Winter 2019

Math 657: **Nonlinear Partial Differential Equations**

TTh 2:30 – 4:00 PM,

Instructor: Zaher Hani, (*Office:* 5834 East Hall (EH)), *Email:* zhani@umich.edu,
Website: https://sites.lsa.umich.edu/zhani/.*Office hours:* Tuesday 1:00-2:00pm and by appointment.

Prerequisites: Ideal requirements for this course would be Math 656 and a solid background in graduate real analysis. Please consult instructor otherwise.

Course Coordinates: TTh 2:30–4:00 pm. On Tuesday, we meet in EH1060 East Hall (MH), and on Thursday in EH3254 East Hall (MH) (in the Psychology part of the building).

Textbook: No textbook is required.

Reference and resources: Here are some references on Fourier Analysis (first three), Fluids (forth and fifth), and dispersive PDE (sixth).

1. H. Bahouri, J-Y. Chemin, R. Danchin, *Fourier Analysis and Nonlinear Partial Differential Equations*. Springer. ISBN 978-3-642-16829-1.

2. J. Duoandikoetxea , *Fourier Analysis*. Graduate Studies in Mathematics, AMS. ISBN: 978-0821821725.

3. E. Stein and G. Weiss, *Introduction to Fourier Analysis on Euclidean Spaces*. Princeton University Press. ISBN-13: 978-0691080789

4. A. Majda, A. Bertozzi, *Vorticity and incompressible flow*. Cambridge texts in Applied Mathematics. ISBN: ISBN 0 521 63948 4.

5. Terry Tao's lecture notes on Fluid Equations available of his "What's new" blog: https://terrytao.wordpress.com/category/teaching/254a-incompressible-fluid-equations.

6. T. Tao, *Nonlinear Dispersive Equations: local and global analysis*. CBMS Regional Conference Series in Mathematics. ISBN: 978-0-8218-4143-3.

Homework: There will be a few homework sets throughout the semester.

Grading: Grading will be based on homework.

Course Description: Partial differential equations are at the core of models in science, engineering, economics, and related fields. These equations and their solutions have interesting structures that are studied by a beautiful combinations of methods from analysis, geometry, probability and other mathematical fields.

This course is an introduction to nonlinear partial differential equations for a diverse audience in pure and applied mathematics. It will start by reviewing and introducing some analytical techniques that are central to the modern study of nonlinear PDEs like Fourier analysis, Sobolev spaces, and Littlewood-Paley theory. Afterwards, the course will present various techniques to construct and analyze solutions of nonlinear PDE. The focus will be on models coming from fluids (Euler and Navier-Stokes equations) and nonlinear dispersive/hyperbolic equations (the nonlinear Schrödinger and wave equations). Other topics and equations will be covered time permitting.

Outline:

A) Harmonic Analysis Prerequisites

- Introduction to the Fourier transform
- Introduction to Littlewood-Paley theory
- Sobolev spaces and embeddings

B) Introduction to Fluid Equations

- The Navier-Stokes and Euler's equations: basic properties
- Construction of solutions via energy methods
- Weak solutions
- Euler's equation: Existence theory and blowup criteria

C) Introduction to nonlinear dispersive/hyperbolic equations

- What are dispersive PDE: Basic features and properties
- Linear Equations and their estimates
- Nonlinear theory: local vs global existence.

Important Dates

Jan 10	First day of classes
March 2-10	Spring Break- No Class
April 23	Last day of classes.