Four Number Game: A Lesson Plan

Content Standards

8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

HSG-CA.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

HSS-CP.B.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

As with many open-ended problem-solving explorations, there may be other content standards that arise. Conversely, you can only guarantee these particular ones will arise if you steer students towards particular solution paths; that is not necessarily recommended.

Standards of Mathematical Practice

SMP.3 Construct Viable Arguments and Critique the Reasoning of Others

SMP.5 Use Appropriate Tools Strategically

SMP.6 Attend to Precision

SMP.7 Look For and Make Use of Structure

SMP.8 Look For and Express Regularity in Repeated Reasoning

Learning Objectives

- Students fluently and accurately practice routine subtraction.
- Students generate accurate new versions of the game using the same four numbers and develop a system for recording their findings.
- Students reason about why some versions of the four number game generate the same numbers and the same number of steps. (Their reasoning may rely on physical models such as square pieces of paper that they reflect and rotate to demonstrate why certain games are the same.)
- Students develop arguments for how many arrangements of the four starting numbers are equivalent to the original using systematic record-keeping, symmetries of the square, and precise argumentation.
- Students develop arguments for identifying how many non-equivalent games there are using systematic record-keeping, symmetries of the square, and precise argumentation.

Materials

- 10 pieces of square paper per student. Students could make this themselves from regular $8\frac{1}{2} \times 11$ paper if they have scissors.
- Paper for recoding findings.
- Pencils.

Squares made of overhead transparencies may also help students to work with reflections more easily.

Sample Outline

- *Find 4 numbers ahead of time that will give games that are not too long and not too short.*
- Introduce students to the game and have them do some examples on their own (using the same four numbers), in pairs, or in groups.
- Have them check the accuracy of each others' work and emphasize record-keeping skills like always starting from the upper left corner when naming the numbers and accurately counting the number of steps.
- Record some findings on the board for the class to verify. Record the game played (e.g. 1, 4, 5, 6) and the number of steps it took.
- If students don't notice on their own, ask them how we might know that THIS game and THAT game give the same number of steps (where THIS game and THAT game are two games that are related by reflections and/or rotations).
- Now pose a few questions for them to consider: how many different ways do we have of labeling the square, starting in the upper left-hand corner? How many games can you find that are equivalent to the game we started with? How many non-equivalent games are there?

This provides an opportunity to see if students are counting the steps correctly and to see that not that many different numbers of steps come up. Inaccurate counting of steps will certainly come up—ask students in other groups if they agree.

Formative Assessment

- What do we mean that two games will be "equivalent"?
- If I give a game with the square labeled like THIS, can you give me more games that will be equivalent to it?
- Here are two games that I think are equivalent. Explain how I can show that THIS one is equivalent to THAT one.
- Here are two games that I think might be different. Are they? Can you explain how you know they're different?
- If I start with a 7 in the upper left corner, how many ways do I have of placing a 3, 2 and 4 on the remaining corners?

Follow-up Explorations

If students finish early or need more of a challenge, there are many ways to extend this problem. Here are a few examples:

- If we start with the numbers 5, 1, 1, 1, how many non-equivalent games can I find? What if I start with the numbers 5, 2, 1, 1?
- Why will the Four Numbers game always end in all zeros?
- What is the longest Four Numbers Game you can find?
- What if we played a Three Numbers Game, or Five Numbers Game, or Eight Numbers Game. Will these games always end in all zeros?
- What if we could use fractions to begin with? Could we find four numbers to start with so that the game would never end?

Pedagogical Resources

There are many challenging aspects of facilitating an open-ended problem or one that requires sophisticated student reasoning or explaining: maintaining the cognitive demand of the problem by not over-scaffolding, eliciting student thinking, understanding student thinking, asking the right questions, having students share their work in ways that keeps the class productive, etc.

Two resources we particularly recommend for grappling with some of these issues are the following:

Smith, Margaret E. and Mary Kay Stein. *Five Practices for Orchestrating Productive Mathematics Discussions.* Reston, VA: National Council of Teachers of Mathematics, 2011.

Stein, Mary Kay, ed. *Implementing standards-based mathematics instruction: A casebook for professional development.* Teachers College Press, 2000.