Fall 2019

Math 756: **Advanced Partial Differential Equations**

TTh 1:00 – 2:30 PM,

Instructor: Zaher Hani, (*Office:* 5834 East Hall (EH)), *Email:* zhani@umich.edu,
Website: https://sites.lsa.umich.edu/zhani/. *Office hours:* Tuesday 2:30-3:30pm and by appointment.

Prerequisites: Ideal requirements for this course would be Math 656-657 and a solid background in graduate real analysis. Practically, some previous exposure to undergraduate PDE and Fourier analysis would be useful. We will review some required material from Fourier analysis in the course.

Course Coordinates: TTh 1:00–2:30 pm in EH2866 East Hall (MH).

Textbook: No textbook is required.

Reference and resources: The main reference for this course is Tao's book on dispersive PDE. The following textbooks mentioned below could be consulted for some topics to be covered in class.

1. T. Tao, *Nonlinear Dispersive Equations: local and global analysis.* CBMS Regional Conference Series in Mathematics. ISBN: 978-0-8218-4143-3.

2. Jalal Shatah and Michael Struwe, *Geometric wave equations. Courant Lecture Notes in Mathematics*, 2. New York University, Courant Institute of Mathematical Sciences, New York; American Mathematical Society, Providence, RI, 1998. viii+153 pp. ISBN: 0-9658703-1-6; 0-8218-2749-9.

3. Thierry Cazenave, *Semilinear Schrödinger equations.* Courant Lecture Notes in Mathematics, 10. New York University, Courant Institute of Mathematical Sciences, New York; American Mathematical Society, Providence, RI, 2003. xiv+323 pp. ISBN: 0-8218-3399-5.

4. J. Duoandikoetxea , *Fourier Analysis.* Graduate Studies in Mathematics, AMS. ISBN: 978-0821821725.

5. E. Stein and G. Weiss, *Introduction to Fourier Analysis on Euclidean Spaces.* Princeton University Press. ISBN-13: 978-0691080789

Course Description: This course will survey some topics on one of the most fundamental and rich classes of partial differential equations (PDE), namely nonlinear dispersive and wave equations. Such equations arise in numerous areas of physics and engineering, ranging from quantum mechanics, field theories, nonlinear optics, plasma physics, all the way to ocean science and general relativity. The underlying phenomenon in all these equations/systems is the nonlinear interaction of waves/frequencies, the study of which is commonly referred to as nonlinear wave theory.

The mathematical study of dispersive PDE has witnessed an explosion of activity in the past thirty years, as it featured a beautiful combination of ideas and tools from various fields of mathematics, most notably harmonic analysis, dynamical systems, probability, and math physics.

In this course, we will give an introduction to this rich topic, and try to convey how tools from analysis and Hamiltonian dynamics are used in the deterministic study of such equations, and (time permitting) how tools from probability theory can be useful to understand questions that are too complex to answer deterministically.

Outline:

A) Harmonic Analysis Prerequisites (Fourier transform; Littlewood-Paley theory; Sobolev spaces and embeddings).

B) Dispersive Equations on $\mathbb{R}^d$:

- Linear theory and Strichartz estimates
- Local well-posedness
- Elements of global well-posedness theory

C) Introduction to dispersive Equations on bounded domains

- Linear estimates
- Nonlinear theory: local and global existence.

Important Dates

Sep 3	First day of classes
October 14-15	Fall Break- No Class
November 28-29	Thanksgiving Break
December 10	Last day of class.