system. Nevertheless, we believe that examining the mechanical effect of multiple vote systems is important for a number of reasons. First, it demonstrates that such systems do not behave identically in all countries, but nevertheless that countries have broad similarities as Table 1 indicates. Second, it

---

9 We selected this size so that with a single vote this party would not have been represented in the Budestag, and it would not have altered the results of a single vote system.
is upon these broad similarities that strategic calculations of voters and parties will be based. One may argue that voters have personal, social, or cultural misgivings about voting for an extreme party (like, say, a fascist or communist party). In addition, parties are constrained in their ability to adjust their ideological positions in a rapid fashion. Activists within the party would likely resist such changes, and voters may respond poorly to drastic changes in the ideological “brand” associated with the party. Thus, while future research may account for important strategic considerations faced by voters and parties, these considerations have to be based on the mechanical effects of the multiple vote system in the same way as Duverger’s (1951) “psychological effect” was grafted upon the “mechanical effect” of the plurality system.

In this paper, we have used simulations in order to calculate the mechanical effects of multiple voting systems (whether they are applied to single or multiple member constituencies). Our findings confirm the ones of Cox in a single dimension. He was able to prove his results, because the combination of single peaked preferences with a single dimension leads to an equilibrium (the median voter). However, in multiple dimensions the equilibrium disappears (and so do the formal proofs), generating the need for simulations. The lack of equilibrium in multiple dimensions leads us to a different logic for our investigation. While Cox’s model leaves the parties free to move in the one dimensional space and determines whether (in equilibrium) they cluster in the middle or disperse all over the (one-dimensional) space in order to maximize the number of votes, we keep the parties in their initial location and have the voters select the parties that are closer to them (as the different parameters of the model permit). The outcome of our model is that centrist parties get better results with multiple votes. So, our model demonstrates that the single dimension is not a necessary condition for convergence, but that the single peaked preferences of the voters is.

However, beyond this mechanical effect, we believe that the adoption of this multiple vote system may imply several additional long-term changes. With respect to voters, this system presents an exponential increase in the number of voting alternatives. Indeed, if we permit voters to have number of votes equal to half the number of parties, the number of choices is:

$$\sum_{i=1}^{N/2} \binom{N}{i} + 1$$

where N equals the total number of parties in a country. This increase of choices is likely to reduce the number of abstentions (since it reduces abstention from indifference (e.g., Plane and Gershtenson 2004, Adams, Dow and Merrill 2006, Llavador 2006)). Indeed, a voter who does not know if she should vote for party A or B in a multiparty system may now vote for both. Moreover, she may do so without confronting the cognitively taxing task of ranking candidates: all votes in this setting are “worth” the same.
In addition to its potential for decreasing abstention, we believe that a multiple vote system may help to increase voter information. In order to evaluate different candidates under such a system, voters will have to pay attention to the positions of a larger number of parties or candidates—understanding that they will ultimately be voting for more than a single party.

Moreover, understanding that actually casting multiple votes increases their impact on the outcome, voters face incentives both to cast more votes and improve their information in the process. We are hopeful that particular feature of the multiple voting system will have a significant impact on the voting habits of the public. With respect to parties, our results—particularly in the perturbation exercise—suggest that the total number of parties will multiply, since there is no reason for any political entrepreneur not to create their own party and try their chances. This is particularly true given that they can reasonably expect many second or third votes from major parties around them (if the party is situated appropriately). In order to reduce this tendency, countries may consider strict rules of which parties are allowed to compete should be enforced (for example, parties have to exist 6 months before the election, and a large number of signatures is required for the creation of a new party). These restrictions will enable voters to know the positions of the parties in competition, and choose them according to their preferences.

