Review: Double Slit / Two-hole interferometer (1-D)

Single:

\[ p(x) = \prod_{\pm}(x) \rightarrow p(k) = \text{sinc} k \]

The two-hole is the convolution of the hole shape with delta functions at the hole locations

\[ p(x) * [\delta(x-x_1) + \delta(x-x_2)] \rightarrow p(w) [e^{-ik(x-x_1)} + e^{-ik(x-x_2)}] \]

For example, \(x_1 = -x_2\) (symmetric) gives cosine transform

Image \(\propto E^2\)

\[ \| p(k) [e^{-ik(x-x_1)} + e^{-ik(x-x_2)}] \|^2 = p^2(k) [e^{-ik(x-x_1)} e^{ik(x-x_2)} + e^{-ik(x-x_1)} e^{ik(x-x_2)} + e^{-ik(x-x_1)} e^{-ik(x-x_2)} + e^{ik(x-x_1)} e^{-ik(x-x_2)}] \]

\[ = p^2(k) [2 + 2 \cos(k(x_2-x_1))] \quad \text{All real, non-negative} \]
Two-hole interferometer: binary source signal

From principles of Long Baseline Stellar Interferometry:

$$V_{\text{binary}} = e^{-2\pi i (u\alpha + v\beta)} \frac{|V_1 + r|V_2|}{1 + r} e^{-2\pi i (u\Delta\alpha + v\Delta\beta)}$$

\[ r = \text{contrast ratio} \]

$$\Rightarrow \frac{1 + r e^{-2\pi i (u\Delta\alpha + v\Delta\beta)}}{1 + r} \quad (u, v) = \vec{x}_2 - \vec{x}_1$$

Notice that the signal is baseline-dependent.

What can confuse this signal? Unknown phase errors.

- **Phase delay**
  - **Field:** \(p(k) \left[ e^{i\phi_1} e^{-i(k(x-x_1))} + e^{-i\phi_2} e^{-i(k(x-x_2))} \right] \)
  - **PSF:** \(p^2(k) \left[ 2 e^{i(k(x_2-x_1)+\phi_2-\phi_1)} + e^{2i(k(x_2-x_1)+\Delta\phi)} \right] \)

**BUT** \(\phi_1's\) don't care about baseline length!

**L** NRM can be useful for measuring hole-dependent errors!
Suppose three-hole interferometer

\[ \text{Pupil} = P(x) \ast \left[ \delta(x-x_1) + \delta(x-x_2) + \delta(x-x_3) \right] \text{ in general} \]

\[ = p(x) \left[ e^{-ik(x-x_1)} + e^{-ik(x-x_2)} + e^{-ik(x-x_3)} \right] \]

\[ \text{PSF} = p^2(x) \left[ 3 + e^{ik(x_2-x_1)} + e^{-ik(x_2-x_1)} + e^{ik(x_3-x_1)} + e^{-ik(x_3-x_1)} + e^{ik(x_1-x_3)} + e^{-ik(x_1-x_3)} \right] \]

Again, cosine fringes \( \frac{N(N-1)}{2} \) unique fringe phases

Attach constant phases to each hole

For a point source, measured phases:
\[ \phi_2 - \phi_1 \]
\[ \phi_3 - \phi_2 \]
\[ \phi_1 - \phi_3 \]

in general how do we go from pupil phases to fringe phases?

Call this matrix **A**

\[ \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\phi_3 
\end{bmatrix} =
\begin{bmatrix}
\phi_2 - \phi_1 \\
\phi_3 - \phi_2 \\
\phi_1 - \phi_3 
\end{bmatrix} \]

If these are phase "errors" how can we get rid of them?

In other words, is there a matrix **K** such that:
\[ K \cdot A = 0 \]
(leaving only physical signal?)
3-hole interferometer

Closure phase = \( \phi_{12} + \phi_{23} + \phi_{31} \) = \( \phi_2 - \phi_1 + \phi_3 - \phi_2 + \phi_1 - \phi_3 \)

(hole phase errors only) = \( \phi_2 - \phi_2 + \phi_1 - \phi_1 + \phi_3 - \phi_3 = 0 \)

Only 1 closure phase for three holes

\[
\begin{bmatrix}
1 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
\phi_2 \\
\phi_3 \\
\phi_3
\end{bmatrix} = 0
\]

(3) Closure phases

In observations:

\( \Theta_{\text{measured}} = A \cdot \phi_{\text{holes}} + \Theta_{\text{physical}} \)

\( K \cdot \Theta_{\text{measured}} = K \cdot A \phi_{\text{holes}} + K \cdot \Theta_{\text{physical}} \)

\[\uparrow\]

closure phases – measure physical structure

What does the physical signal look like?

\[ V_{\text{binary}} = \frac{1 + r e^{-2\pi i (u \Delta \alpha + v \Delta \beta)}}{1 + r} \]

1-D in general, \( r = 0.5 \) example:

\[ V_1 = 1 + \frac{1}{2} e^{-2\pi i \Delta \alpha U_{12}} \]
\[ V_2 = 1 + \frac{1}{2} e^{-2\pi i \Delta \alpha U_{23}} \]
\[ V_3 = 1 + \frac{1}{2} e^{-2\pi i \Delta \alpha U_{31}} \]

\[ V_1 V_2 V_3 = \left[ 1 + \frac{1}{2} e^{-2\pi i \Delta \alpha U_{12}} + \frac{1}{2} e^{-2\pi i \Delta \alpha U_{23}} + \frac{1}{2} e^{-2\pi i \Delta \alpha U_{31}} \right] / (1 + \frac{1}{2})^3 \]
Redundant pupils?

\[ \Theta_{\text{measured}} = A \cdot \Phi_{\text{holes}} + \Theta_{\text{physical}} \]

Example: partially redundant pupil - annulus

Doubly redundant in all except longest baselines

Break it into 8 sub-apertures

One row in \( A \):

to define baseline

\[ [0 \quad -0.5 \quad 0.5 \quad 0 \quad 0 \quad 0.5 \quad -0.5 \quad 0] \]

\[ \Theta_{\text{measured}} = A \Phi_{\text{subapertures}} + \Theta_{\text{physical}} \]

Choose \( K \) so \( K \cdot A = 0 \)

\[ K \Theta_{\text{measured}} = \Theta_{\text{kernel}} = K \Theta_{\text{physical}} \]

For arbitrary pupil shapes Kernel phase is a self-calibrating quantity.