Winter 2020

Math 650: **Harmonic Analysis**

MW 2:30 – 4:00 PM,

Instructor: Zaher Hani, (*Office:* 5834 East Hall (EH)), *Email:* zhani@umich.edu,
Website: https://sites.lsa.umich.edu/zhani/.*Office hours:* TBA and by appointment.

Prerequisites: A necessary prerequisite for this course is a solid background in graduate real analysis (measure theory, L^p spaces, Hardy-Littlewood maximal function, etc). Previous exposure to the Fourier transform in also useful, but we shall review that at the beginning of the course.

Course Coordinates: MW 2:30–4:00 pm in MH (Mason Hall) 1437.

Textbook: No textbook is required.

Reference and resources: The following textbooks mentioned below can be consulted for some topics to be covered in class.

1. E. Stein, *Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals*, Princeton University Press, ISBN-13: 978-0691032160.

2. E. Stein and G. Weiss, *Introduction to Fourier Analysis on Euclidean Spaces.* Princeton University Press. ISBN-13: 978-0691080789

3. J. Duoandikoetxea , *Fourier Analysis.* Graduate Studies in Mathematics, AMS. ISBN: 978-0821821725.

4. T. Tao, Lecture notes for Math 247 A and B available at the following two links Math 247A and Math 247B

Homework: There will be a few homework sets throughout the semester.

Grading: Grading will be based completely on homework.

Course Description: Harmonic analysis grew out of the study of Fourier analysis into a broader set of ideas and problems in hard analysis aimed at the quantitative understanding of operators (linear and nonlinear), singularities, oscillations that often arise in mathematics and physics.

In addition to being important in its own right, harmonic analysis proved to be a powerful and useful tool in the modern study of partial differential equations, analytic number theory, probability, and many other fields. In this course, we will survey several topics in classical and modern Harmonic analysis such as singular integral operators, Littlewood-Paley theory, pseudo-differential and para-differential operators, maximal functions, etc. Time permitting we will discuss some applications to other fields.

Outline:

A) Quick review of preliminaries

- The Fourier transform
- Advanced L^p theory (interpolation, Schur's test, Young's inequality, compact subsets of L^p).
- Maximal functions and covering lemma.

B) Calderon-Zygmund theory:

- Calderon-Zygmund operators
- Littlewood-Paley theory and Sobolev spaces
- Pseudo-differential operators

C) Paradifferential operators

D) Oscillatory Integrals

Important Dates

Jan 8	First day of classes
Jan 20	Martin Luther King Jr. Day - No classes
March 2-7	Winter Break-No Class
April 20	Last day of class.