Third, with respect to the party positions, we showed that centrist parties are privileged in a multiple vote system. However, these advantages are attenuated in cases where party clusters emerge in large distance from each other (like the case of Germany in our examples). Still, when the system is applied several times, the emergence of a centrist party—or the convergence of existing parties close to the multidimensional median—is likely, because political entrepreneurs will understand the potential for success of such a party. This is a similar argument with the one in the report of the American Academy of Arts and Sciences, in defense of the ranked-choice electoral system that it proposes for the US case: “Because second and third choices matter in the ranked-choice model, candidates have an incentive to speak to a broader group of voters. The result: more moderate candidates and campaigns, a more welcoming environment for third-party candidates, and greater confidence among voters that their votes are not being wasted or distorting the outcome” (American Academy of Arts & Sciences 2020).

Finally, perhaps the most important consequence of such a voting system (although not directly demonstrated in this paper) is the potential promotion of a critical attitude of voters vis a vis parties, as opposed to an identification attitude. That is, instead of voters trying to find a party to identify with, they can be more critical and express their preferences more fully (if they so wish). This result carries with it both pros and cons. On one hand, party identification may fulfill a variety of positive societal functions, such as increasing voter turnout, serving as a policy evaluation heuristic, and encouraging other types of political participation (see Dalton 2016 for a review). On the other, as Lavine, Johnston, and Steenbergen (2012) and others have underscored, intense partisan identification can lead to narrow-mindedness on the part of partisans. Indeed, such identifications may lead partisans to disregard important information that does not confirm
their partisan biases. Doing so could empower demagogic leaders or create partisan informational asymmetries and fracture a society according to partisan identifications.

Finally, the centripetal effects of the electoral system are likely to have important ramifications for the governing coalitions of each country. In fact, ideological proximity is a key feature of coalition formation (Warwick 1996, 1998; Tsebelis and Ha 2014), and the multiple vote electoral system will lead to governments with more uniformly centrist composition. The policy implications of such a transformation will be significant, given parliamentary governments’ control of the legislative agenda. As a result, such coalitions’ proposals and legislation will lie in closer correspondence with the aspirations and desires of a broader portion of the public.
Works Cited


Grose, Christian R. 2014. The adoption of electoral reforms and ideological change in the California State Legislature. *Schwarzenegger Institute Report*, University of Southern California, Price School of Public Policy.


Tsebelis, G. 2014. The Greek Constitution from a Political Science Point of View”, *Greek Political Science Review* (July; 42): 145-72


Figure 1. Estimated Voter Locations in the Netherlands

Simulations for the Netherlands, with four total ballots (m=4), acceptability (a = 12), and error term (r= 0.35). Locations depicted in Lowe et al.’s (2011) two-dimensional reduction of the Manifesto Project data. Party platform locations are depicted by the location of the textual abbreviation for each respective party.
Figure 2. Ideological Centrism and Gains from $m$ vote System; Coefficient Plot

_Pooled fixed-effects regression from Table 1._

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>All (w/ FEs)</th>
<th>Netherlands</th>
<th>Belgium</th>
<th>Germany</th>
<th>Romania</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distance from Center</strong></td>
<td>$-0.270^{***}$</td>
<td>$-0.640^{***}$</td>
<td>$-0.625^{***}$</td>
<td>$-0.093^{***}$</td>
<td>$-0.137^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.017)</td>
<td>(0.031)</td>
<td>(0.007)</td>
<td>(0.010)</td>
</tr>
<tr>
<td><strong>Initial Party Size</strong></td>
<td>$-8.288^{***}$</td>
<td>$-14.769^{***}$</td>
<td>$-37.352^{***}$</td>
<td>$-6.862^{***}$</td>
<td>$-1.644^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.104)</td>
<td>(0.217)</td>
<td>(0.736)</td>
<td>(0.081)</td>
<td>(0.116)</td>
</tr>
<tr>
<td><strong>Acceptability Parameter</strong></td>
<td>0.011</td>
<td>0.052$^{***}$</td>
<td>0.004</td>
<td>$-0.010$</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.012)</td>
<td>(0.018)</td>
<td>(0.008)</td>
<td>(0.002)</td>
</tr>
<tr>
<td><strong>Number of Votes</strong></td>
<td>$0.191^{***}$</td>
<td>$0.090^{***}$</td>
<td>$0.355^{***}$</td>
<td>$0.082^{***}$</td>
<td>$0.117^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.017)</td>
<td>(0.027)</td>
<td>(0.016)</td>
<td>(0.007)</td>
</tr>
<tr>
<td><strong>Proximity Voting</strong></td>
<td>$-0.160^*$</td>
<td>0.440$^{***}$</td>
<td>0.020</td>
<td>$-0.546^{***}$</td>
<td>$-0.057^*$</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.159)</td>
<td>(0.247)</td>
<td>(0.105)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>Observations</td>
<td>25,350</td>
<td>6,000</td>
<td>6,600</td>
<td>6,750</td>
<td>6,000</td>
</tr>
<tr>
<td>Adjusted R$^2$</td>
<td>0.238</td>
<td>0.473</td>
<td>0.309</td>
<td>0.519</td>
<td>0.074</td>
</tr>
</tbody>
</table>

_{Note:} $^*$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01

Table 1. Country-Specific $m$ vote Regression Results

(Center of Gravity)
Figure 3. Two-Dimensional Depictions of Parties, Gains and Losses

Two-dimensional projections of parties’ Manifesto scores, depicted with gains and losses in an m vote \((n = 3; \ a = 12)\) setting. Here, the larger the plus sign, the larger the gains for a party, whereas the larger the minus sign the larger the losses. As the figures depict, centrally located parties generally benefit in the m vote setting, relative to peripheral parties.
Figure 4. Two-Dimensional Depictions of Parties, Gains and Losses

Two-dimensional projections of parties’ Manifesto scores, depicted with gains and losses in an m vote \((n = 3; a = 12)\) setting. Here, the “Centrist Party” has been added, and its somewhat central location enhances the centripetal nature of the hypothetical results in Germany.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Gains from n-vote System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from Center</td>
<td>(-0.250^{***}) (0.018)</td>
</tr>
<tr>
<td>Initial Party Size</td>
<td>(-6.750^{***}) (0.079)</td>
</tr>
<tr>
<td>Acceptability Parameter</td>
<td>0.0002 (0.006)</td>
</tr>
<tr>
<td>Number of Votes</td>
<td>0.172^{***} (0.013)</td>
</tr>
<tr>
<td>Proximity Voting</td>
<td>0.193^{**} (0.091)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.482^{***} (0.108)</td>
</tr>
<tr>
<td>Observations</td>
<td>8,100</td>
</tr>
<tr>
<td>Adjusted R(^2)</td>
<td>0.491</td>
</tr>
</tbody>
</table>

Note: \(*p<0.1; **p<0.05; ***p<0.01\)

Table 2. Simulation Results in Germany, Following Addition of Populist Party
APPENDIX

R Function Used to Calculate New Vote Share

```r
require("plyr")
require("Gmedian")
require(SDMTools)
require("ggrepel")
require(ggthemes)
require("jtools")

COG_ndim <- function(ideology, parties){
  M <- sum(parties[,2])
  output_mean <- NA
  output.sd <- NA
  for(i in 1:ncol(ideology)){
    assign(paste("COG", i, sep=""), wt.mean(ideology[,i], parties[,2]))
    assign(paste("COG.sd", i, sep=""), wt.sd(ideology[,i], parties[,2]))
    output_mean[i] <- get(paste("COG",i,sep=""))
    output.sd[i] <- get(paste("COG.sd", i, sep=""))
  }
  data.frame(cbind(output_mean, output.sd))
}

result <- function(population, parties, ideology, acceptability, ballots, rational){
  voters <- c()
  lengths <- round_preserve_sum(parties$percentages, 3)
  for(i in parties$names){ # creates the party-voter vector
    subvector <- c() 
    length = population*lengths[i]
    length[is.na(length)] <- 0
    subvector <- c(rep(i, times=length))
    voters <- append(voters, subvector)
  }
  vote.mat <- matrix(ncol = ncol(ideology)) #dimensionality
  for(i in voters){
    vote.mat <- rbind(vote.mat, ideology[i,])
  }
  ### VOTES ###
  # First Vote
  votes <- c()
  vote1 <- c()
  for(i in 2:nrow(vote.mat)){
    distances <- c()
    for(j in 1:nrow(ideology)){
      distances <- append(distances, dist(rbind(vote.mat[i,], ideology[j,])))
    }
    vote1 <- append(vote1, which.min(distances))
  }
  votes <- vote1
  # Second vote
  vote2 <- c()
  for(i in 2:nrow(vote.mat)){
    if(runif(1) <= rational){
      distances <- c()
      for(j in 1:nrow(ideology)){
```
distances <- append(distances, dist(rbind(vote.mat[i,], ideology[j,])))
ifelse(sort(unique(distances))[2] < acceptability, vote2 <-
append(vote2, which(distances == sort(unique(distances))[2])), vote2 <-
append(vote2, NA))
} else{vote2 <- append(vote2, sample(parties$names, 1))
}
}
votes <- append(vote1, vote2)
recorded <- cbind(vote1, vote2)
# 3rd vote and beyond
if(balls > 2){
for(b in 3:balls){
voters <- NA
vote.vec.init <- votes
vote.total <- data.frame(table(vote.vec.init, useNA = "always"))
vote.vec.perc <-
round_preserve_sum(vote.total$Freq/sum(vote.total$Freq), 3)
vote.total <- cbind(vote.total, vote.vec.perc)
lengths <- round_preserve_sum(vote.total$vote.vec.init, 3)
for(i in 1:length(as.character(vote.total$vote.vec.init))){ # creates
the party-voter vector
subvector <- c()
length = round_preserve_sum(sum(vote.total$Freq)/(b-1)*lengths[i], 3)
simulate(vote.total$Freq)
length[is.na(length)] <- 0
subvector <- c(rep(as.character(vote.total$vote.vec.init[i]),
times=length))
voters <- append(voters, subvector)
}
voters <- as.numeric(voters[2:length(voters)])
vote.mat.init <- matrix(ncol = ncol(ideology))
for(i in voters){
vote.mat.init <- rbind(vote.mat.init, ideology[i,])
}
vote.mat.init <- vote.mat.init[2:nrow(vote.mat.init),]
assign(paste("vote", b, sep=""), c())
for(i in 1:nrow(vote.mat.init)){
distances <- c()
loopnums <- 1:nrow(ideology)
loopideo <- subset(loopnums, loopnums%in%recorded[i]==F)
if(runif(1) <= rational){
for(j in loopideo){
distances <- append(distances, dist(rbind(vote.mat.init[i,], ideology[j,])))
}
ifelse(is.na(sort(unique(distances))[1])==F &
sort(unique(distances))[1] < acceptability, assign(paste("vote", b, sep=""),
append(get(paste("vote", b, sep="")), loopideo[which.min(distances)]))),
assign(paste("vote", b, sep=""), append(get(paste("vote", b,
sep="")), NA)))
} else{assign(paste("vote", b, sep=""), append(get(paste("vote", b,
sep="")), sample(1:1:length(loopideo), 1)))
}
}
votes <- append(votes, get(paste("vote", b, sep="")))
recorded <- cbind(recorded, get(paste("vote", b, sep="")))
```r
{NA}
counts <- data.frame(table(votes))
percentage <- (counts$Freq)/sum(counts$Freq)
vote1count <- data.frame(table(vote1))
votelcount <- (votelcount$Freq)/sum(votelcount$Freq)
distance_from_center <- c()
for(i in 1:nrow(ideology)){
    distance_from_center <- append(distance_from_center,
        dist(rbind(ideology[i,],COG_ndim(ideology, parties)$output_mean)))
}
results <- data.frame(cbind(counts$votes, percentage, vote1count, distance_from_center))
names(results) <- c("Party", "Votes", "Initial Vote", "Distance from Center")
results
}````